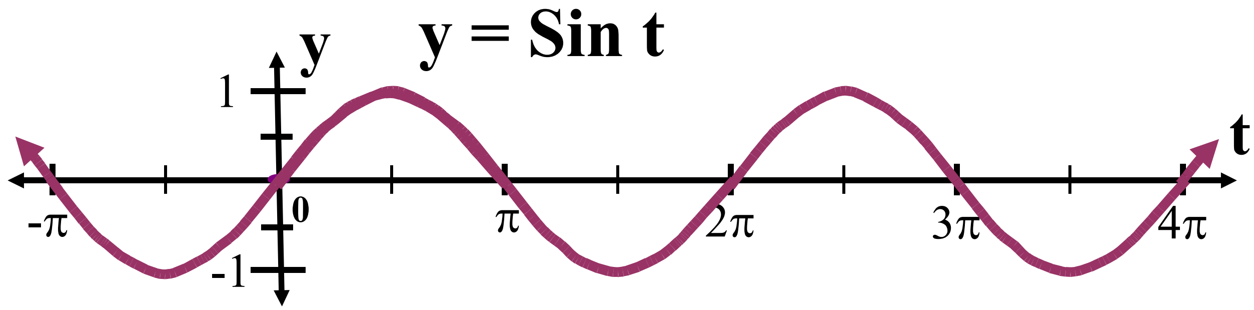
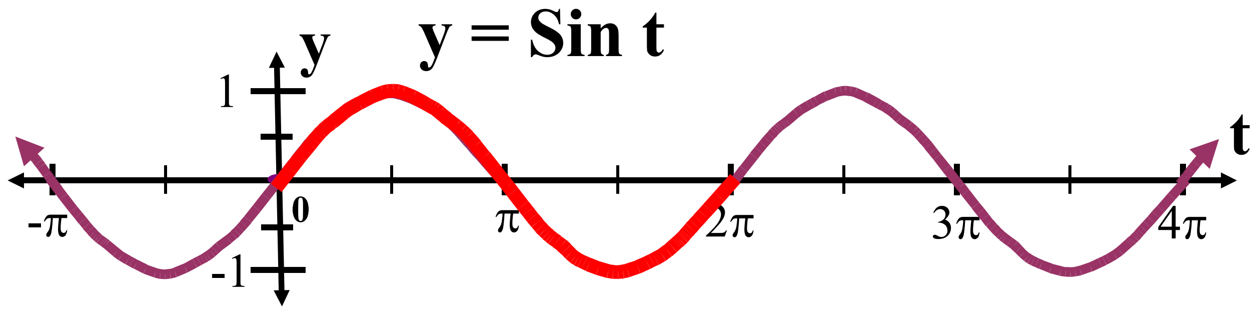
Variations of the Sine Function

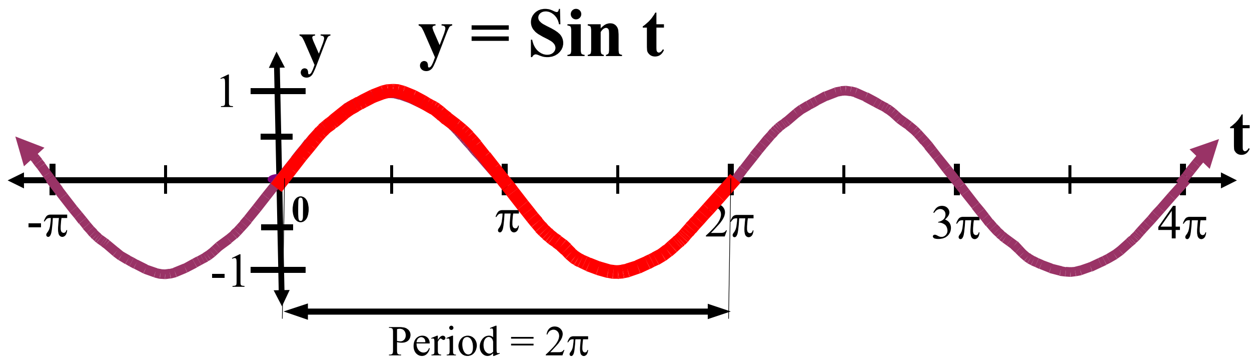


Variations of the Sine Function



This is the 'basic cycle' of the sine function.

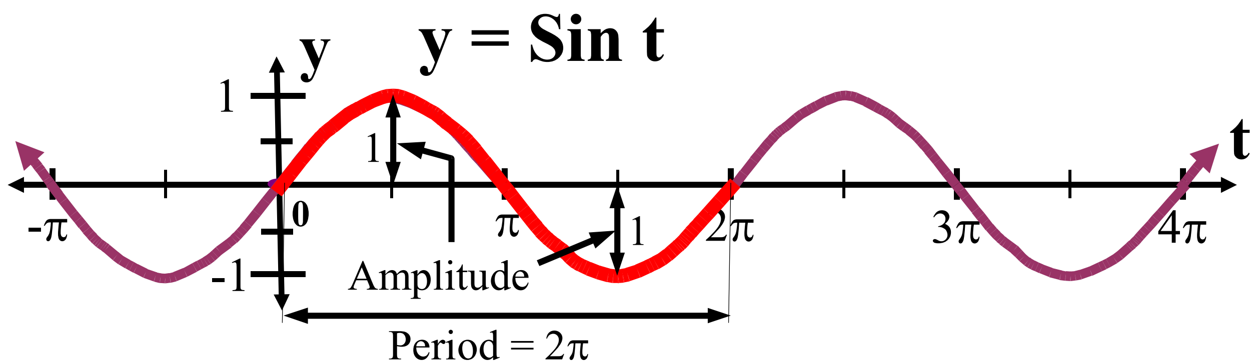
Variations of the Sine Function



This is the 'basic cycle' of the sine function.

Its period is 2π units.

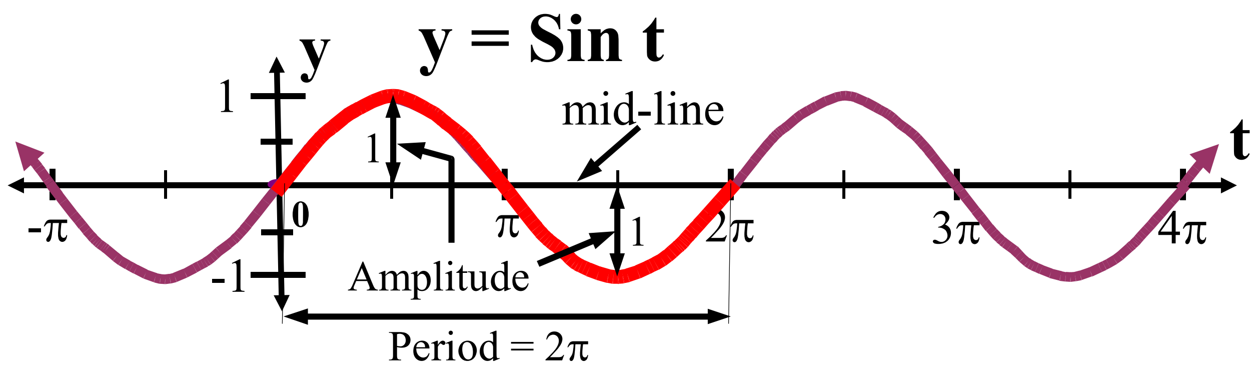
Variations of the Sine Function



This is the 'basic cycle' of the sine function.

Its <u>period</u> is 2π units. Its <u>amplitude</u> is 1 unit.

Variations of the Sine Function

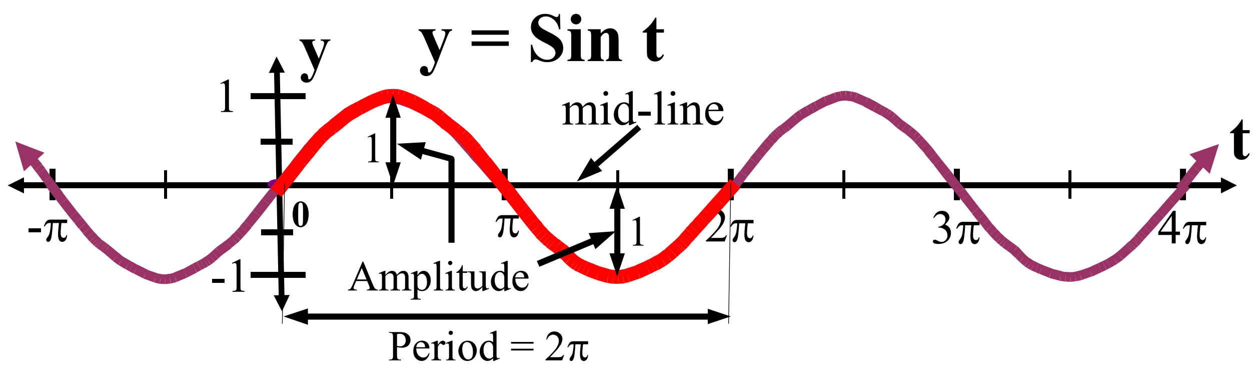


This is the 'basic cycle' of the sine function.

Its period is 2π units. Its amplitude is 1 unit.

The line y = 0, the t axis, is the mid-line of the curve.

Variations of the Sine Function



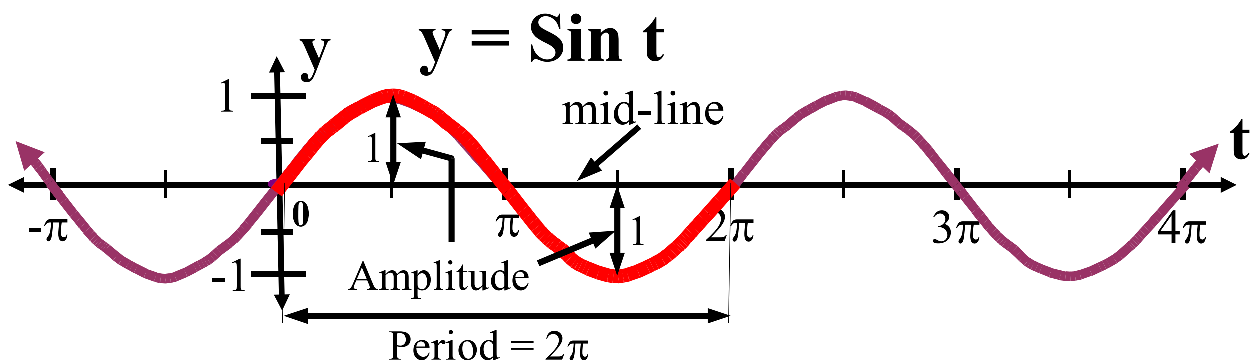
This is the 'basic cycle' of the sine function.

Its period is 2π units. Its amplitude is 1 unit.

The line y = 0, the t axis, is the mid-line of the curve.

Consider the equation $y = A\,\mathrm{Sin}(Bt + C) + D$.

Variations of the Sine Function



This is the 'basic cycle' of the sine function.

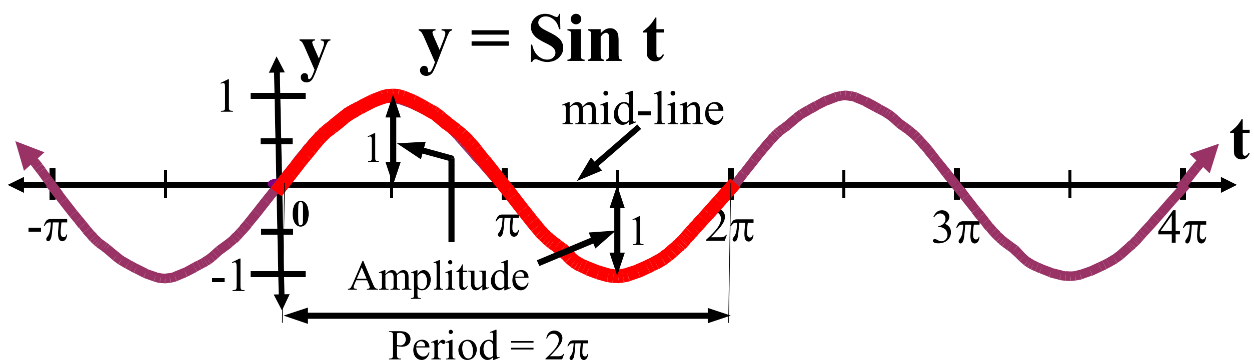
Its period is 2π units. Its amplitude is 1 unit.

The line y = 0, the t axis, is the mid-line of the curve.

Consider the equation $y = A\,\mathrm{Sin}(Bt + C) + D$.

We will consider the significance of each of the constants A, B, C, and D.

Variations of the Sine Function



This is the 'basic cycle' of the sine function.

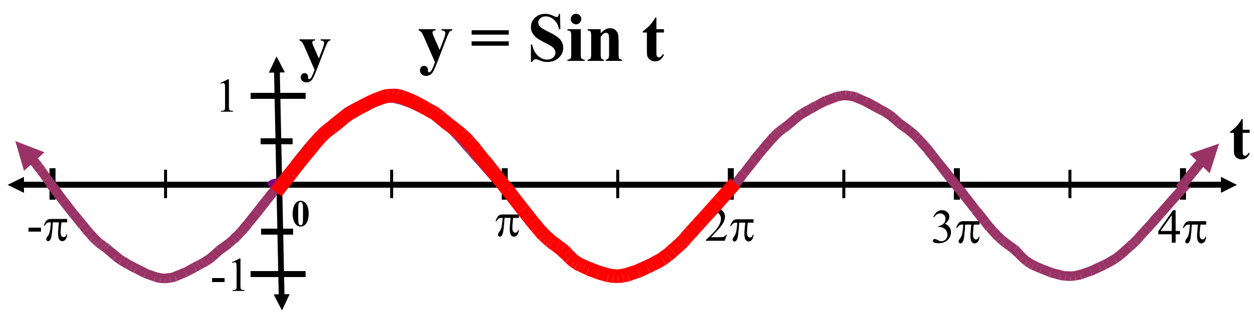
Its <u>period</u> is 2π units. Its <u>amplitude</u> is 1 unit.

The line $y = 0$, the t axis, is the <u>mid-line</u> of the curve.

Consider the equation $y = A\,\text{Sin}(Bt + C) + D$.

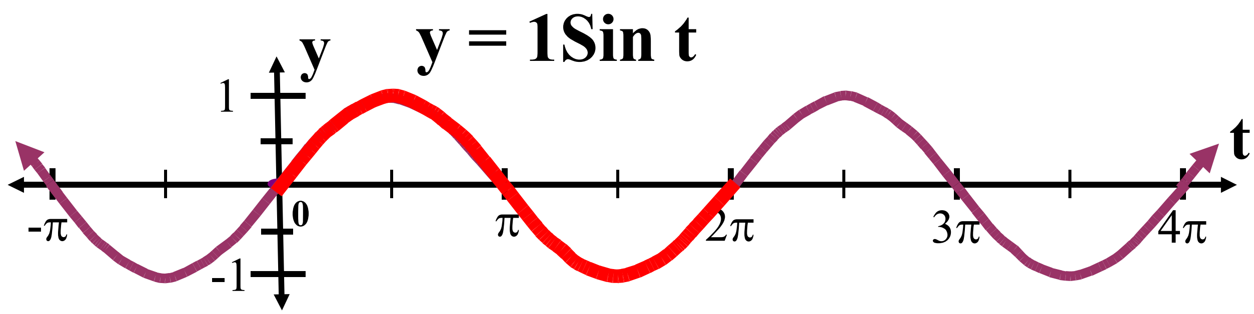
We will consider the significance of each of the constants A, B, C, and D, starting with A.

Variations of the Sine Function



We will start with equations of the form y = ASin(t)

Variations of the Sine Function



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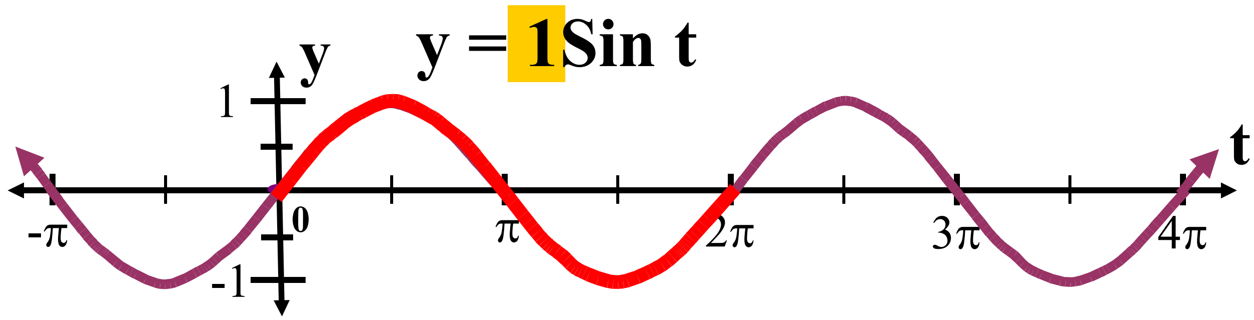

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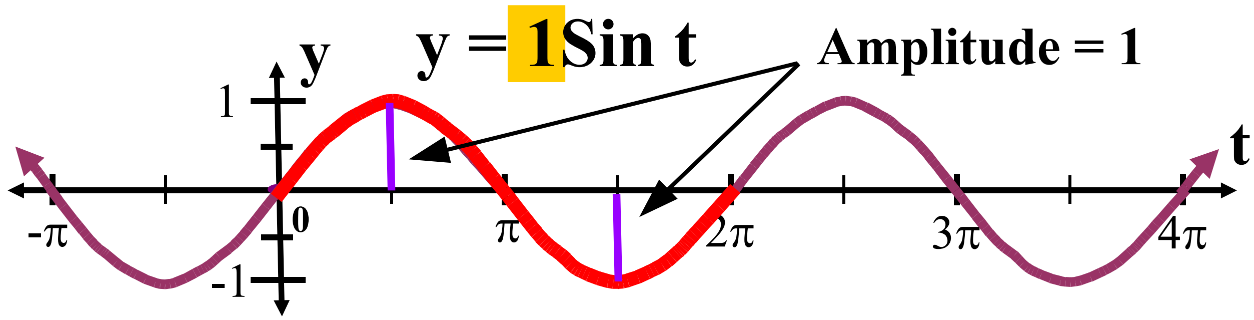

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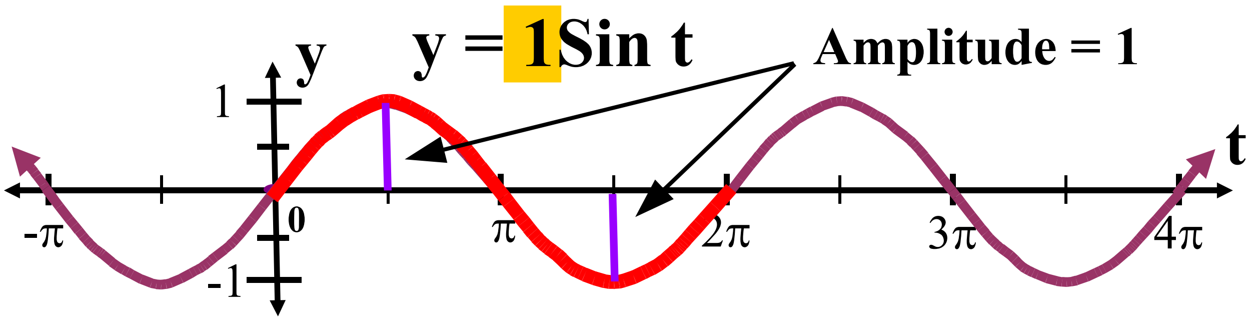
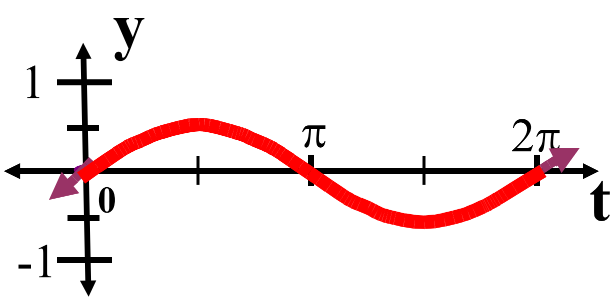


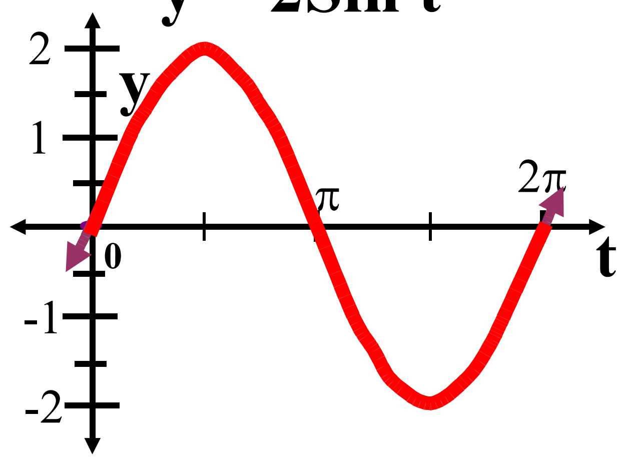
We will start with equations of the form y = ASin(t)

Here are two other examples (showing the 'basic cycle' only).

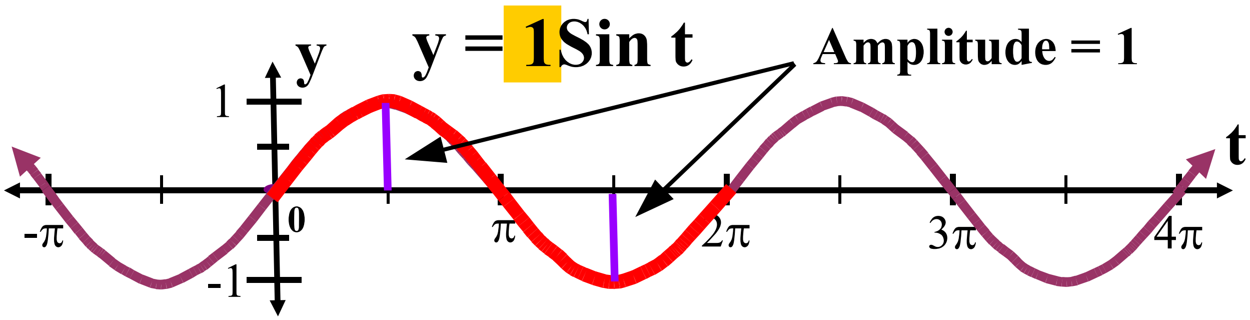
y = (1/2)Sin t



y = 2Sin t



Variations of the Sine Function



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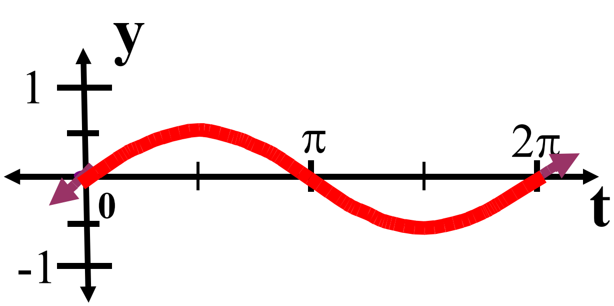

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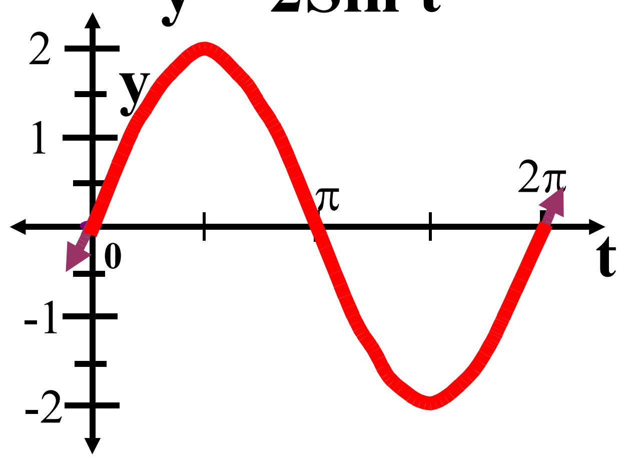

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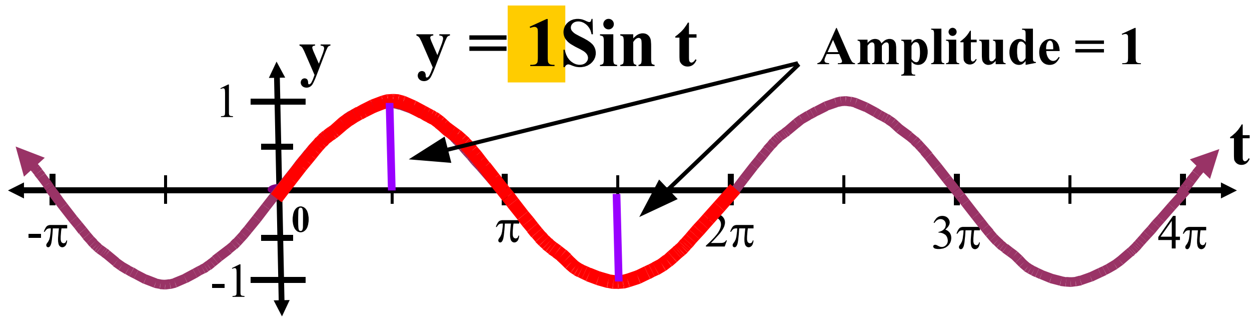

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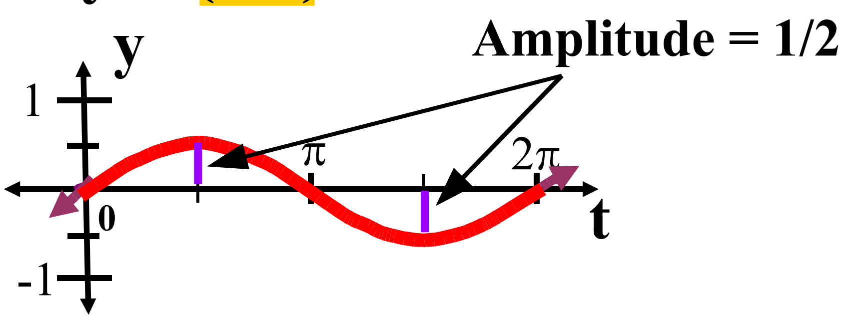

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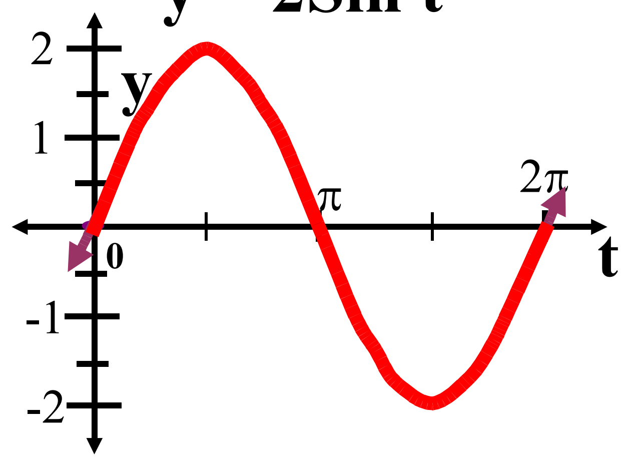
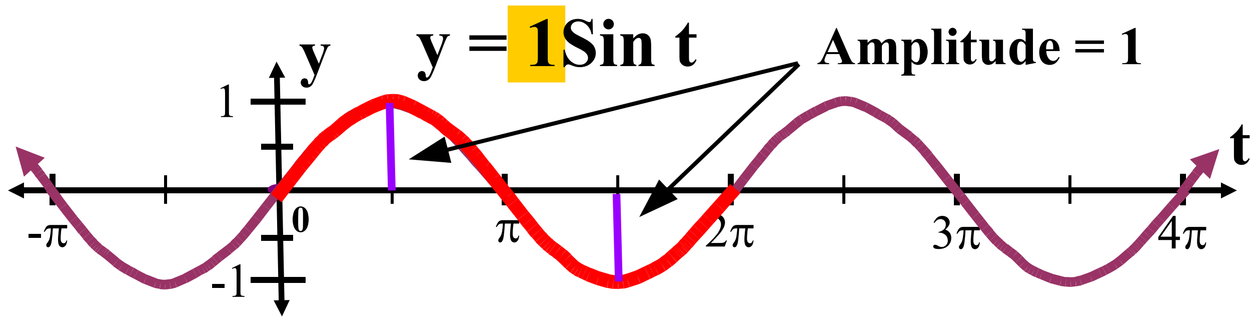


Variations of the Sine Function



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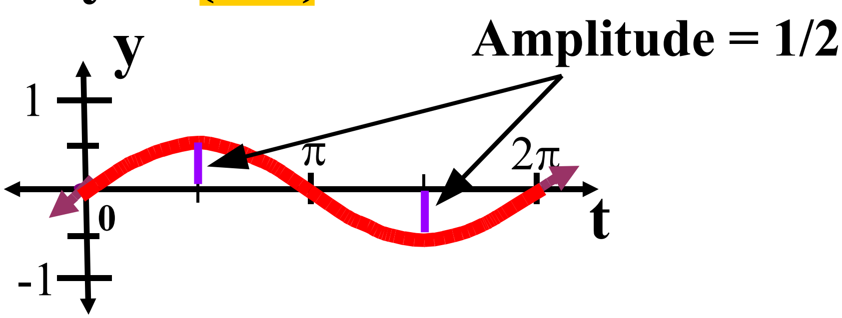

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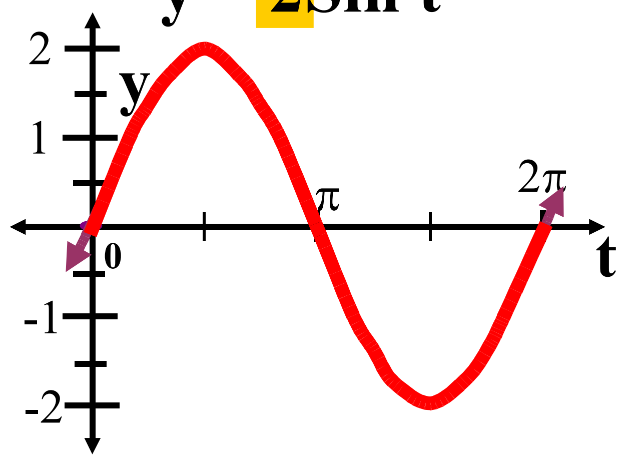
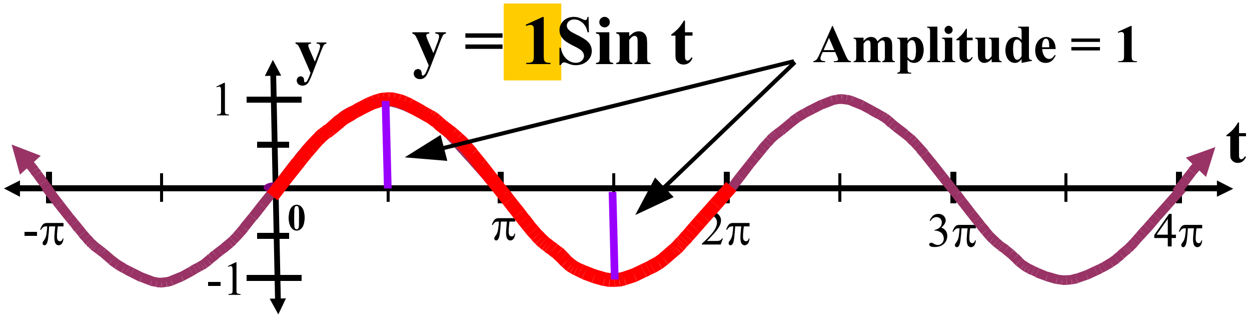


Variations of the Sine Function

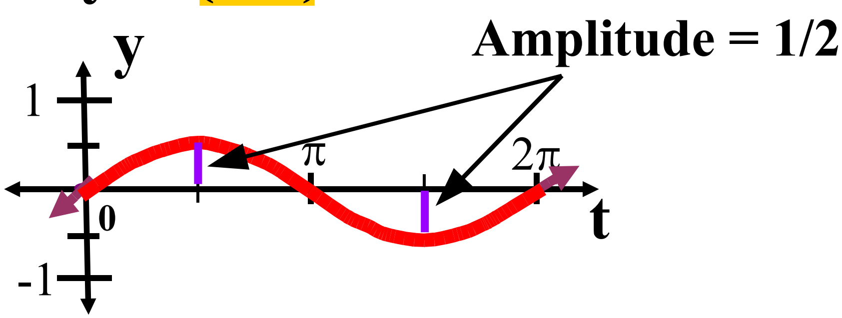
y = 1Sin t



We will start with equations of the form y = ASin(t)

Here are two other examples (showing the 'basic cycle' only).

y = (1/2)Sin t



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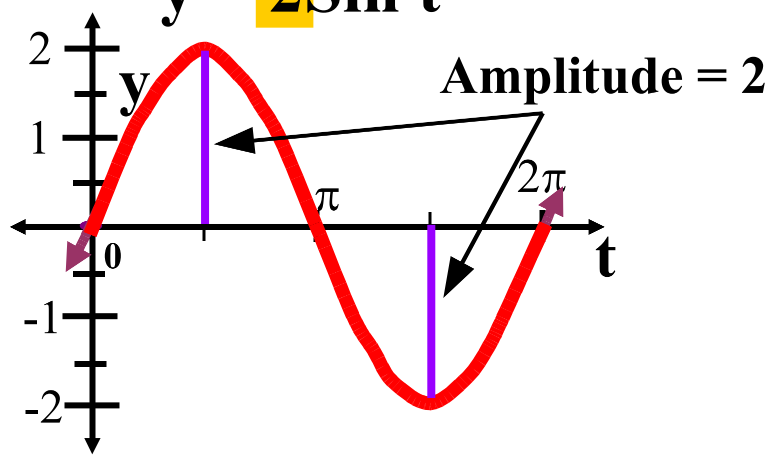
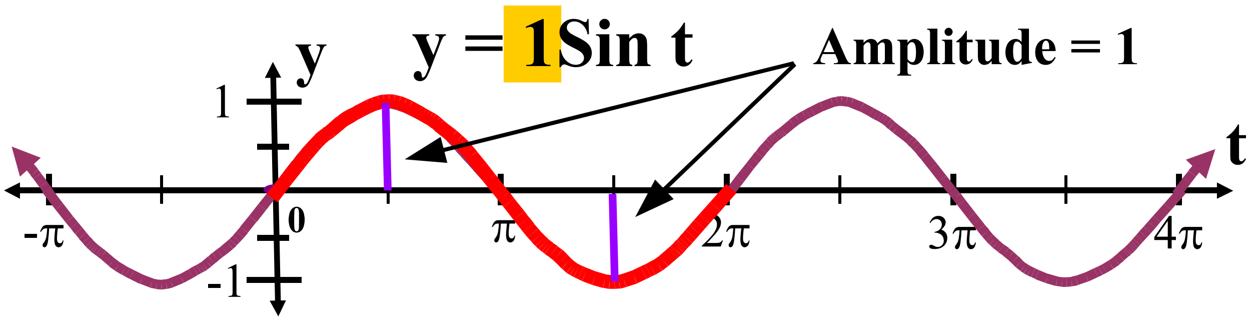
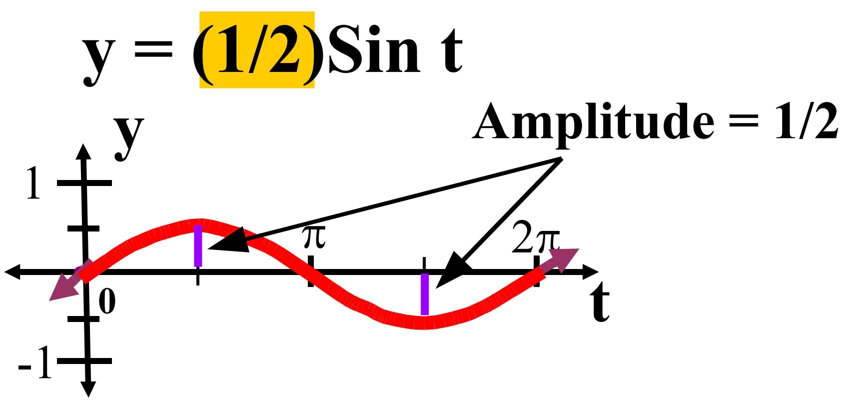


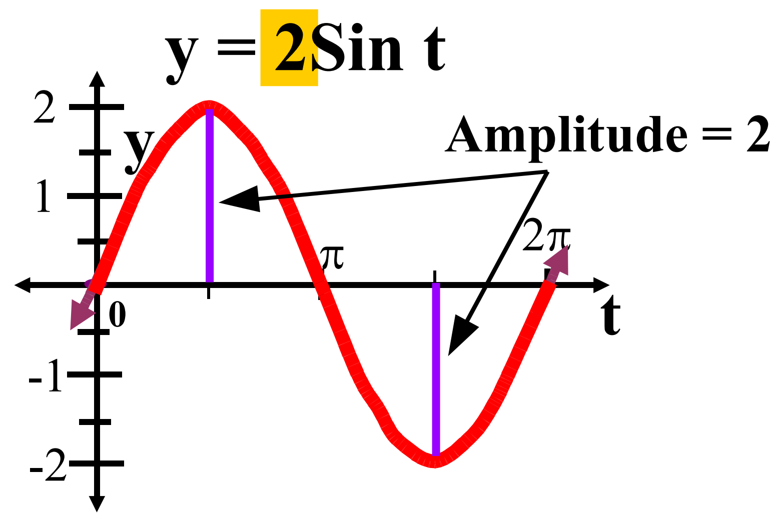
Variations of the Sine Function



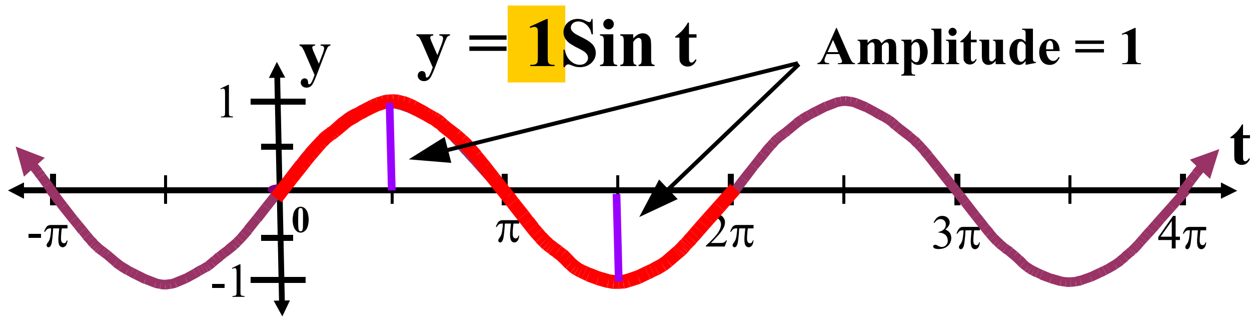
We will start with equations of the form y = ASin(t)

In these examples, the amplitude = A.



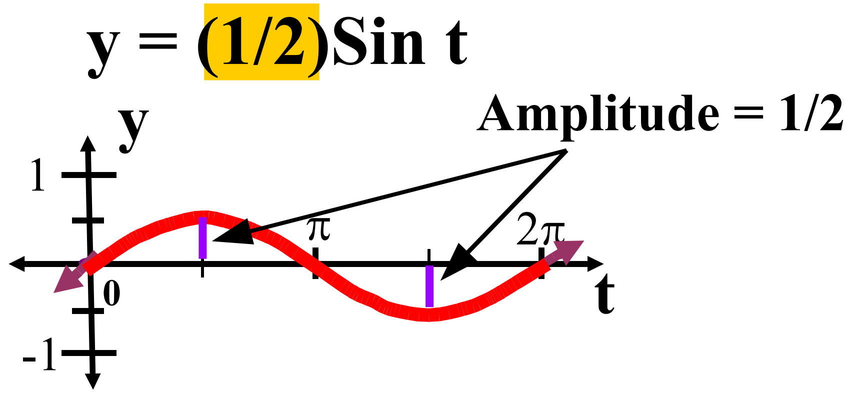


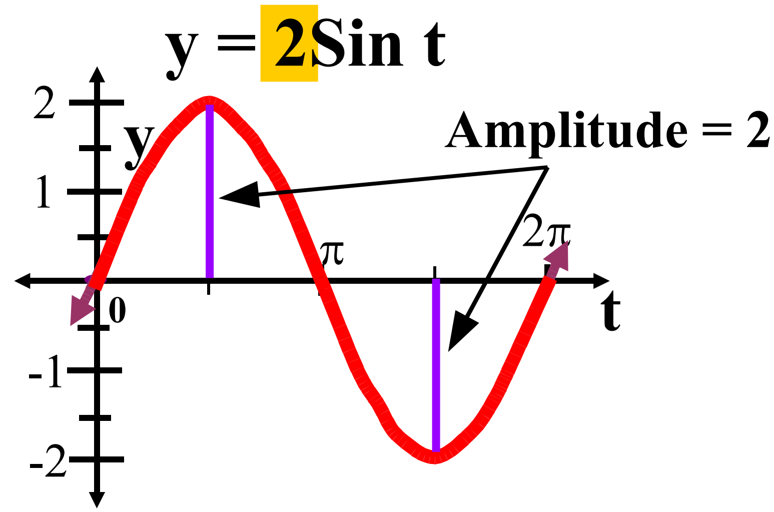
Variations of the Sine Function



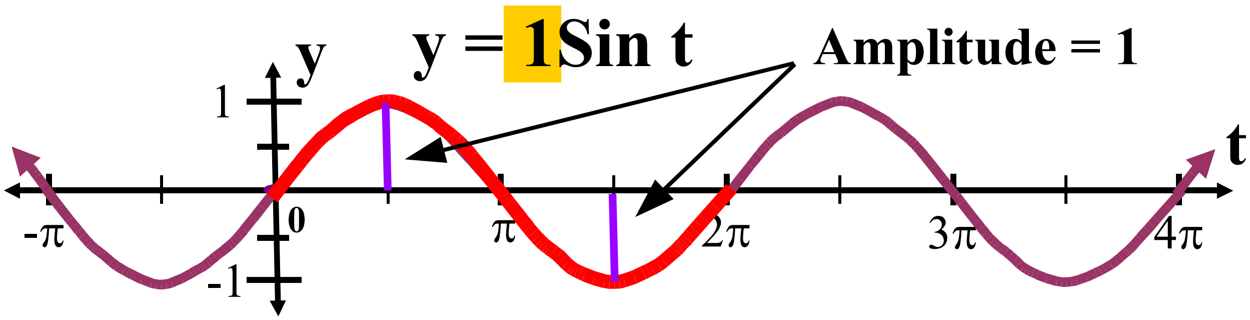
We will start with equations of the form y = ASin(t)

In these examples, the amplitude = A. What if A < 0?





Variations of the Sine Function



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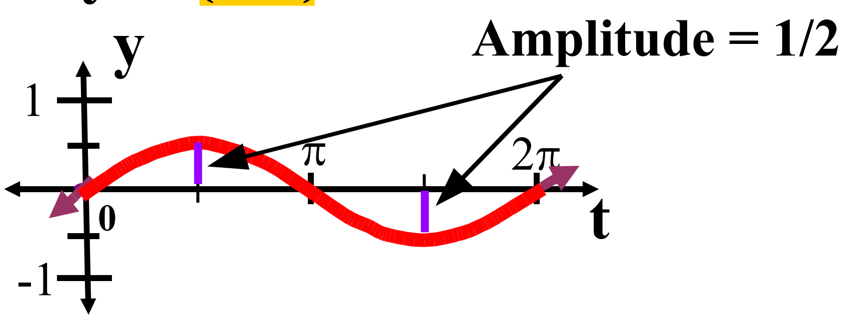
In these examples, the amplitude = A. What if A < 0?

y = (1/2)Sin t



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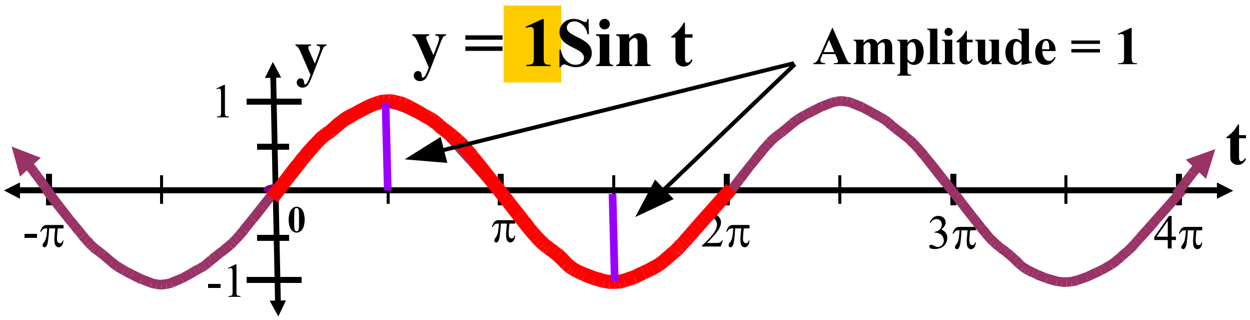

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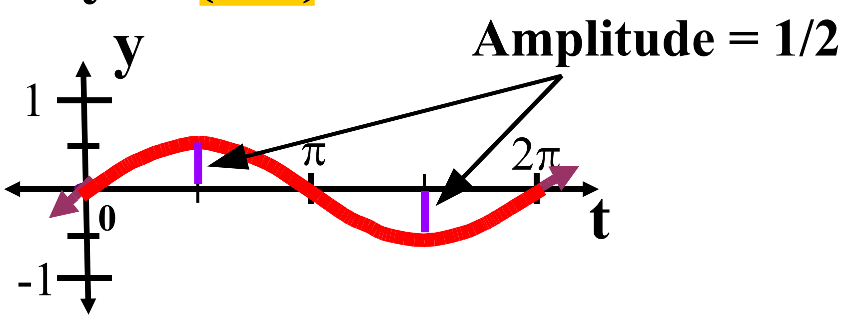
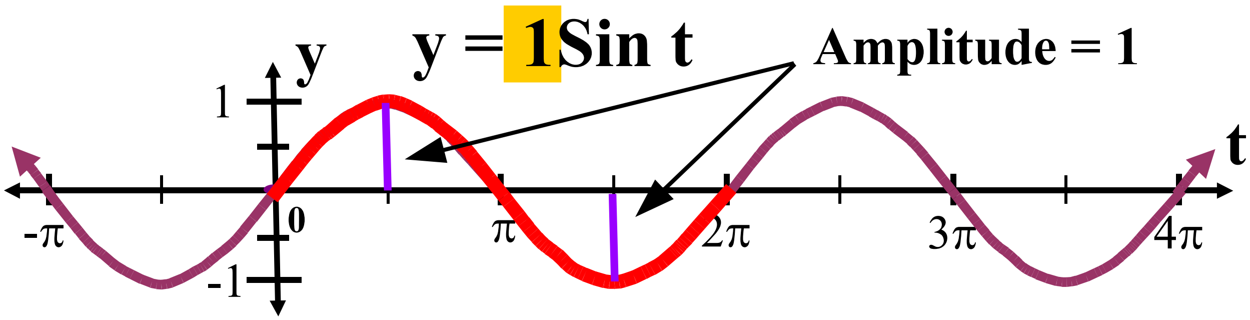
y = (1/2)Sin t **y = (-1/2)Sin t**



Variations of the Sine Function



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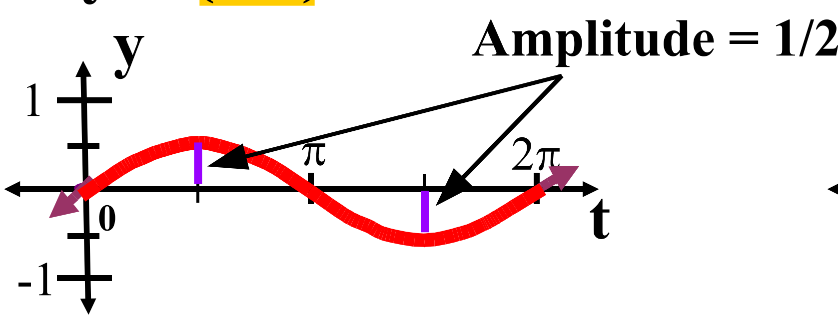

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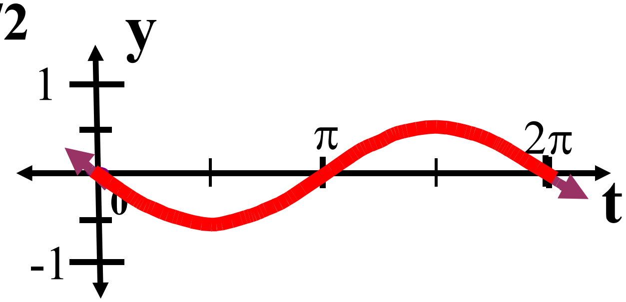
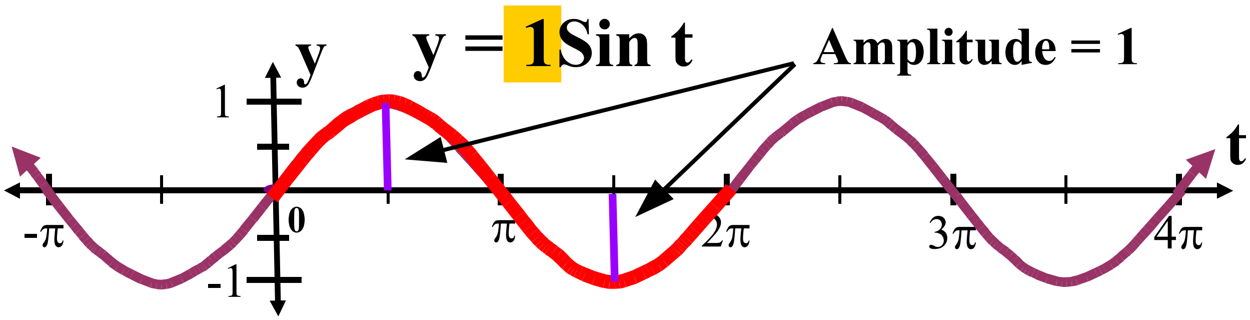


Variations of the Sine Function



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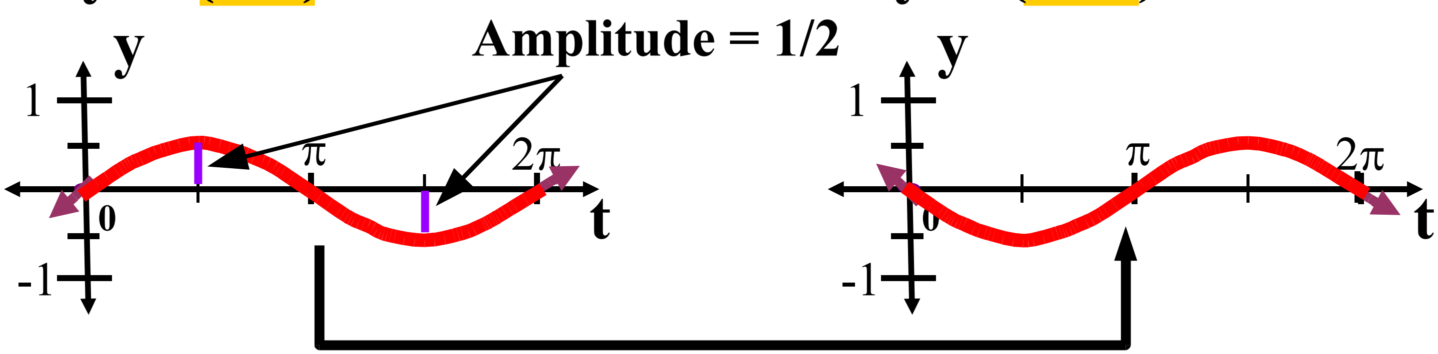
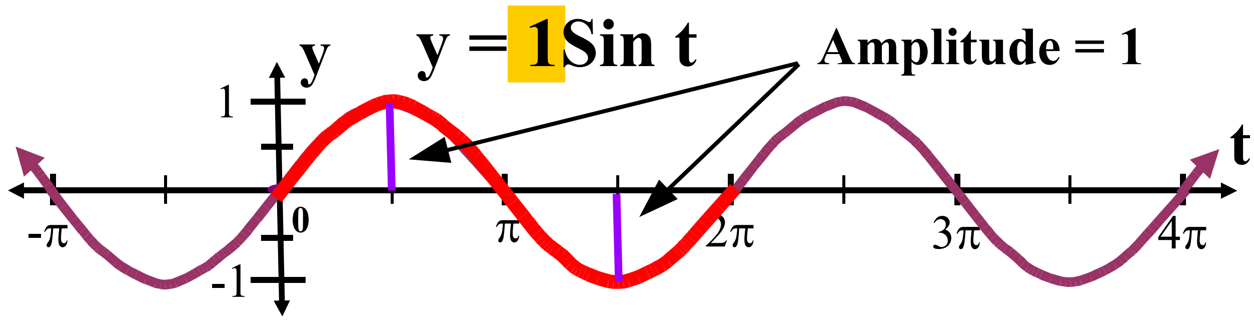


If A < 0, then the graph 'flips' over the mid-line.

Variations of the Sine Function



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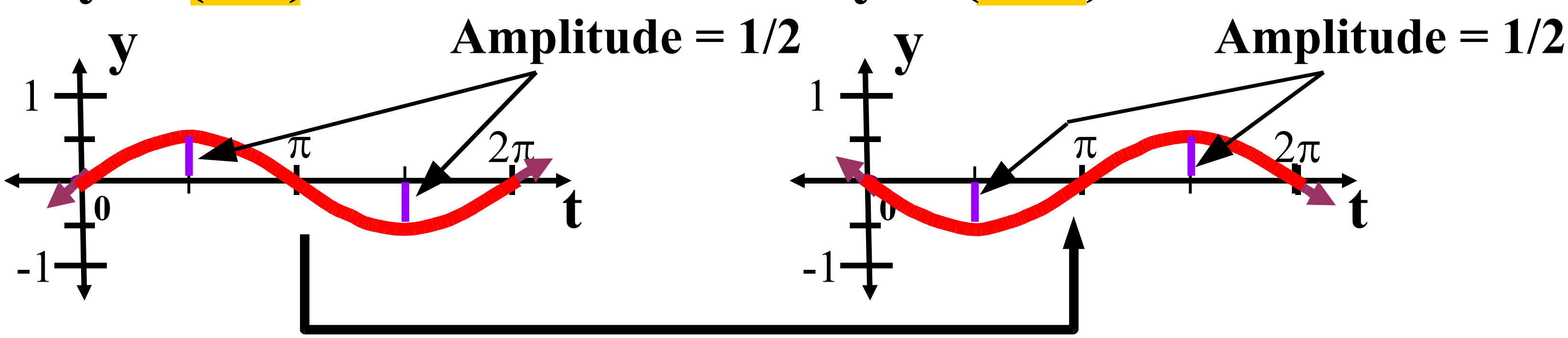

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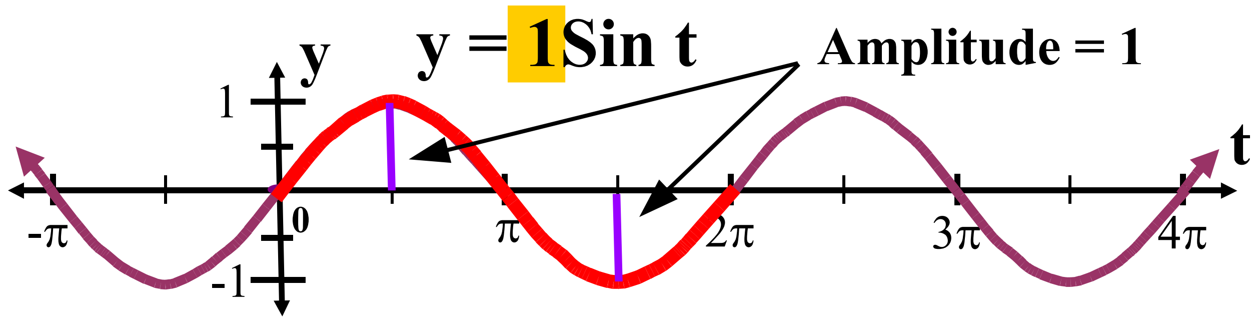
Variations of the Sine Function



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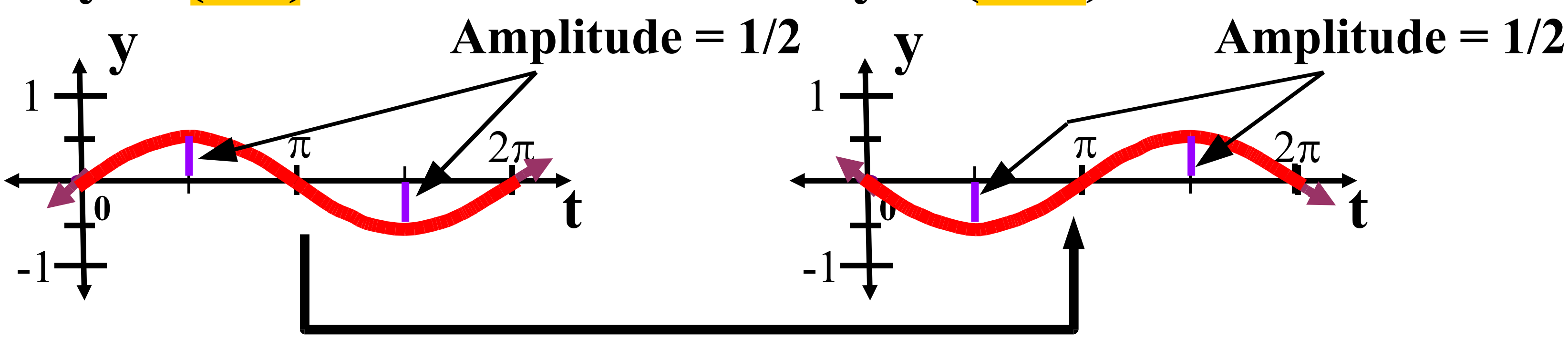


If A < 0, then the graph 'flips' over the mid-line.

The amplitude is equal to the absolute value of A.

Variations of the Sine Function

Consider the equation $y = A\sin(Bt + C) + D$.

Variations of the Sine Function

Consider the equation $y = A\sin(Bt + C) + D$.

Variations of the Sine Function

Consider the equation $y = A\sin(Bt + C) + D$.

(1) The amplitude of the 'sine wave' is the absolute value of A.

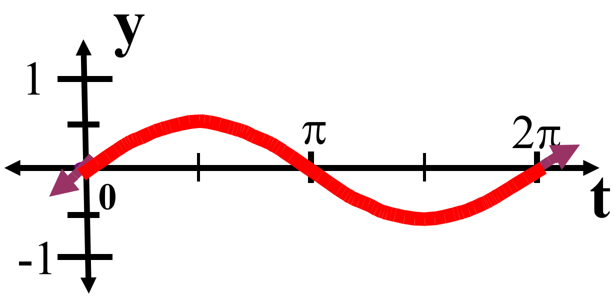
Variations of the Sine Function

Consider the equation y = Asin(Bt + C) + D.

(1) The amplitude of the 'sine wave' is the absolute value of A.

(2) If A > 0,

y = (1/2)Sin t



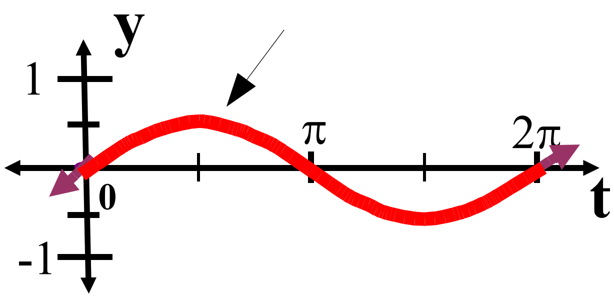
Variations of the Sine Function

Consider the equation y = Asin(Bt + C) + D.

(1) The amplitude of the 'sine wave' is the absolute value of A.

(2) If A > 0, then the basic cycle is 'above the mid-line' for the first half of the cycle

y = (1/2)Sin t



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Consider the equation y = Asin(Bt + C) + D.

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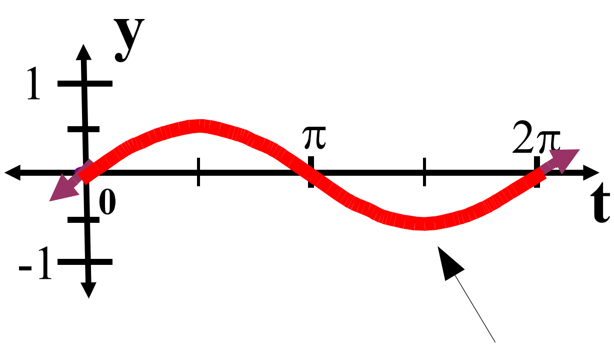

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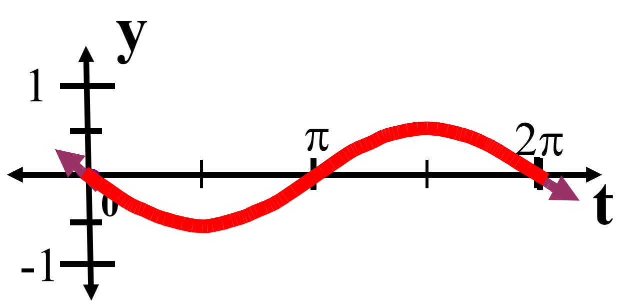
(2) If $A > 0$, then the basic cycle is 'above the mid-line' for the first half of the cycle and 'below the mid-line' for the second half of the cycle.

(3) If $A < 0$,

$y = (-1/2)\text{Sin } t$



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Consider the equation y = Asin(Bt + C) + D.

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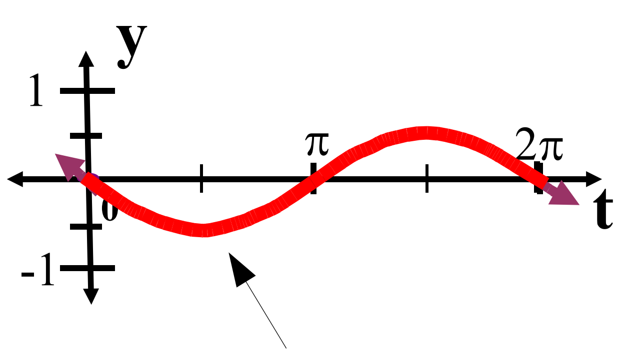

Variations of the Sine Function

Consider the equation y = Asin(Bt + C) + D.

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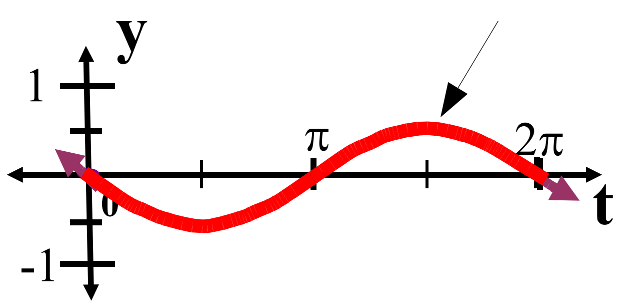
(3) If A < 0, then the basic cycle is 'below the mid-line' for the first half of the cycle and 'above the mid-line' for the second half of the cycle.

y = (-1/2)Sin t



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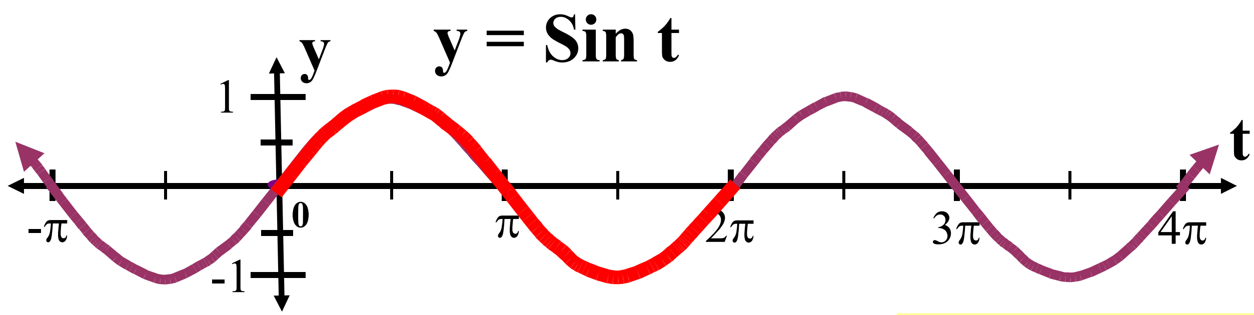
(3) If $A < 0$, then the basic cycle is 'below the mid-line' for the first half of the cycle and 'above the mid-line' for the second half of the cycle.

We will next consider the significance of the constant D.

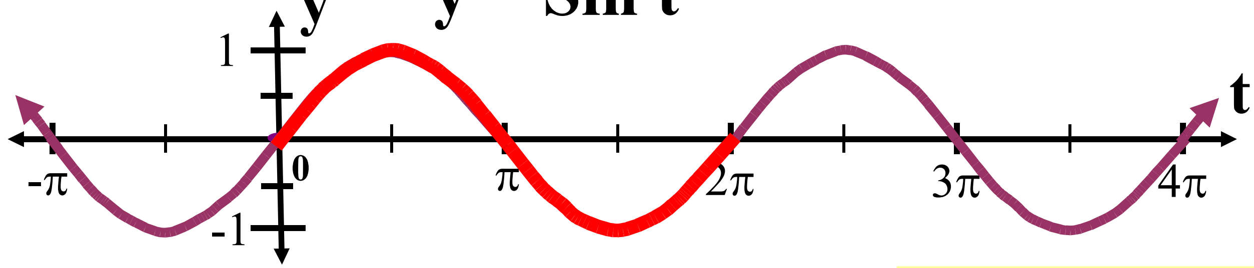
Variations of the Sine Function



We will start with equations of the form y = Asin t + D.

Variations of the Sine Function

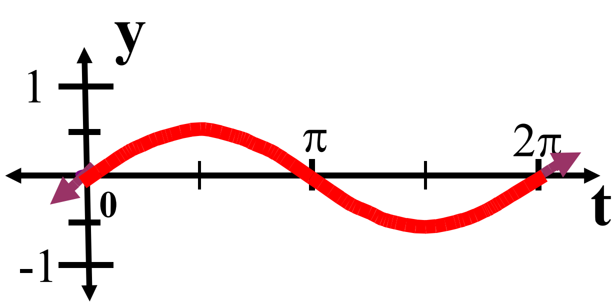
y = Sin t



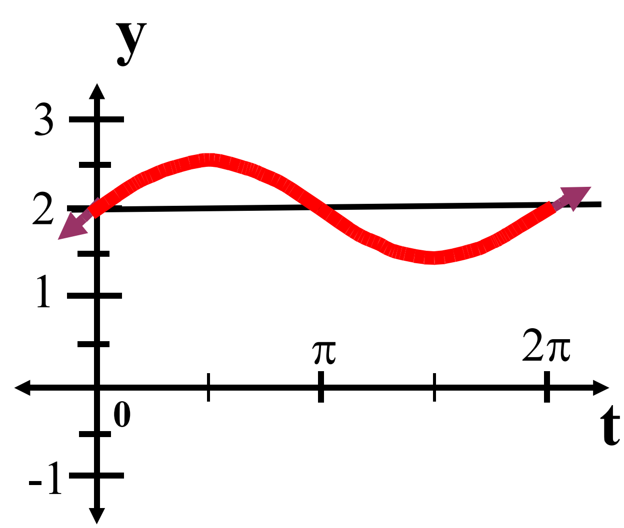
We will start with equations of the form y = Asin t + D.

Here are two examples (showing the 'basic cycle' only).

y = (1/2)Sin t + 0



y = (1/2)Sin t + 2



Variations of the Sine Function

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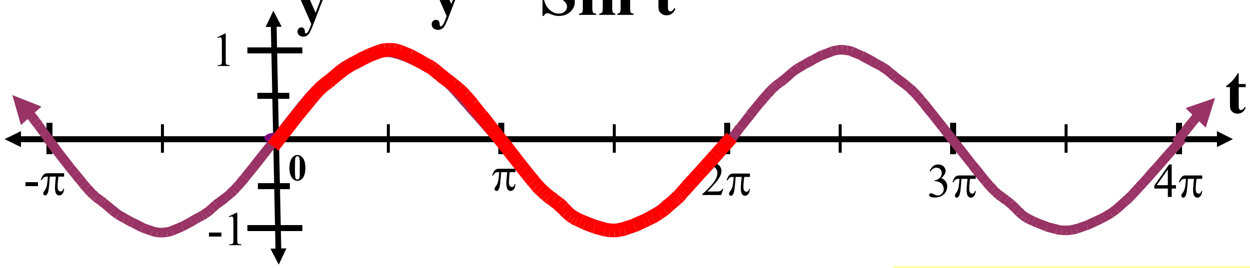

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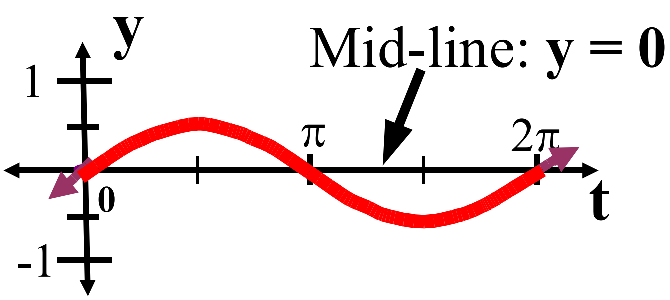

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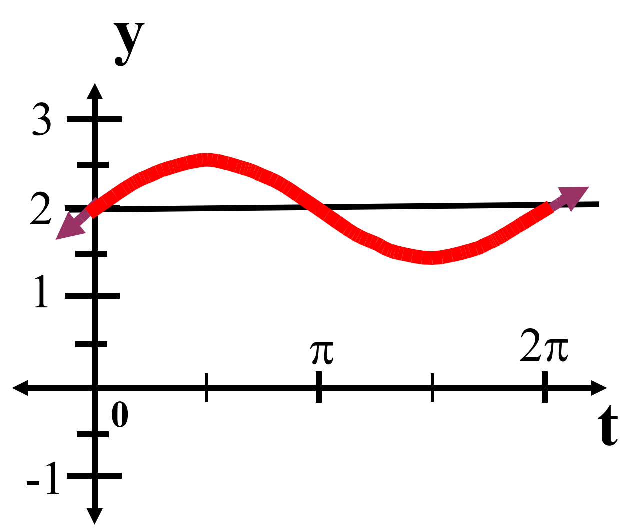

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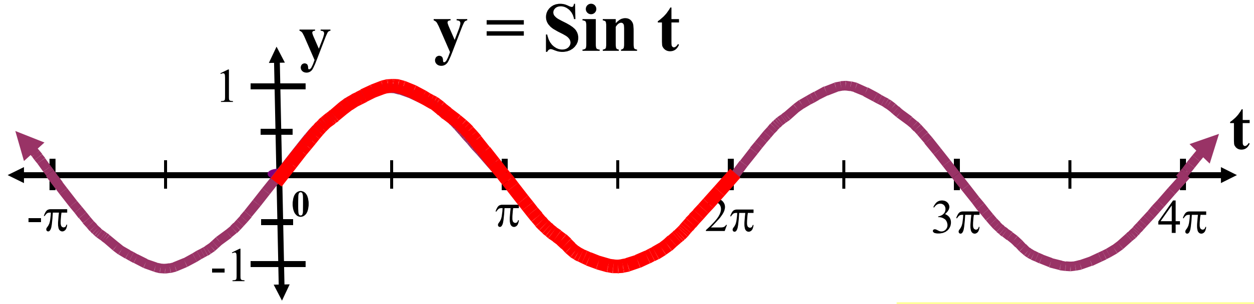

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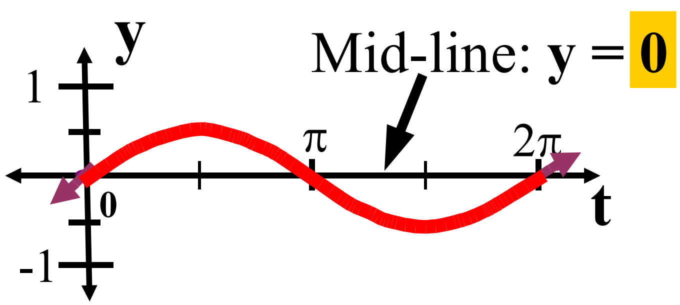

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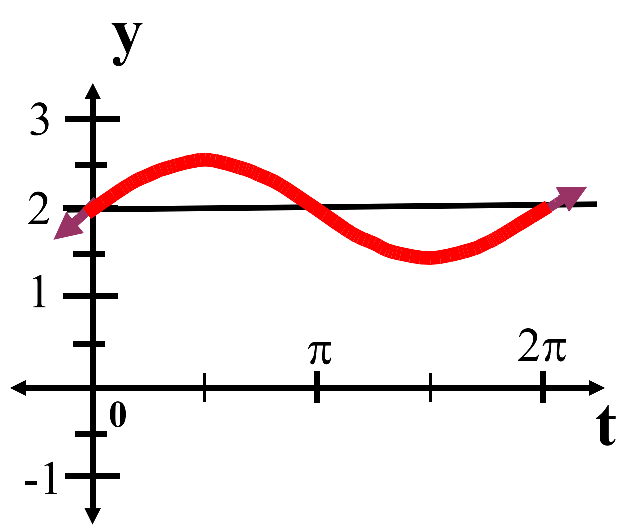

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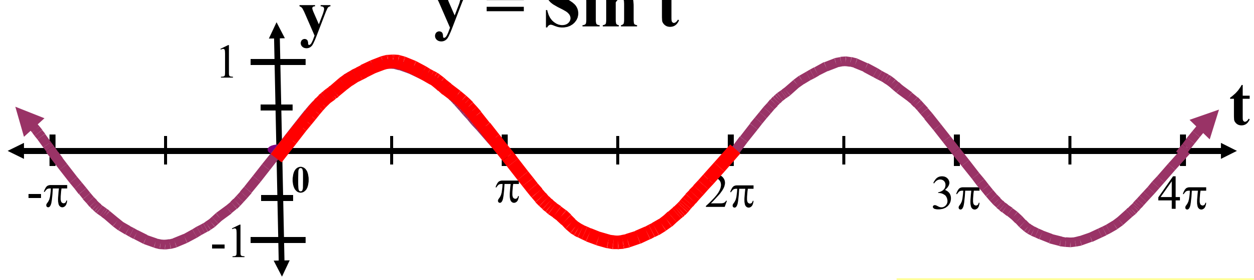

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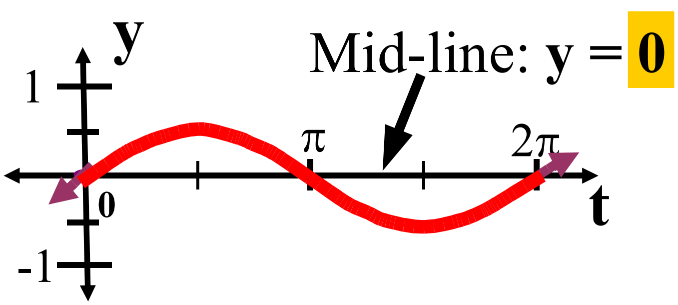

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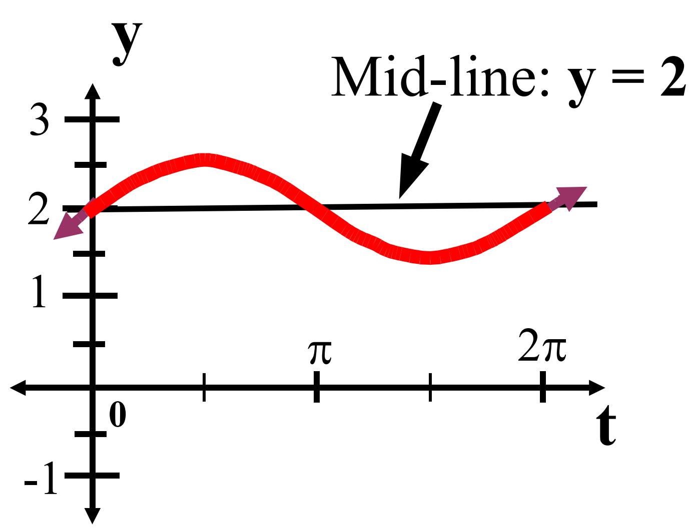

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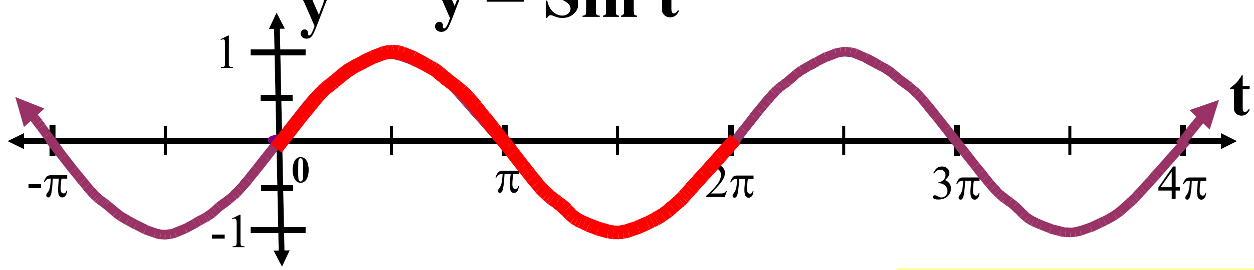

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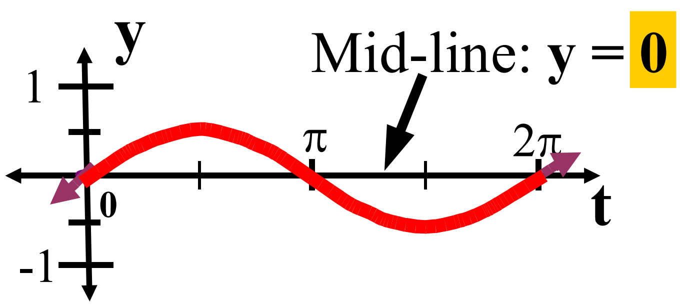

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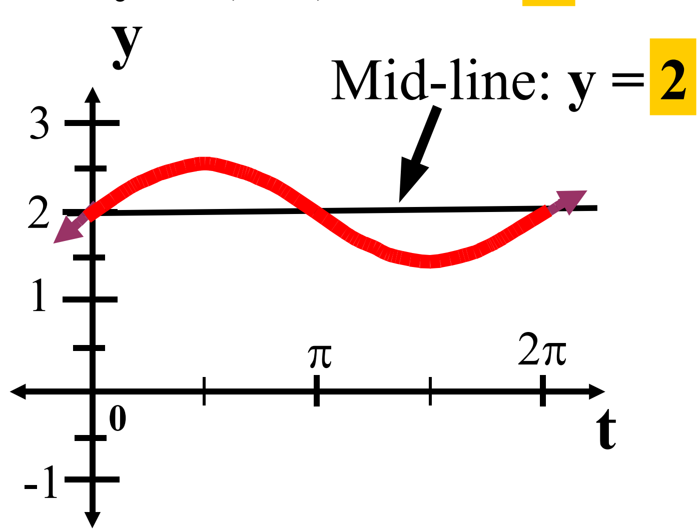
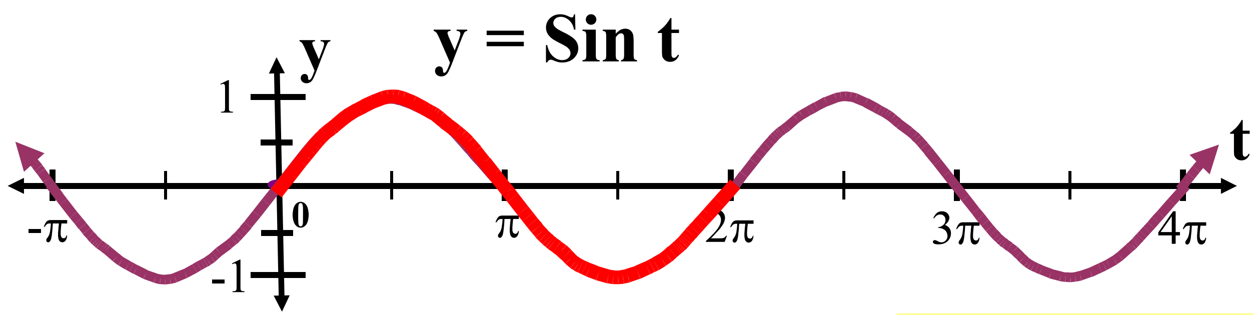


Variations of the Sine Function



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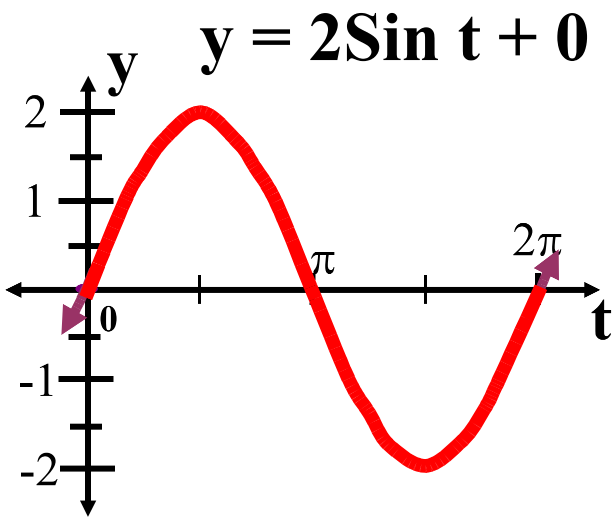

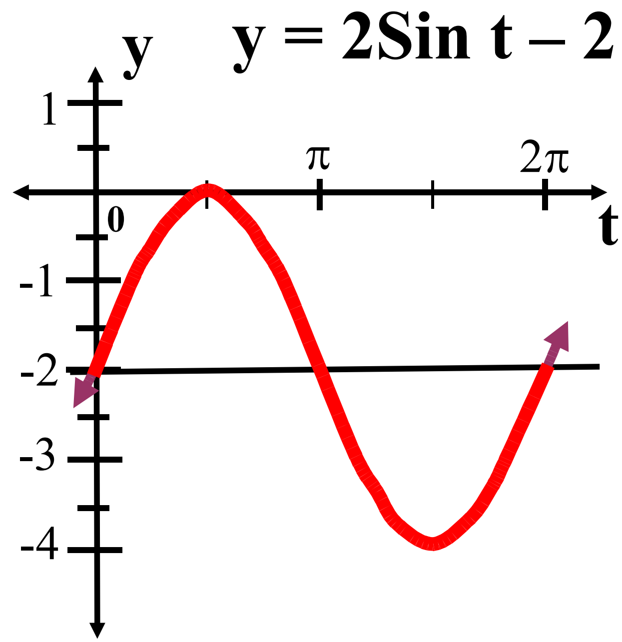

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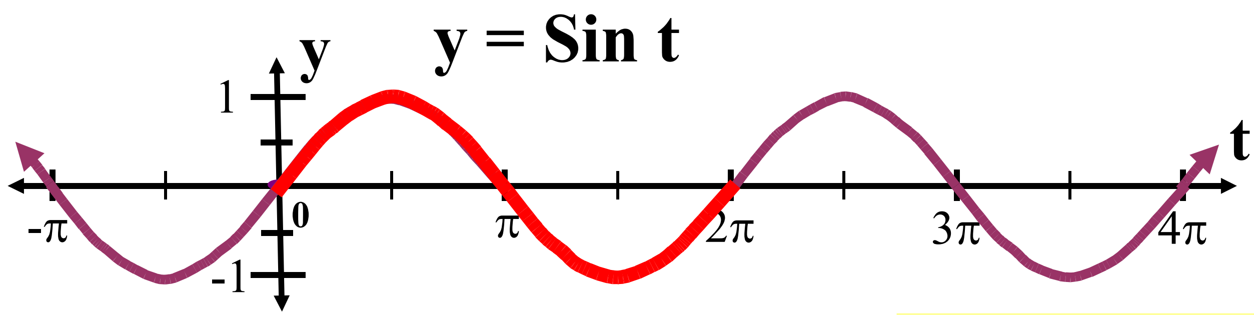

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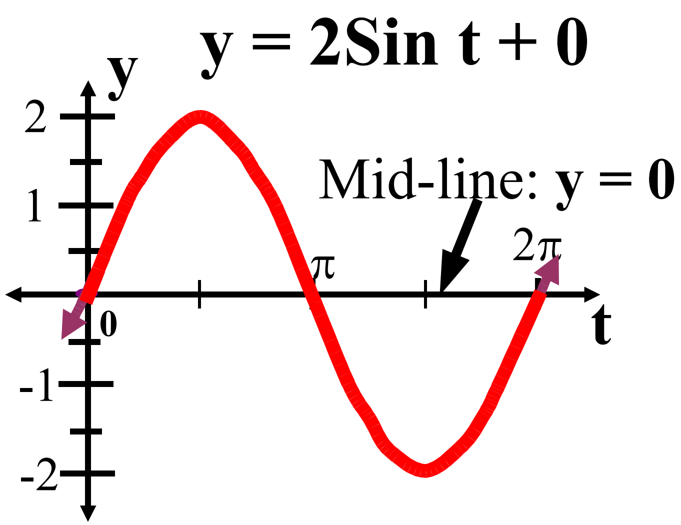

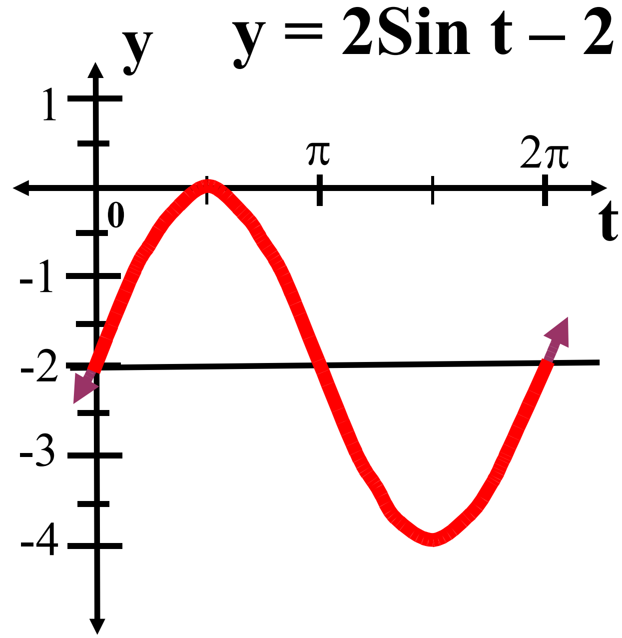

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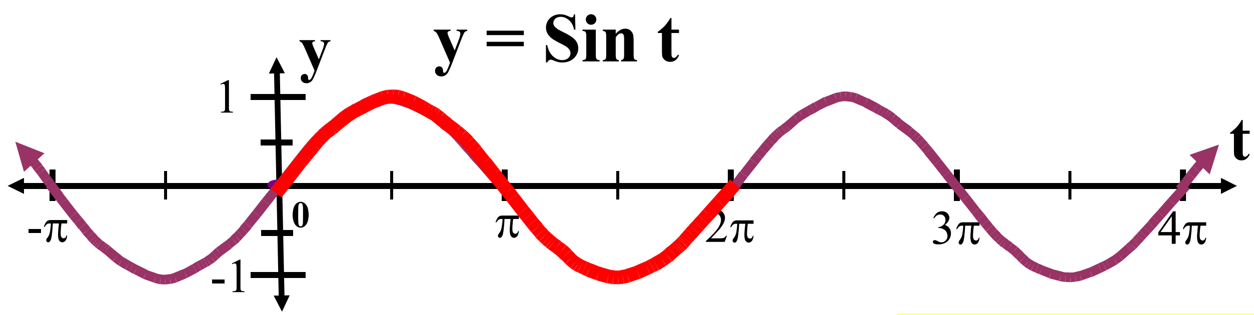

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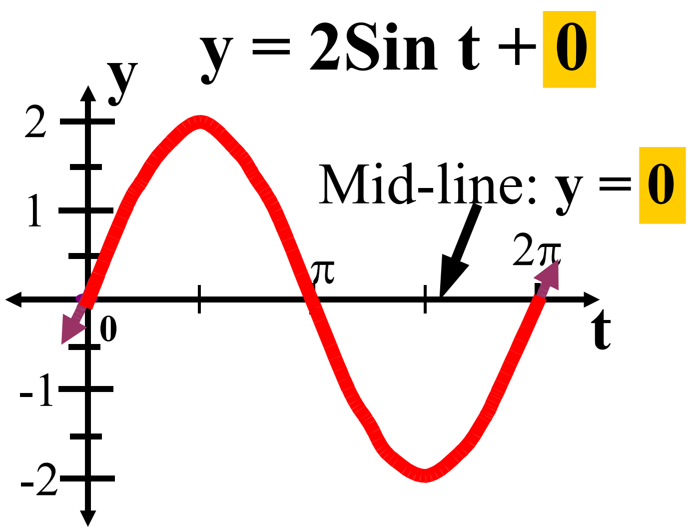
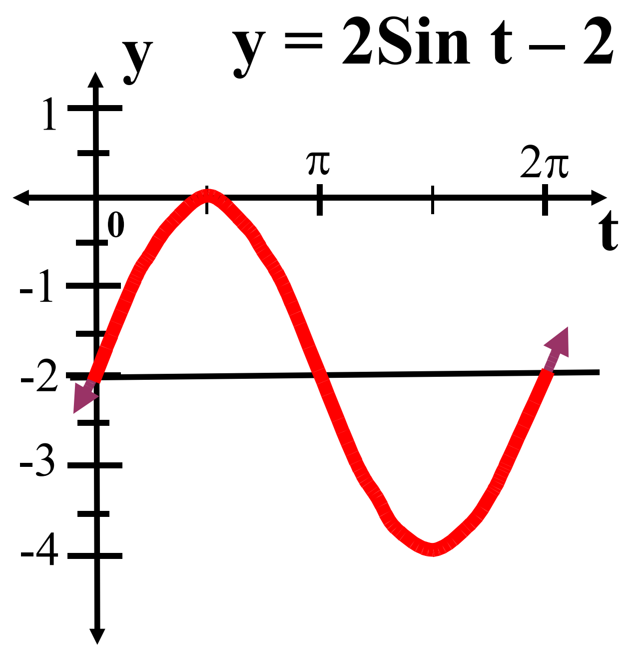
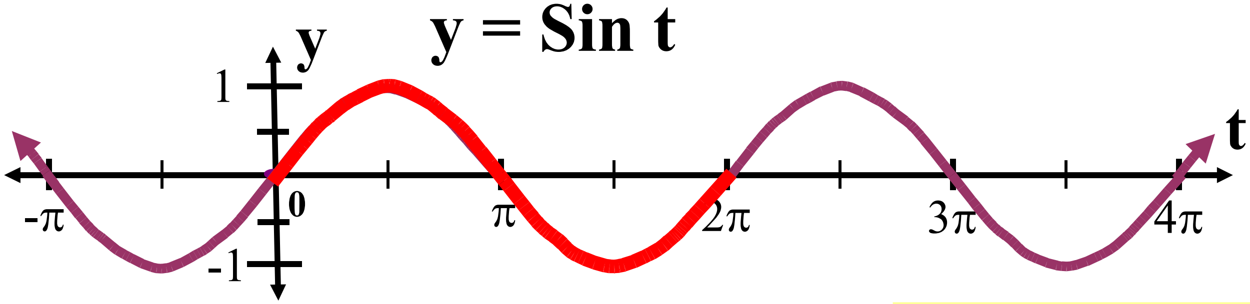




Variations of the Sine Function

y = Sin t



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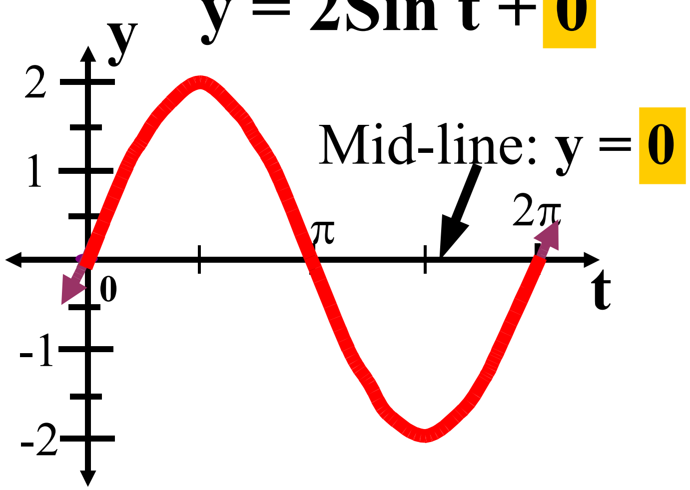
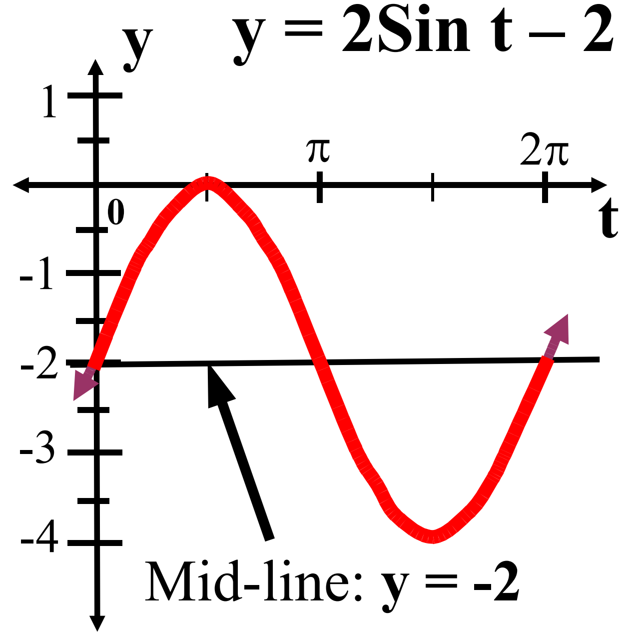
y = 2Sin t + 0



y = 2Sin t – 2



Variations of the Sine Function

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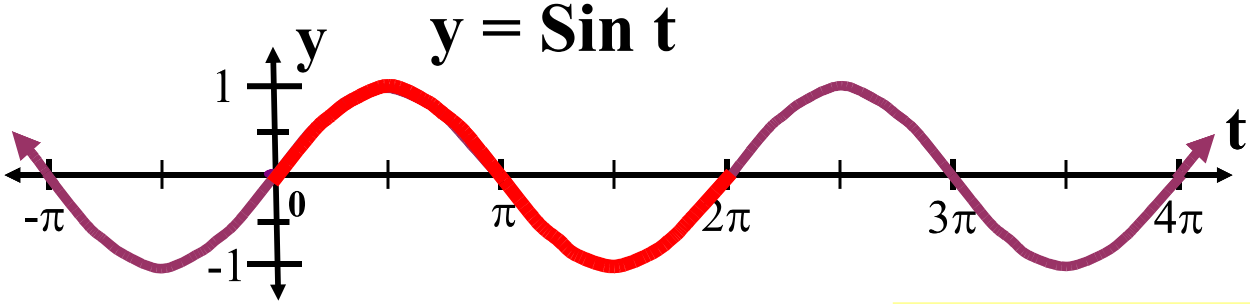

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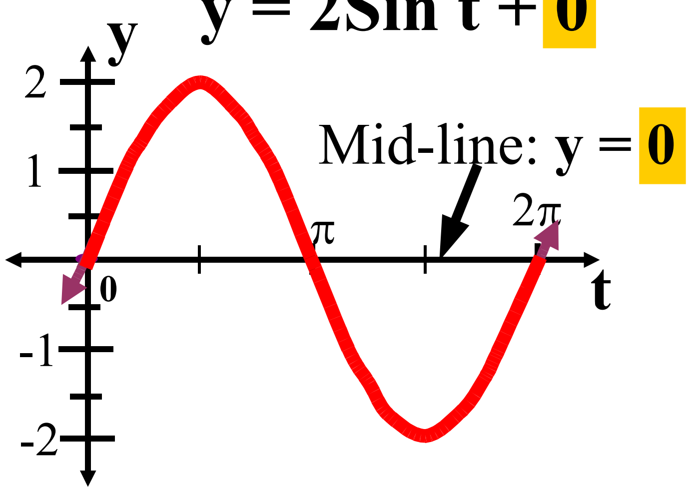

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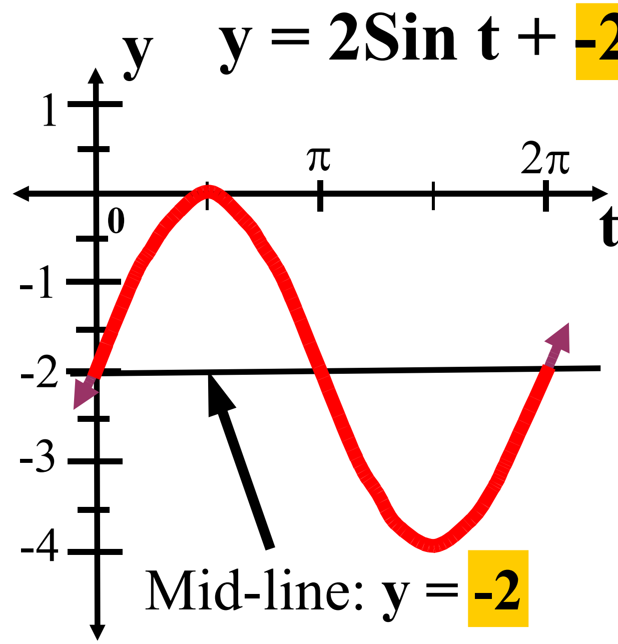

Variations of the Sine Function

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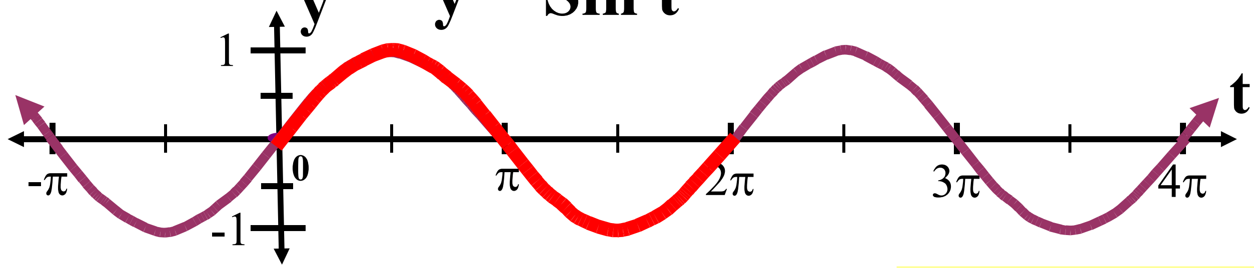

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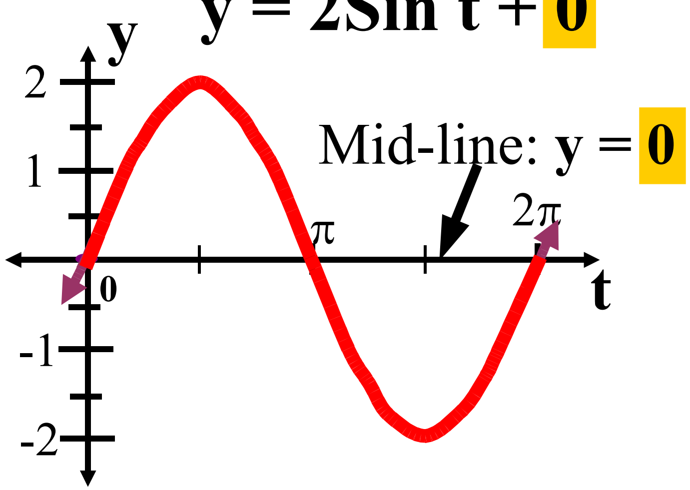

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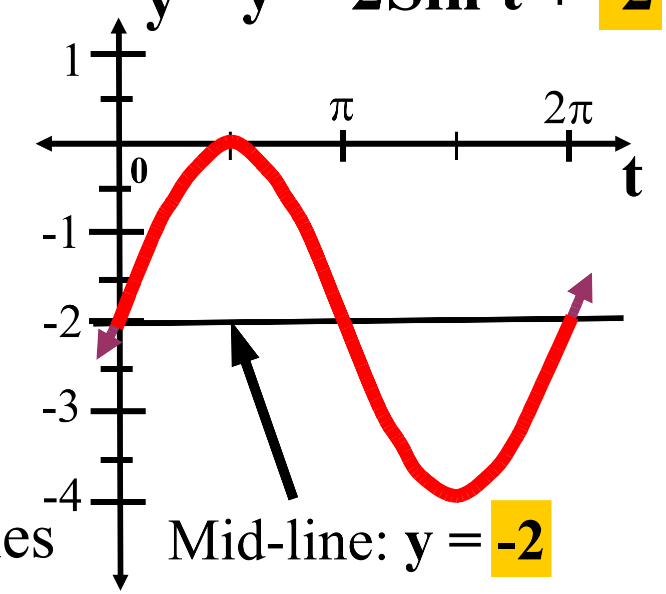

Clearly, the value of D determines the **mid-line** of the graph.

Variations of the Sine Function

Consider the equation $y = A\sin(Bt + C) + D$.

(1) The amplitude of the 'sine wave' is the absolute value of A.

(2) If $A > 0$, then the basic cycle is 'above the mid-line' for the first half of the cycle and below the mid-line for the second half of the cycle.

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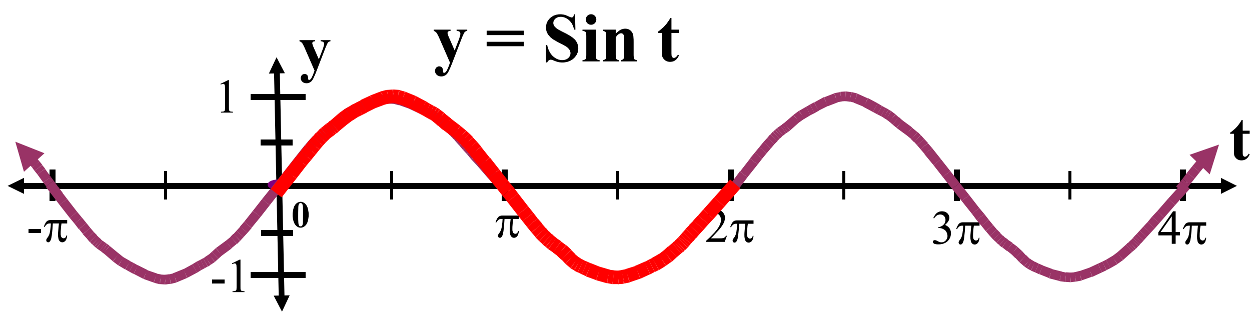
We will next consider the significance of the constants B and C.

Variations of the Sine Function



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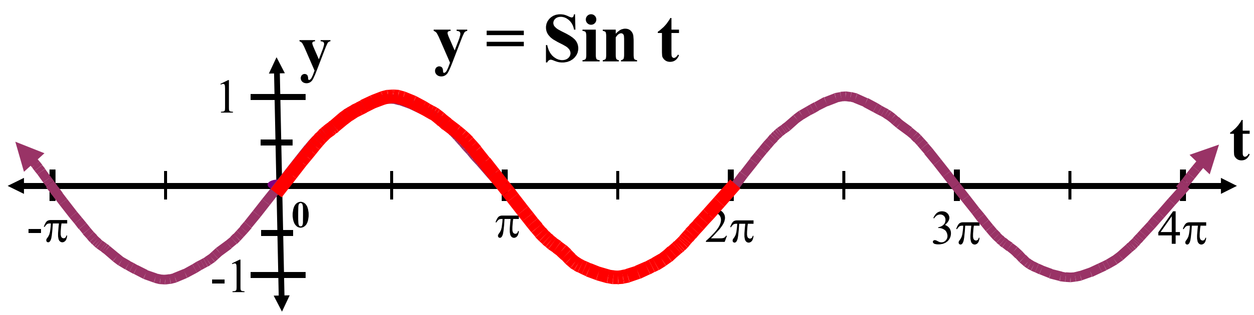
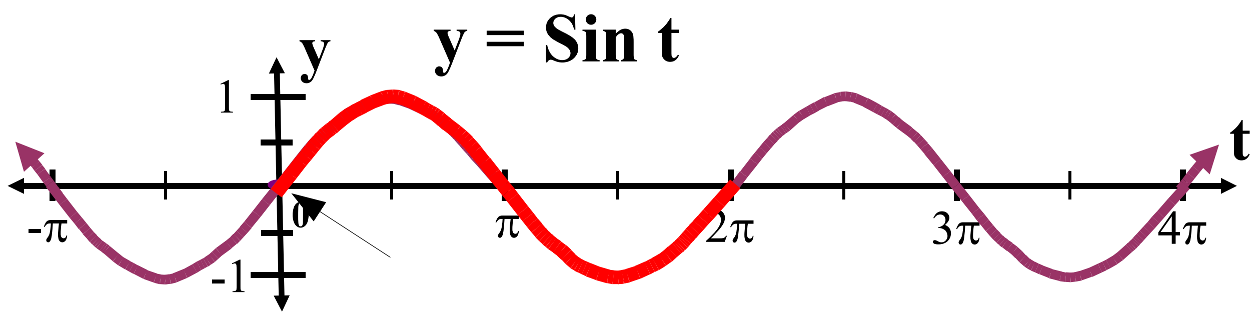


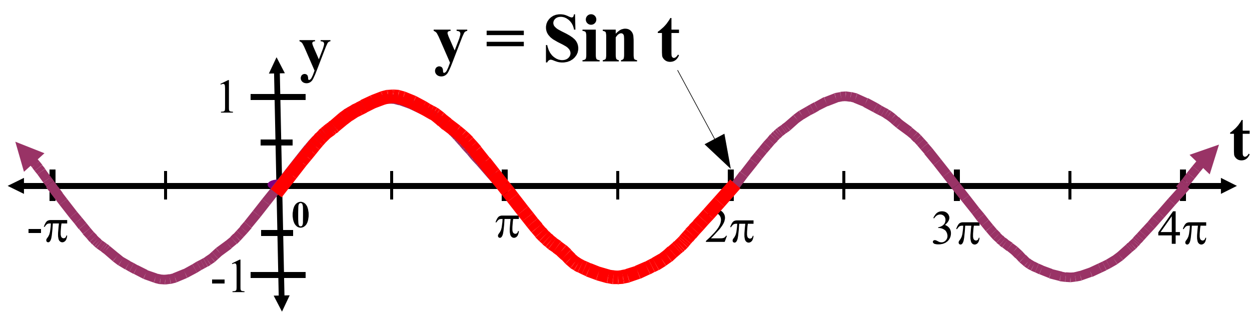
In the above sine graph,

Variations of the Sine Function



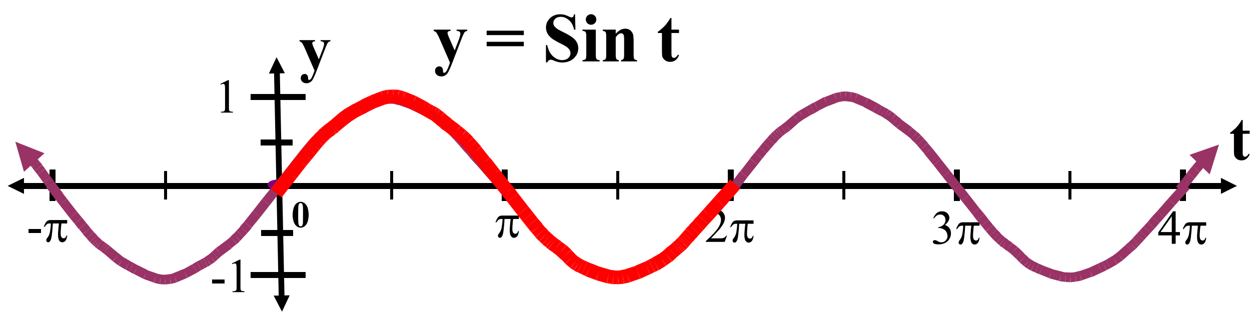
In the above sine graph, the basic cycle starts on the mid-line when t = 0,

Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

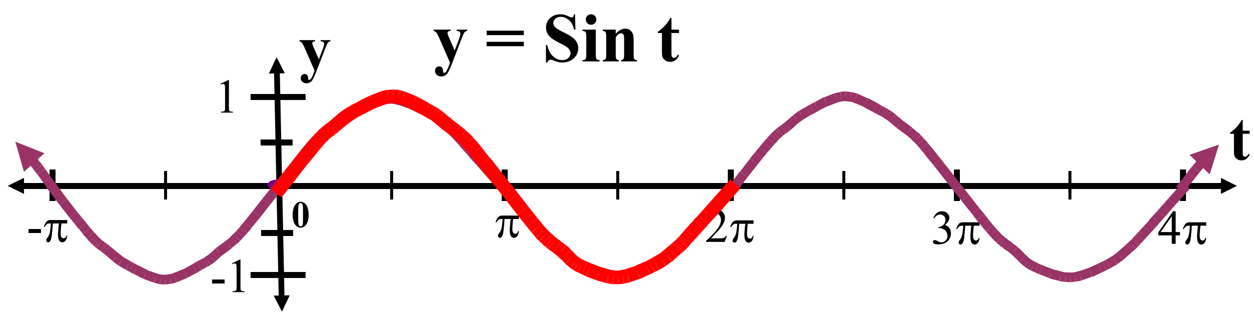
Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = 2Sin(2t - \pi) + 1}$.

Variations of the Sine Function



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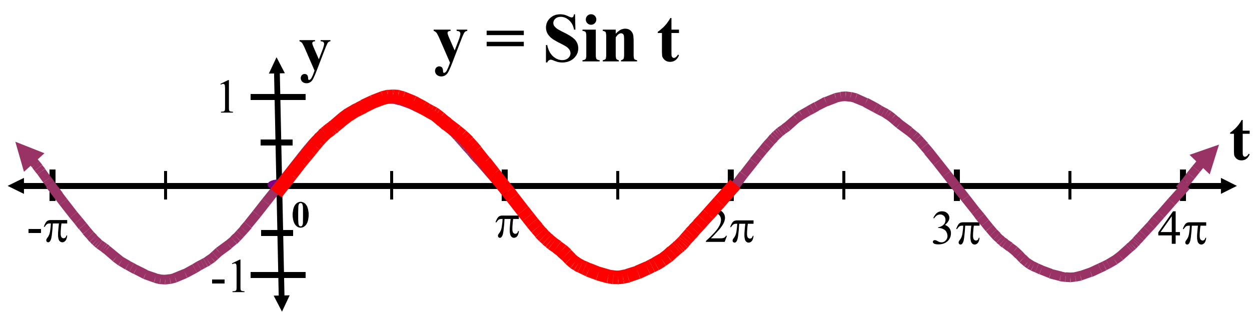
Variations of the Sine Function



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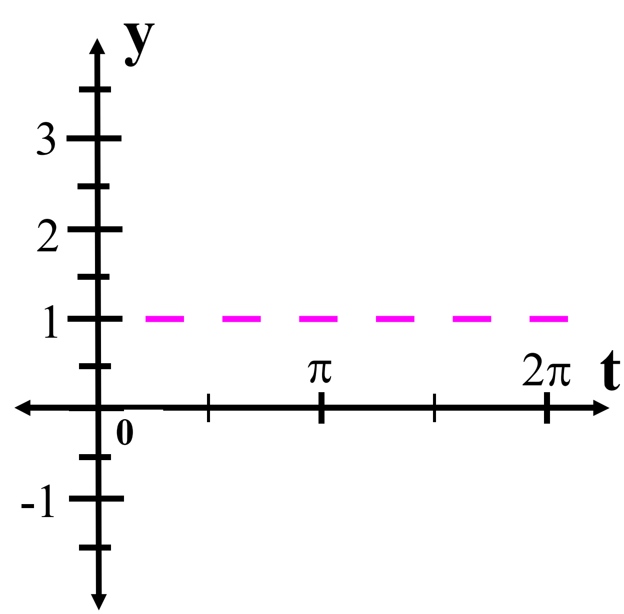

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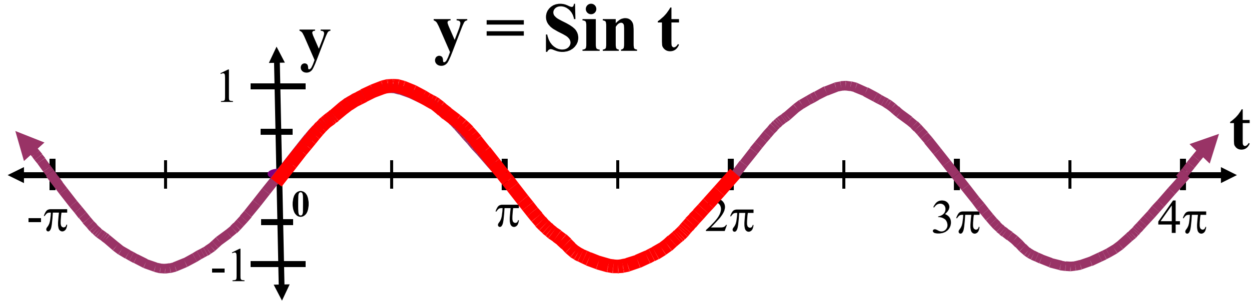

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Now, consider the equation **y = 2Sin(2t – π) + 1**.

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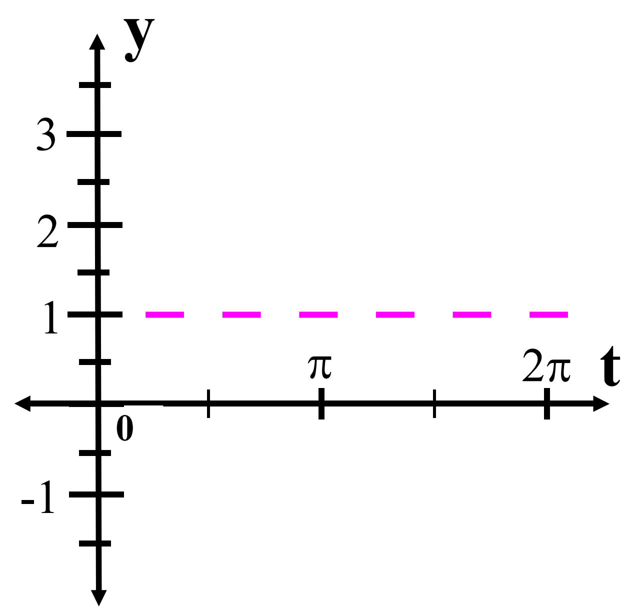

Variations of the Sine Function

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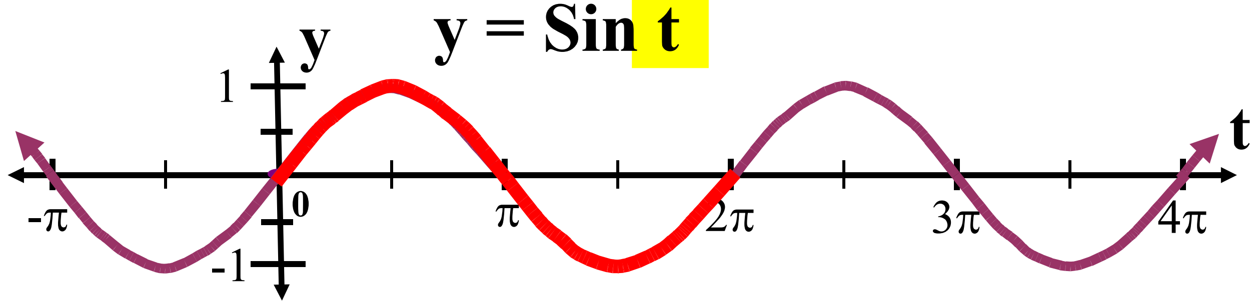

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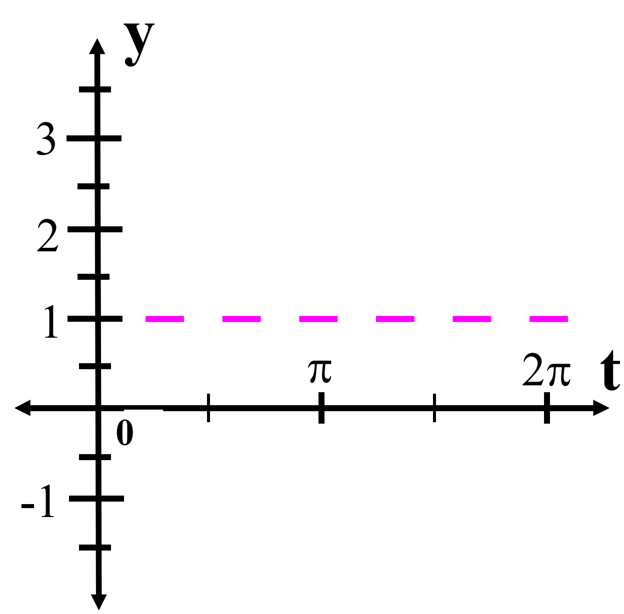

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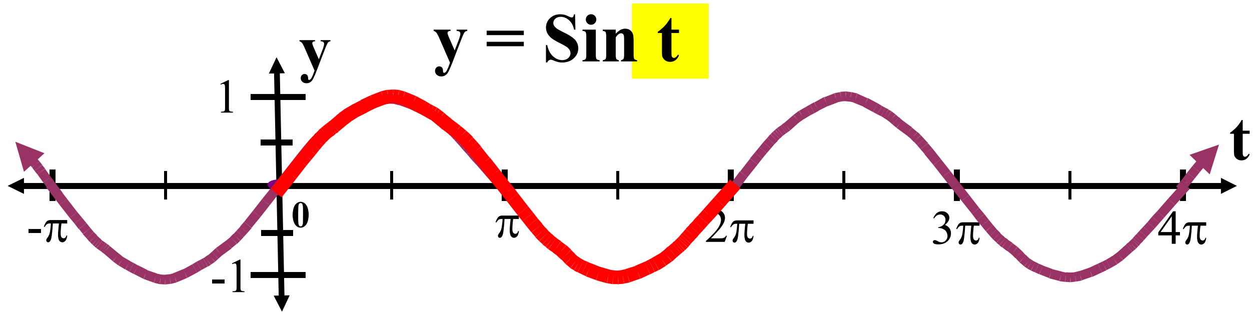

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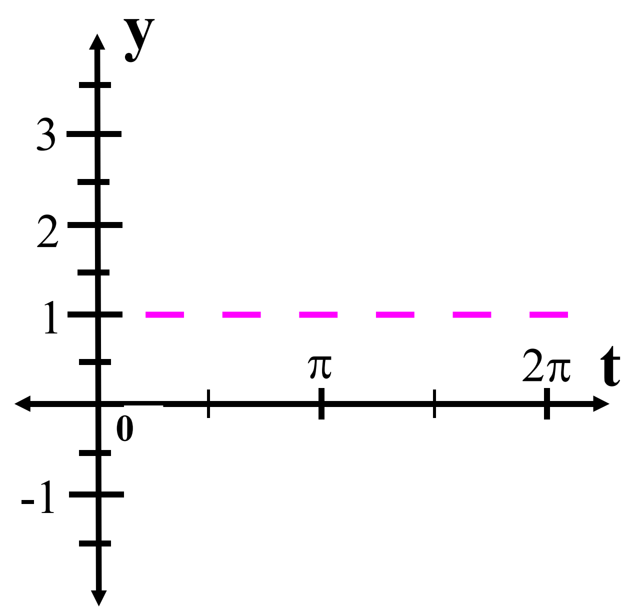

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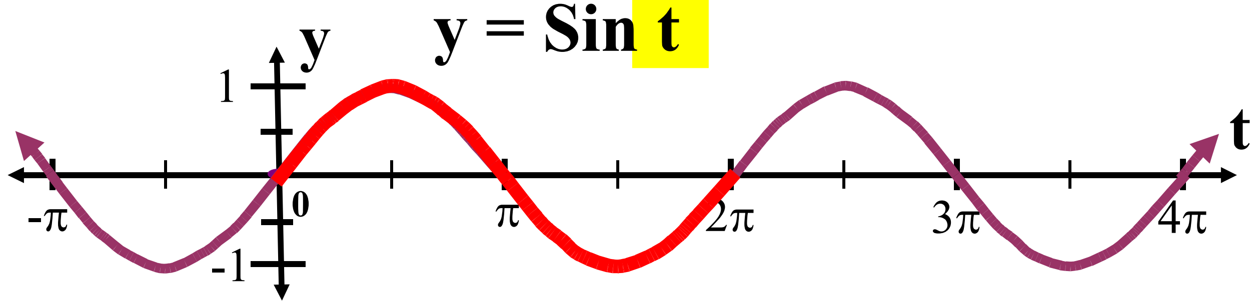

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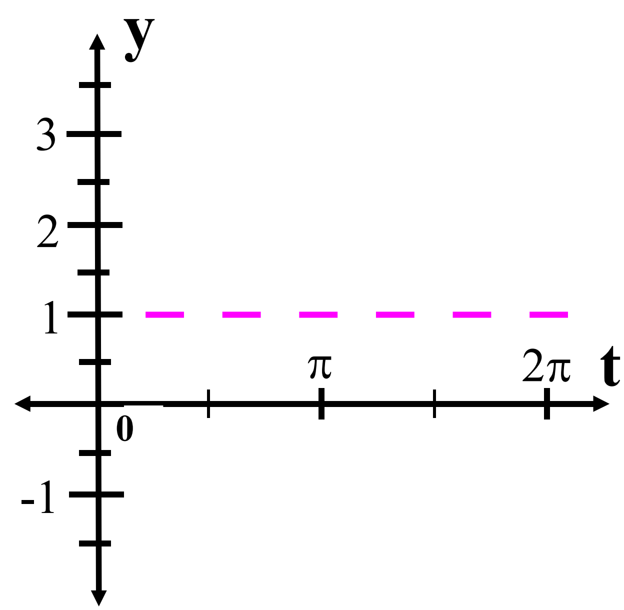

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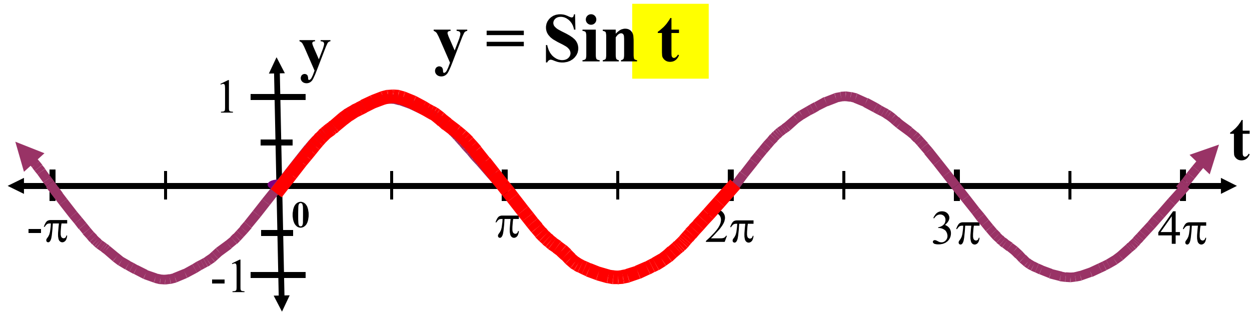

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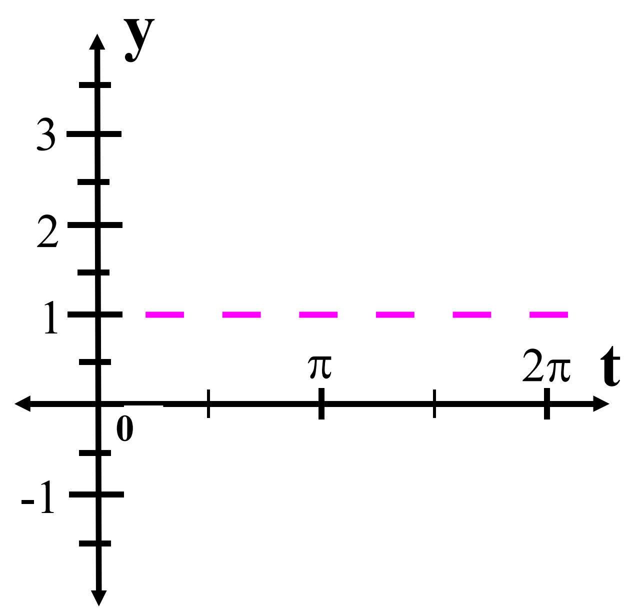

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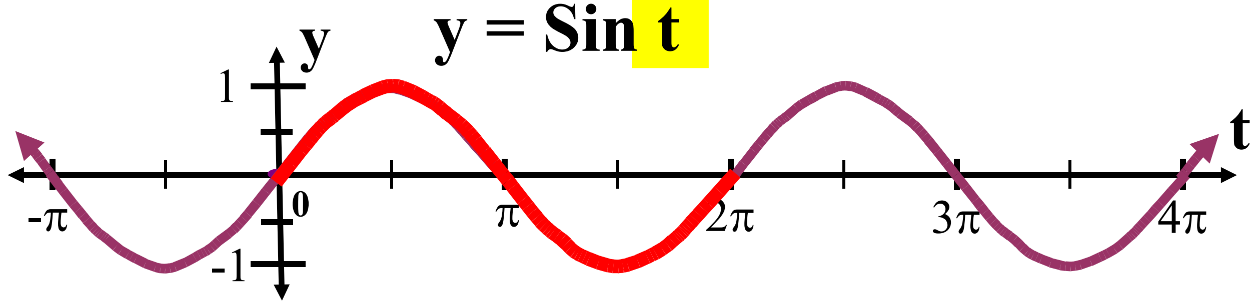

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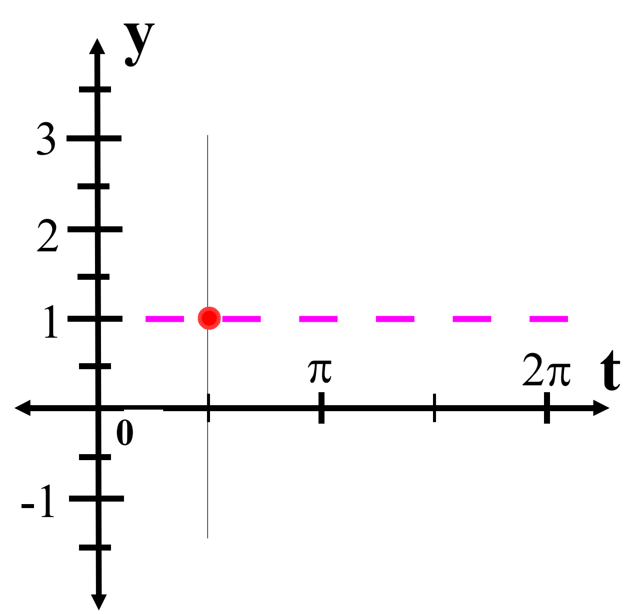

Variations of the Sine Function

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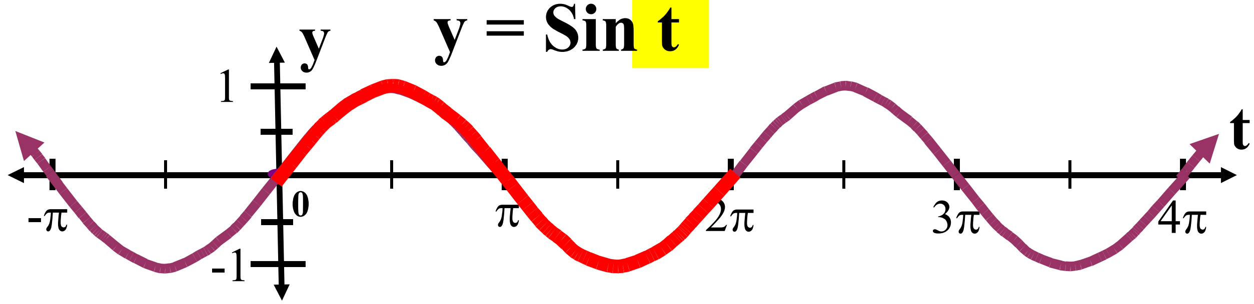

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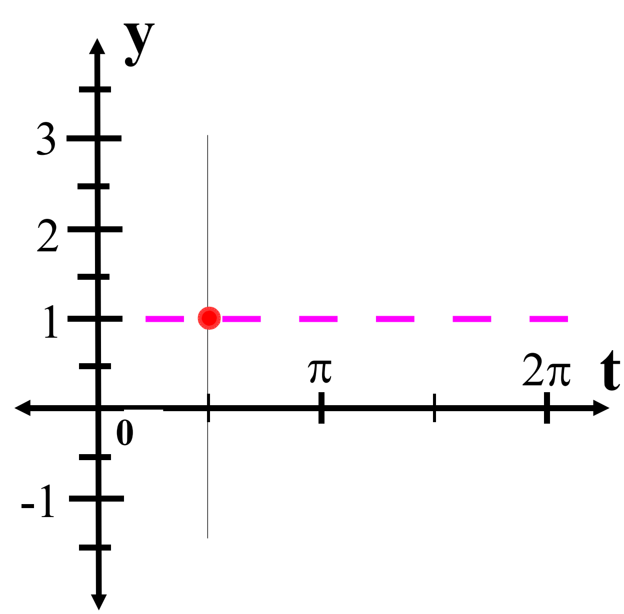
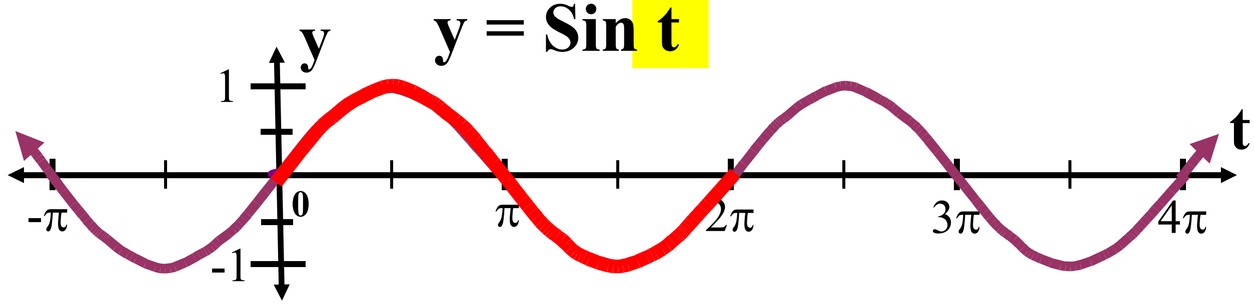


Variations of the Sine Function

y = Sin t



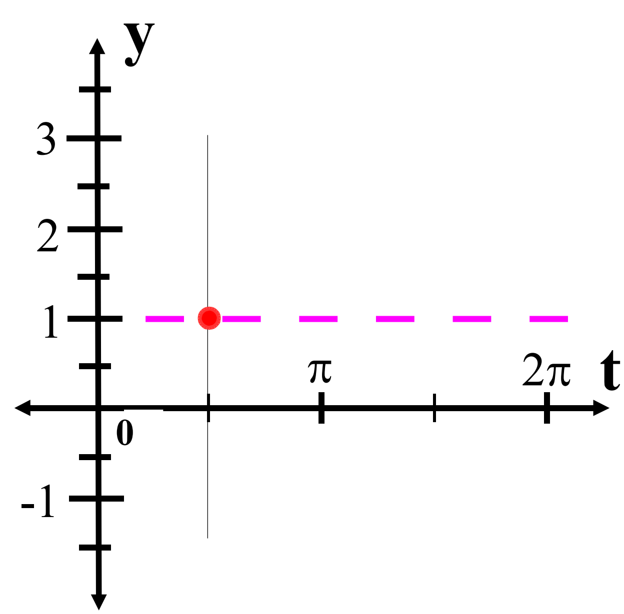
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = 2Sin(2t – π) + 1**.

Mid-line: **y = 1**

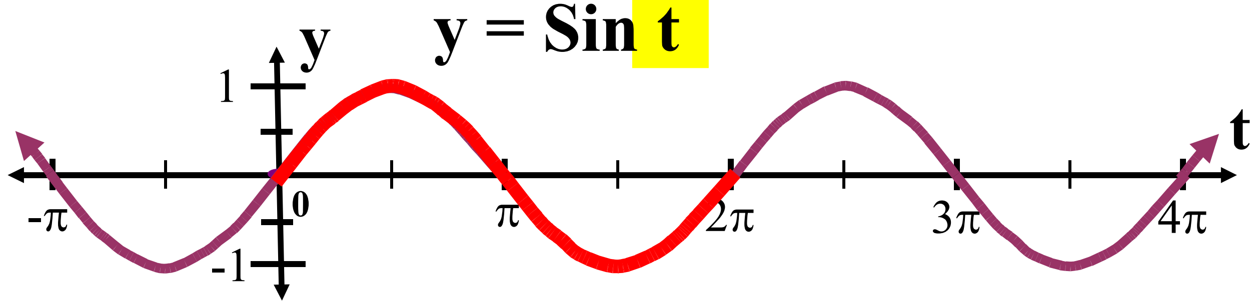
The basic cycle starts on the mid-line when 2t – π = 0. → t = π/2

The basic cycle ends on the mid-line when 2t – π = 2π.



Variations of the Sine Function

$y = \text{Sin } t$



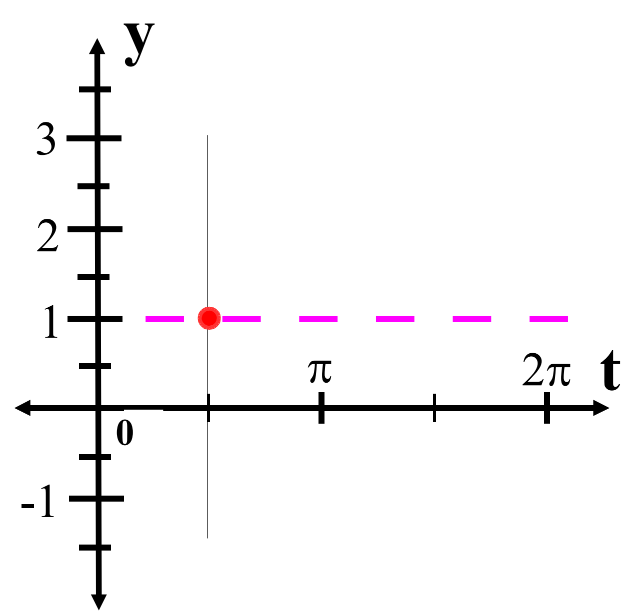
In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = 2Sin(2t - \pi) + 1}$.

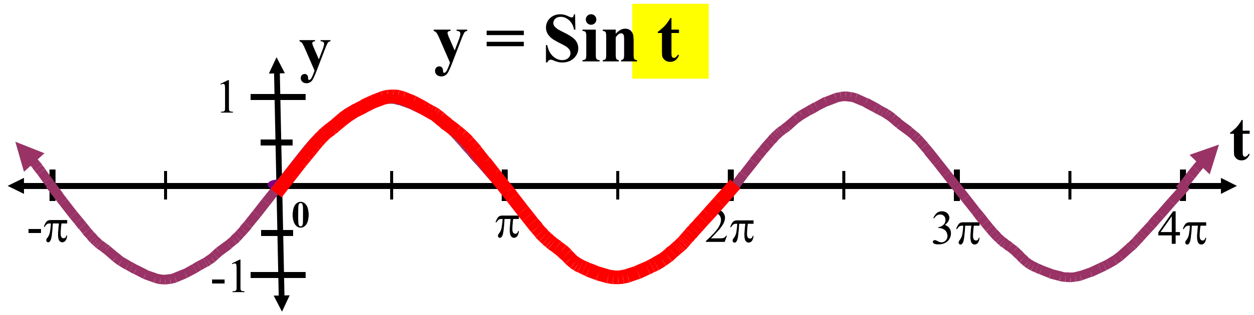
Mid-line: $\mathbf{y = 1}$

The basic cycle starts on the mid-line when $2t - \pi = 0$. → $t = \pi/2$

The basic cycle ends on the mid-line when $2t - \pi = 2\pi$. → $t = 3\pi/2$



Variations of the Sine Function



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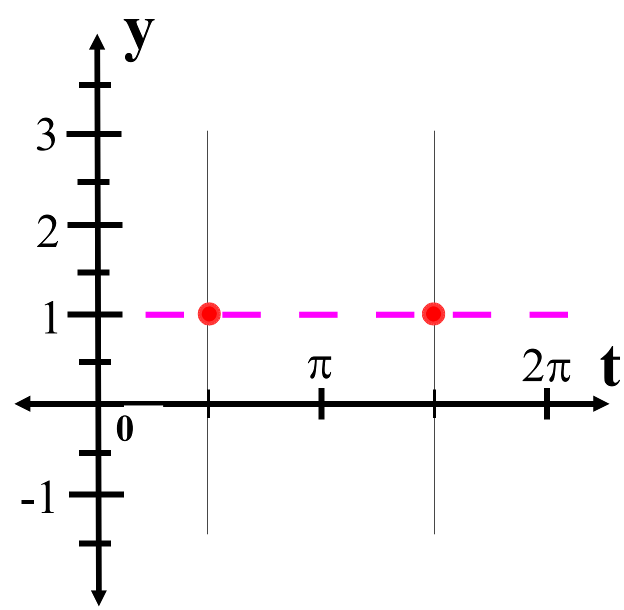
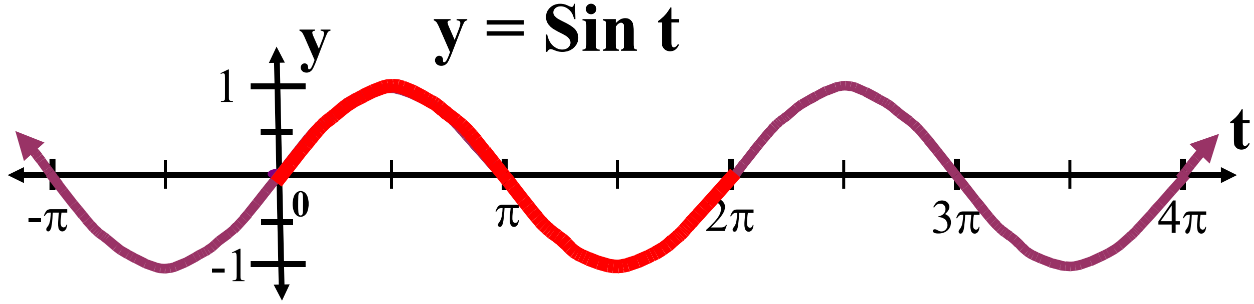


Variations of the Sine Function



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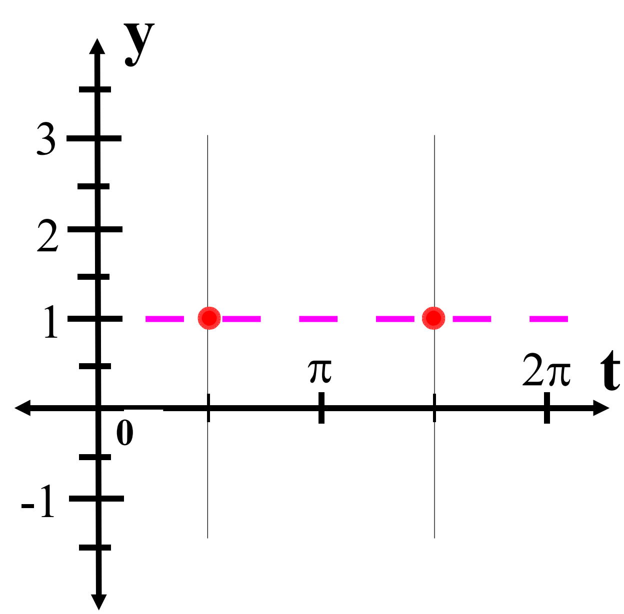
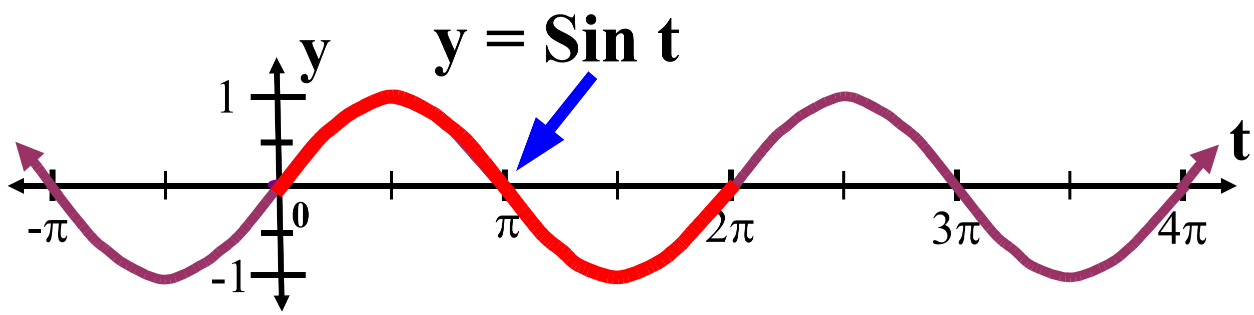


Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

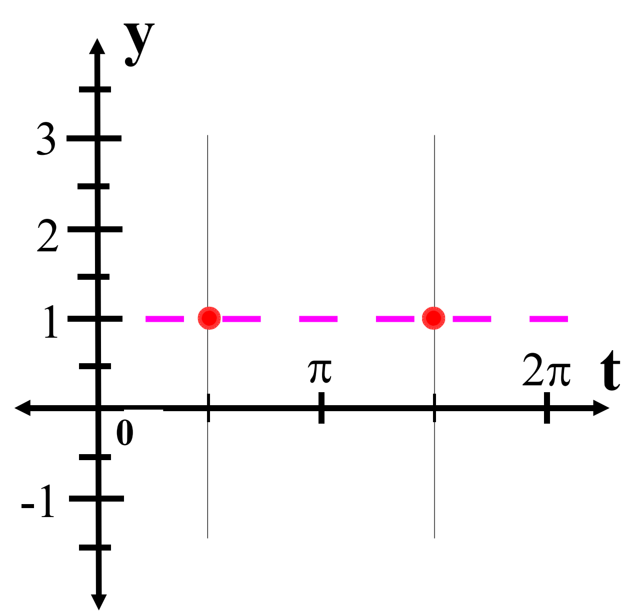
Now, consider the equation y = **2Sin(2t – π) + 1**.

Mid-line: **y = 1**

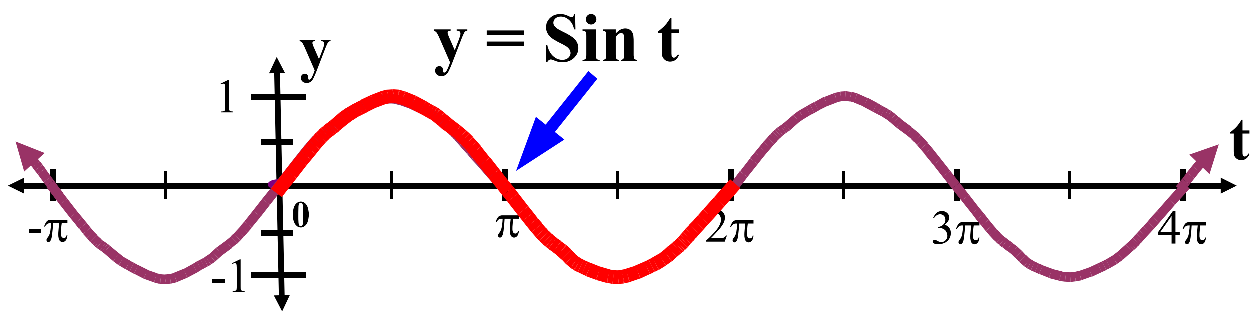
The basic cycle starts on the mid-line when 2t – π = 0. → t = π/2

The basic cycle ends on the mid-line when 2t – π = 2π. → t = 3π/2

The basic cycle 'intersects the mid-line 'half-way' through the cycle.



Variations of the Sine Function



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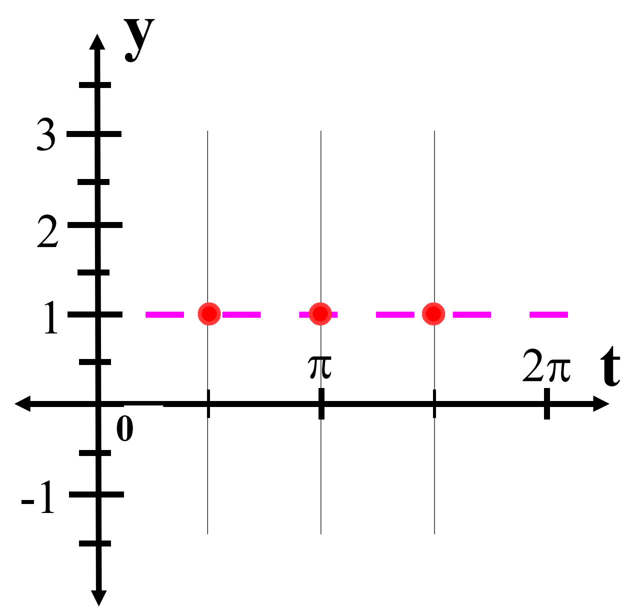
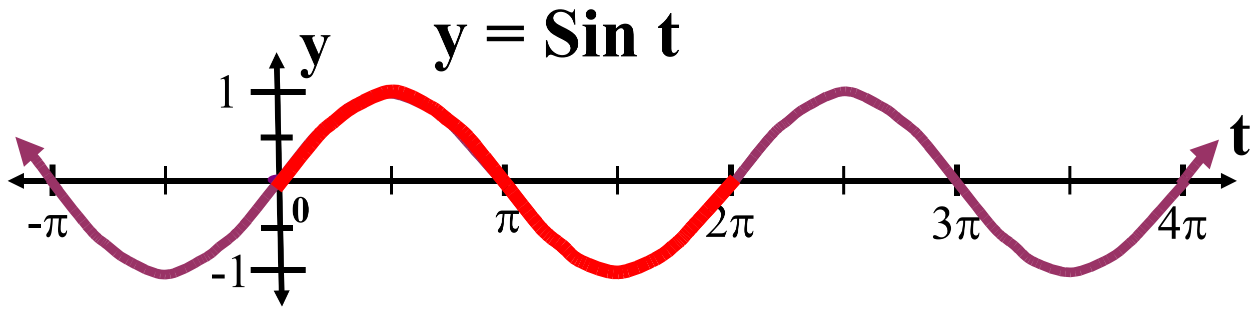


Variations of the Sine Function



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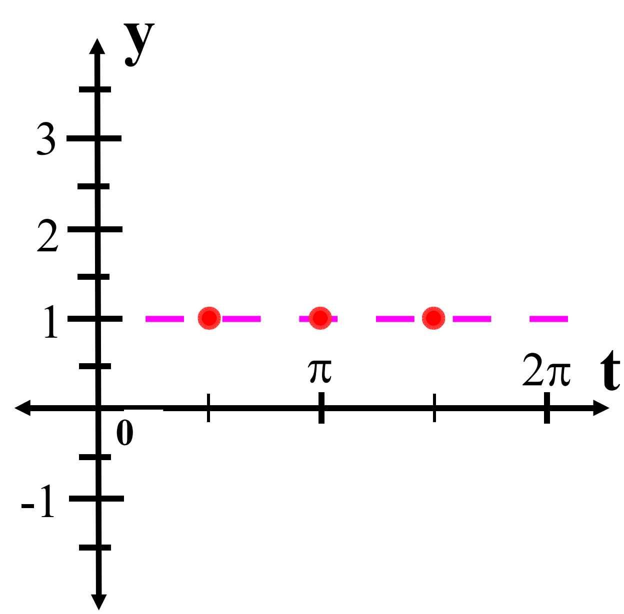
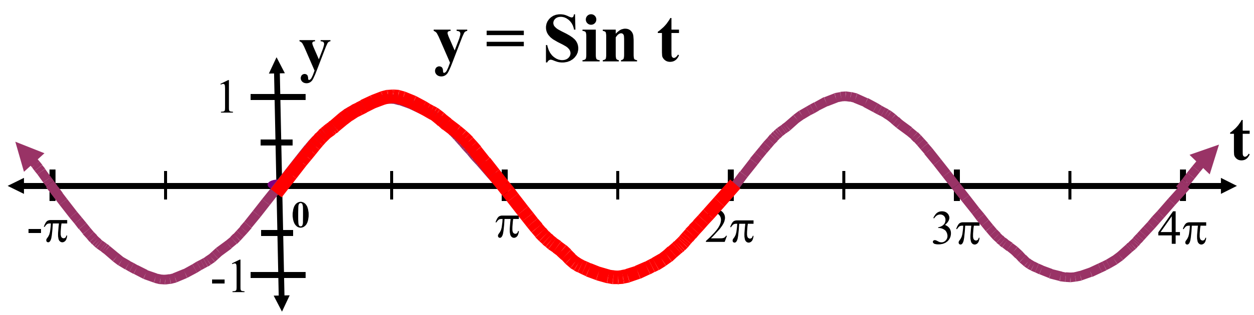


Variations of the Sine Function



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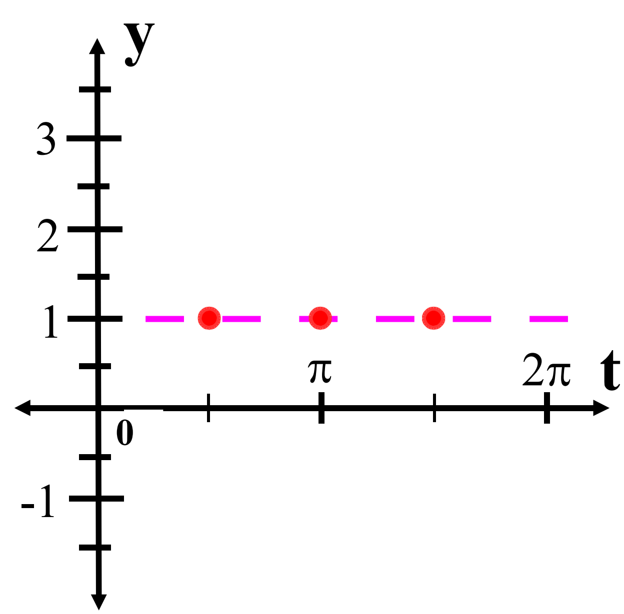
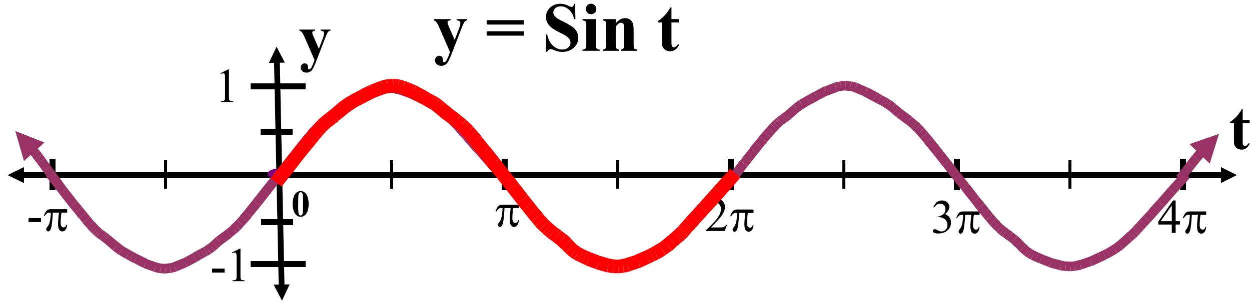


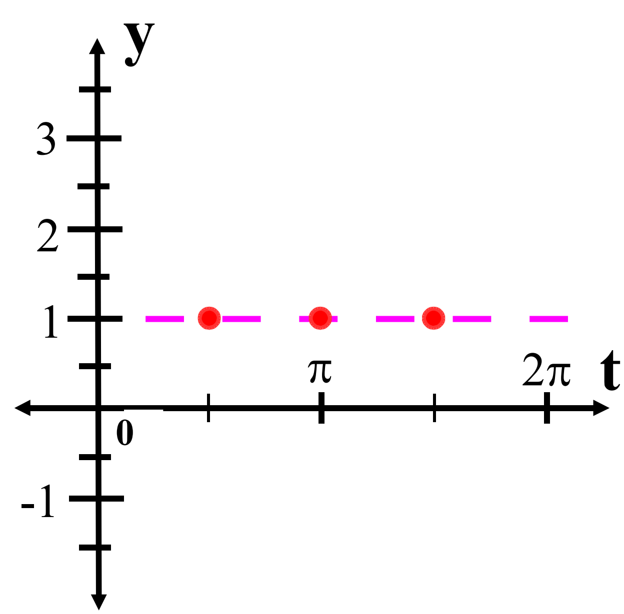
Variations of the Sine Function



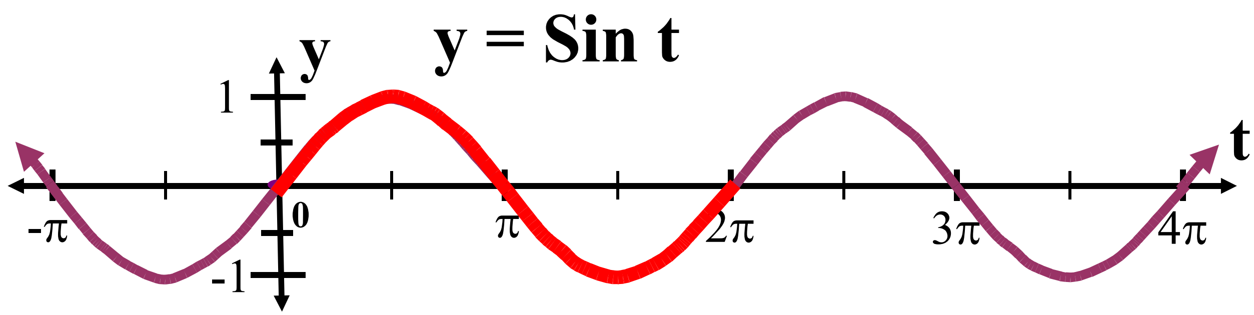
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = 2Sin(2t – π) + 1**.

A = +2



Variations of the Sine Function



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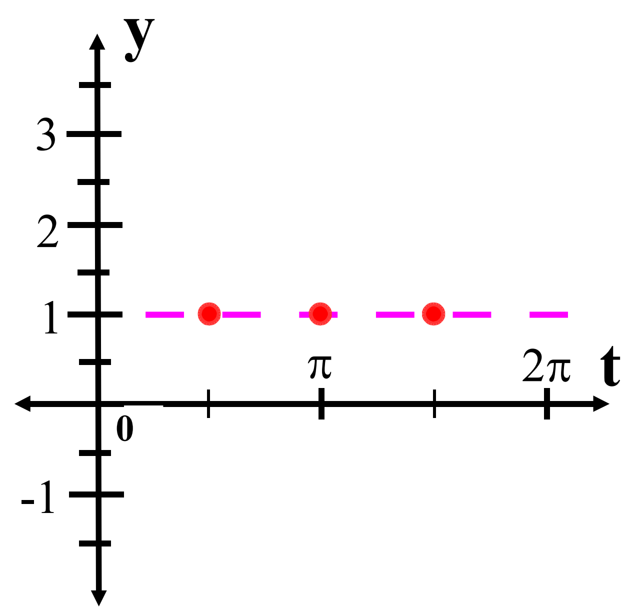
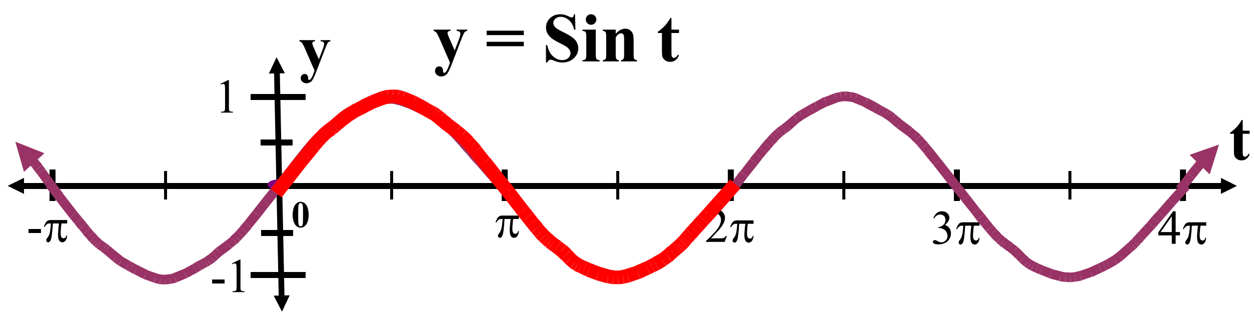
Now, consider the equation **y = 2Sin(2t – π) + 1**.

A = +2

→ The amplitude is 2.



Variations of the Sine Function



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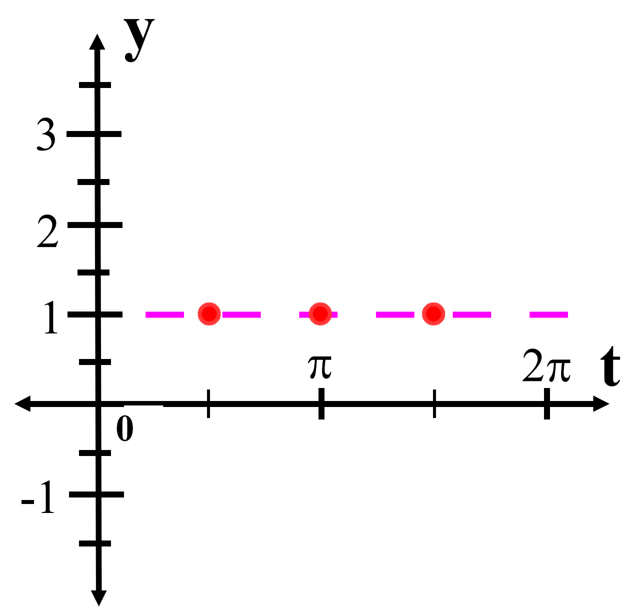
Now, consider the equation **y = 2Sin(2t – π) + 1**.

A = +2

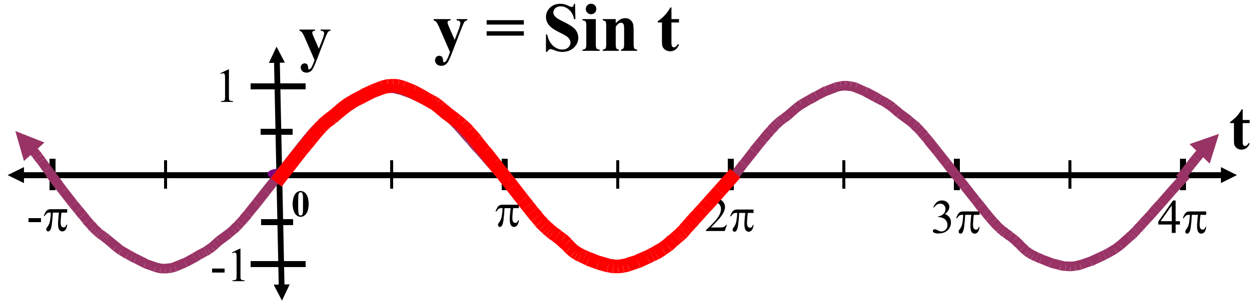
→ The amplitude is 2.

→ The basic cycle is 'above the mid-line' for the first half of the cycle and below the mid-line for the second half of the cycle.



Variations of the Sine Function

y = Sin t



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Now, consider the equation **y = 2Sin(2t – π) + 1**.

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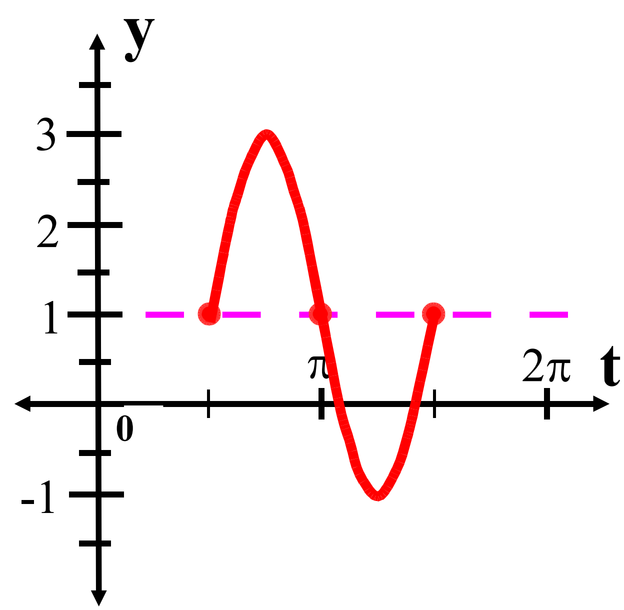
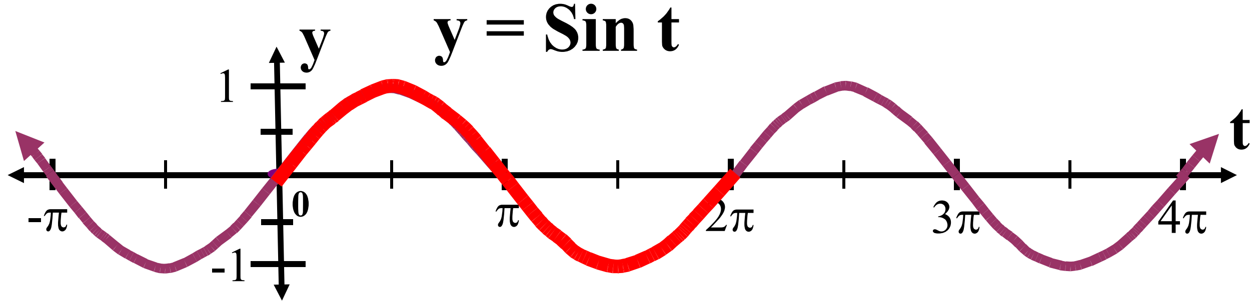


Variations of the Sine Function



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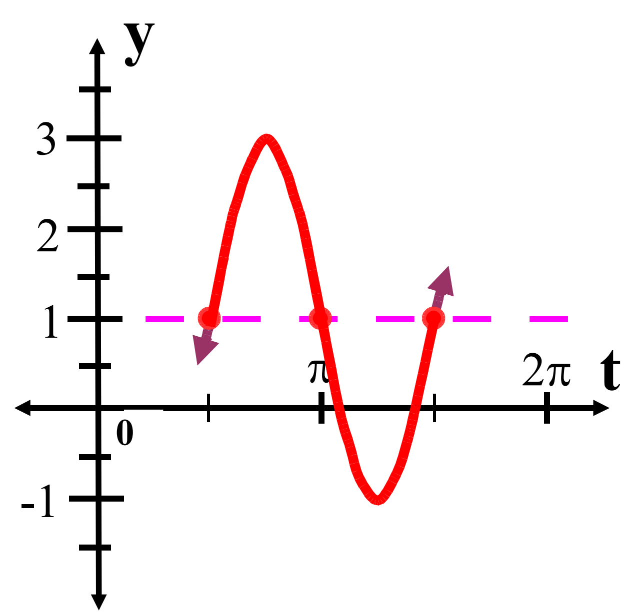
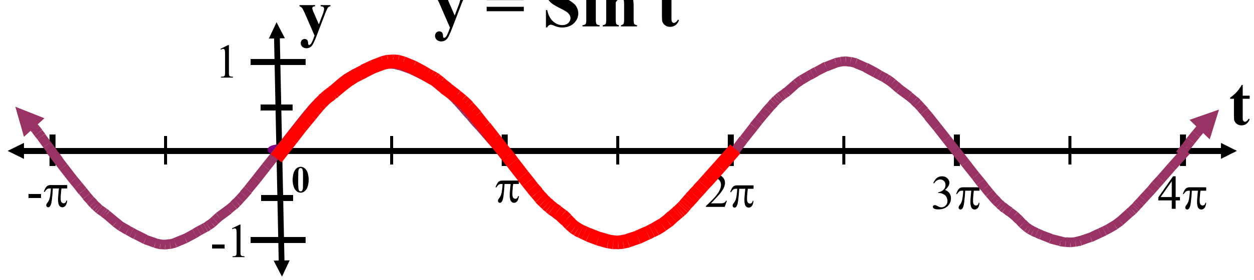


Variations of the Sine Function

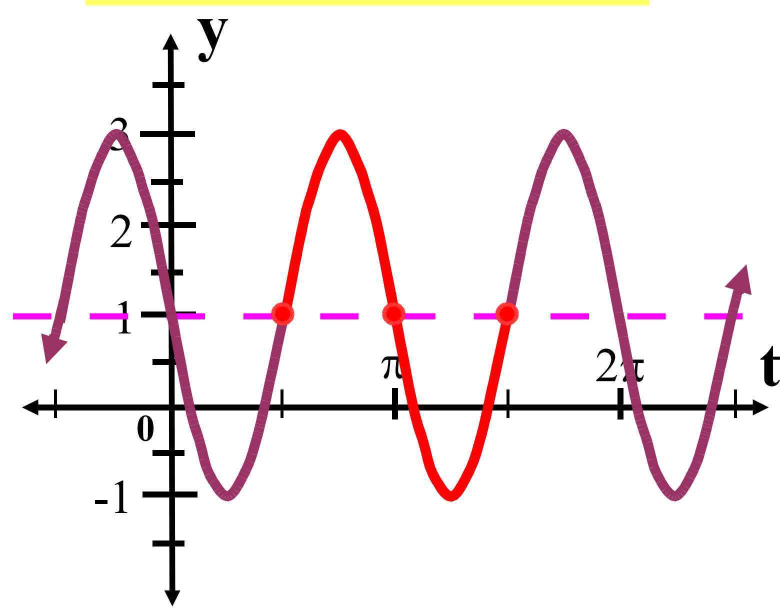
y = Sin t



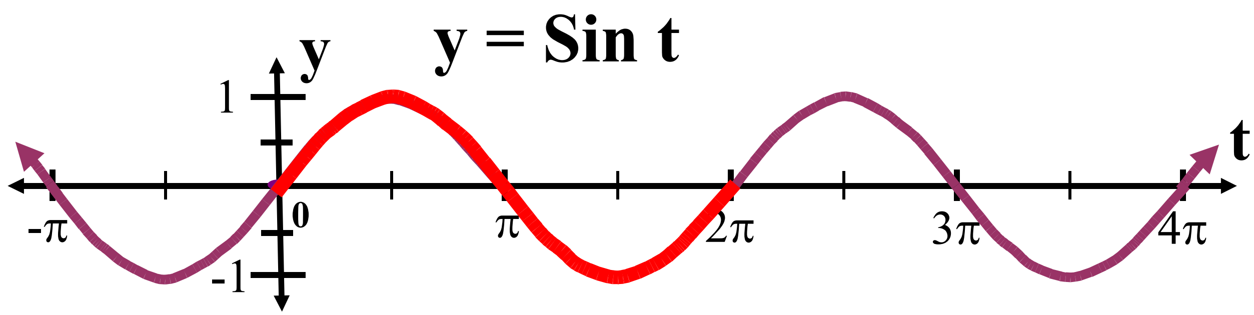
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Here is a more complete graph.

y = 2Sin(2t – π) + 1



Variations of the Sine Function

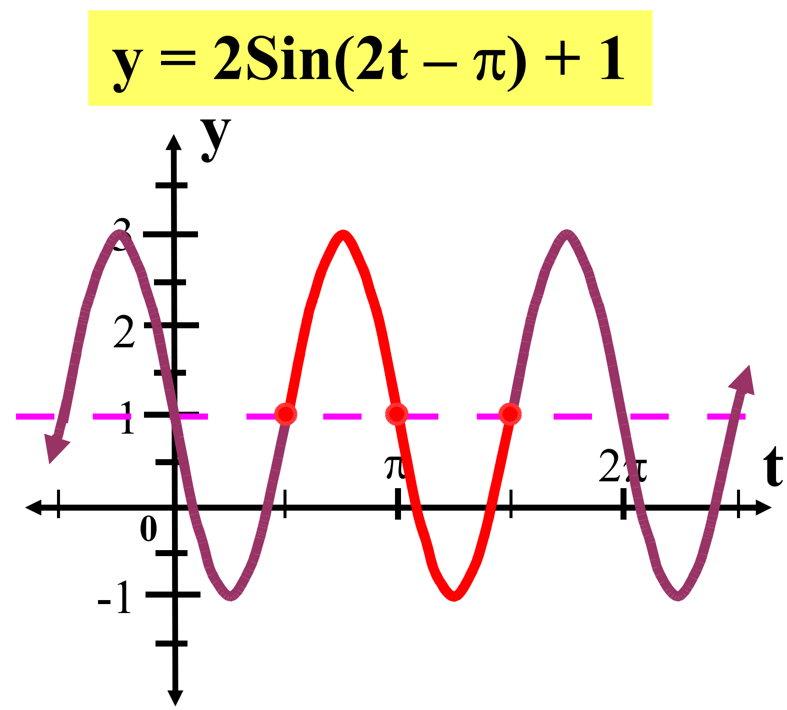


In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

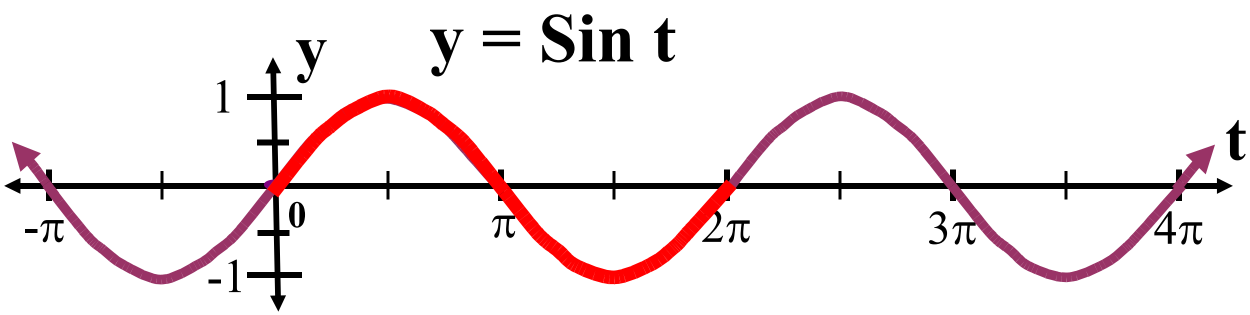
Here is a more complete graph.

Given the equation:

y = ASin(Bt + C) + D



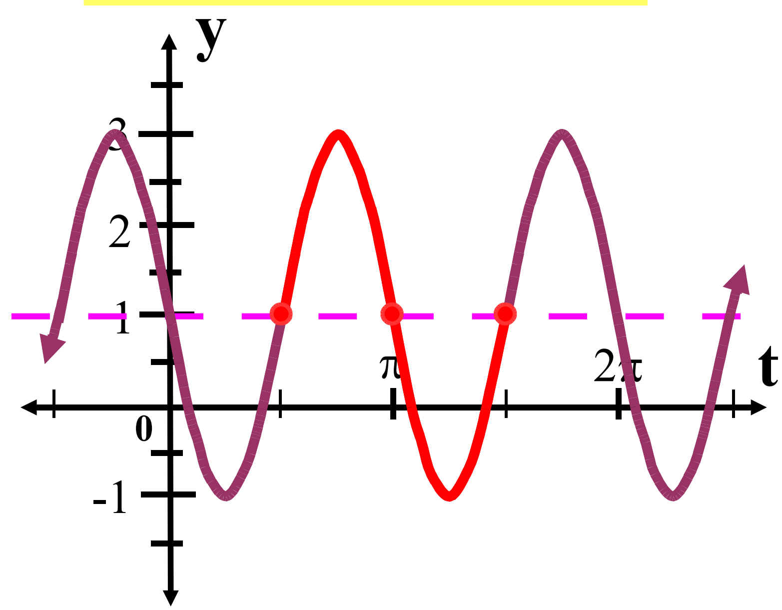
Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Here is a more complete graph.

y = 2Sin(2t – π) + 1



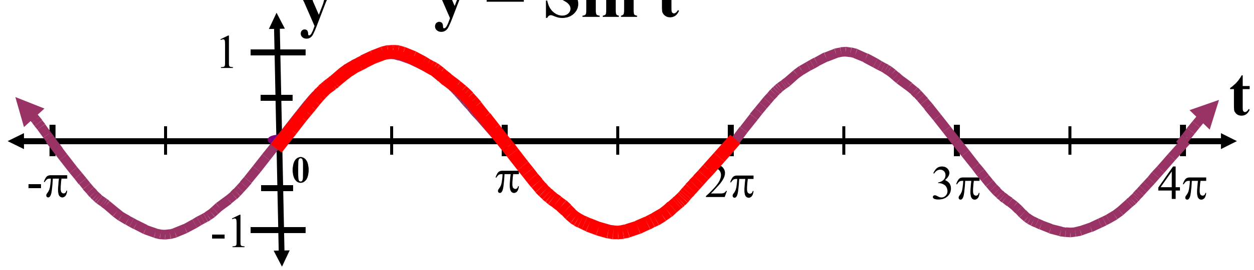
Given the equation:

y = ASin(Bt + C) + D

The 'Basic Cycle' starts on the mid-line when Bt + C = 0

Variations of the Sine Function

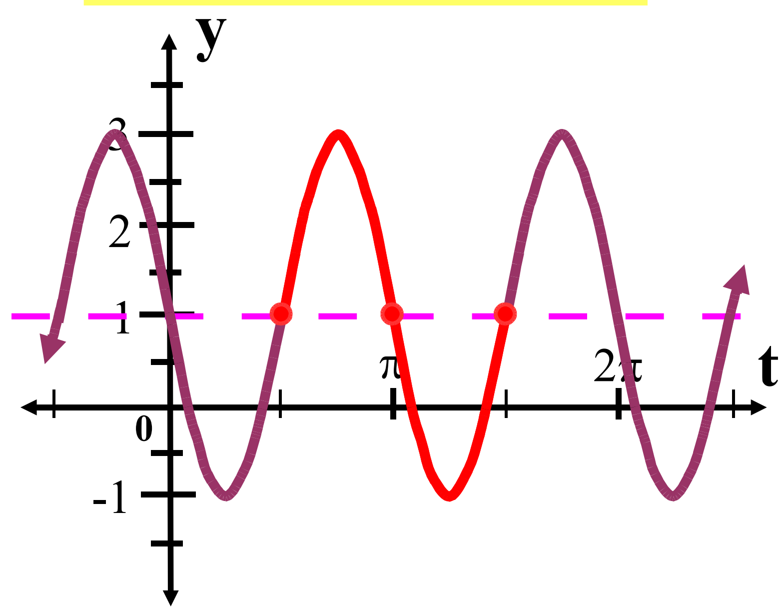
y = Sin t



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Here is a more complete graph.

y = 2Sin(2t – π) + 1



Given the equation:

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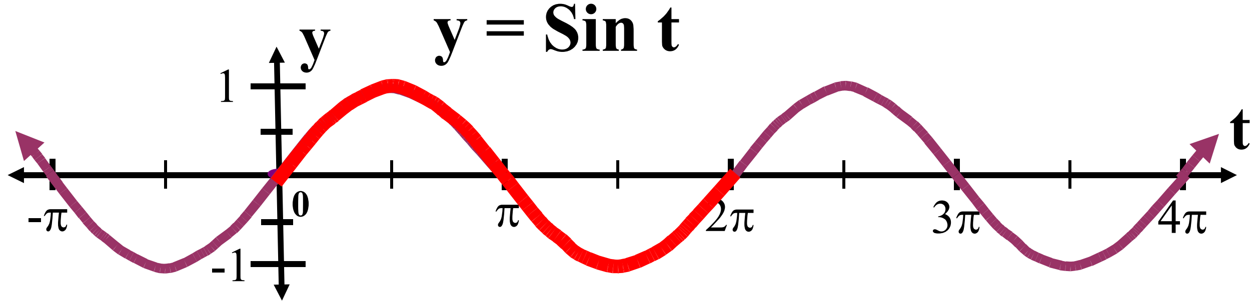
The 'Basic Cycle' starts on the mid-line when Bt + C = 0 and ends on the mid-line when Bt + C = 2π .

Variations of the Sine Function



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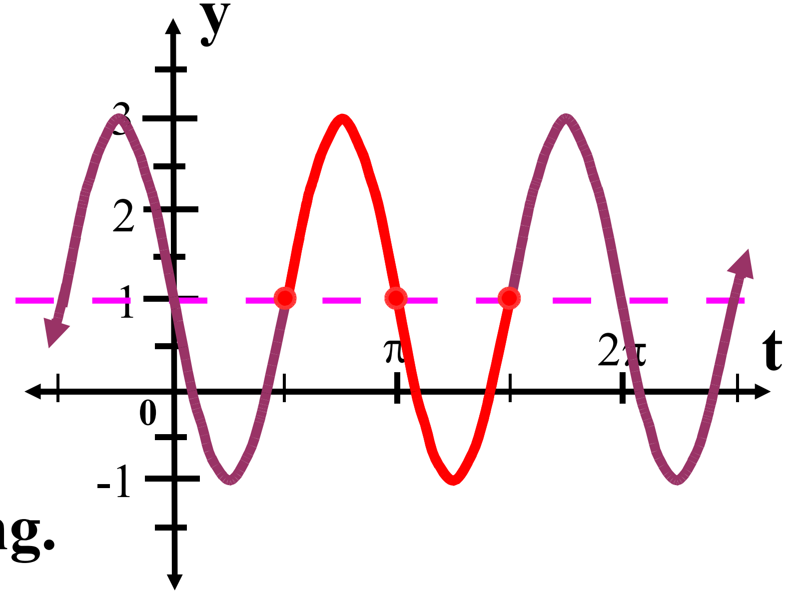
y = 2Sin(2t – π) + 1

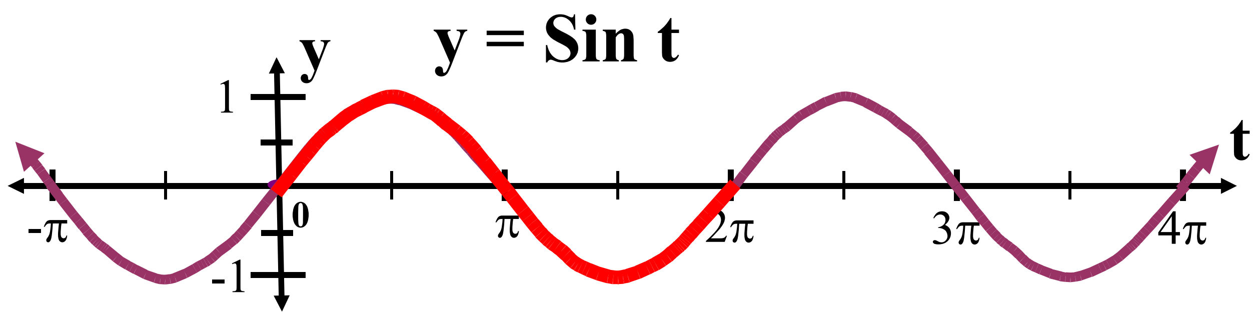


Given the equation:

y = ASin(Bt + C) + D

The 'Basic Cycle' starts on the mid-line when Bt + C = 0 and ends on the mid-line when Bt + C = 2π .
The 'Basic Cycle' is 2π/|B| units long.

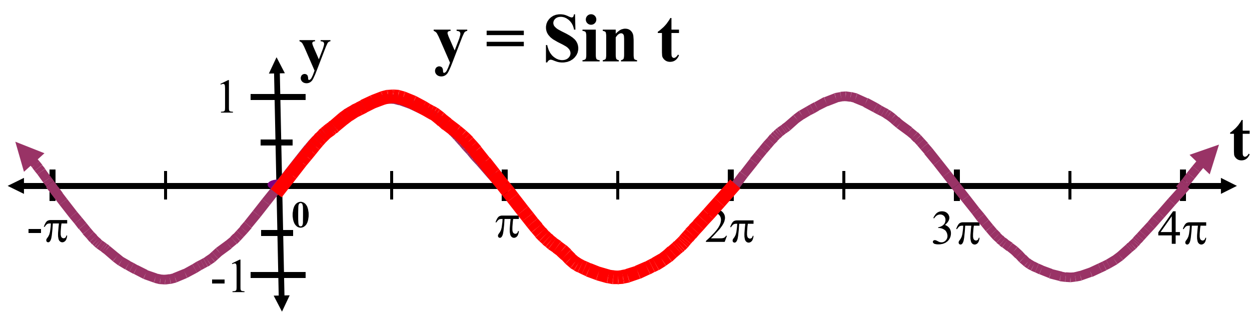
Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

Variations of the Sine Function



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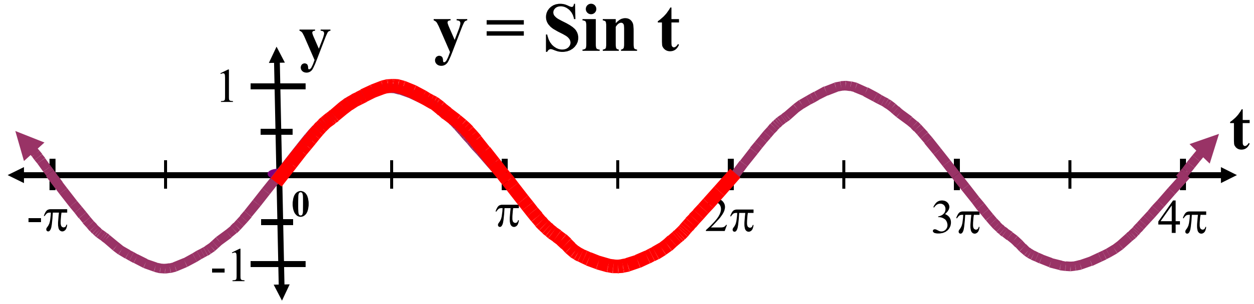
Mid-line:

Variations of the Sine Function

y = Sin t



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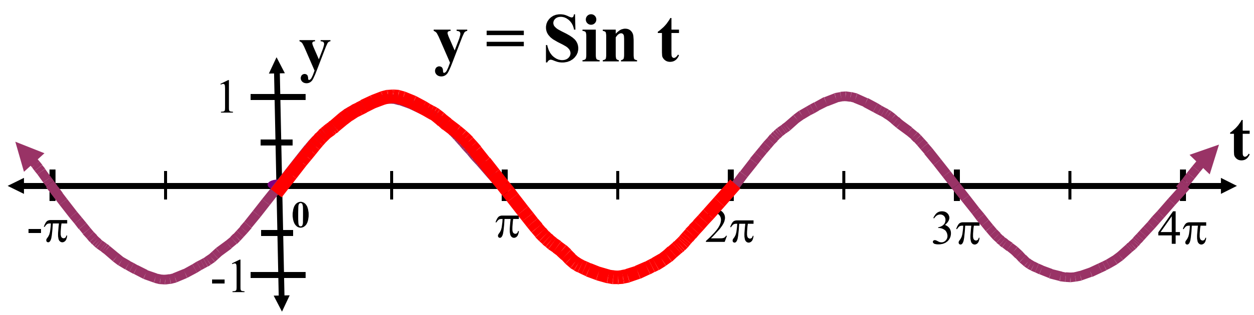
Mid-line: **y = -2**

Variations of the Sine Function



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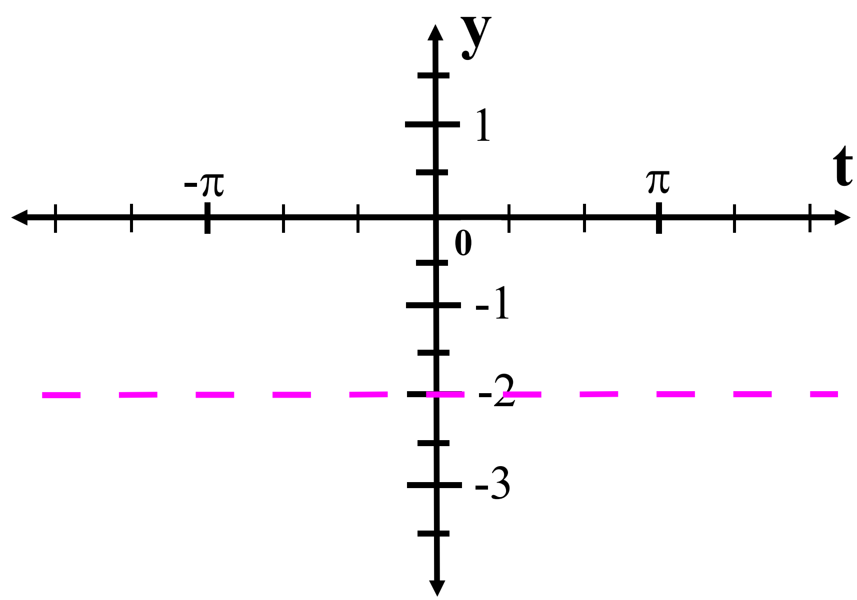
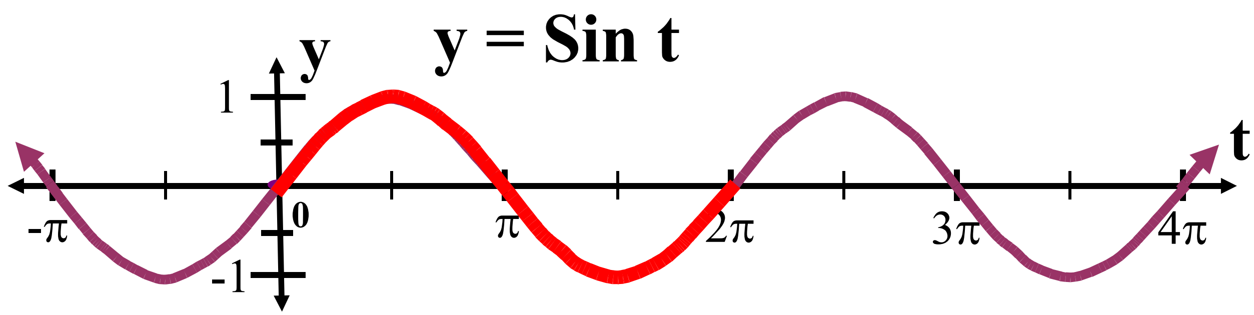
Mid-line: $\mathbf{y = -2}$



Variations of the Sine Function



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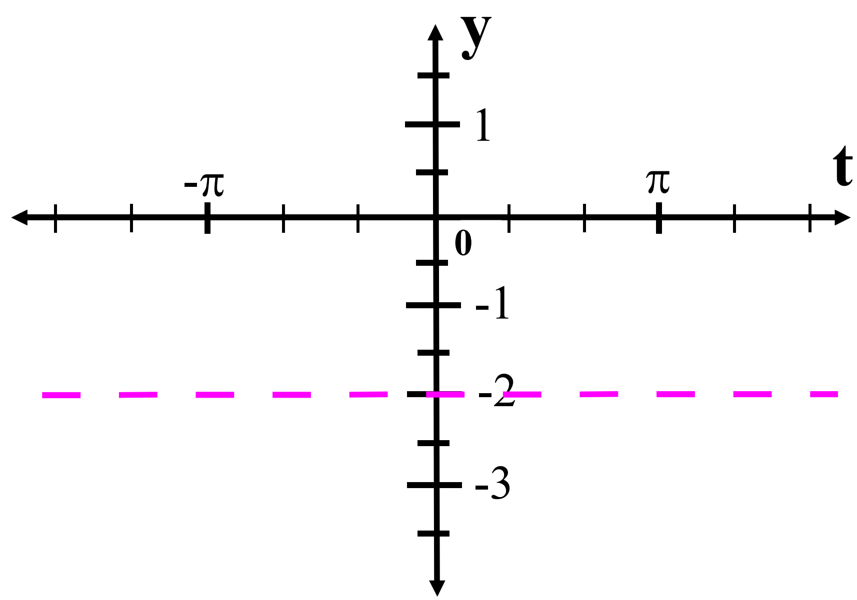
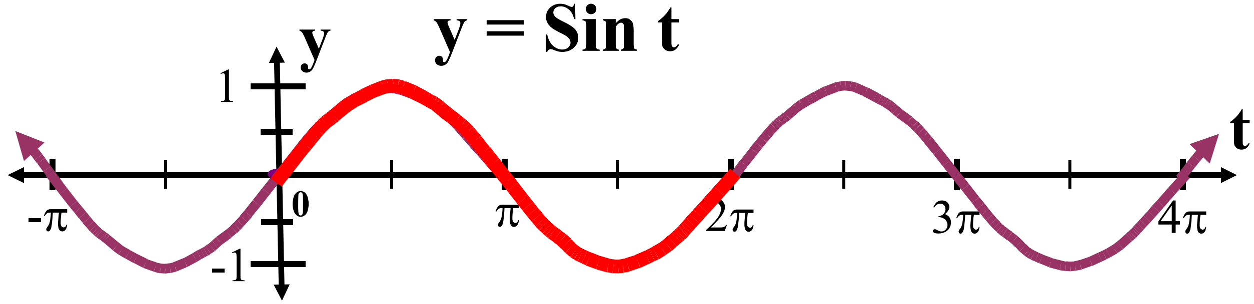


Variations of the Sine Function

y = Sin t



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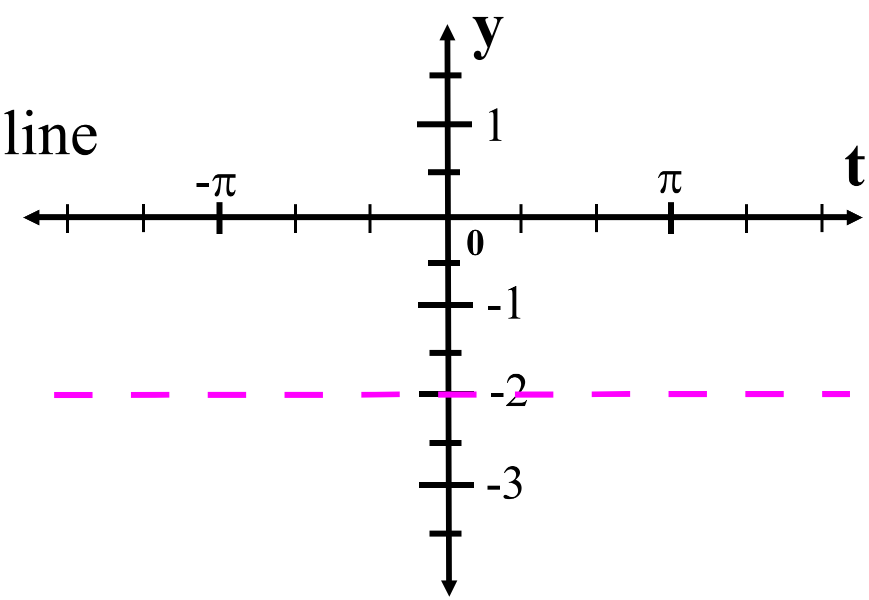
Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

Mid-line: **y = -2**

The basic cycle starts on the mid-line when t + π/3 = 0.



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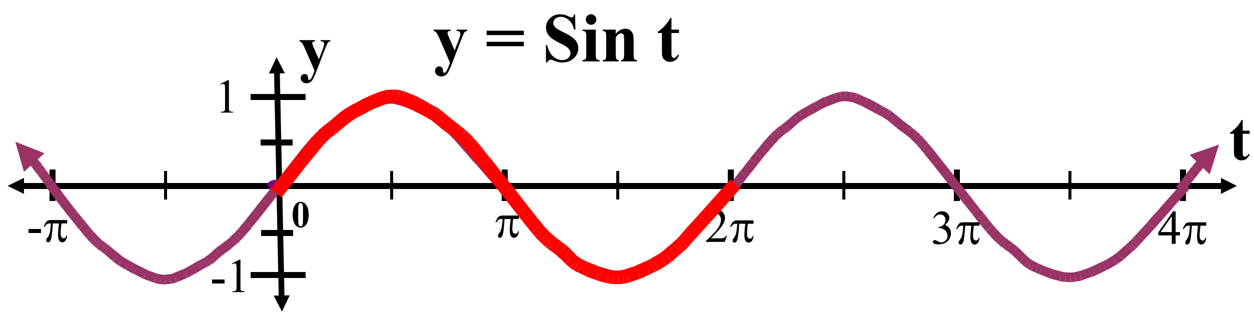
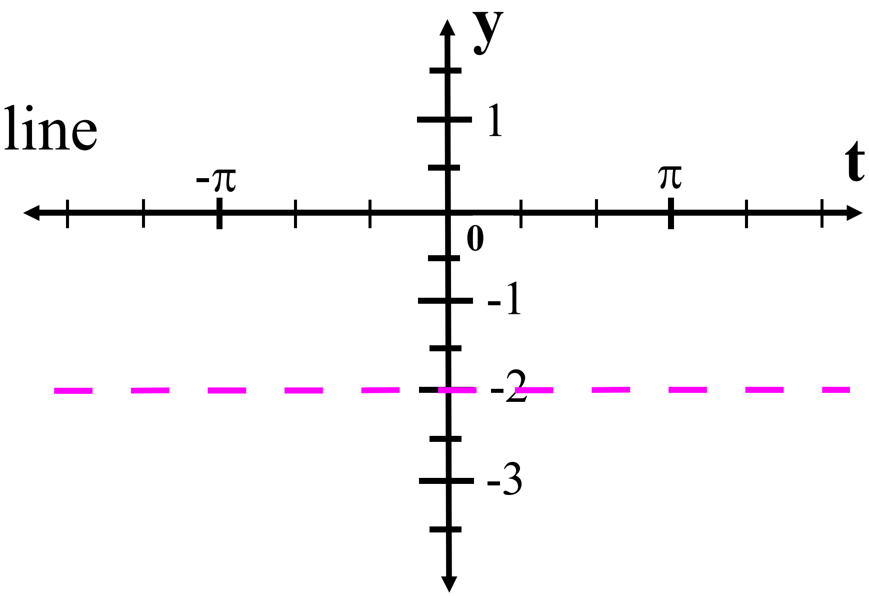


In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = -0.5Sin(t + \pi/3) - 2}$.

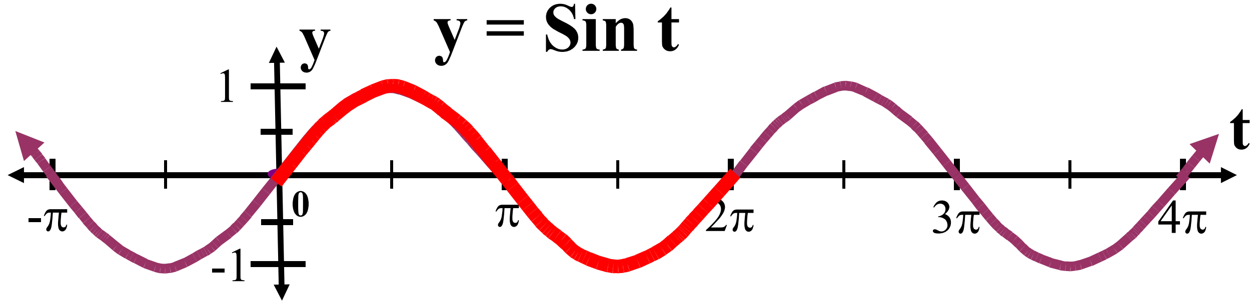
Mid-line: $\mathbf{y = -2}$

The basic cycle starts on the mid-line when $t + \pi/3 = 0$. → $t = -\pi/3$



Variations of the Sine Function

y = Sin t



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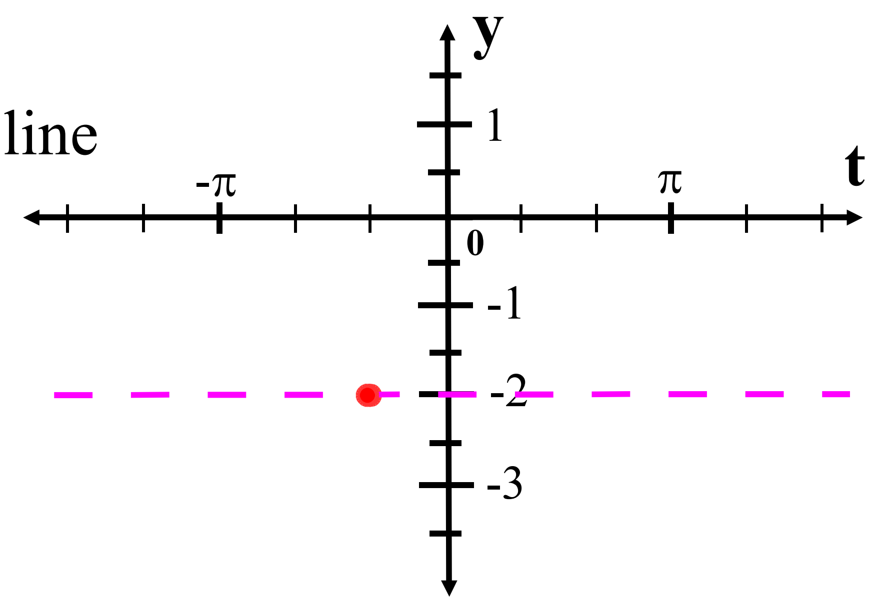
Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

Mid-line: **y = -2**

The basic cycle starts on the mid-line when t + π/3 = 0. → t = -π/3



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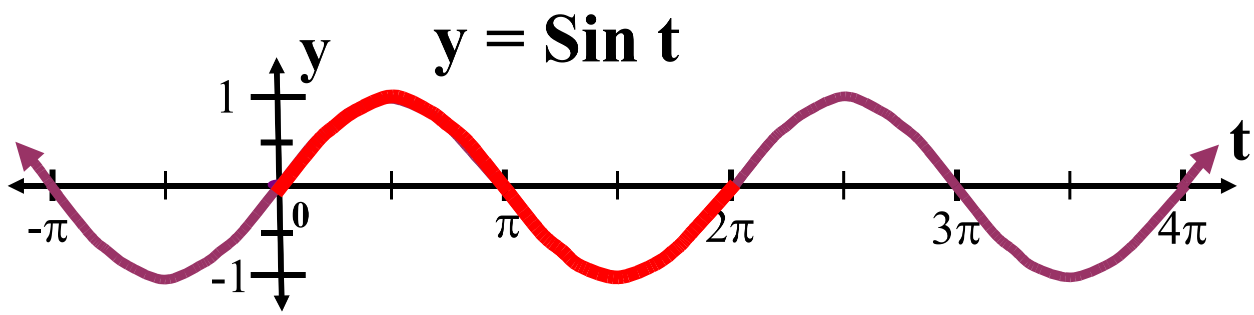

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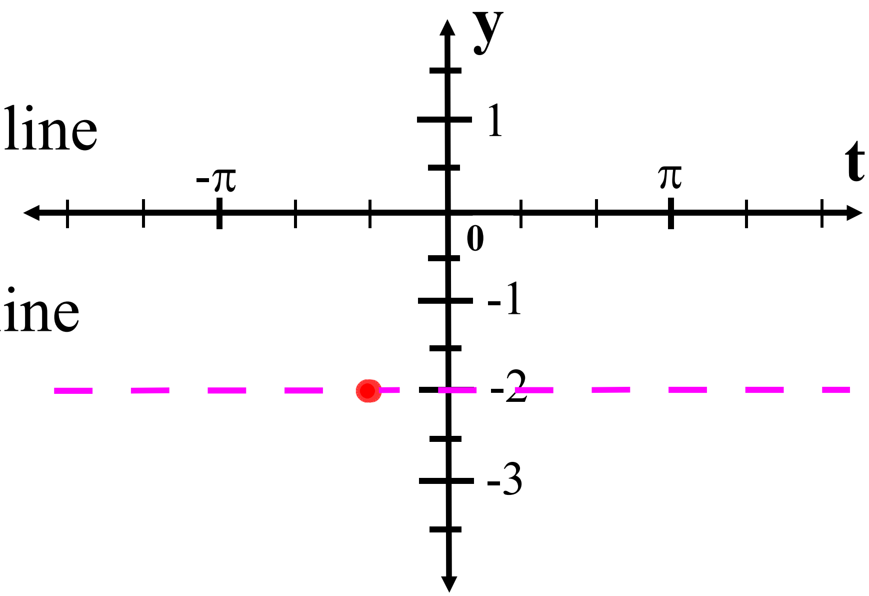
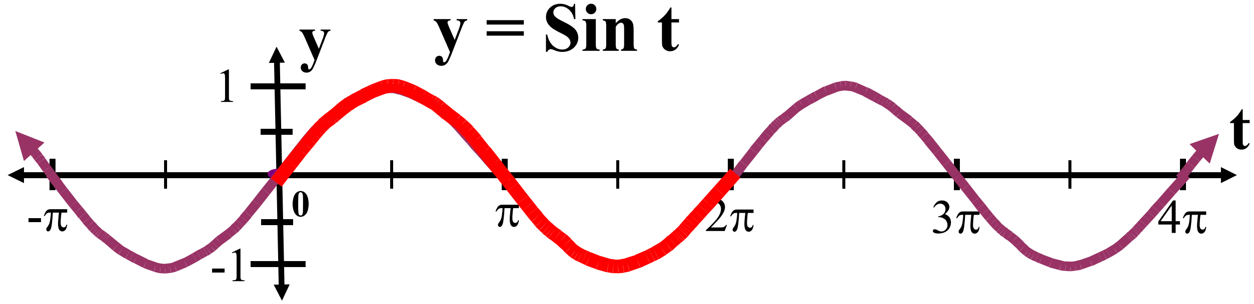
Mid-line: **y = -2**

The basic cycle starts on the mid-line when t + π/3 = 0. → t = -π/3

The basic cycle ends on the mid-line when t + π/3 = 2π.



Variations of the Sine Function



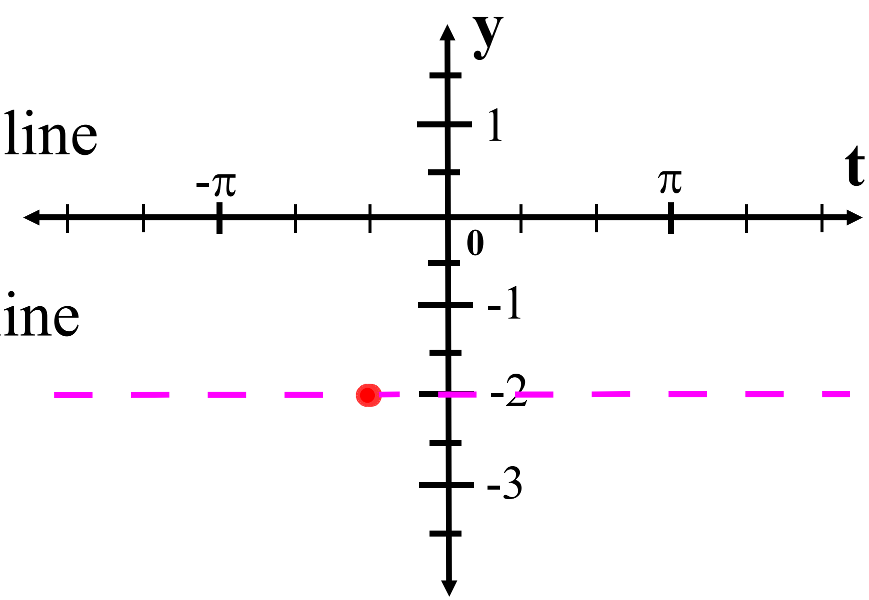
In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = -0.5Sin(t + \pi/3) - 2}$.

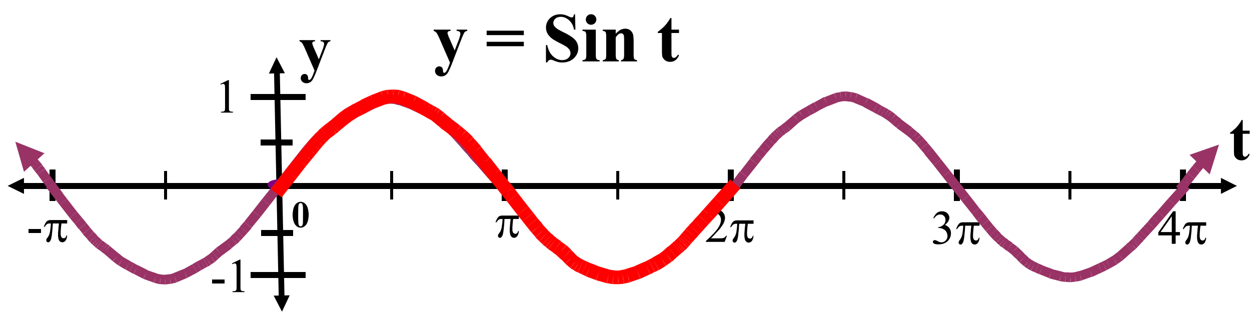
Mid-line: $\mathbf{y = -2}$

The basic cycle starts on the mid-line when $t + \pi/3 = 0$. → $t = -\pi/3$

The basic cycle ends on the mid-line when $t + \pi/3 = 2\pi$. → $t = 5\pi/3$



Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = -0.5Sin(t + \pi/3) - 2}$.

Mid-line: $\mathbf{y = -2}$

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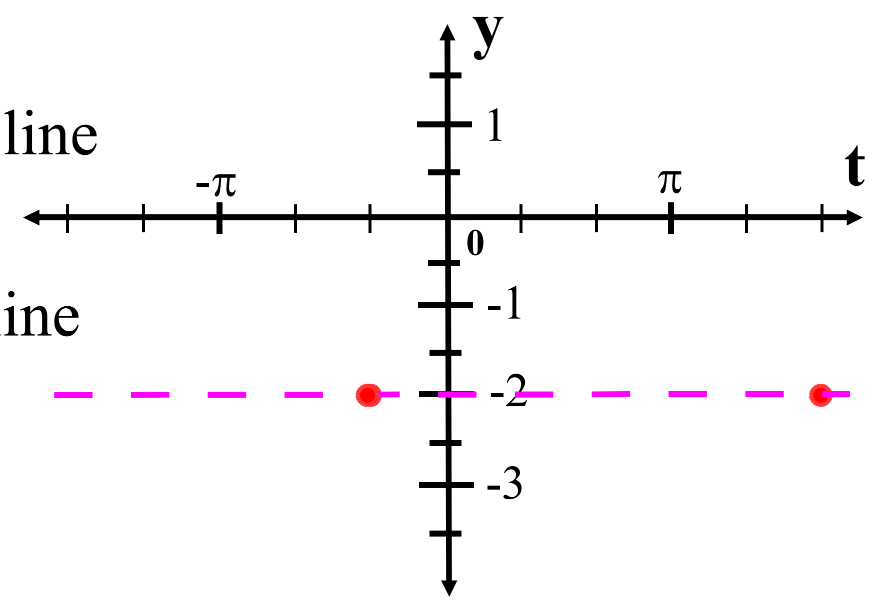
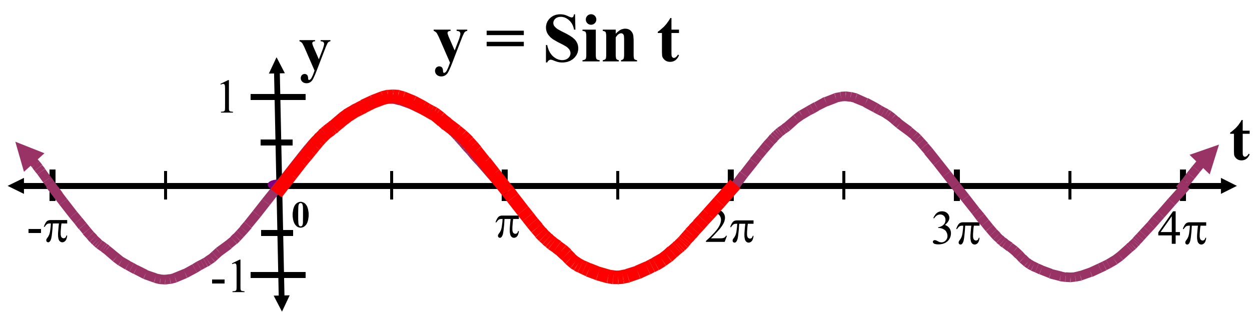
The basic cycle ends on the mid-line when $t + \pi/3 = 2\pi$. → $t = 5\pi/3$



Variations of the Sine Function



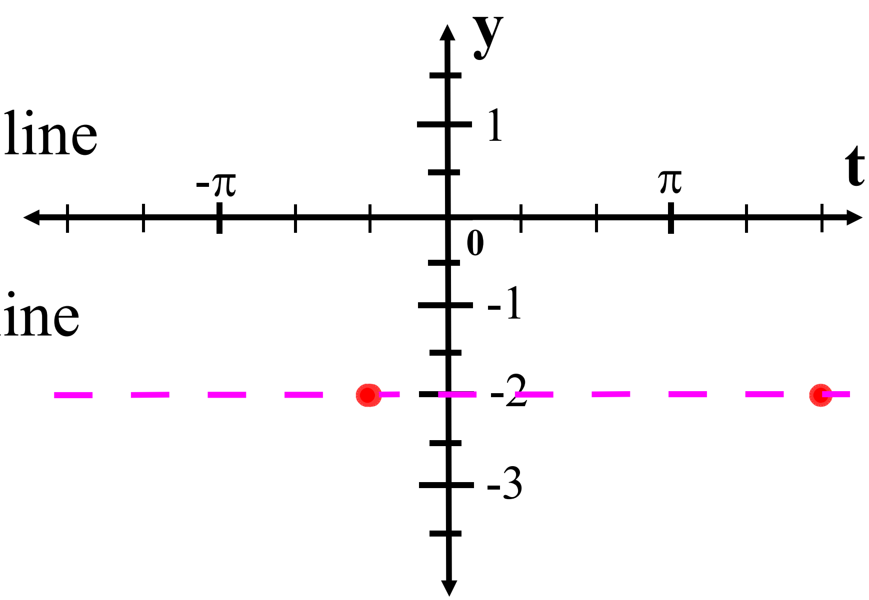
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

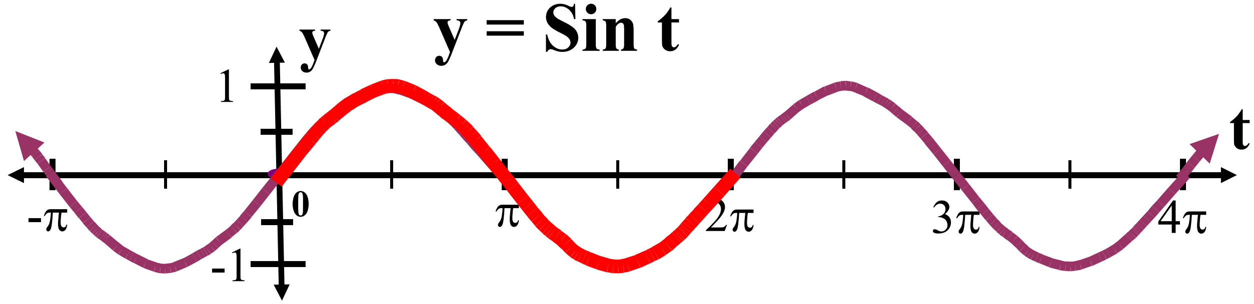
Mid-line: **y = -2**

The basic cycle starts on the mid-line when t + π/3 = 0. → t = -π/3

The basic cycle ends on the mid-line when t + π/3 = 2π. → t = 5π/3



Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

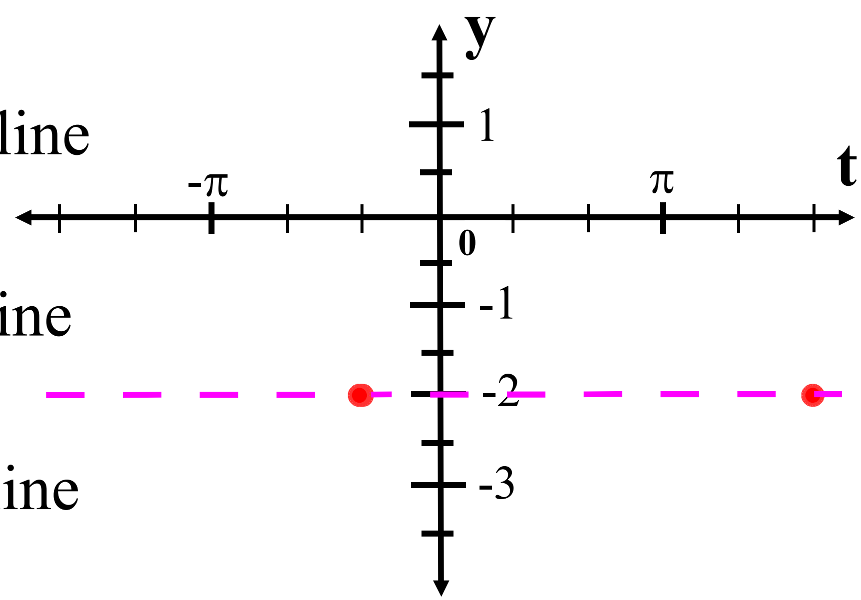
Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

Mid-line: **y = -2**

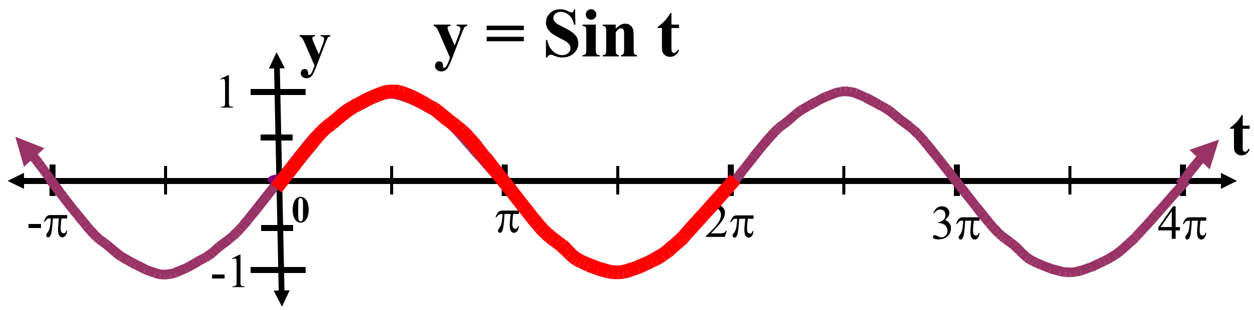
The basic cycle starts on the mid-line when t + π/3 = 0. ⟶ t = -π/3

The basic cycle ends on the mid-line when t + π/3 = 2π. ⟶ t = 5π/3

The basic cycle intersects the mid-line 'half-way' through the cycle.



Variations of the Sine Function



In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

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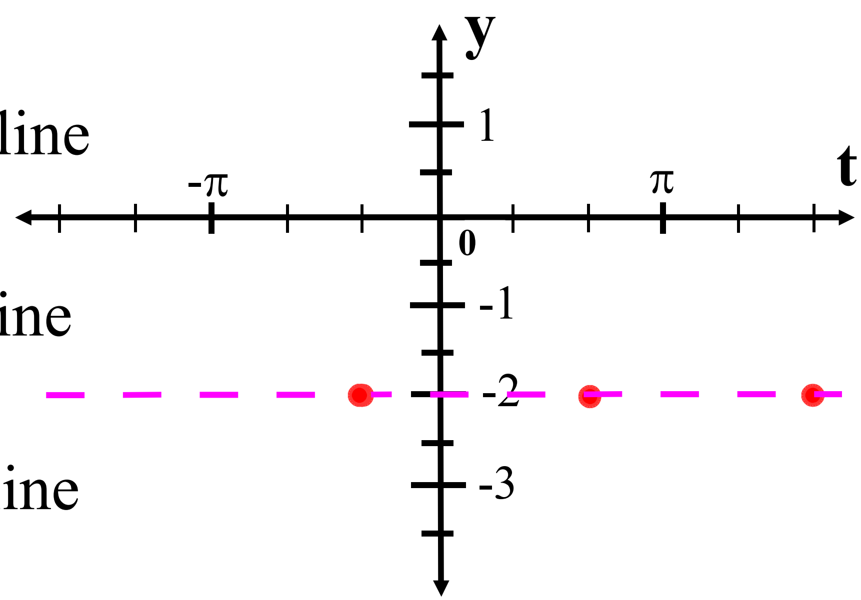
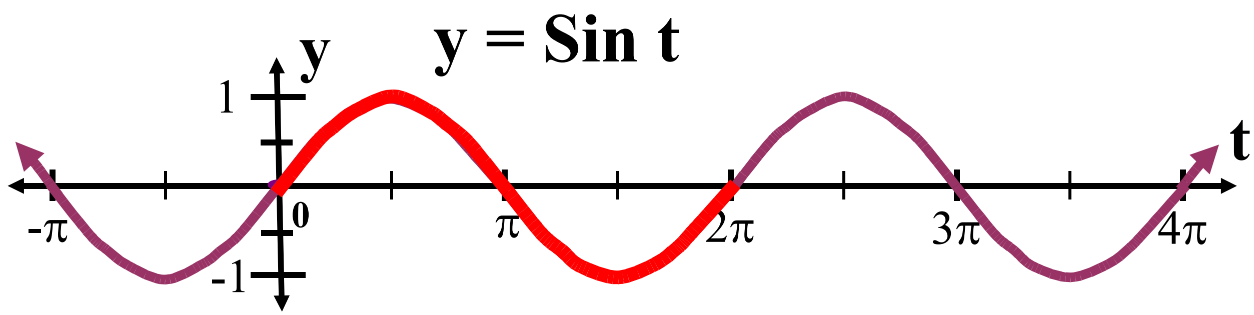
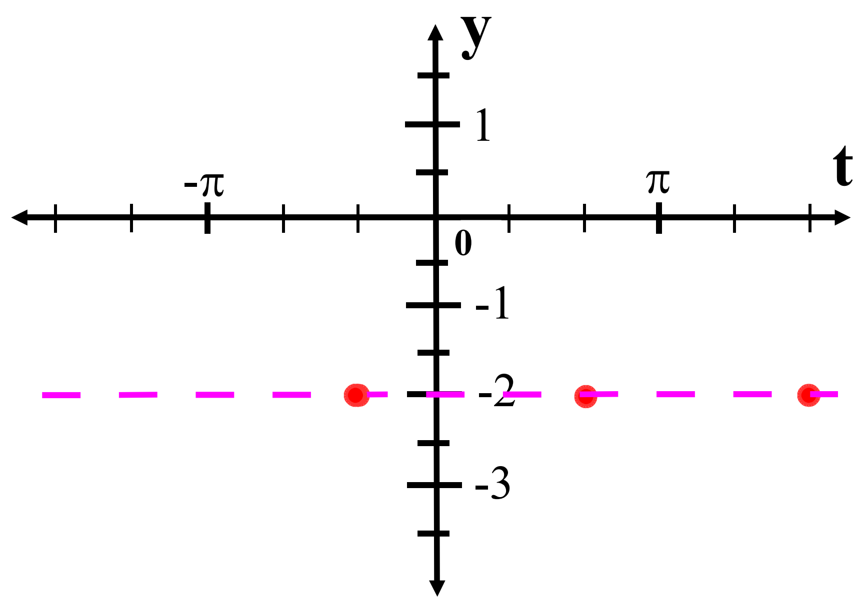


Variations of the Sine Function



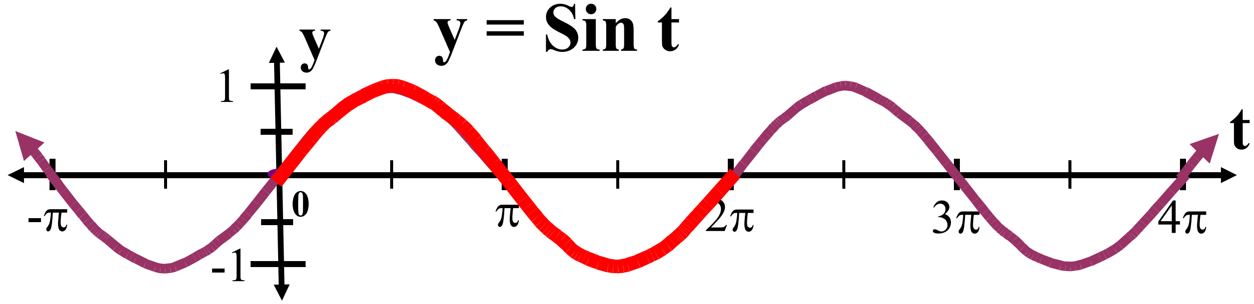
In the above sine graph, the basic cycle starts on the mid-line when $t = 0$, and it ends on the mid-line when $t = 2\pi$.

Now, consider the equation $\mathbf{y = -0.5Sin(t + \pi/3) - 2}$.



Variations of the Sine Function

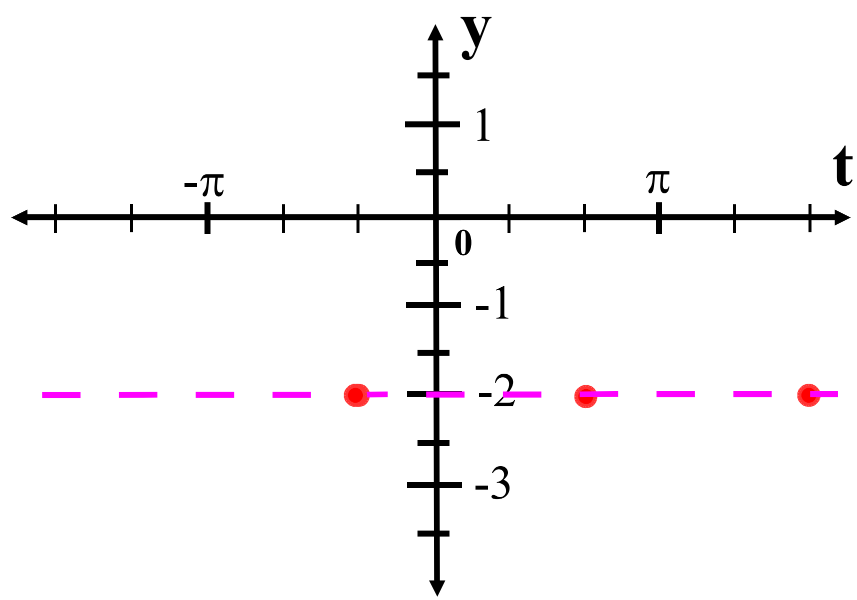
y = Sin t



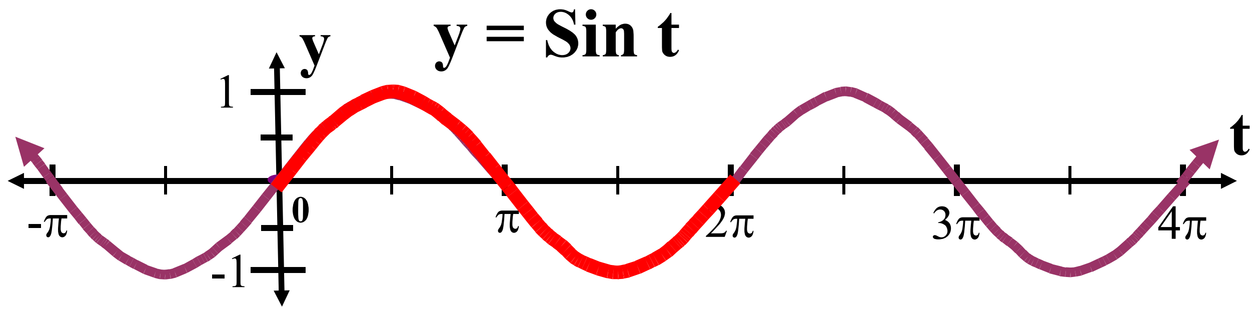
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

A = -0.5



Variations of the Sine Function

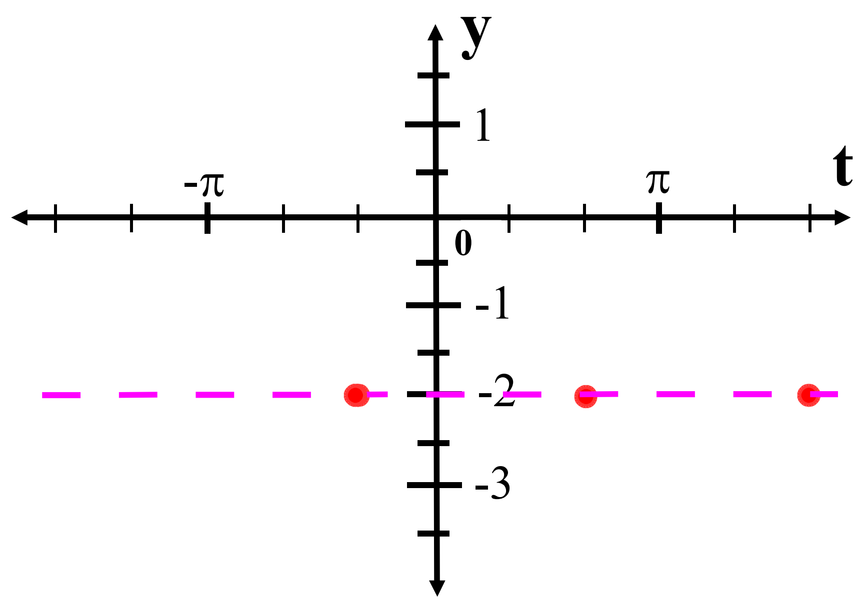


In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

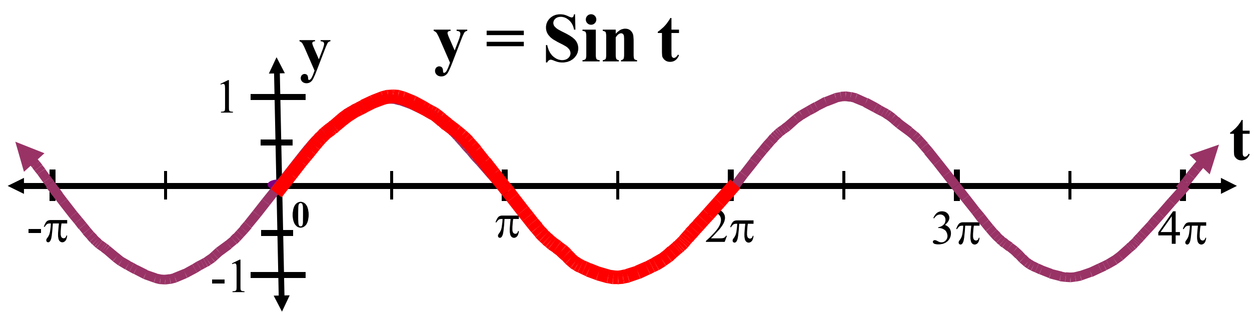
Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

A = -0.5

→ The amplitude is 0.5.



Variations of the Sine Function



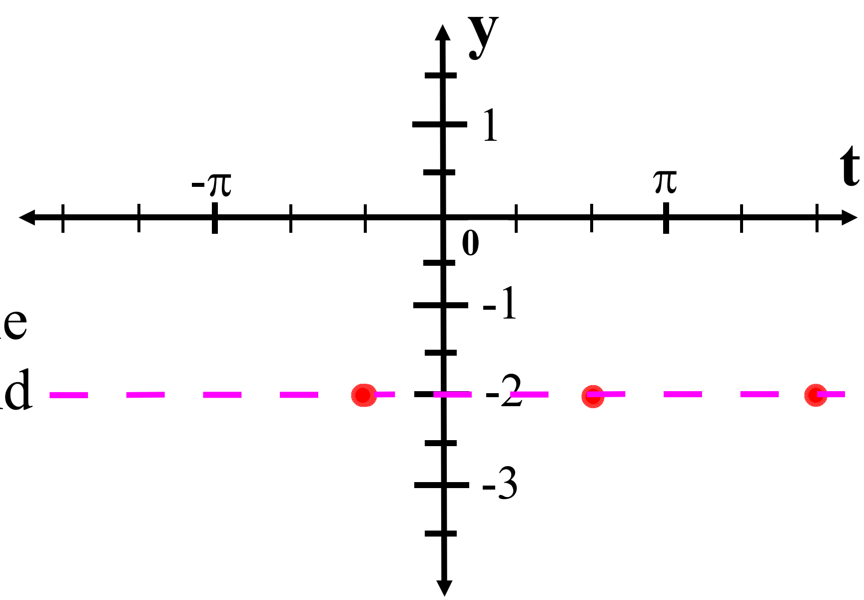
In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

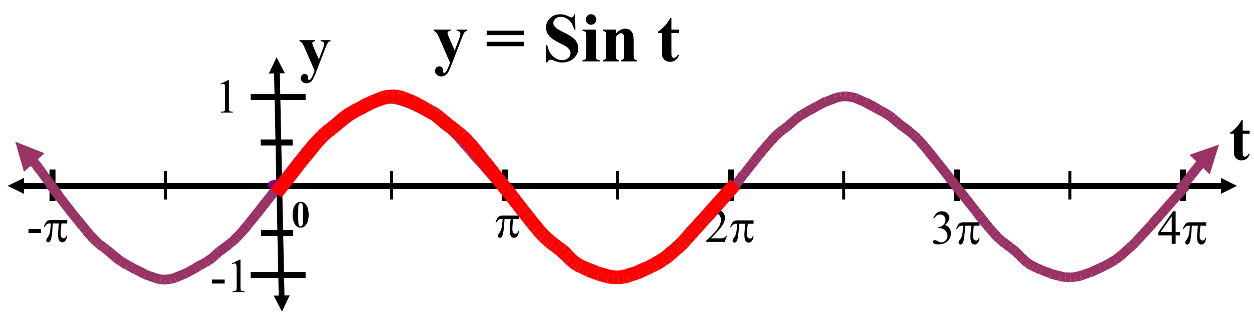
A = -0.5

→ The amplitude is 0.5.

→ The basic cycle is 'below the mid-line' for the first half of the cycle and above the mid-line for the second half of the cycle.



Variations of the Sine Function



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Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

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→ The amplitude is 0.5.

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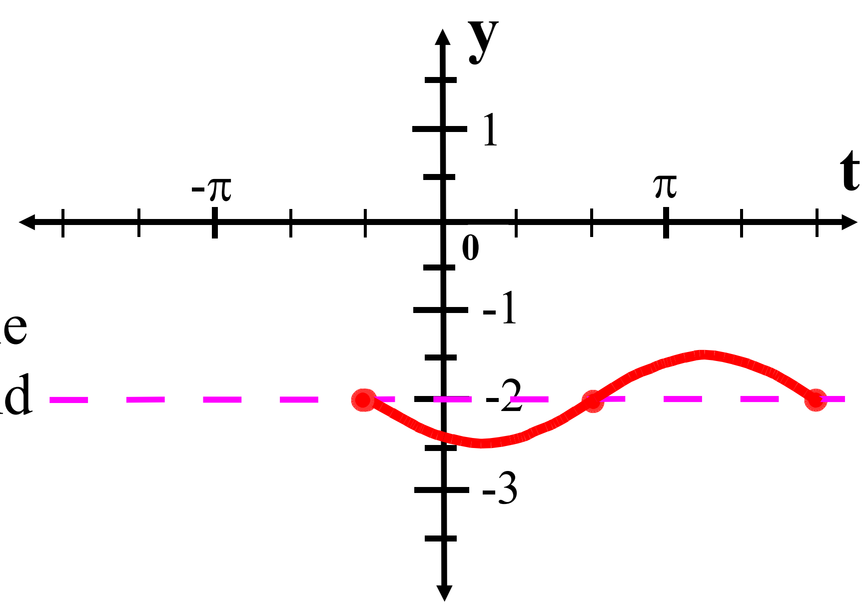
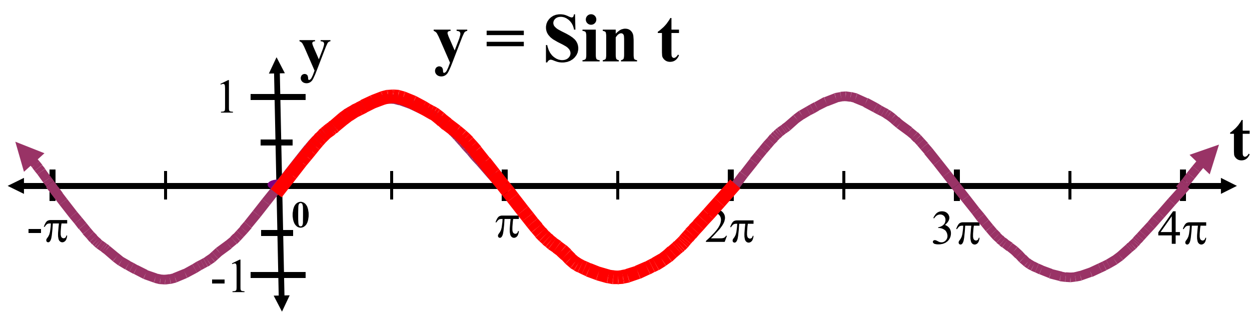
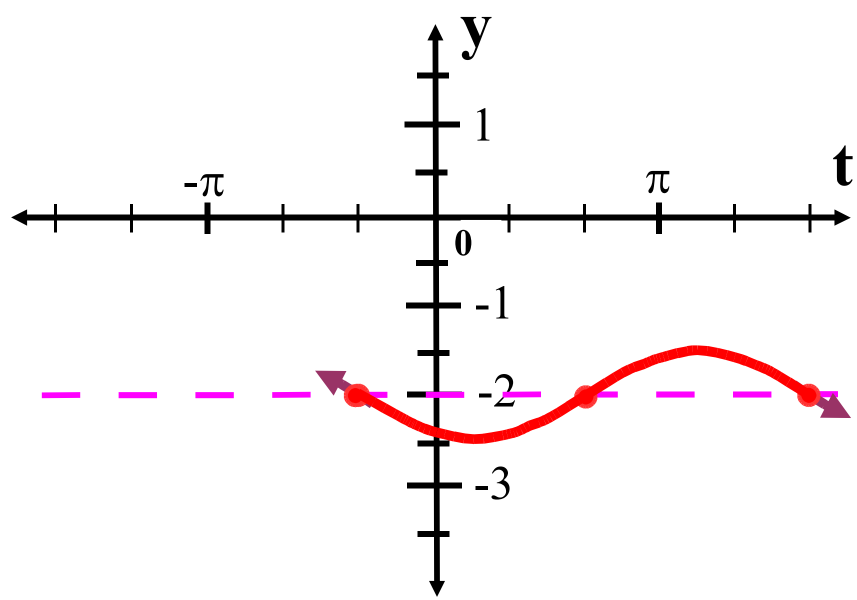


Variations of the Sine Function

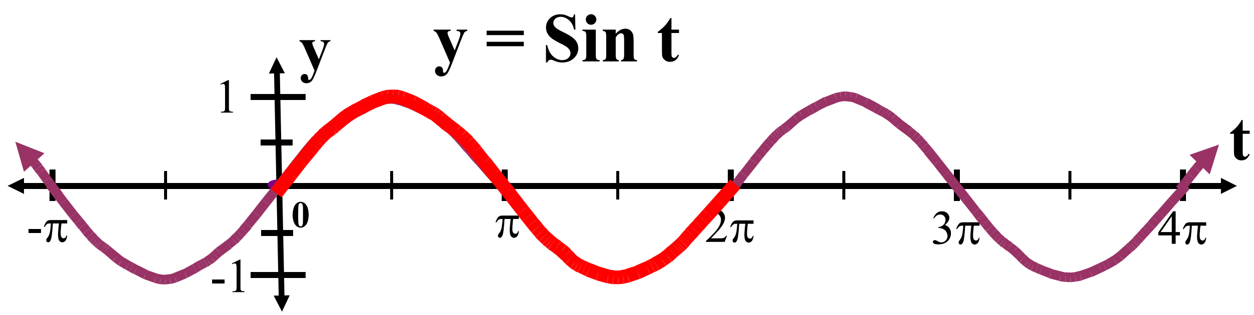


In the above sine graph, the basic cycle starts on the mid-line when t = 0, and it ends on the mid-line when t = 2π.

Now, consider the equation **y = -0.5Sin(t + π/3) – 2**.

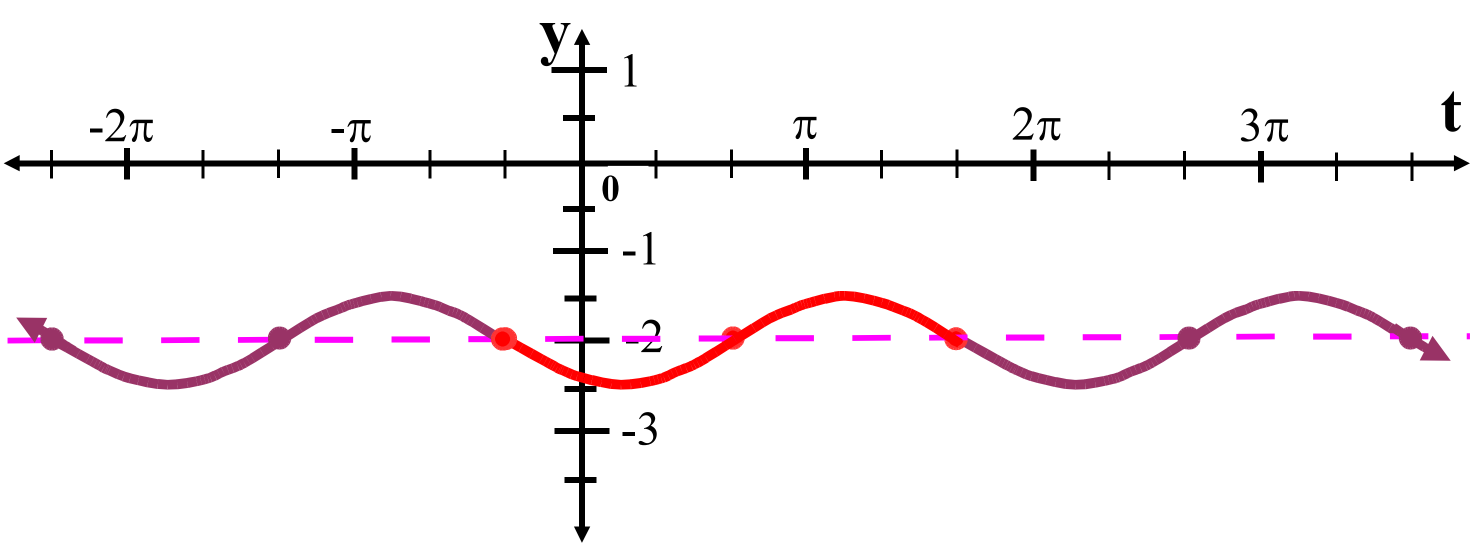


Variations of the Sine Function



Here is a more complete graph.

y = -0.5Sin(t + π/3) – 2



Variations of the Sine Function

Consider the equation $y = A\sin(Bt + C) + D$.

(1) The amplitude of the 'sine wave' is the absolute value of A.

(2) If $A > 0$, then the basic cycle is 'above the mid-line' for the first half of the cycle and below the mid-line for the second half of the cycle.

(3) If $A < 0$, then the basic cycle is 'below the mid-line' for the first half of the cycle and 'above the mid-line' for the second half of the cycle.

(4) The equation of the mid-line is $y = D$.

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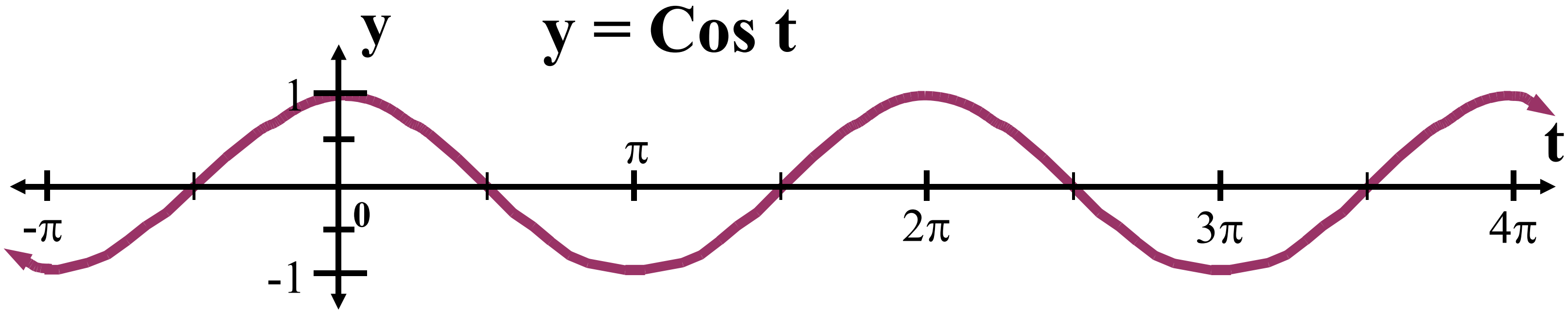
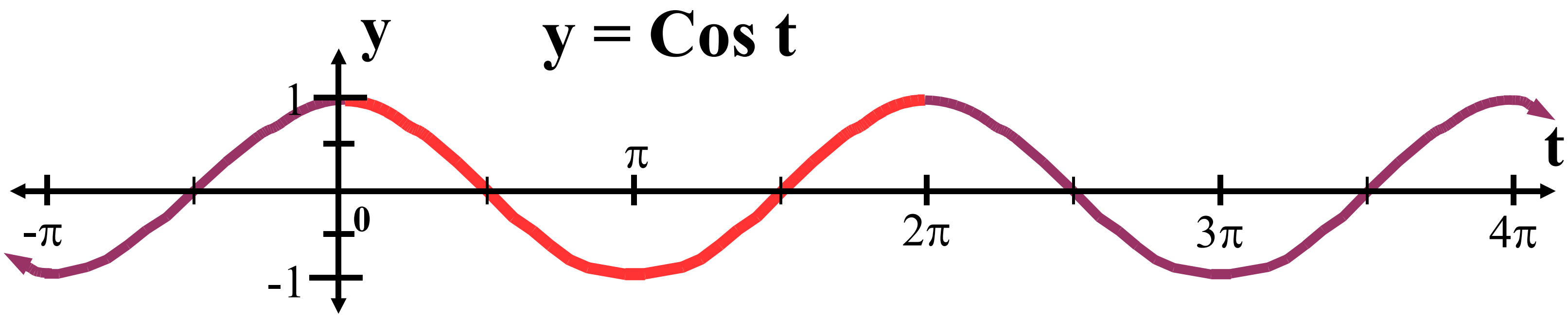
We will now consider **Variations of the Cosine Function**.

Variations of the Cosine Function

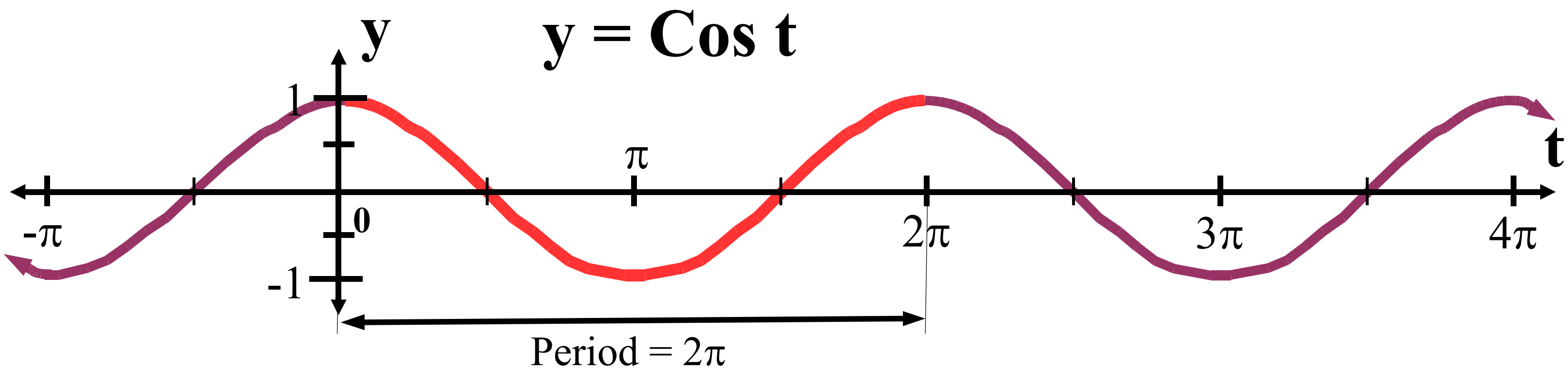


Variations of the Cosine Function



This is the 'basic cycle' of the cosine function.

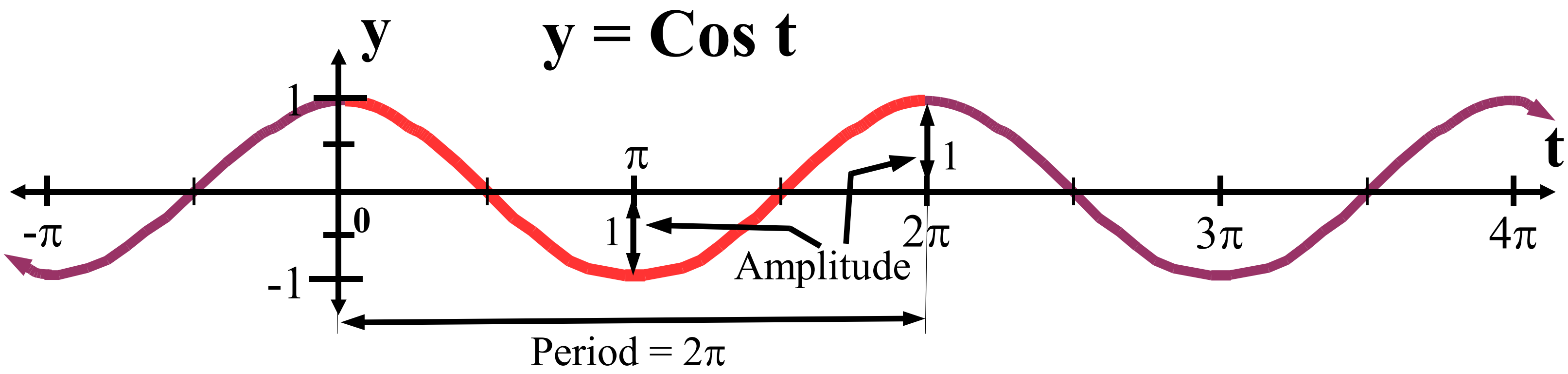
Variations of the Cosine Function



This is the 'basic cycle' of the cosine function.

Its period is 2π units.

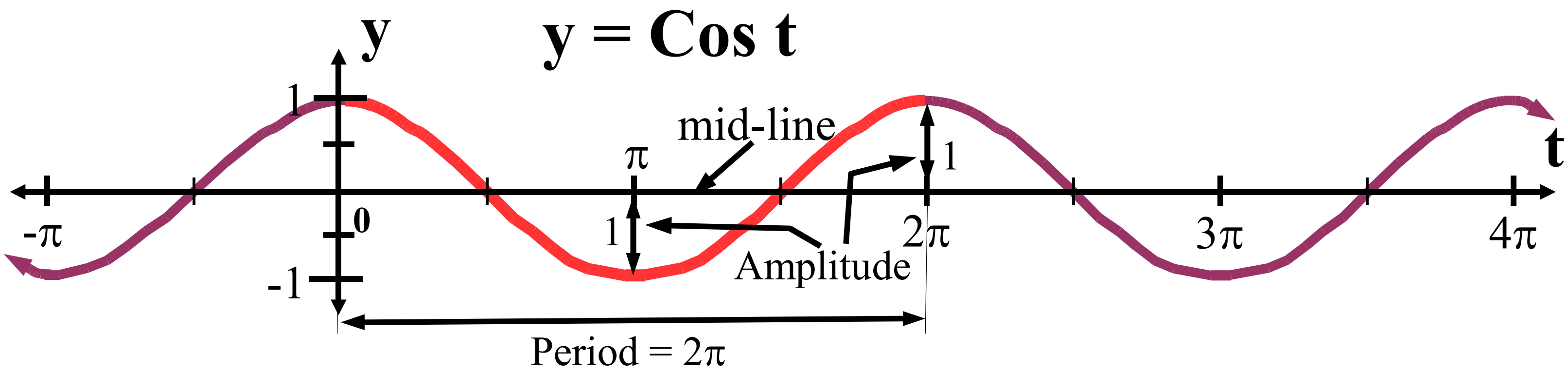
Variations of the Cosine Function



This is the 'basic cycle' of the cosine function.

Its period is 2π units. Its amplitude is 1 unit.

Variations of the Cosine Function

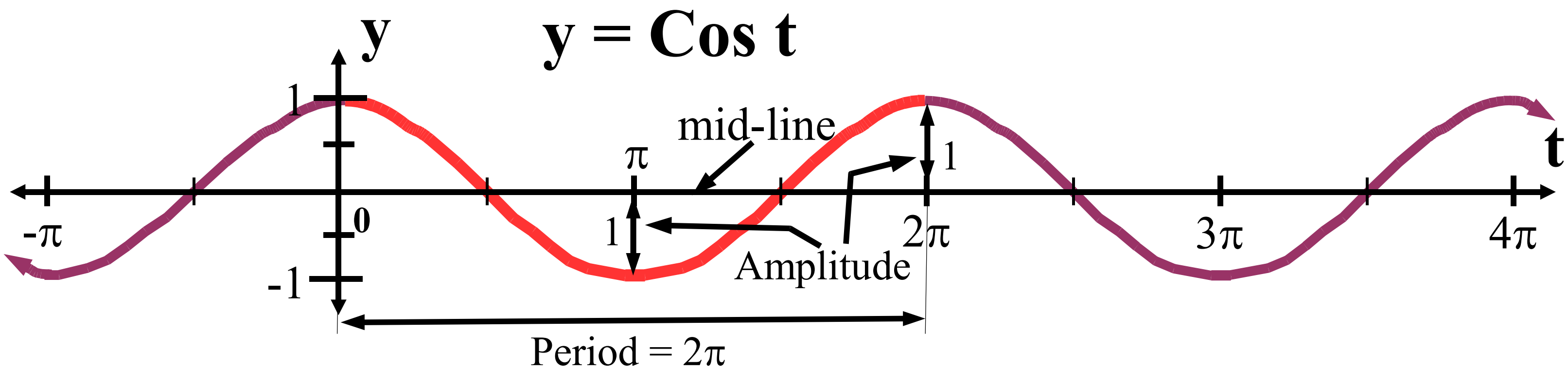


This is the 'basic cycle' of the cosine function.

Its period is 2π units. Its amplitude is 1 unit.

The line y = 0, the t axis, is the mid-line of the curve.

Variations of the Cosine Function



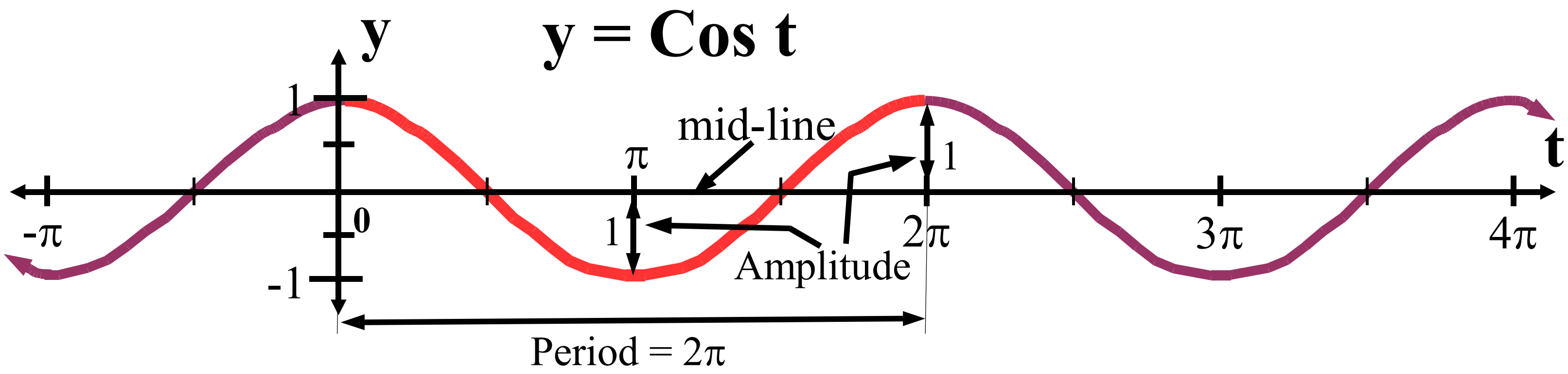
This is the 'basic cycle' of the cosine function.

Its <u>period</u> is 2π units. Its <u>amplitude</u> is 1 unit.

The line y = 0, the t axis, is the <u>mid-line</u> of the curve.

Consider the equation $y = A\,Cos(Bt + C) + D$.

Variations of the Cosine Function



This is the 'basic cycle' of the cosine function.

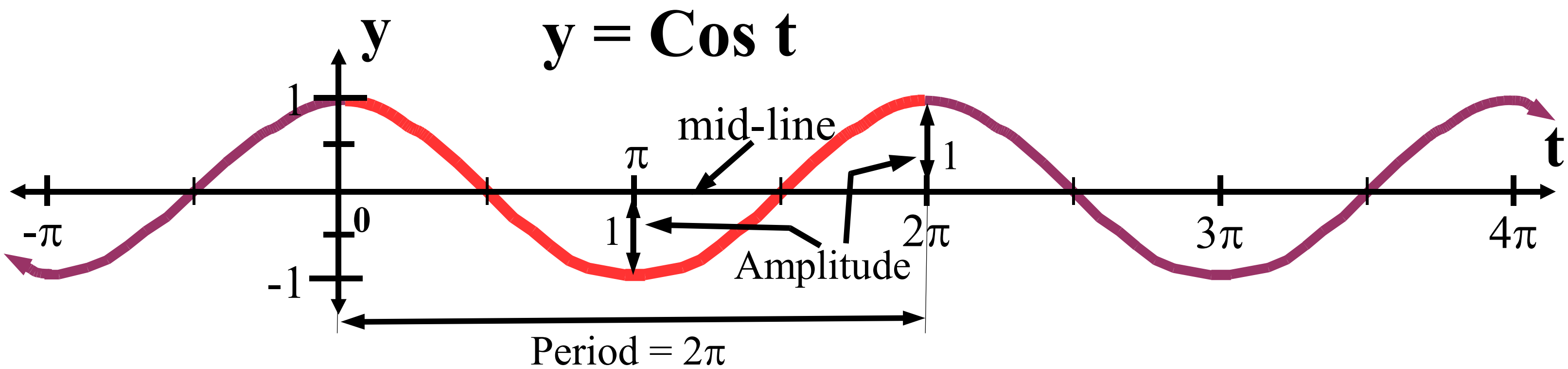
Its <u>period</u> is 2π units. Its <u>amplitude</u> is 1 unit.

The line $y = 0$, the t axis, is the <u>mid-line</u> of the curve.

Consider the equation $y = A\,\text{Cos}(Bt + C) + D$.

We will consider the significance of each of the constants A, B, C, and D.

Variations of the Cosine Function



This is the 'basic cycle' of the cosine function.

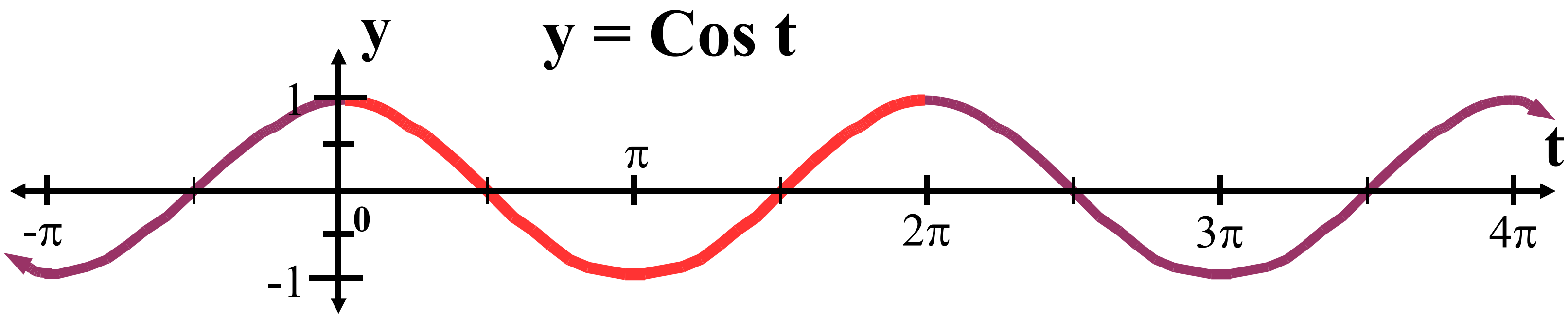
Its period is 2π units. Its amplitude is 1 unit.

The line y = 0, the t axis, is the mid-line of the curve.

Consider the equation $y = A\,Cos(Bt + C) + D$.

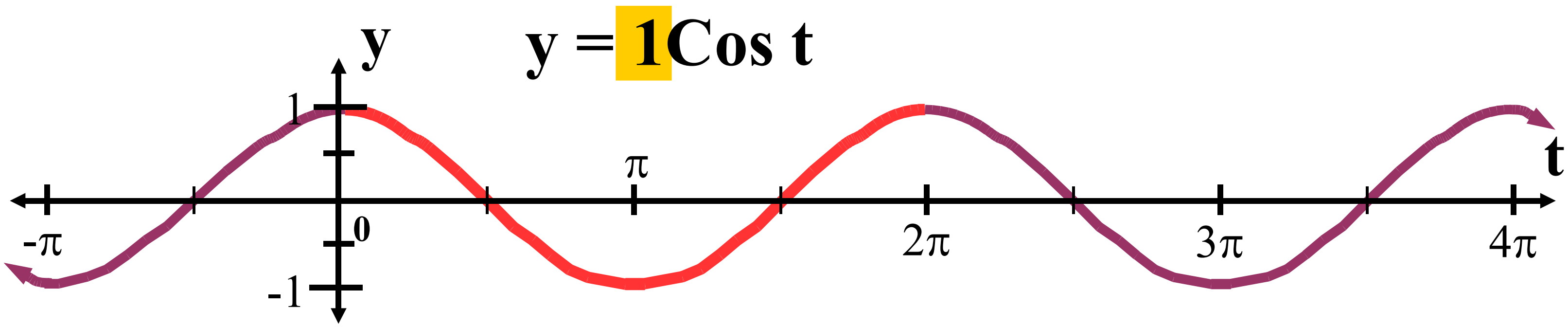
We will consider the significance of each of the constants A, B, C, and D, starting with A.

Variations of the Cosine Function



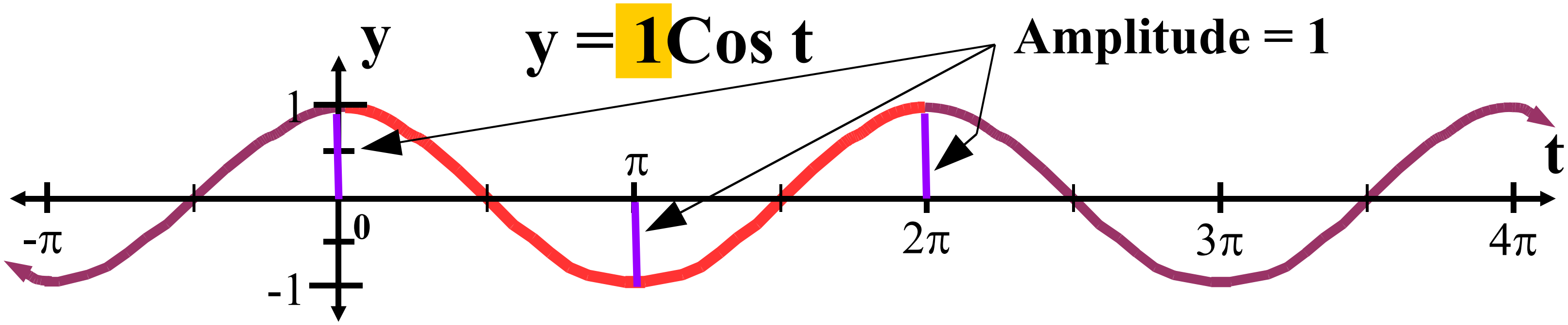
We will start with equations of the form y = ACos(t)

Variations of the Cosine Function



We will start with equations of the form $y = A\text{Cos}(t)$

Variations of the Cosine Function



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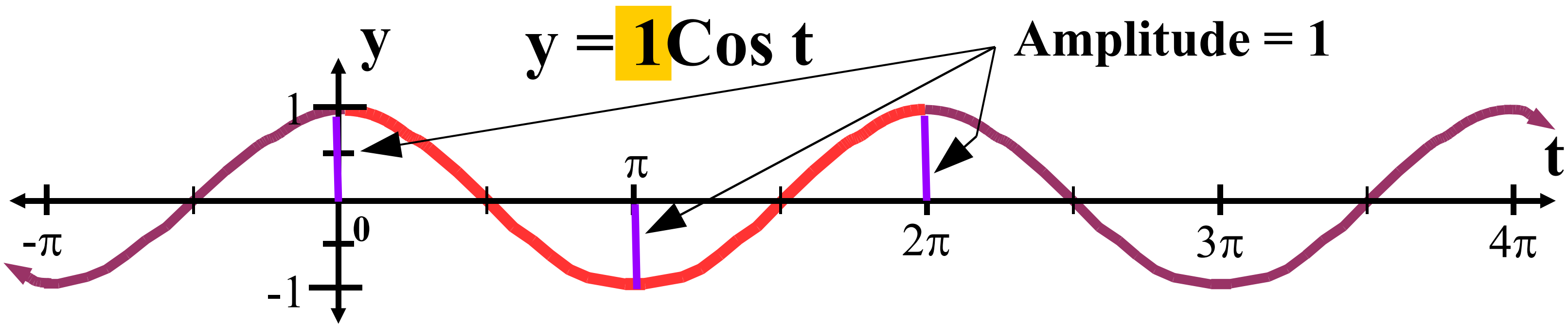
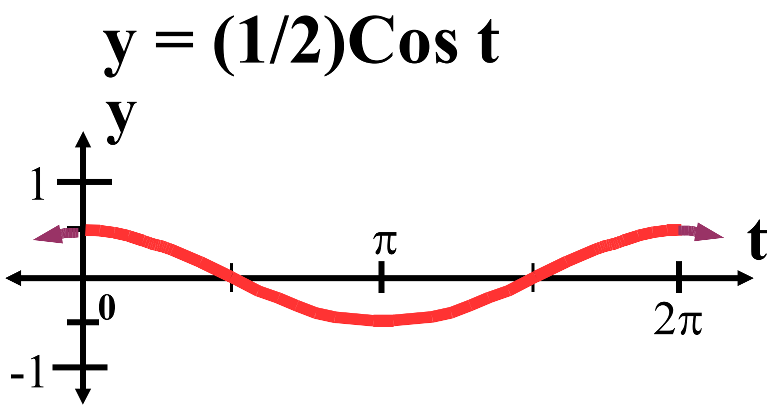
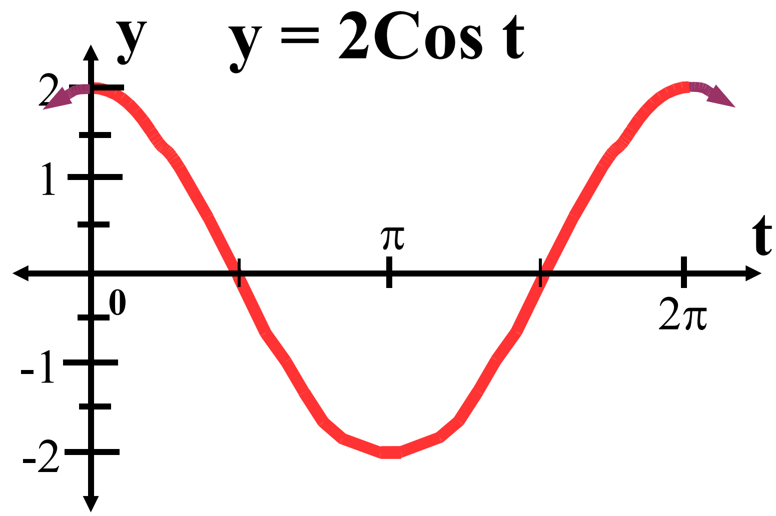


We will start with equations of the form y = ACos(t)

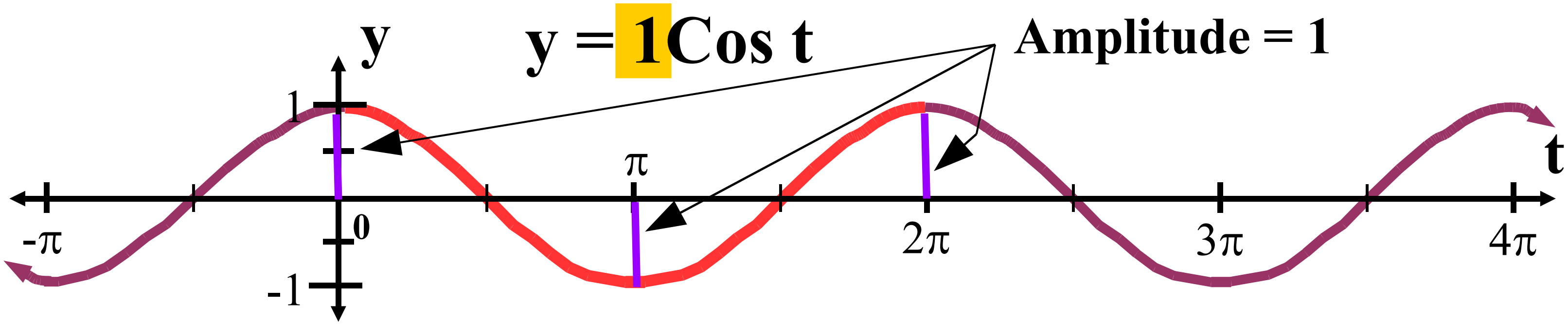
Here are two other examples (showing the 'basic cycle' only).





Variations of the Cosine Function

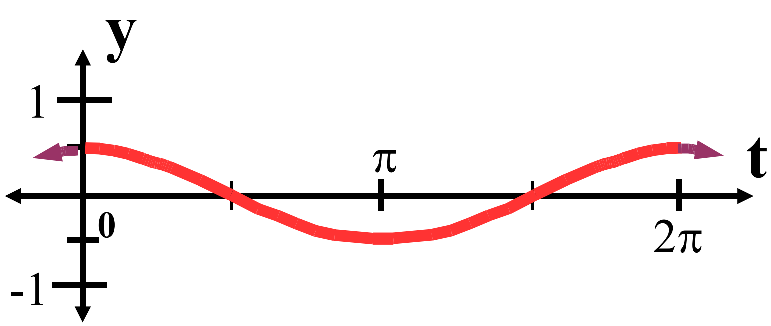
y = 1Cos t



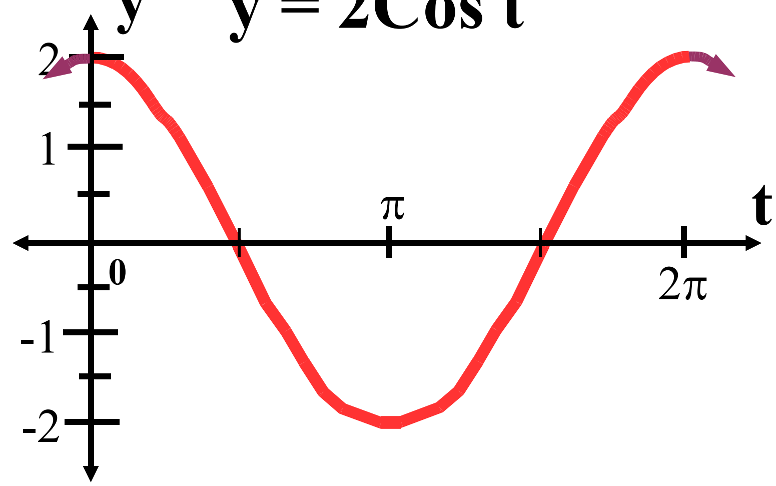
We will start with equations of the form y = ACos(t)

Here are two other examples (showing the 'basic cycle' only).

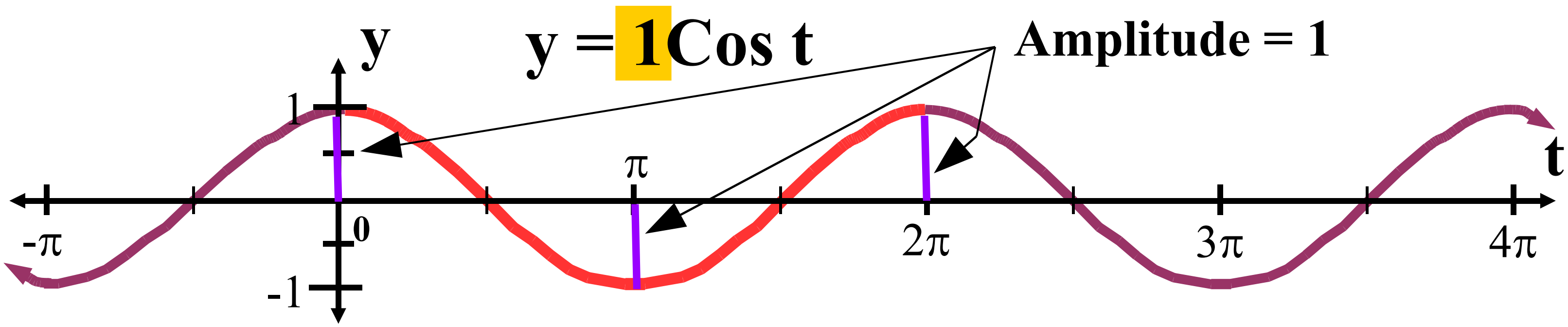
y = (1/2)Cos t



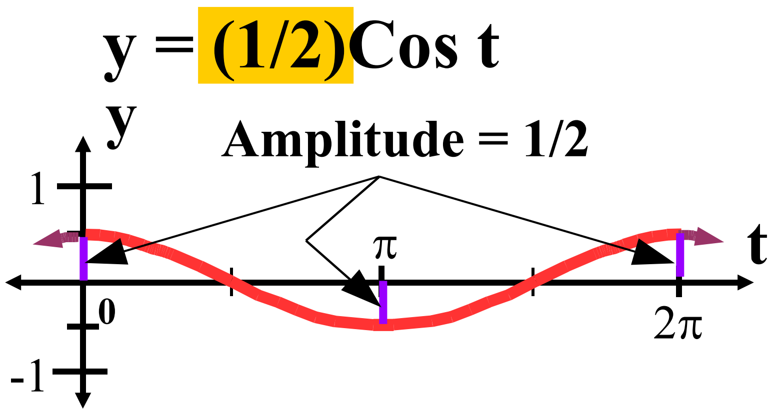
y = 2Cos t

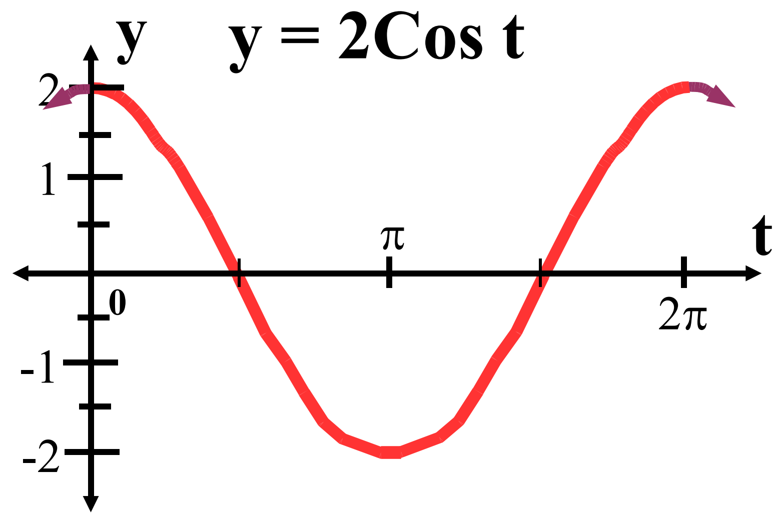


Variations of the Cosine Function

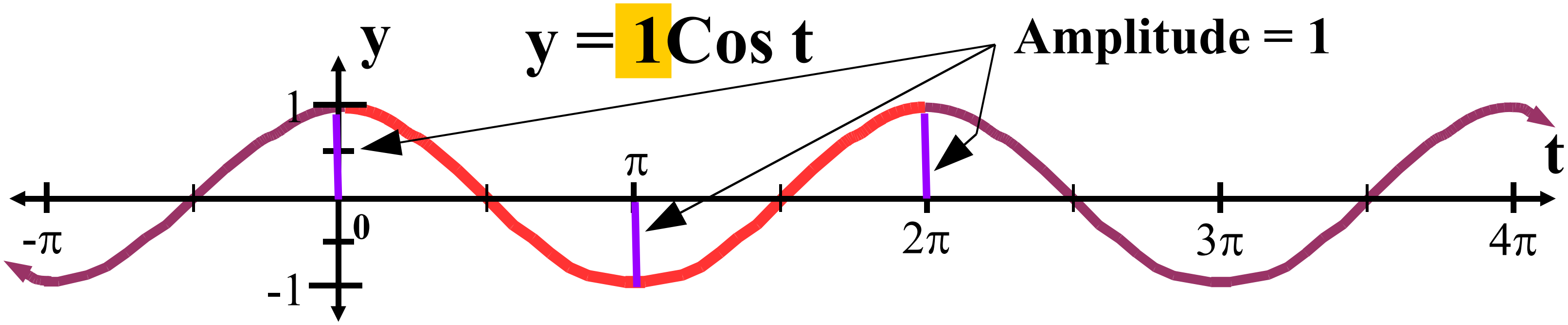


We will start with equations of the form $y = A\text{Cos}(t)$
Here are two other examples (showing the 'basic cycle' only).





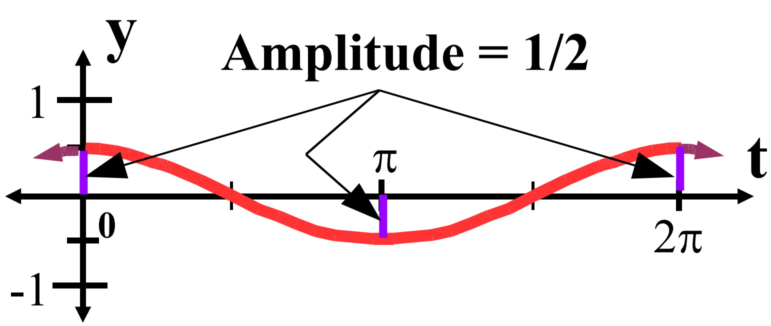
Variations of the Cosine Function

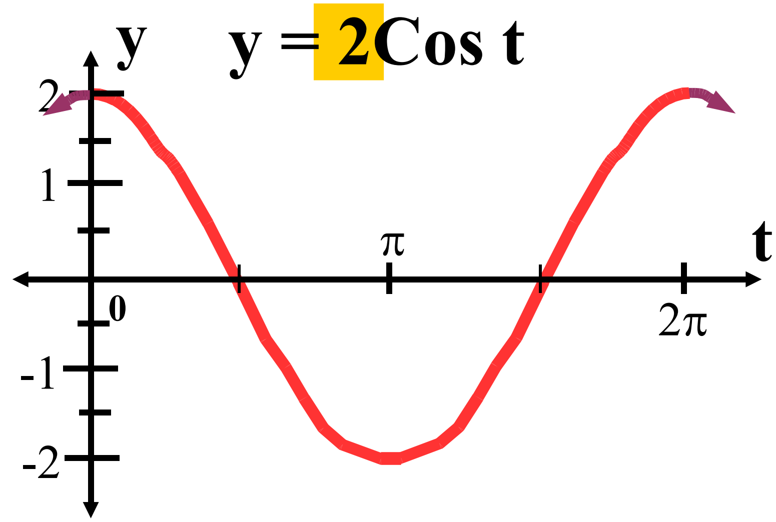


We will start with equations of the form y = ACos(t)

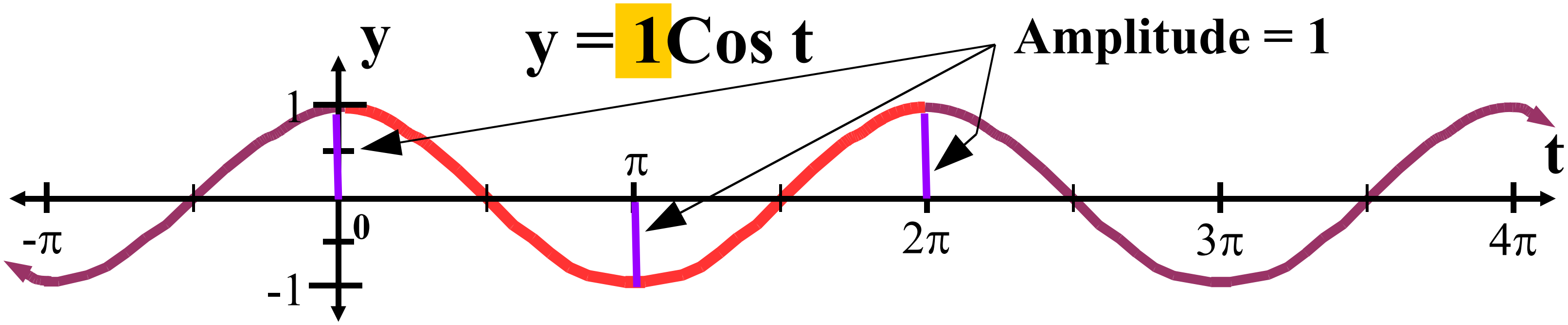
Here are two other examples (showing the 'basic cycle' only).

y = (1/2)Cos t





Variations of the Cosine Function



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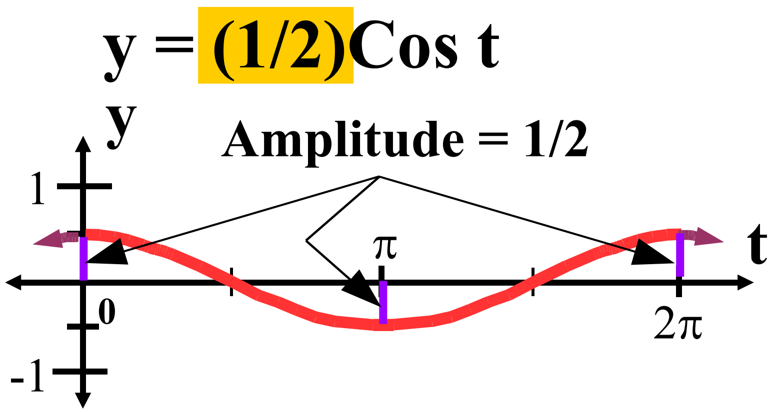
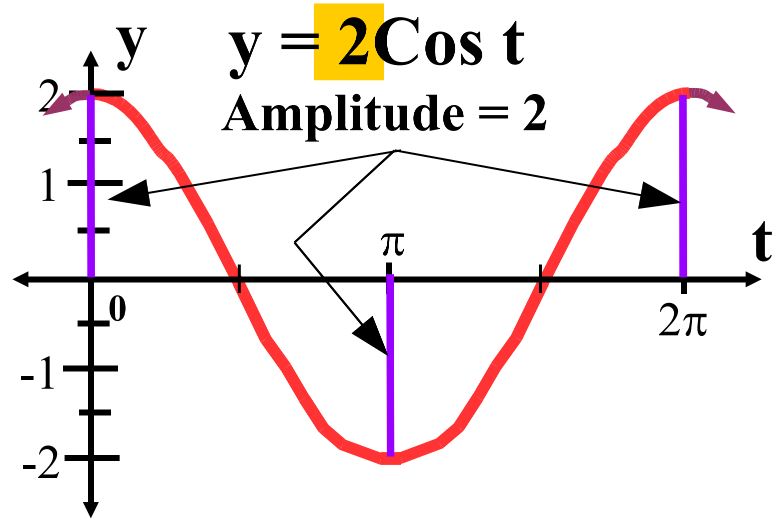
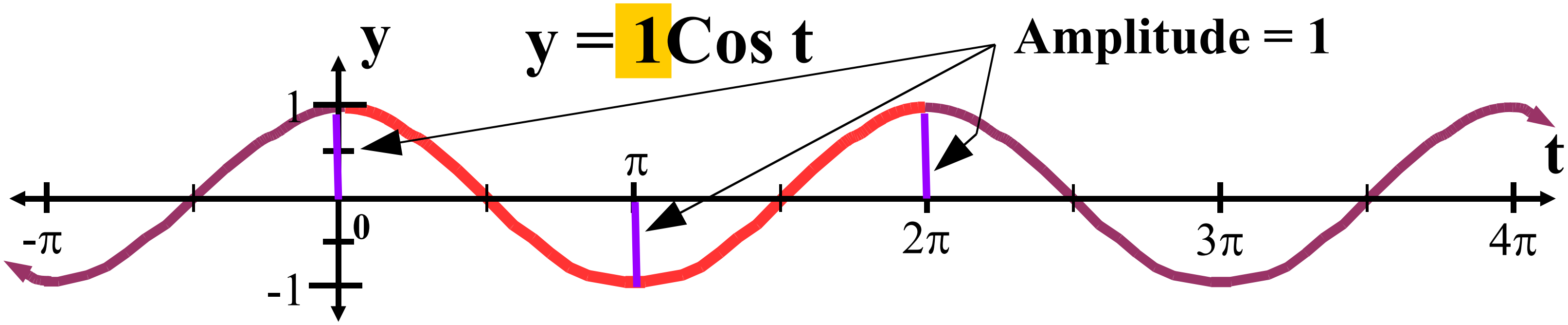




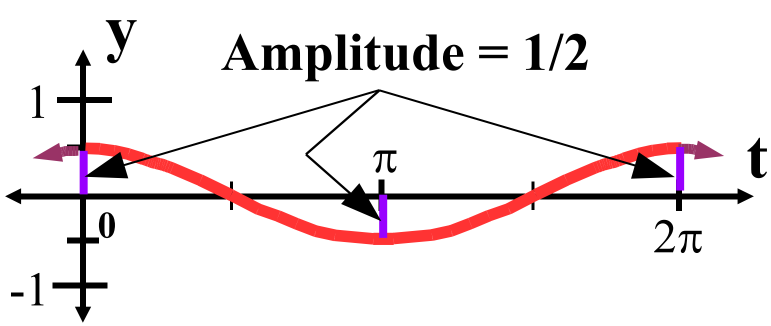
Variations of the Cosine Function

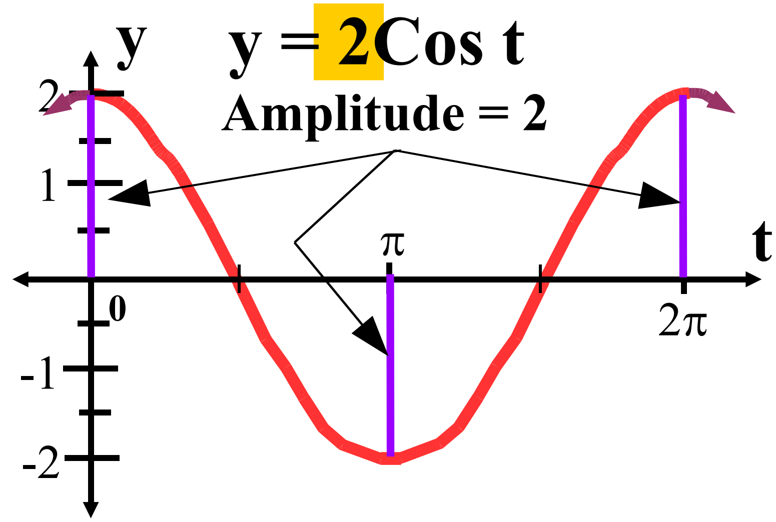


We will start with equations of the form y = ACos(t)

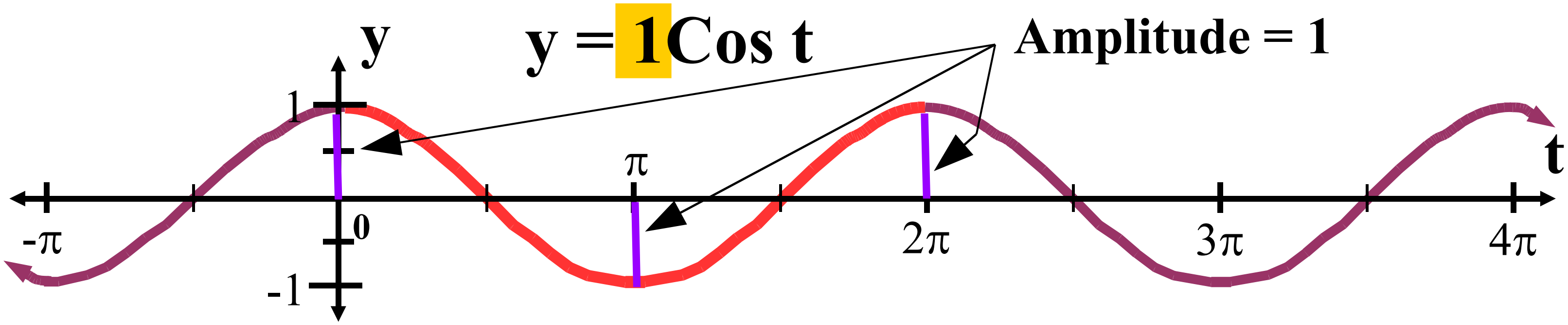
In these examples, the amplitude = A.

y = (1/2)Cos t



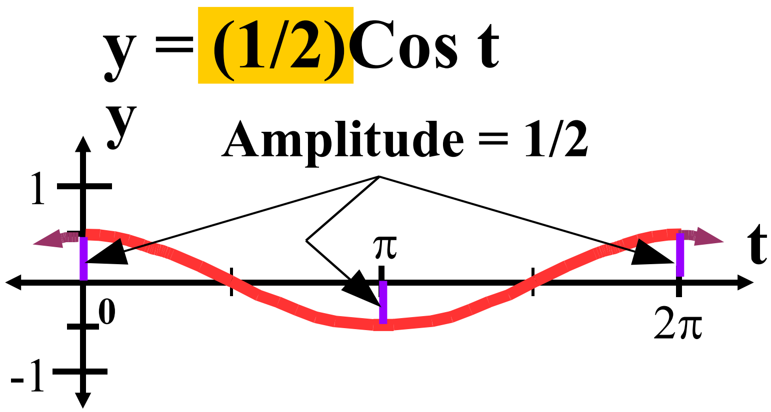


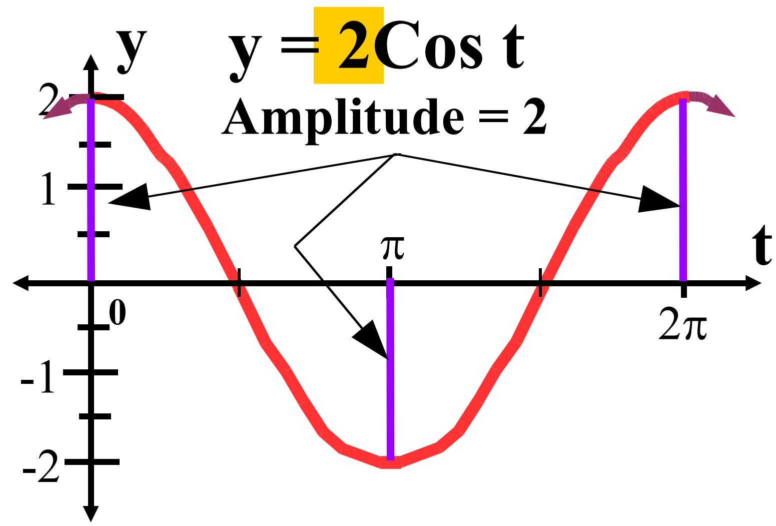
Variations of the Cosine Function



We will start with equations of the form y = ACos(t)

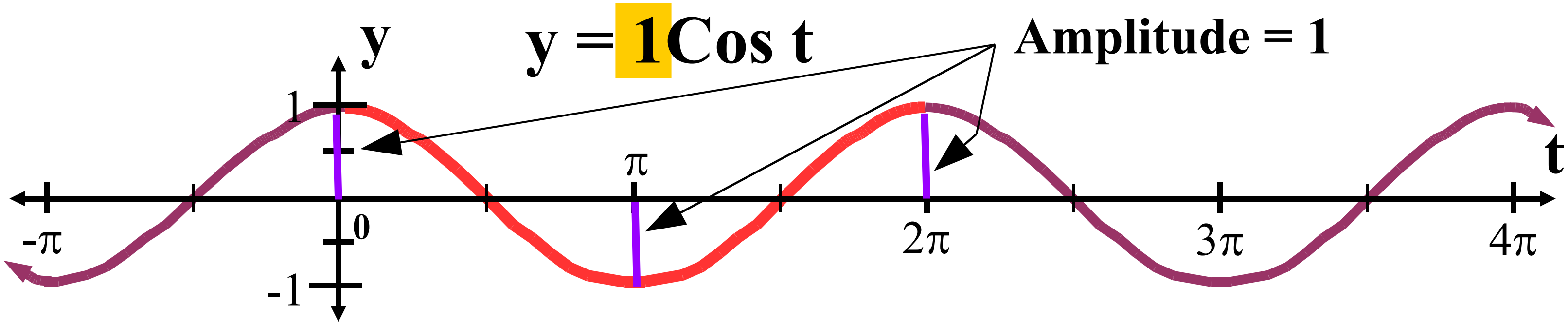
In these examples, the amplitude = A. What if A < 0?





Variations of the Cosine Function

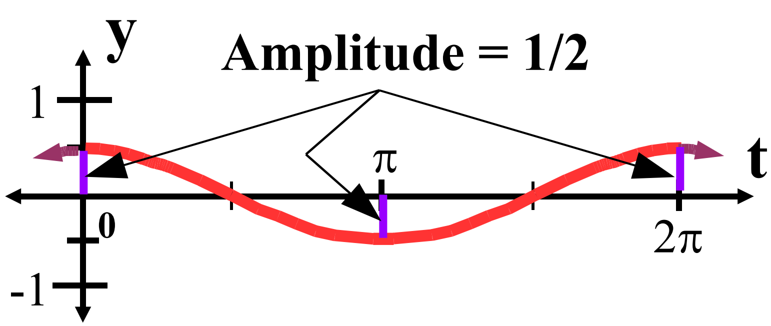
y = 1Cos t



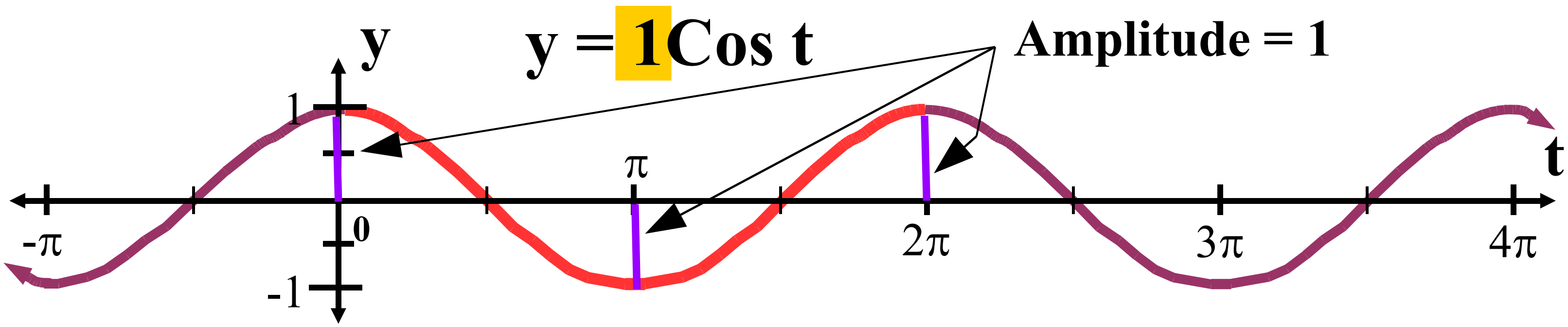
We will start with equations of the form y = ACos(t)
In these examples, the amplitude = A. What if A < 0?

y = (1/2)Cos t

y = (-1/2)Cos t



Variations of the Cosine Function



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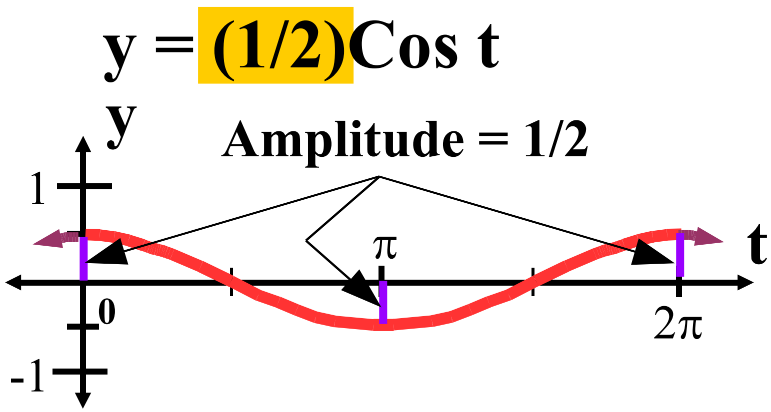
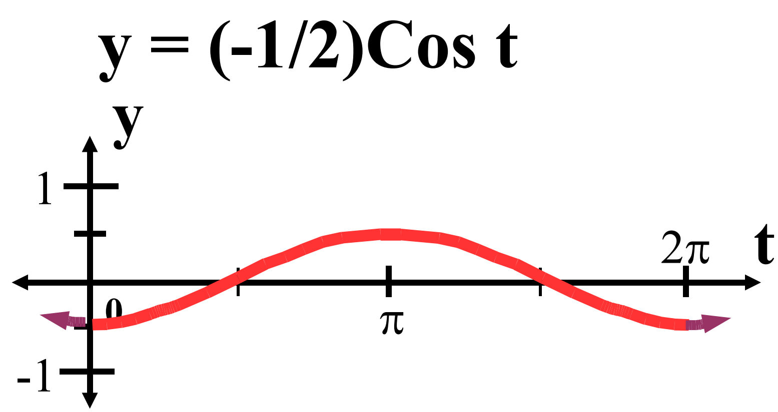
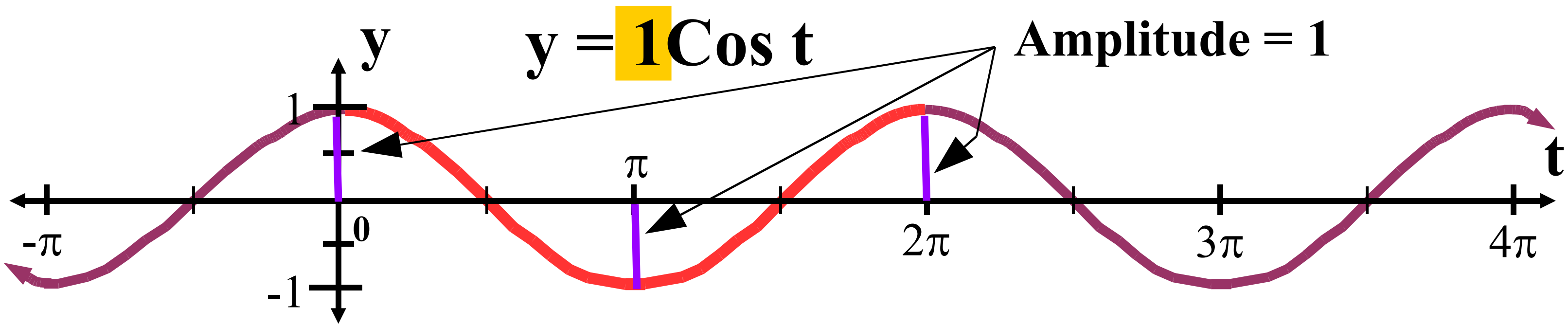




Variations of the Cosine Function



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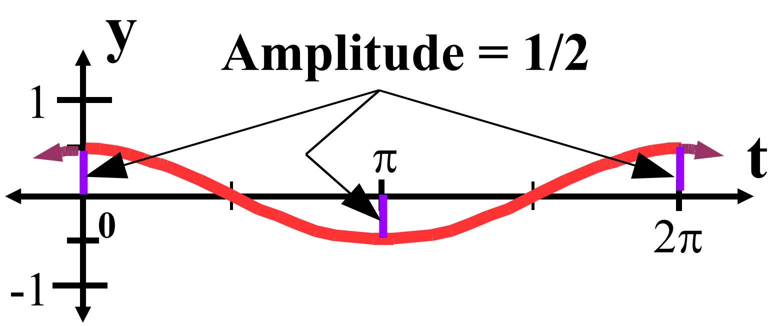
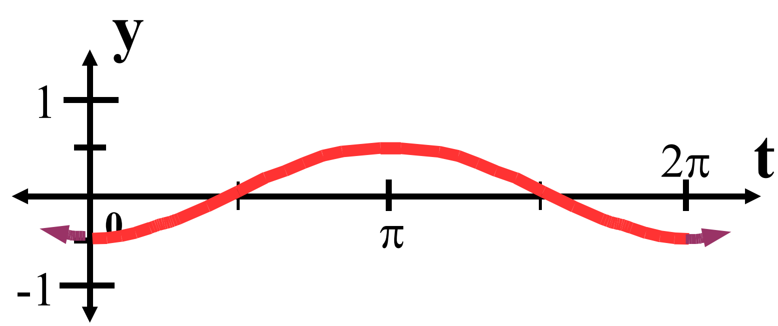
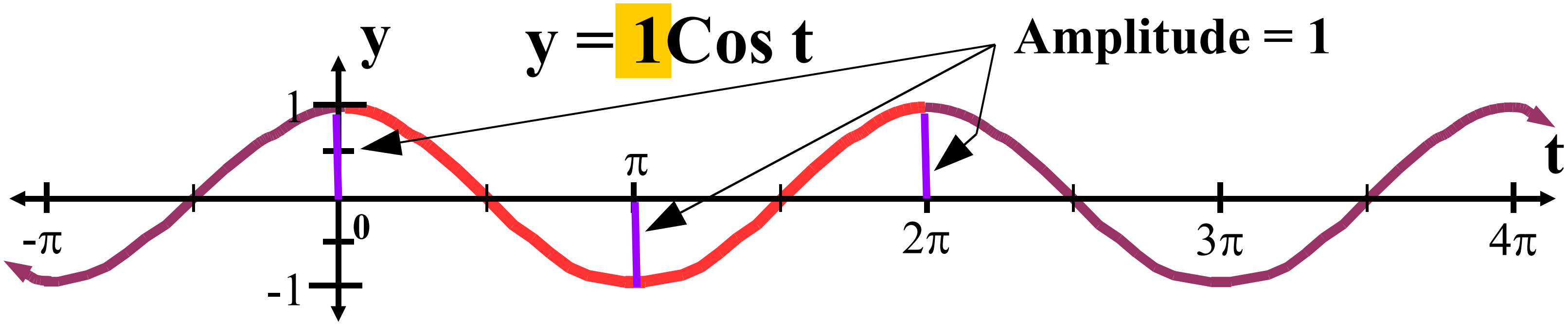
y = (1/2)Cos t



y = (-1/2)Cos t



Variations of the Cosine Function



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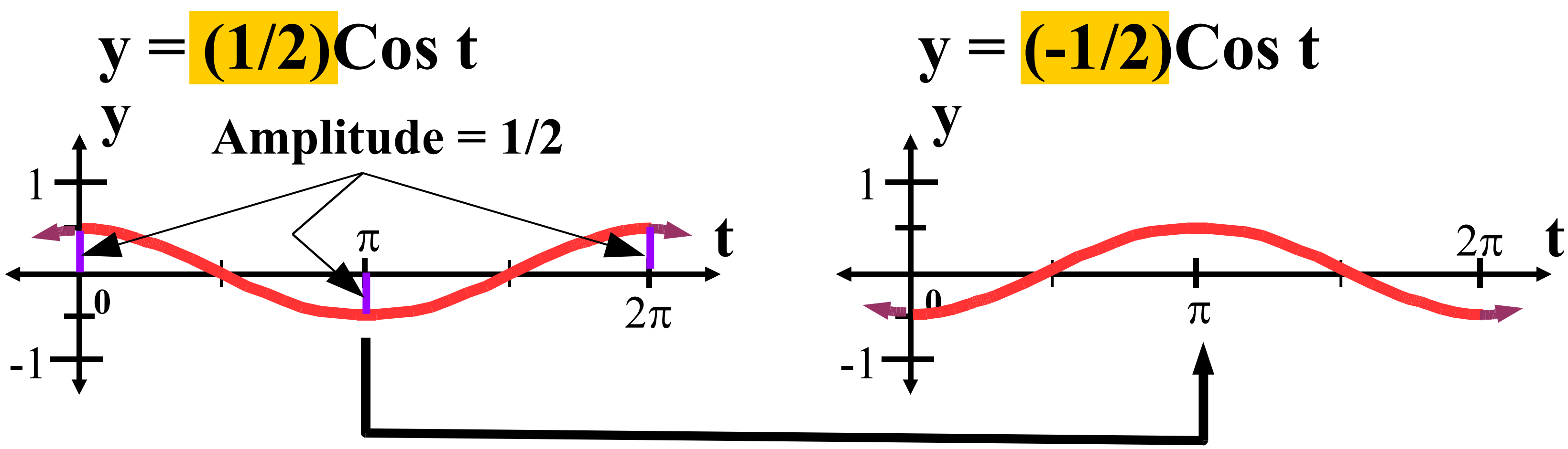
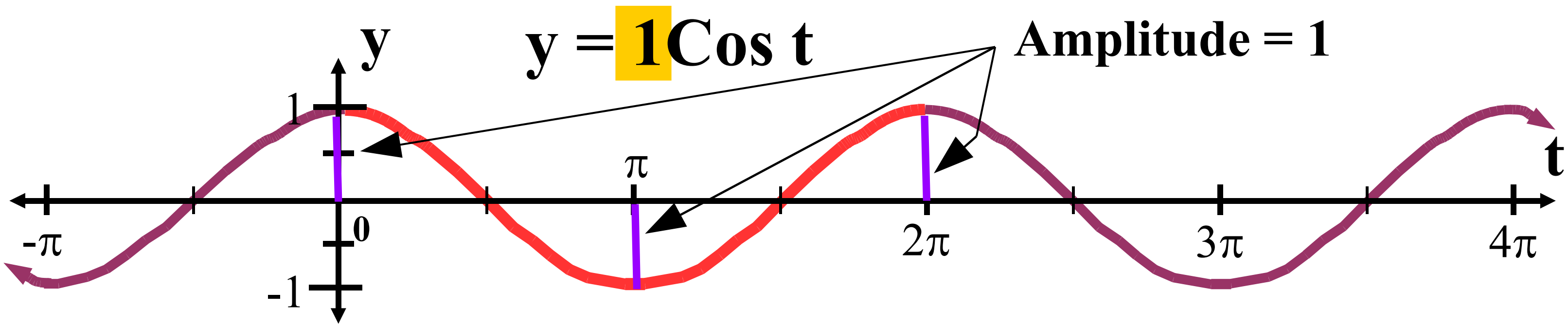


If A < 0, then the graph 'flips' over the mid-line.

Variations of the Cosine Function



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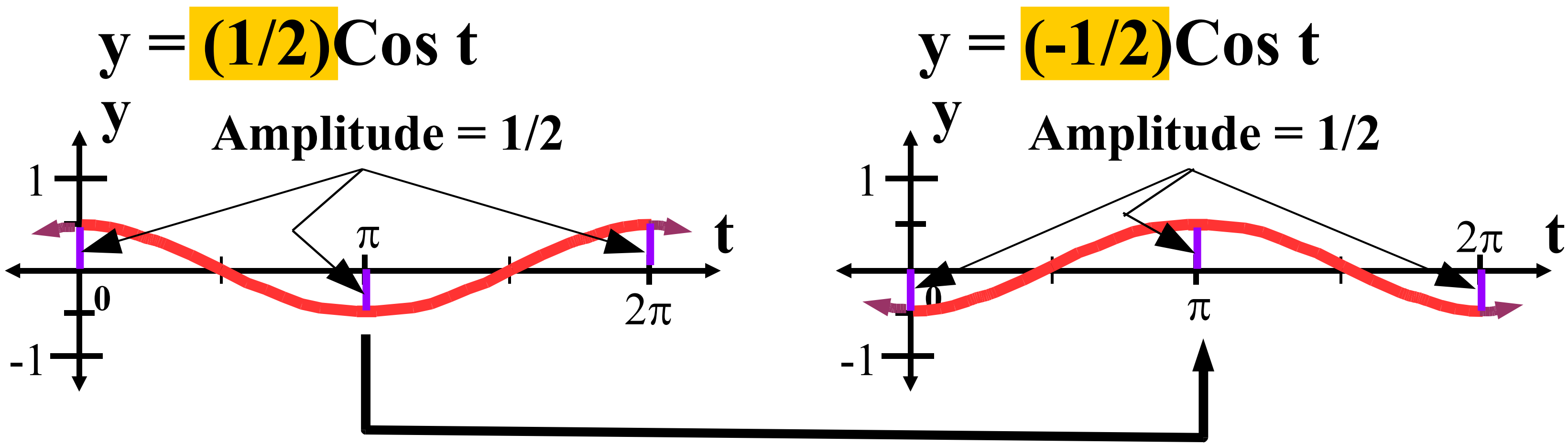

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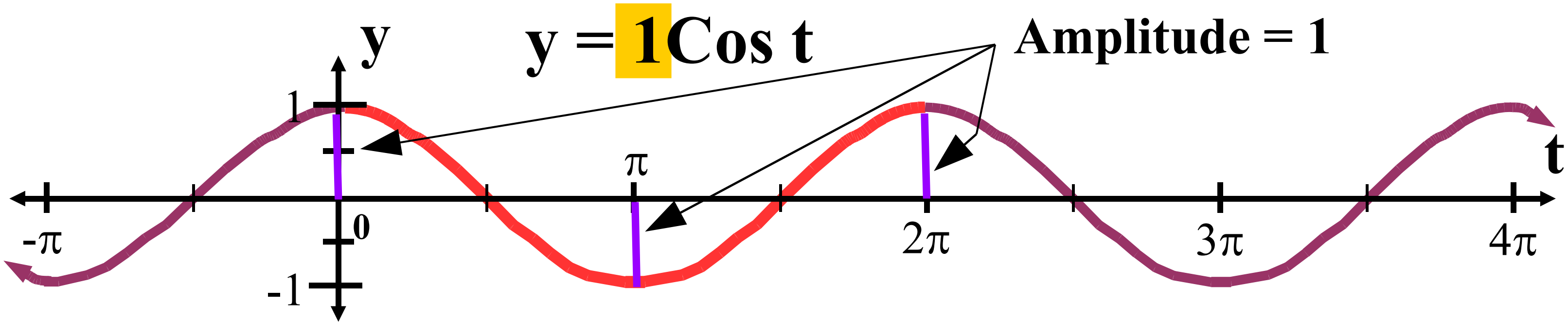

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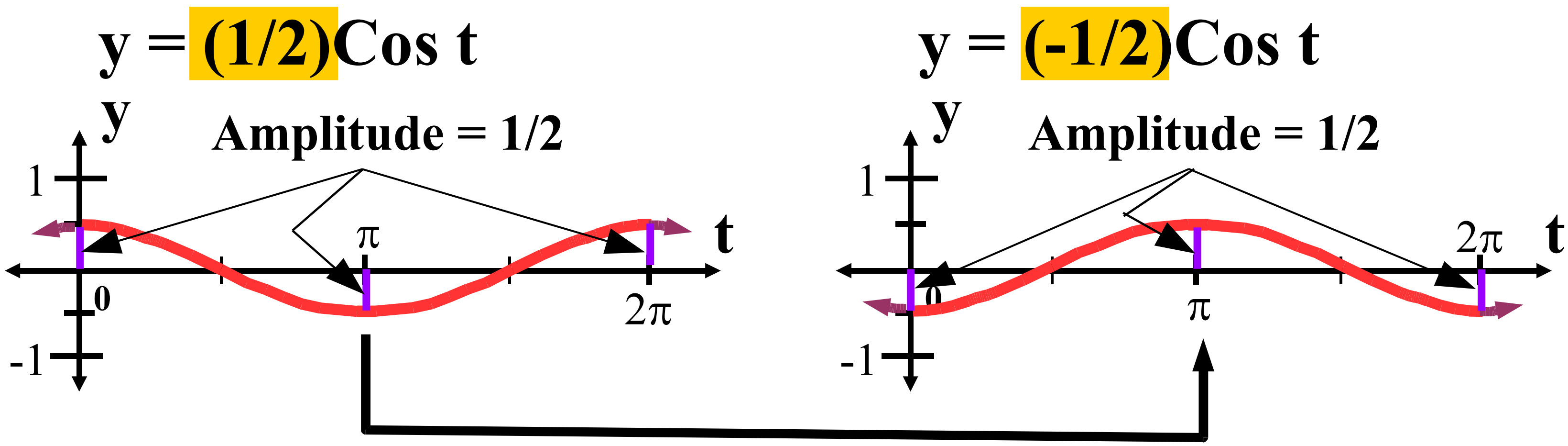

If A < 0, then the graph 'flips' over the mid-line.

The amplitude is equal to the absolute value of A.

Variations of the Cosine Function

Variations of the Cosine Function

Consider the equation $y = A\cos(Bt + C) + D$.

Variations of the Cosine Function

Consider the equation $y = A\cos(Bt + C) + D$.

(1) The amplitude of the 'cosine wave' is the absolute value of A.

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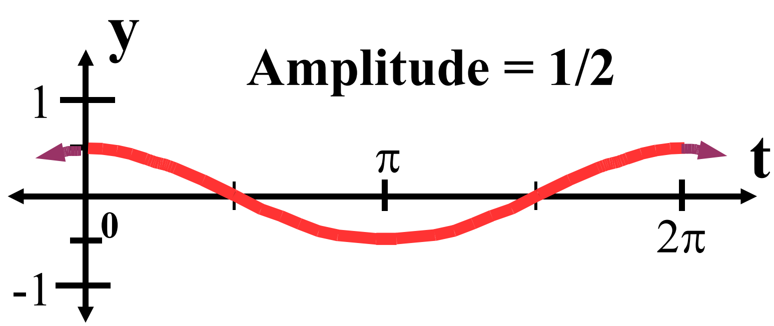
Consider the equation y = Acos(Bt + C) + D.

(1) The amplitude of the 'cosine wave' is the absolute value of A.

(2) If A > 0,

y = (1/2)Cos t



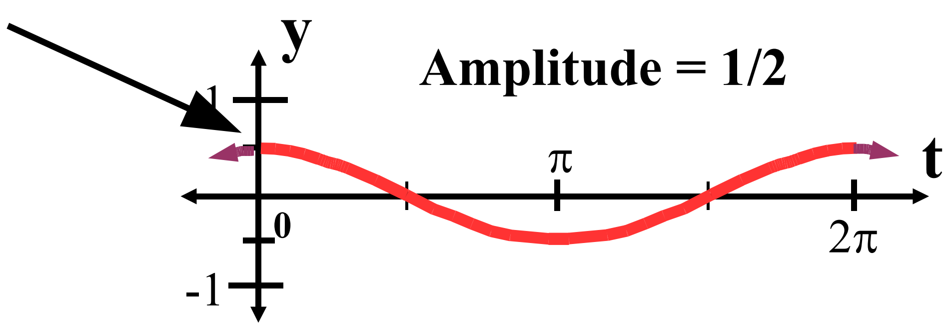
Variations of the Cosine Function

Consider the equation $y = A\cos(Bt + C) + D$.

(1) The amplitude of the 'cosine wave' is the absolute value of A.

(2) If $A > 0$, then the basic cycle starts at its maximum value

$y = (1/2)\text{Cos } t$



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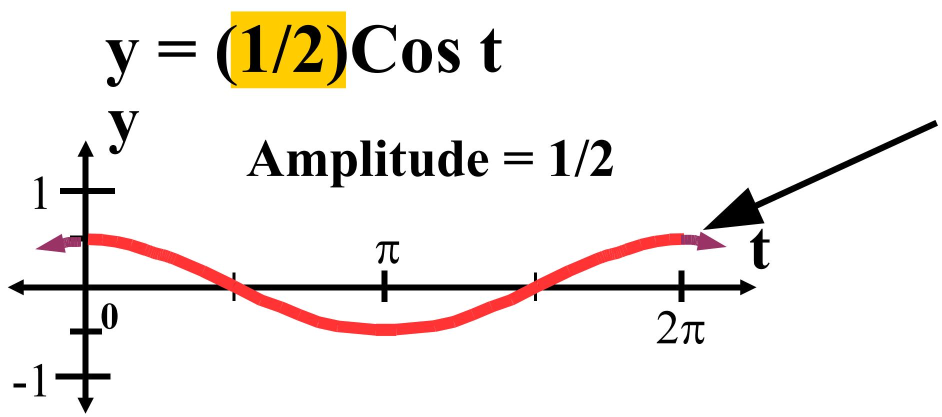

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(1) The amplitude of the 'cosine wave' is the absolute value of A.

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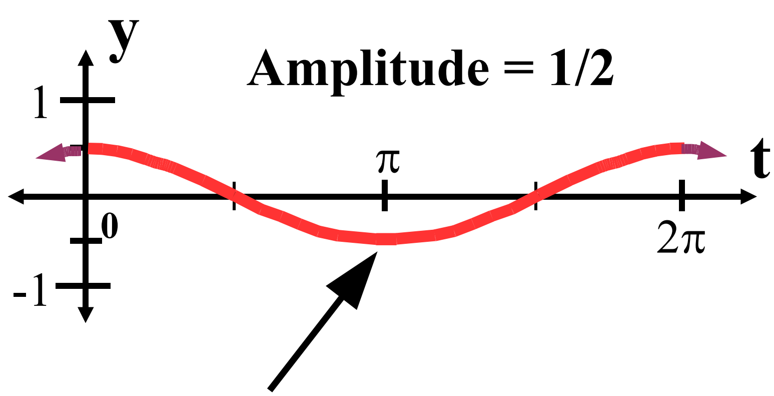
$y = (1/2)\text{Cos } t$



Variations of the Cosine Function

Consider the equation y = Acos(Bt + C) + D.

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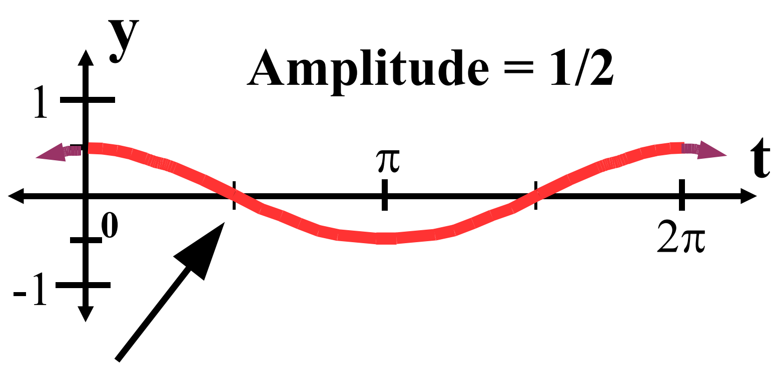
y = (1/2)Cos t



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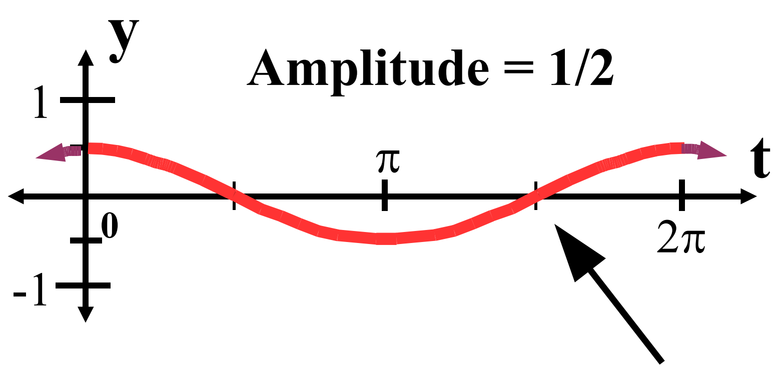

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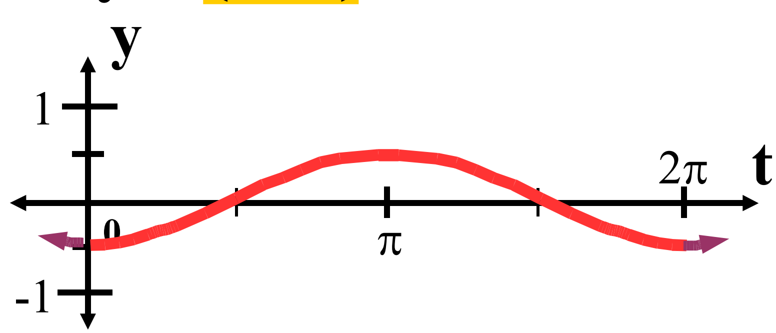
(3) If A < 0,

y = (-1/2)Cos t



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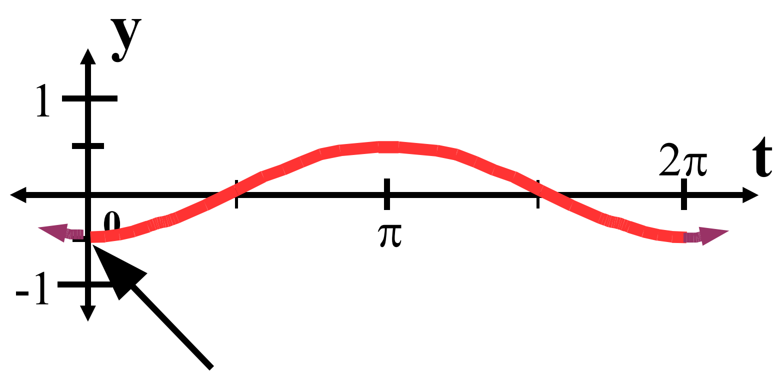
(3) If A < 0, then the basic cycle starts at its minimum value

y = (-1/2)Cos t



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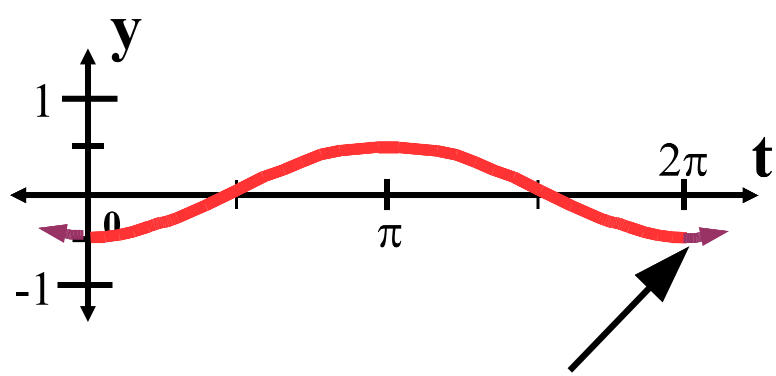

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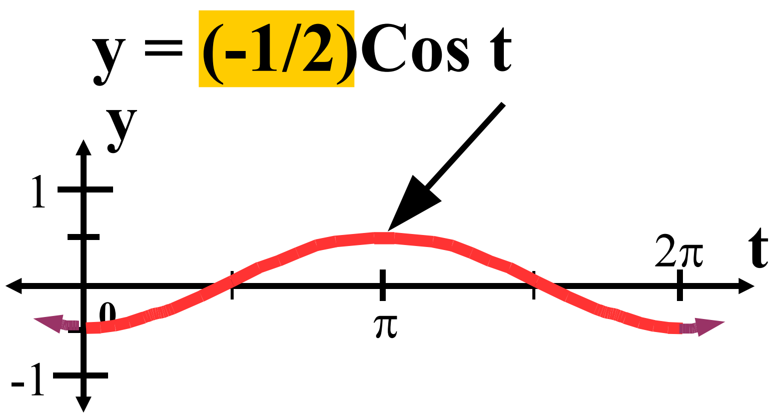

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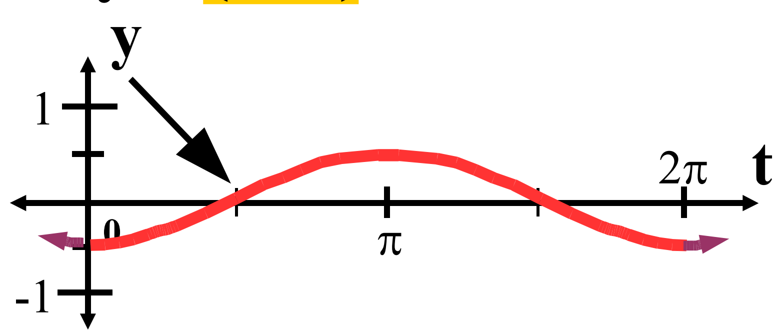
(3) If $A < 0$, then the basic cycle starts at its minimum value and ends at its minimum value. It is at its maximum value “half-way' through the cycle. It crosses the mid-line ¼ way through the cycle

$y = (-1/2)\text{Cos } t$



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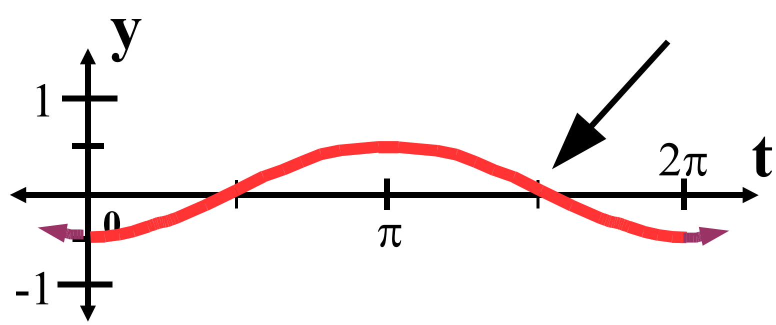

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Variations of the Cosine Function

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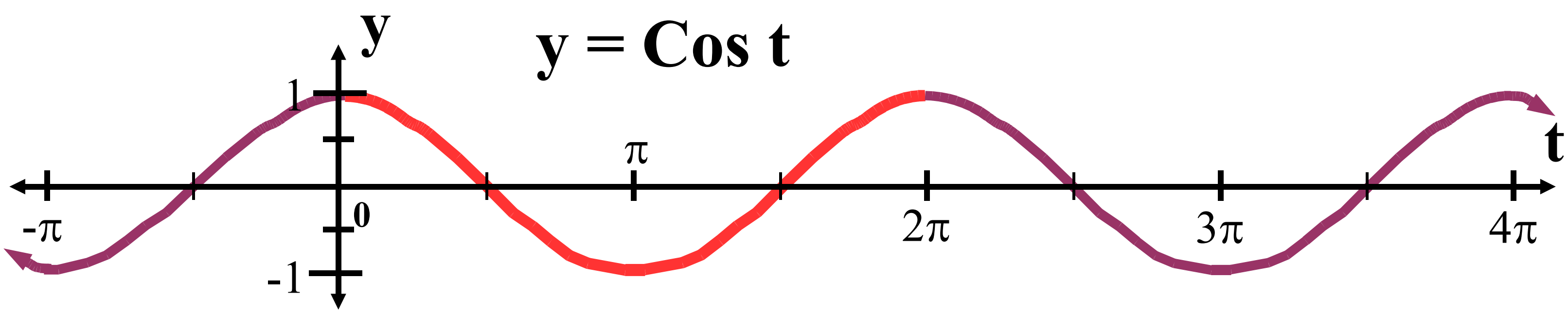
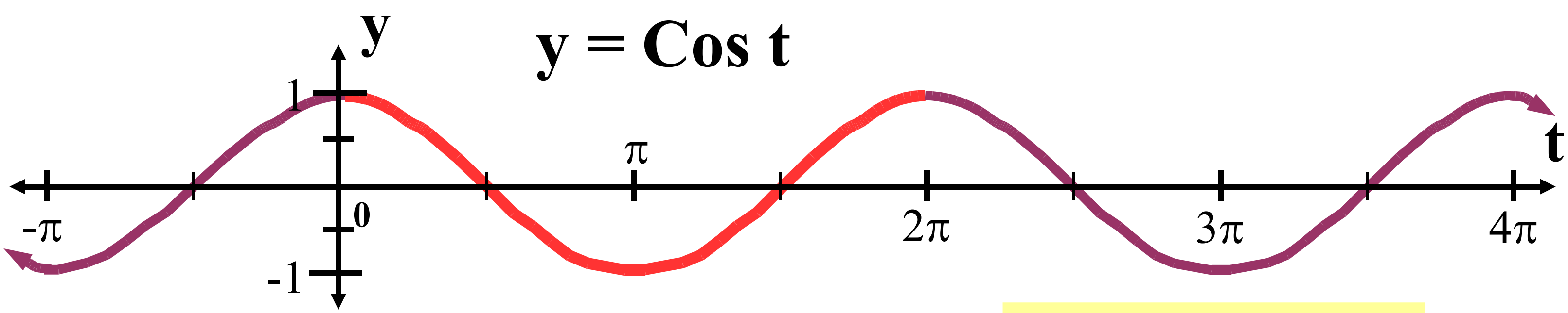
(3) If $A < 0$, then the basic cycle starts at its minimum value and ends at its minimum value. It is at its maximum value “half-way' through the cycle. It crosses the mid-line ¼ way through the cycle and again ¾ way through the cycle.

We will next consider the significance of the constant D.

Variations of the Cosine Function

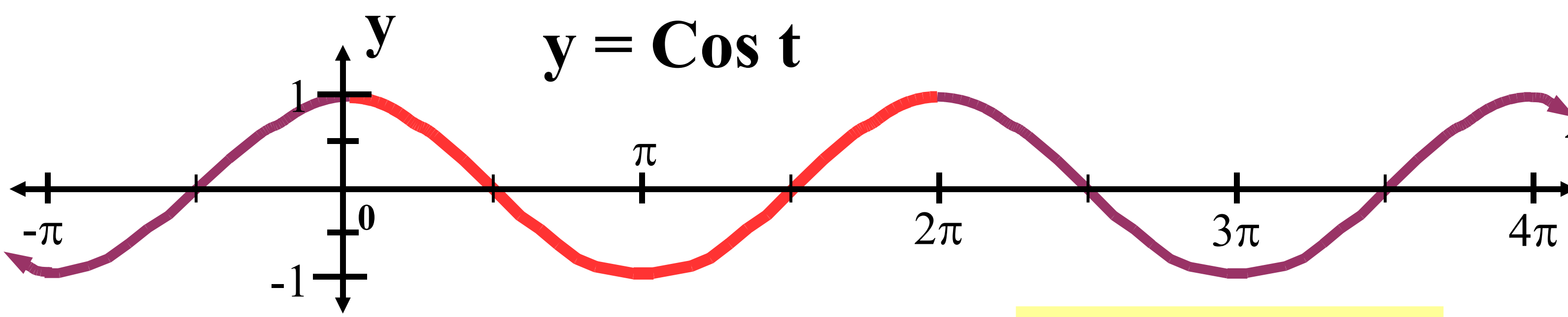


Variations of the Cosine Function



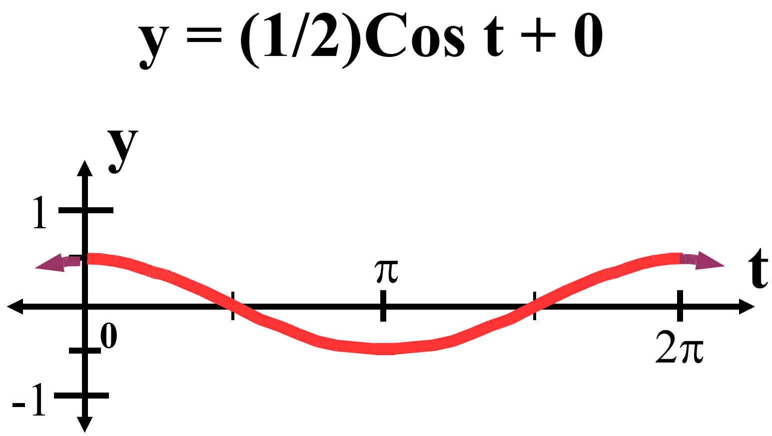
We will start with equations of the form y = ACos t + D.

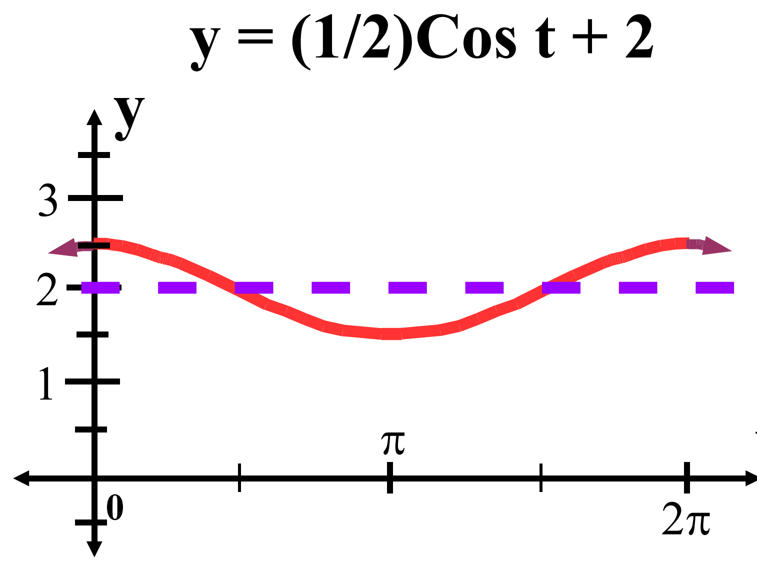
Variations of the Cosine Function



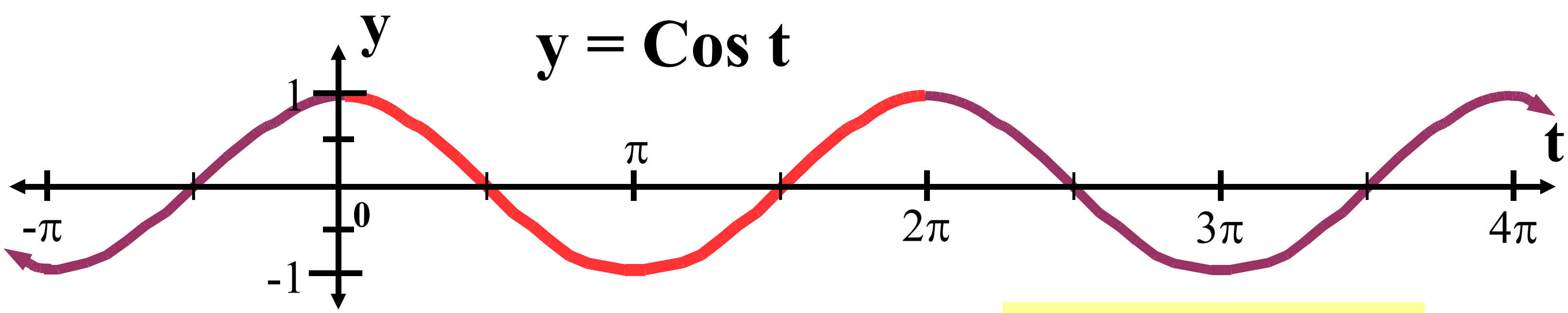
We will start with equations of the form $y = A\text{Cos } t + D$.

Here are two examples (showing the 'basic cycle' only).





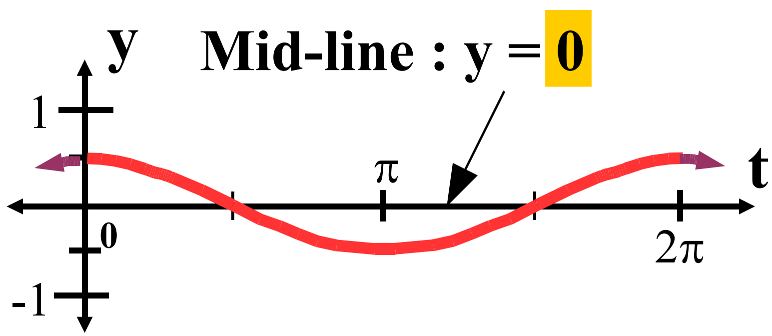
Variations of the Cosine Function



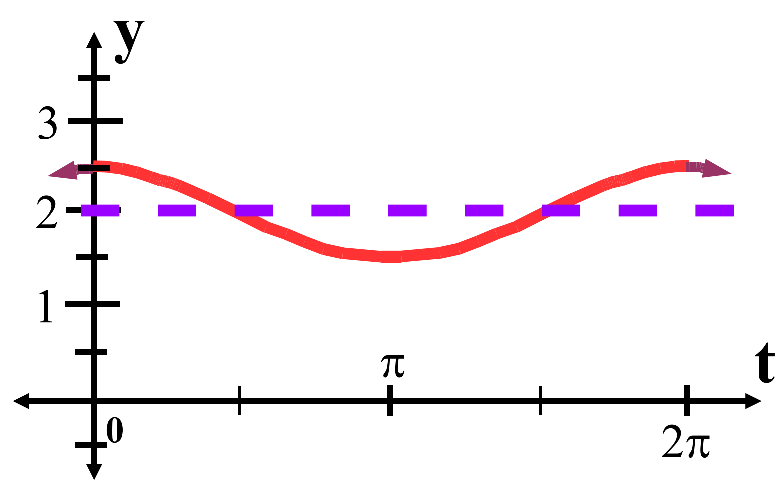
We will start with equations of the form y = ACos t + D.

Here are two examples (showing the 'basic cycle' only).

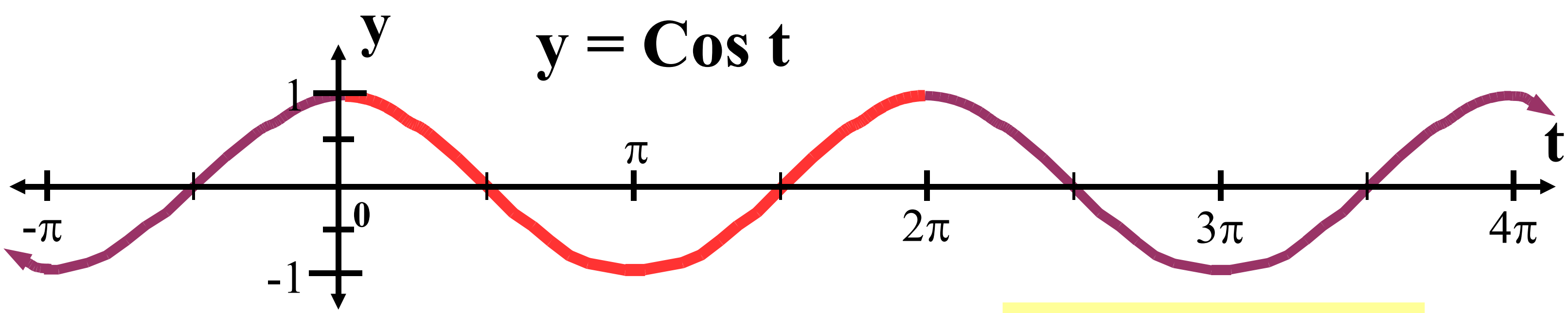
y = (1/2)Cos t + 0



y = (1/2)Cos t + 2



Variations of the Cosine Function



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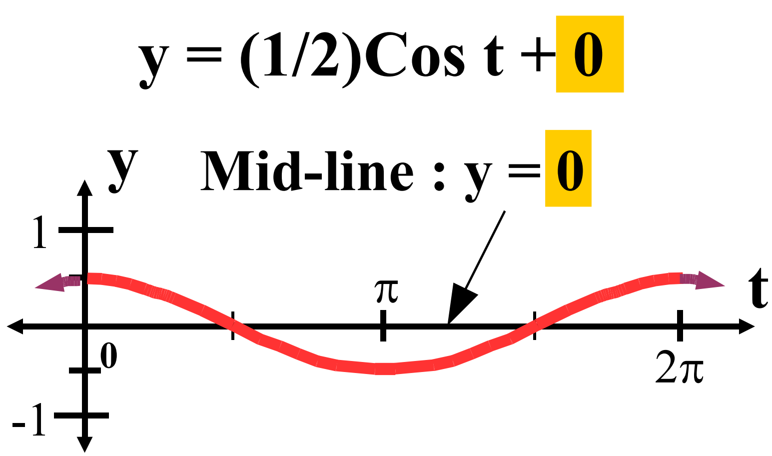
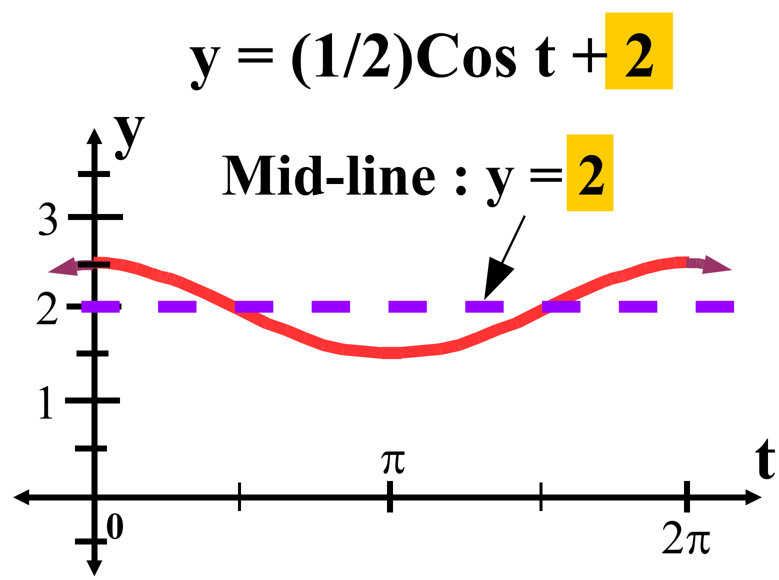
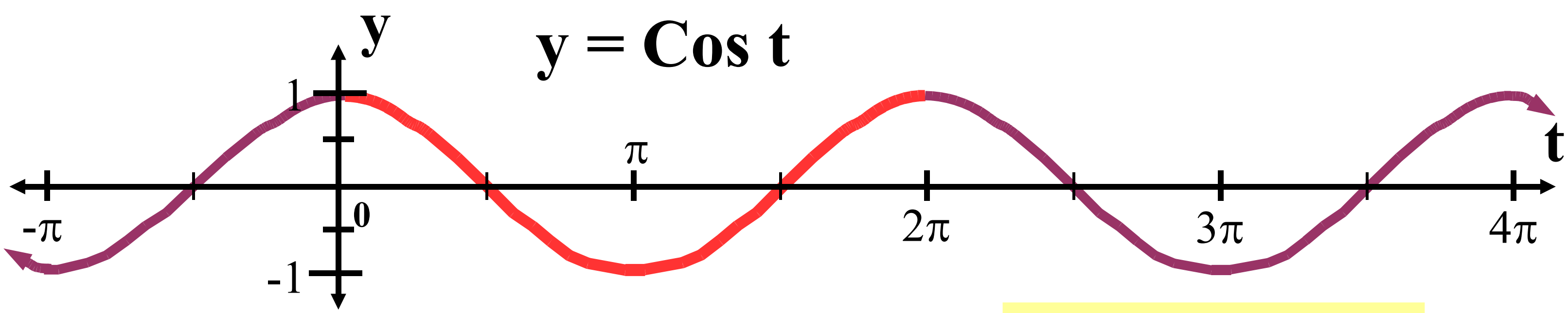


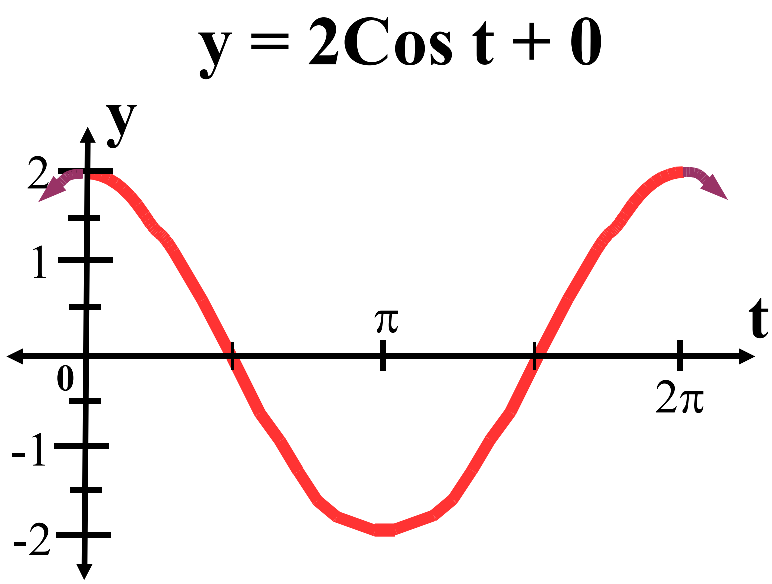


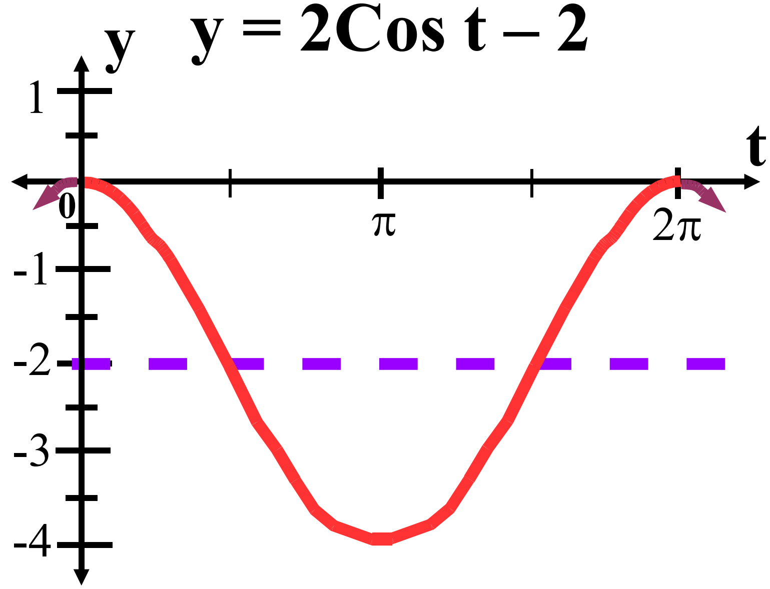
Variations of the Cosine Function



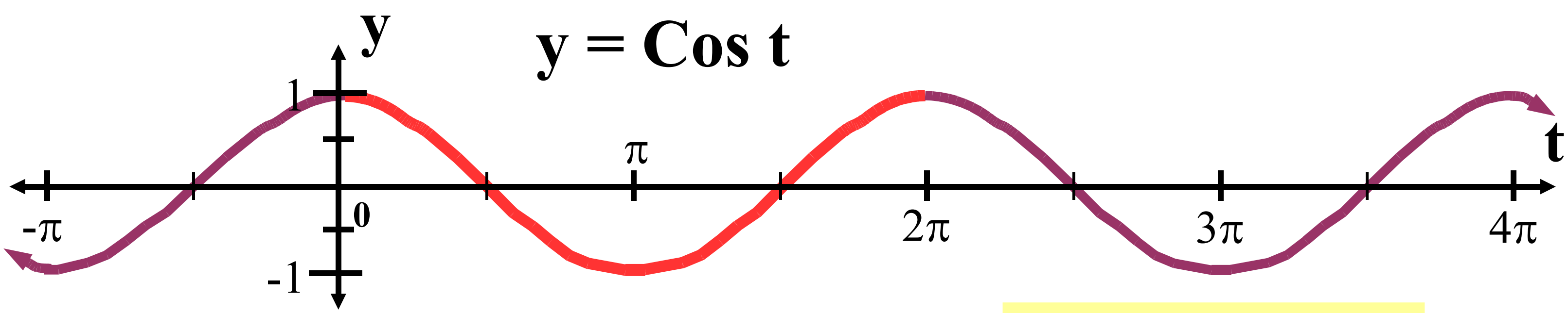
We will start with equations of the form y = ACos t + D.

Here are two more examples (showing the 'basic cycle' only).





Variations of the Cosine Function



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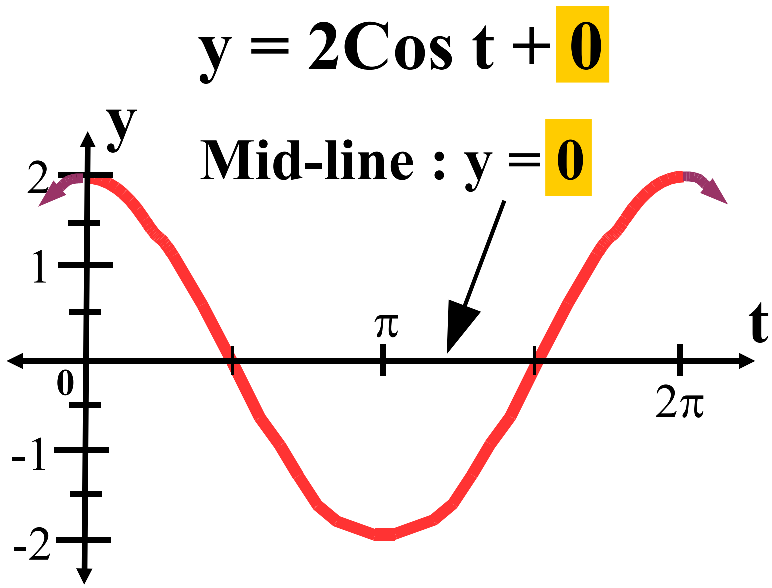
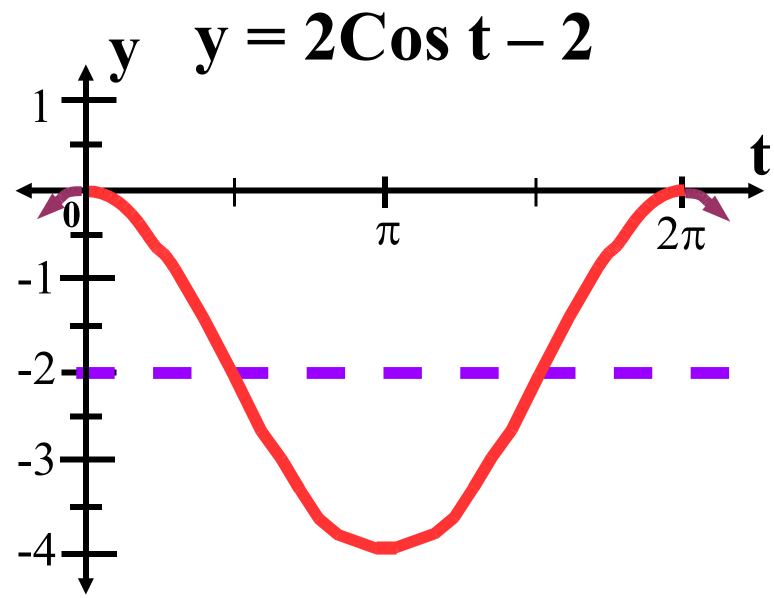
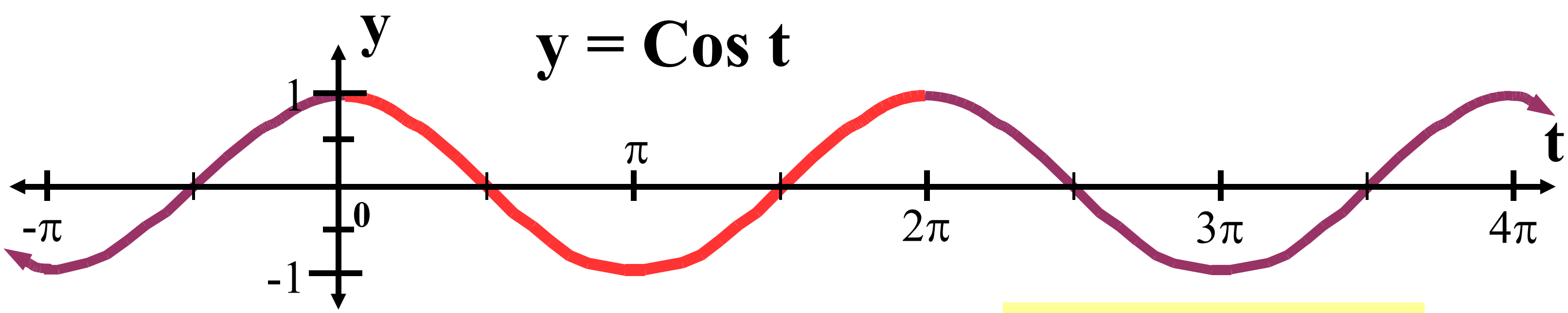




Variations of the Cosine Function



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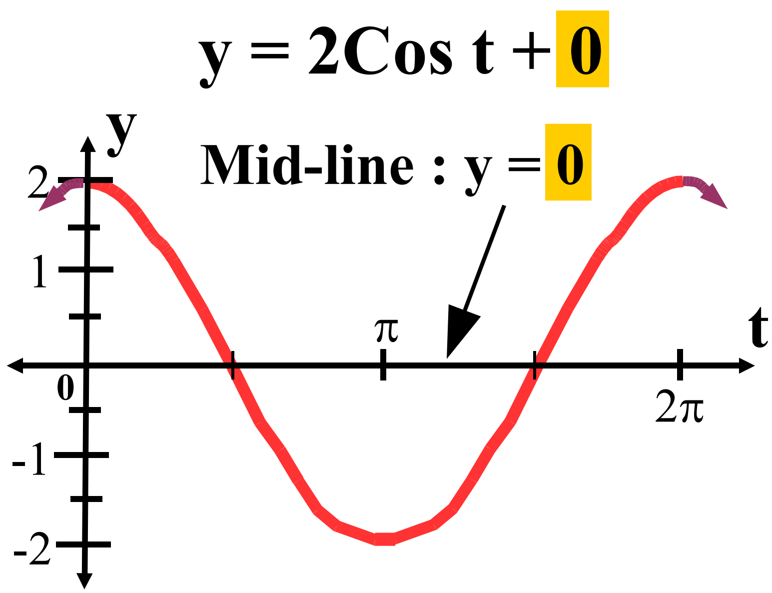
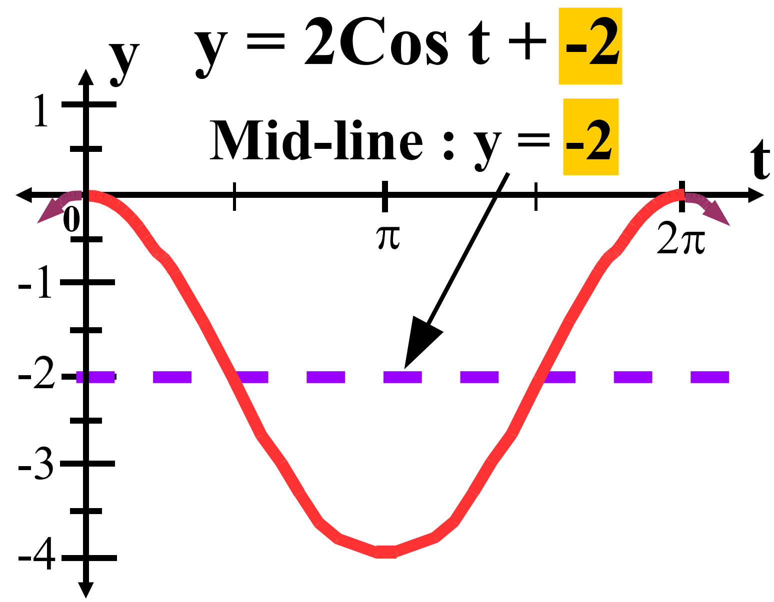
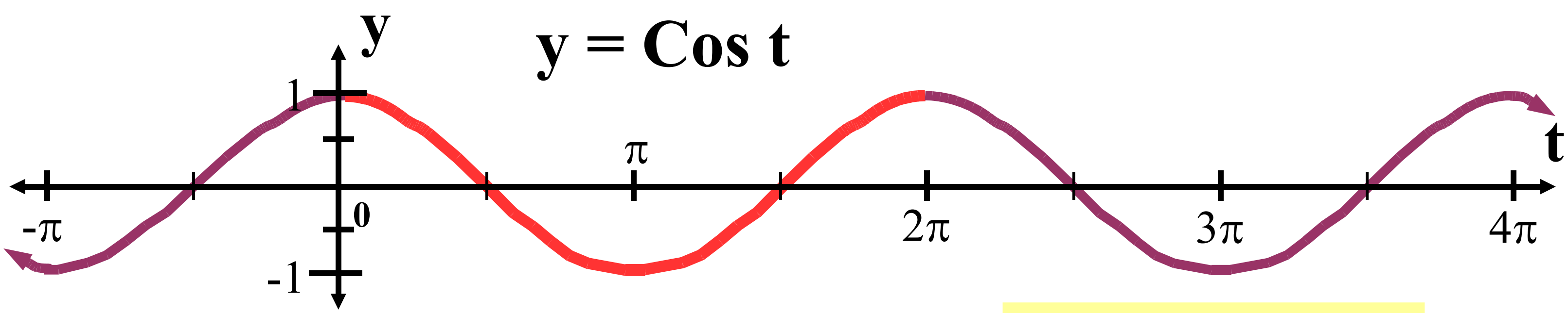




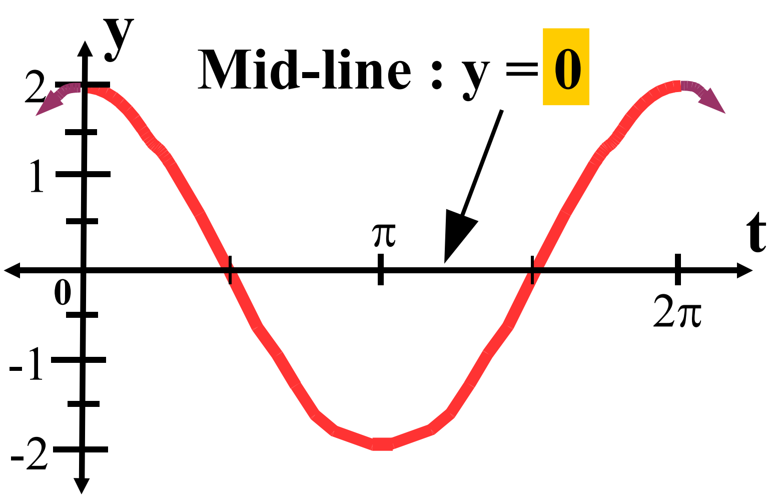
Variations of the Cosine Function



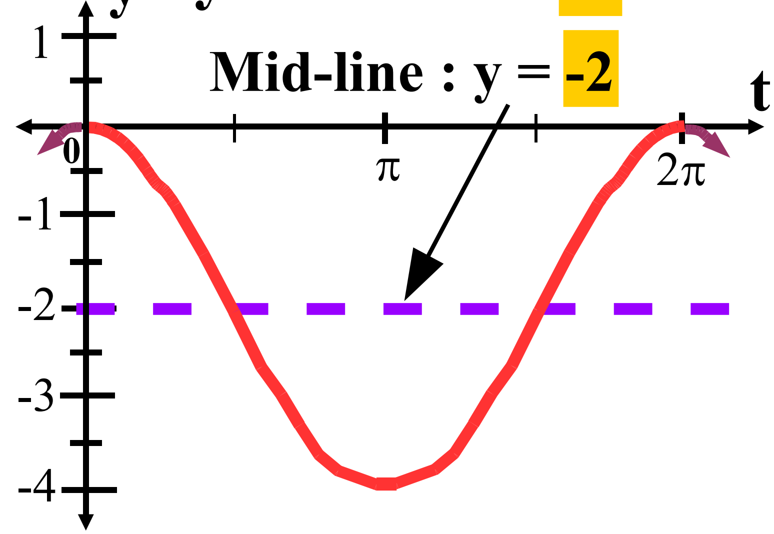
We will start with equations of the form $y = A\text{Cos } t + D$.

Here are two more examples (showing the 'basic cycle' only).

$y = 2\text{Cos } t + 0$



$y = 2\text{Cos } t + -2$



Clearly, the value of D determines the **mid-line** of the graph.

Variations of the Cosine Function

Consider the equation $y = A\cos(Bt + C) + D$.

(1) The amplitude of the 'cosine wave' is the absolute value of A.

(2) If $A > 0$, then the basic cycle starts at its maximum value and ends at its maximum value. It is at its minimum value "half-way' through the cycle. It crosses the mid-line ¼ way through the cycle and again ¾ way through the cycle.

(3) If $A < 0$, then the basic cycle starts at its minimum value and ends at its minimum value. It is at its maximum value "half-way' through the cycle. It crosses the mid-line ¼ way through the cycle and again ¾ way through the cycle.

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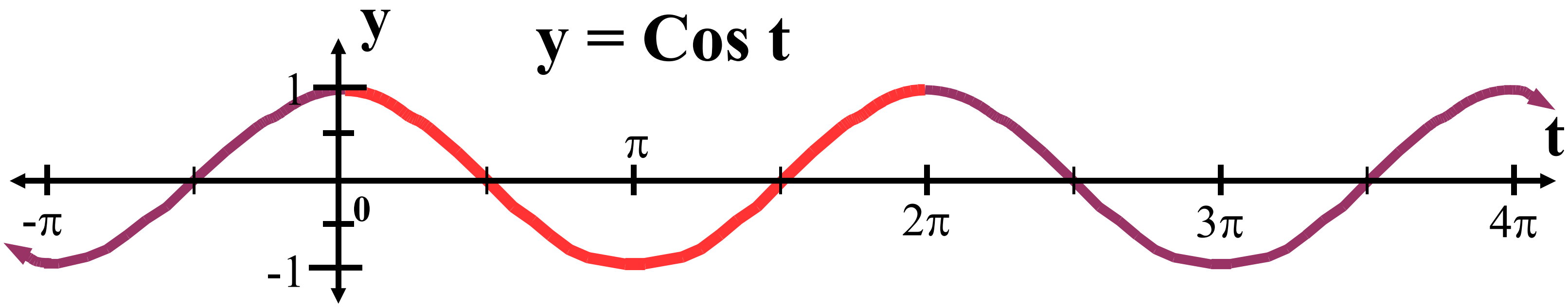
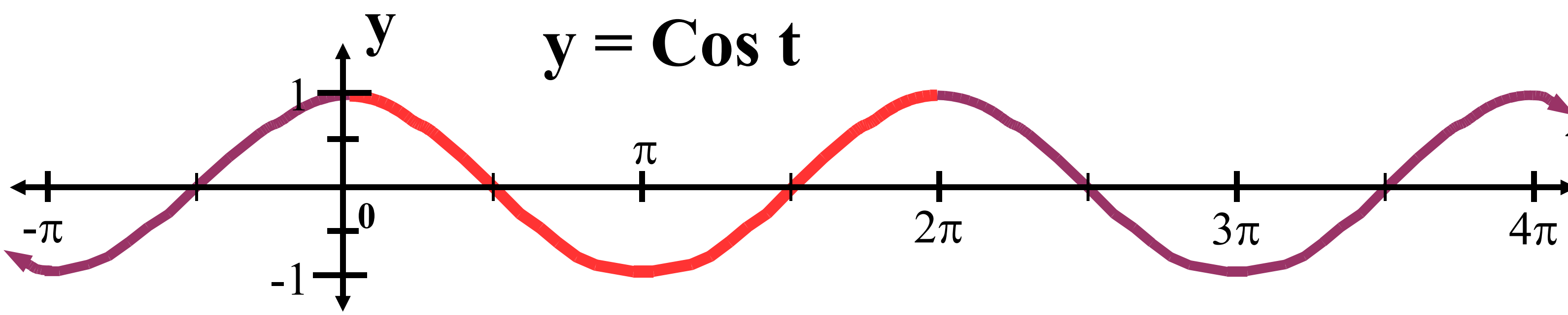
We will next consider the significance of the constants B and C.

Variations of the Cosine Function



In the Cosine graph above, the 'basic cycle' starts when t = 0 and ends when t = 2π.

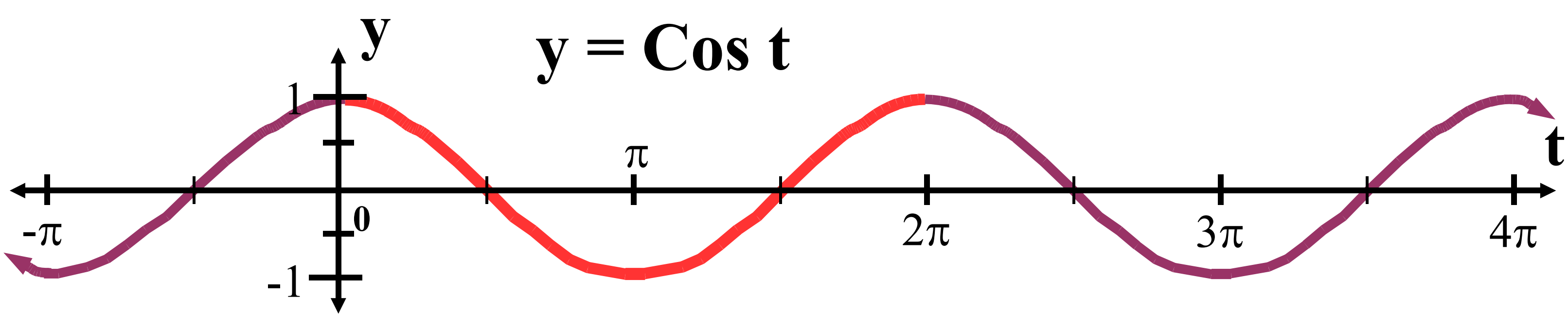
Variations of the Cosine Function



In the Cosine graph above, the 'basic cycle' starts when t = 0 and ends when t = 2π. Consider the more general equation below.

$$y = A\cos(Bt + C) + D.$$

Variations of the Cosine Function



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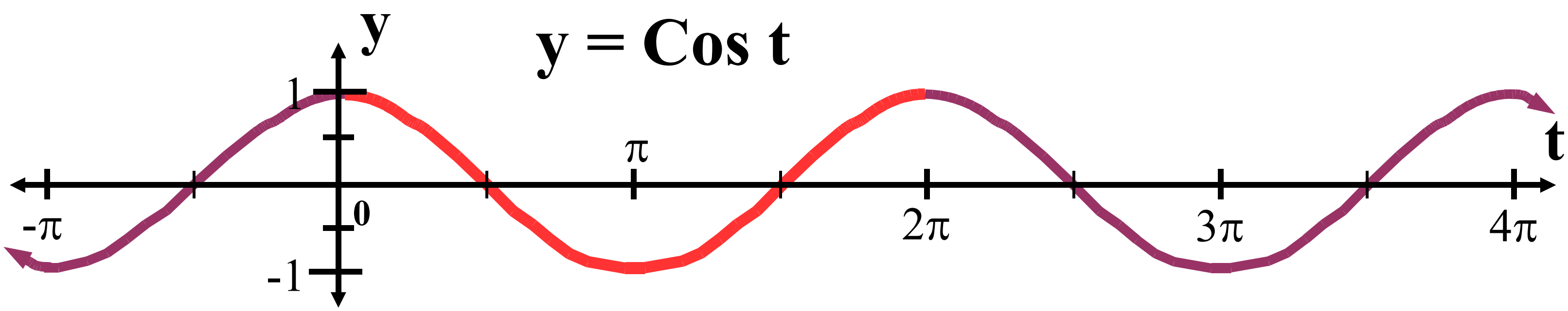
When planning this graph, it is important to understand that the 'basic cycle' starts when **Bt + C = 0** and ends when **Bt + C = 2π**.

Variations of the Cosine Function



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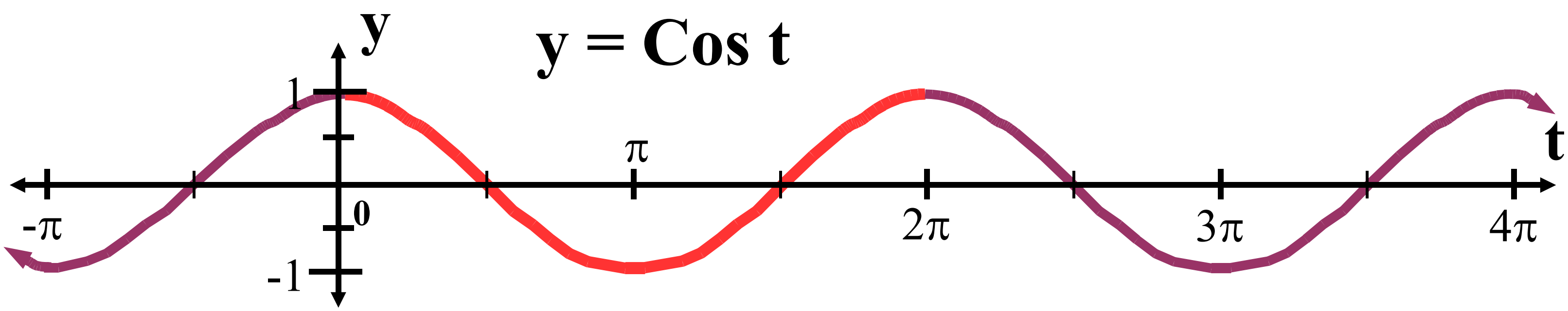
When planning this graph, it is important to understand that the 'basic cycle' starts when **Bt + C = 0** and ends when **Bt + C = 2π**. The 'basic cycle' of the Sine function starts and ends on the mid-line.

Variations of the Cosine Function



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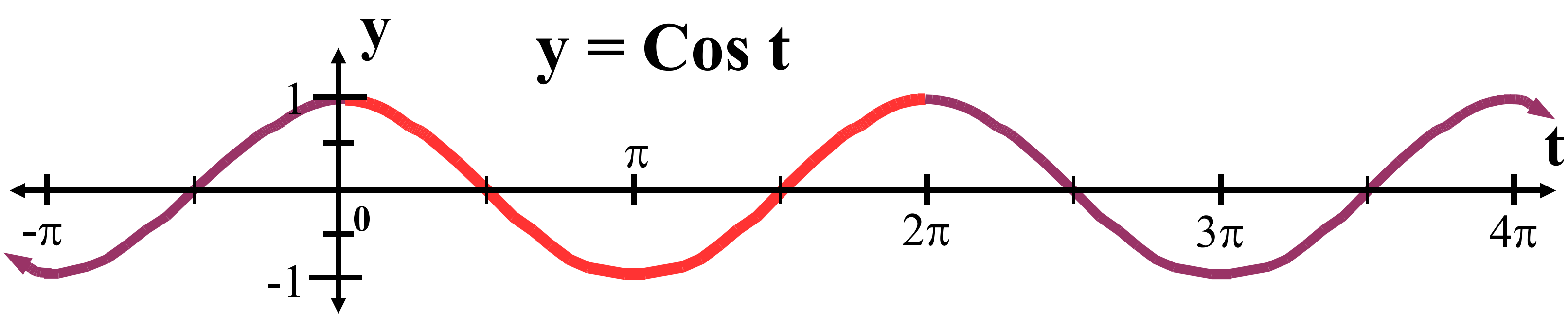
When planning this graph, it is important to understand that the 'basic cycle' starts when **Bt + C = 0** and ends when **Bt + C = 2π**. The 'basic cycle' of the Sine function starts and ends on the mid-line. The Cosine function is different.

Variations of the Cosine Function

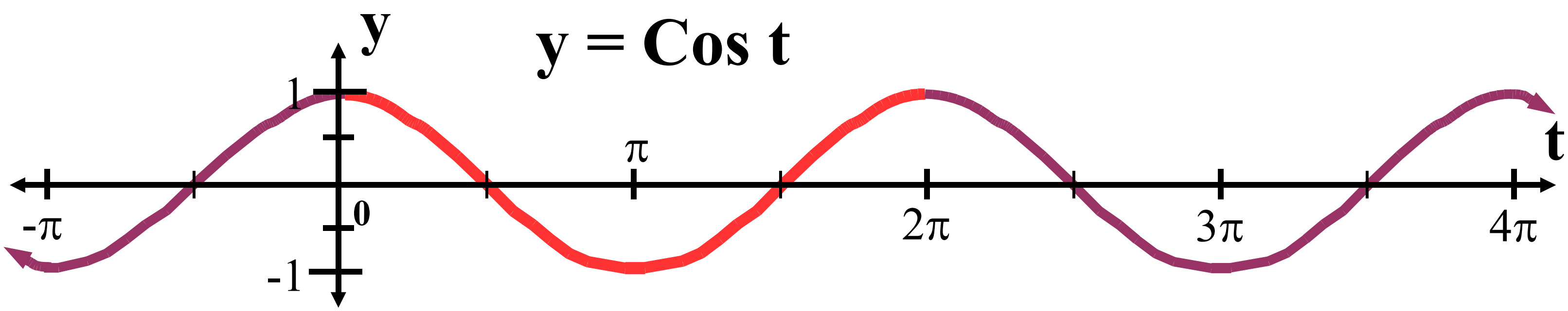


In the Cosine graph above, the 'basic cycle' starts when $t = 0$ and ends when $t = 2\pi$. Consider the more general equation below.

$$y = A\text{Cos}\,(Bt + C) + D.$$

When planning this graph, it is important to understand that the 'basic cycle' starts when **Bt + C = 0** and ends when **Bt + C = 2π**. The 'basic cycle' of the Sine function starts and ends on the mid-line. The Cosine function is different. This makes graphing the Cosine function more complex.

Variations of the Cosine Function



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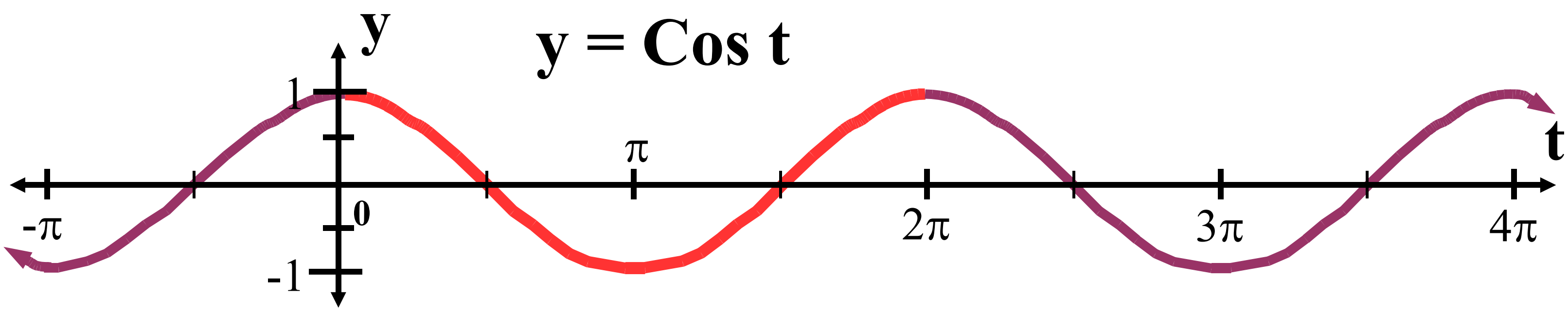
Variations of the Cosine Function



In the Cosine graph above, the 'basic cycle' starts when t = 0 and ends when t = 2π. Consider the more general equation below.

y = ACos (Bt + C) + D.

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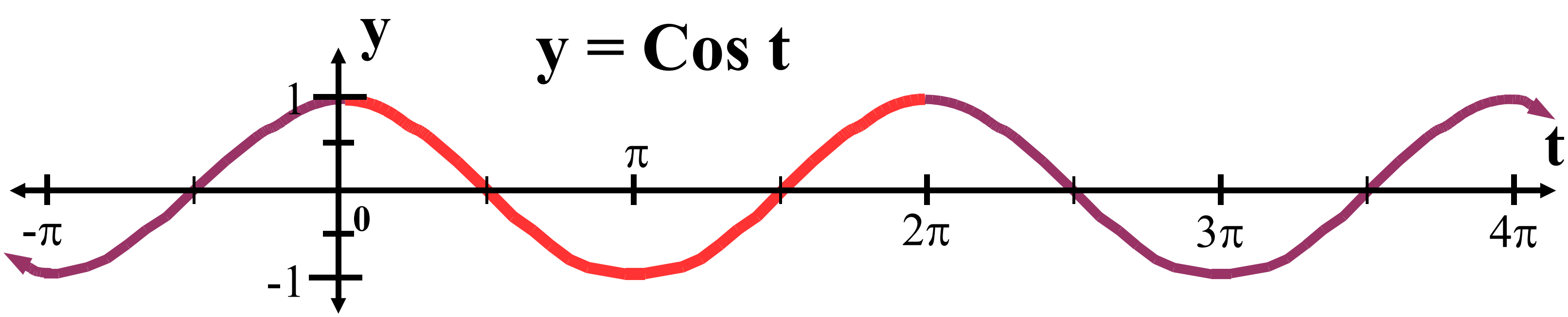
Variations of the Cosine Function



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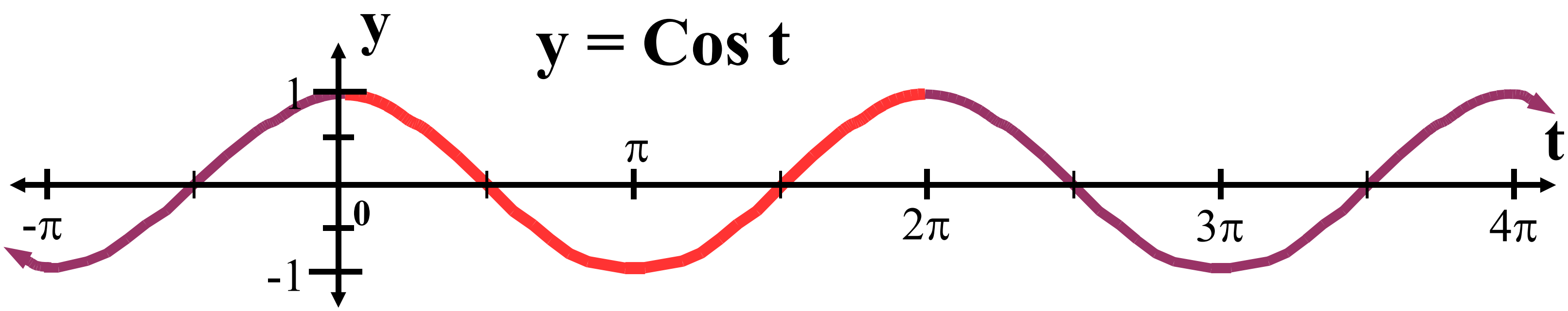
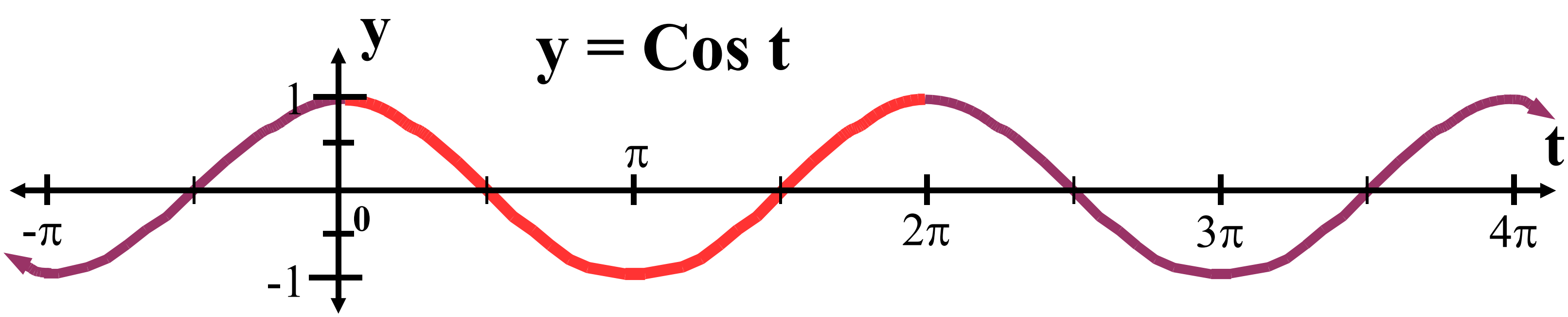
Variations of the Cosine Function



Consider the equation y = **2Cos(2t – π) + 1**.

Variations of the Cosine Function



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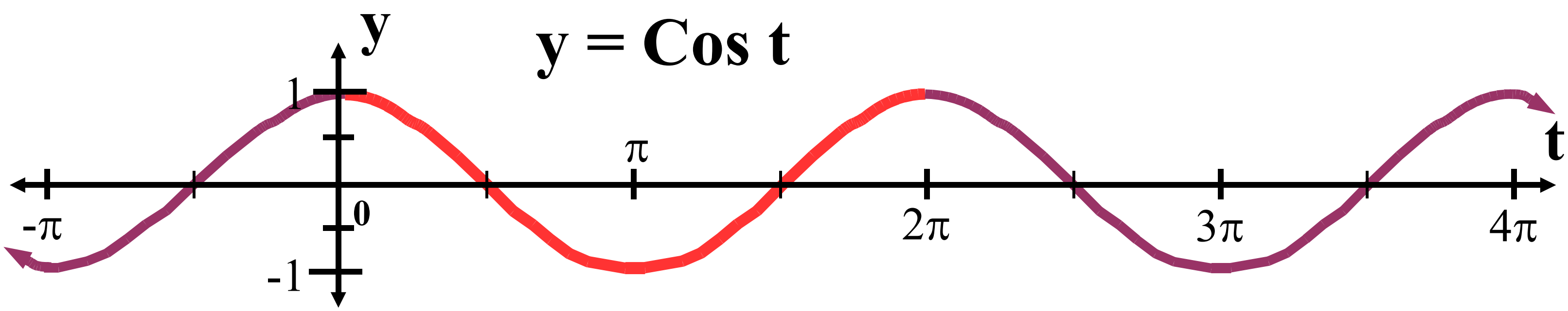

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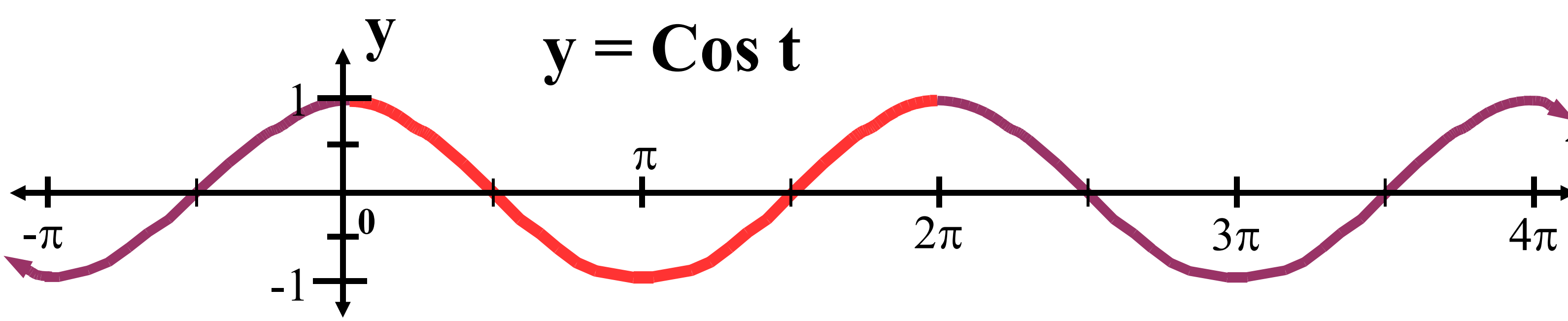
Mid-line: **y = 1**

Variations of the Cosine Function



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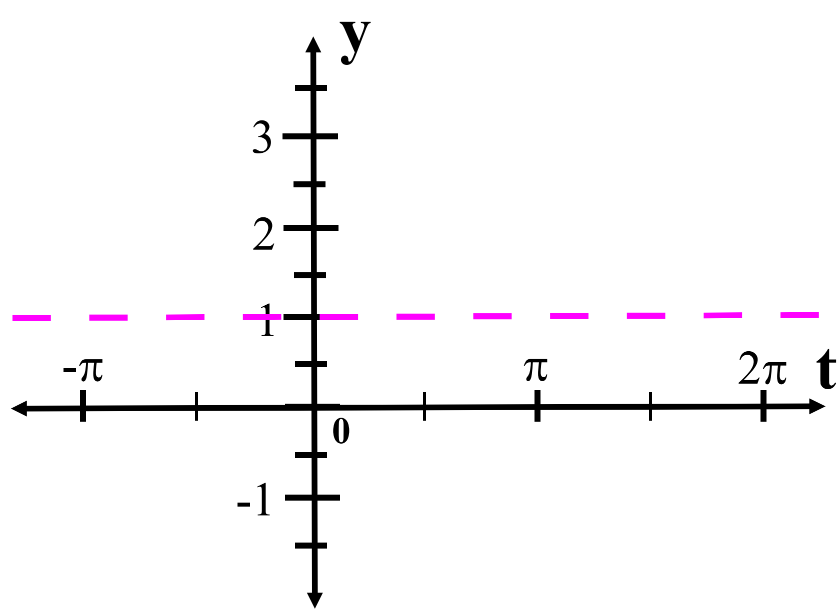

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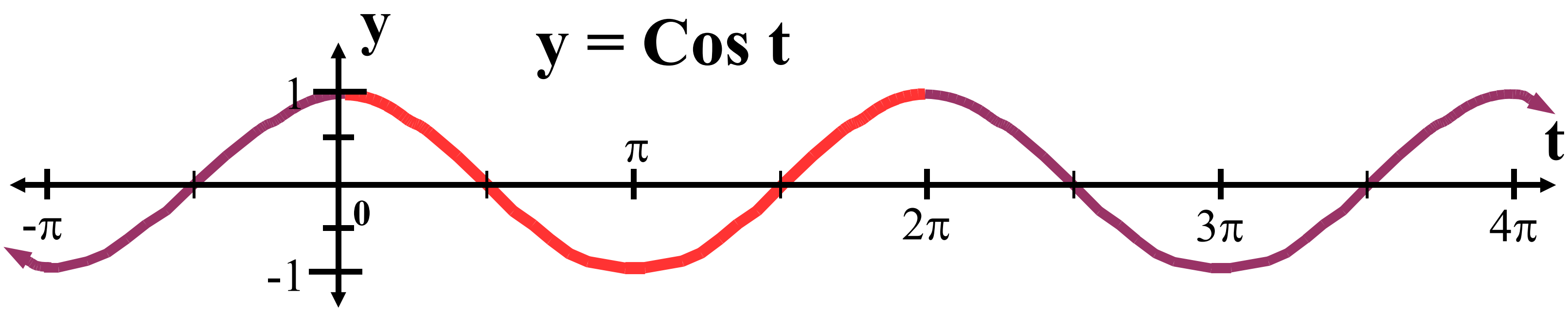

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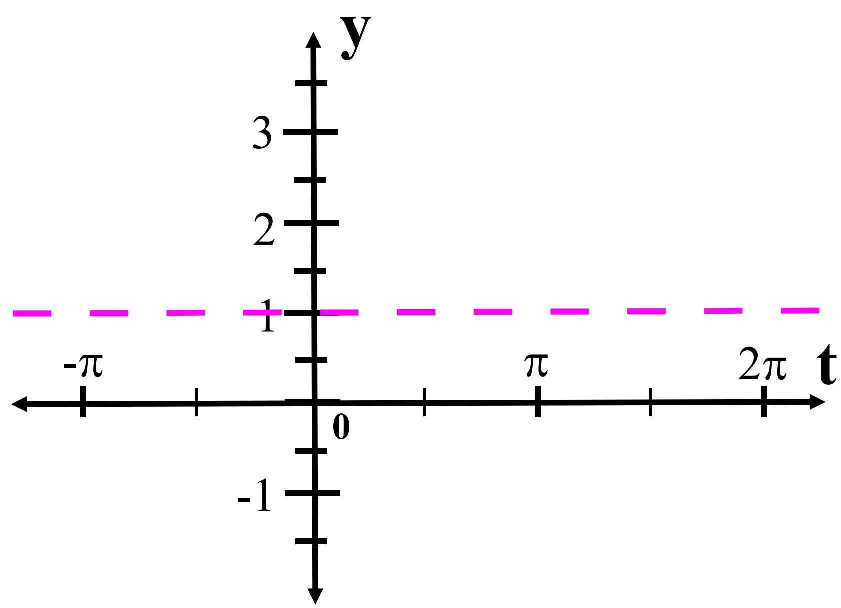

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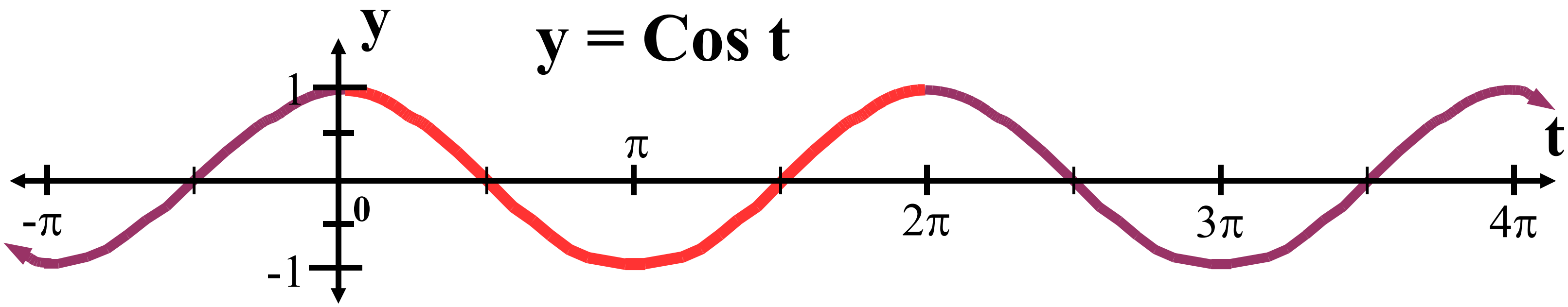

Consider the equation y = **2Cos(2t – π) + 1**.

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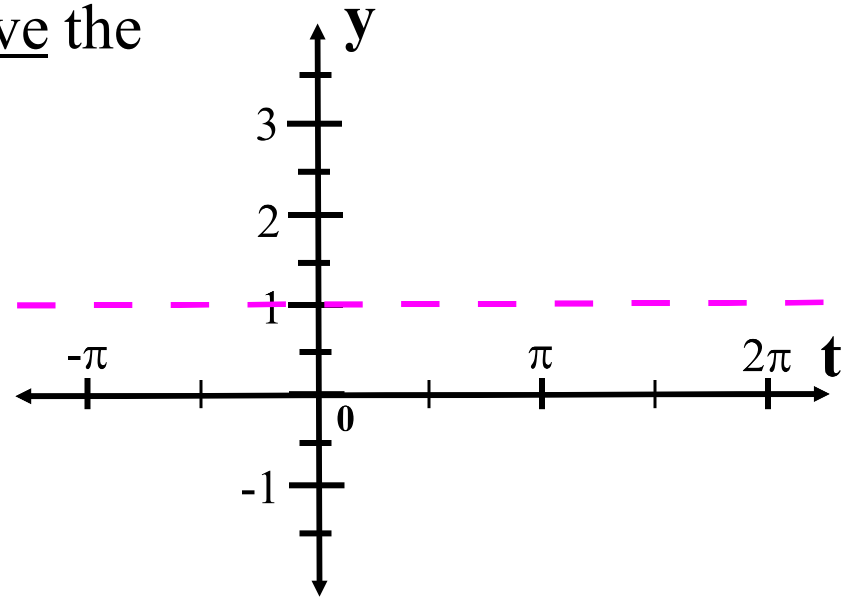
The 'basic cycle' starts 2 units above the mid-line



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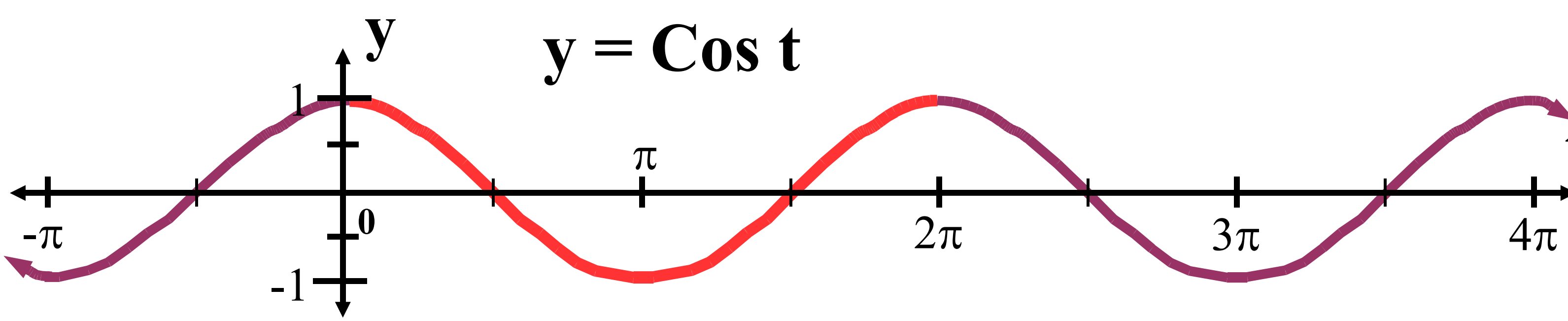
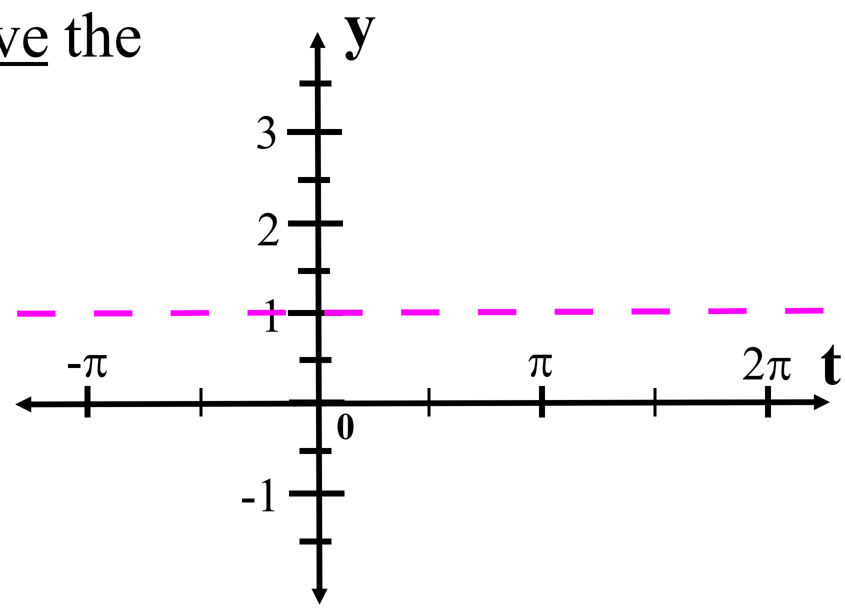


Consider the equation y = **2Cos(2t – π) + 1**.

Mid-line: **y = 1**

The 'basic cycle' starts 2 units above the mid-line when 2t – π = 0.



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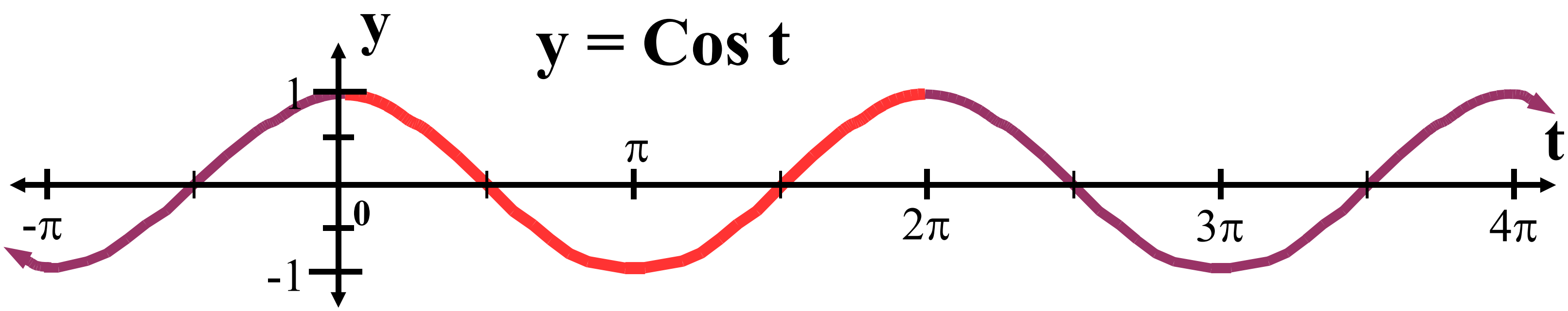
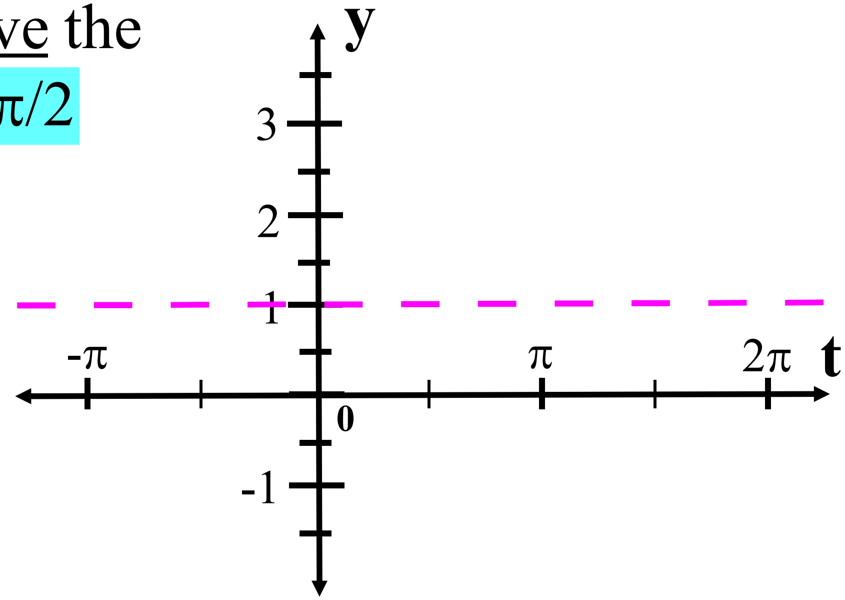


Consider the equation y = **2Cos(2t – π) + 1**.

Mid-line: **y = 1**

The 'basic cycle' starts 2 units above the mid-line when 2t – π = 0. → t = π/2



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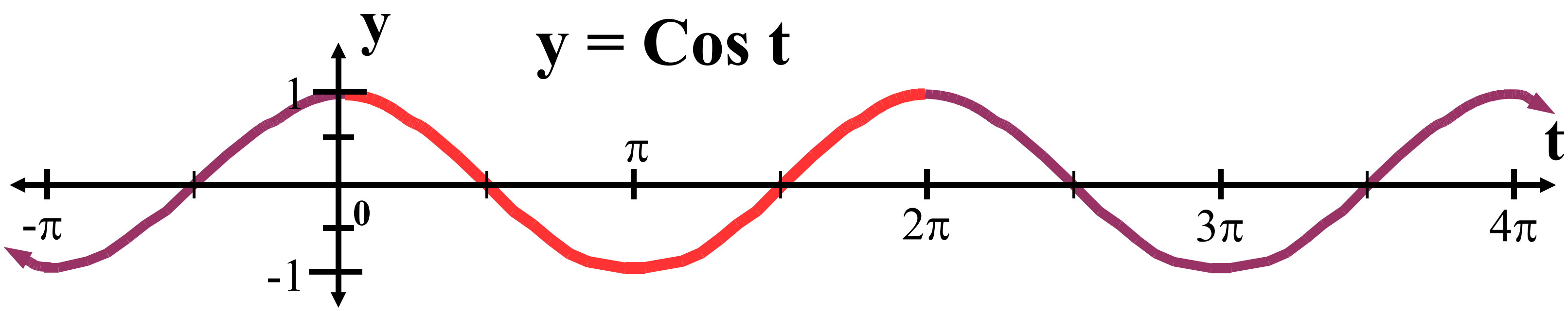

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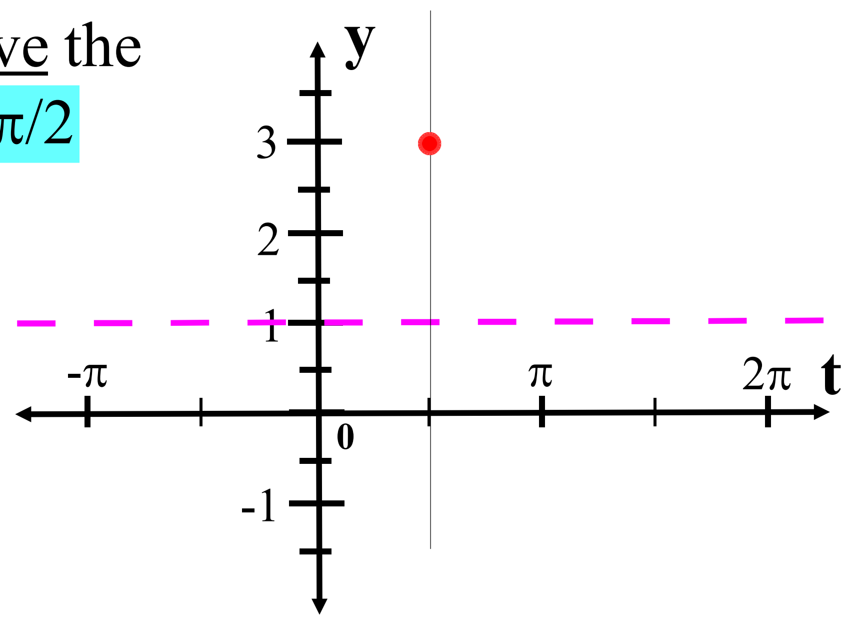

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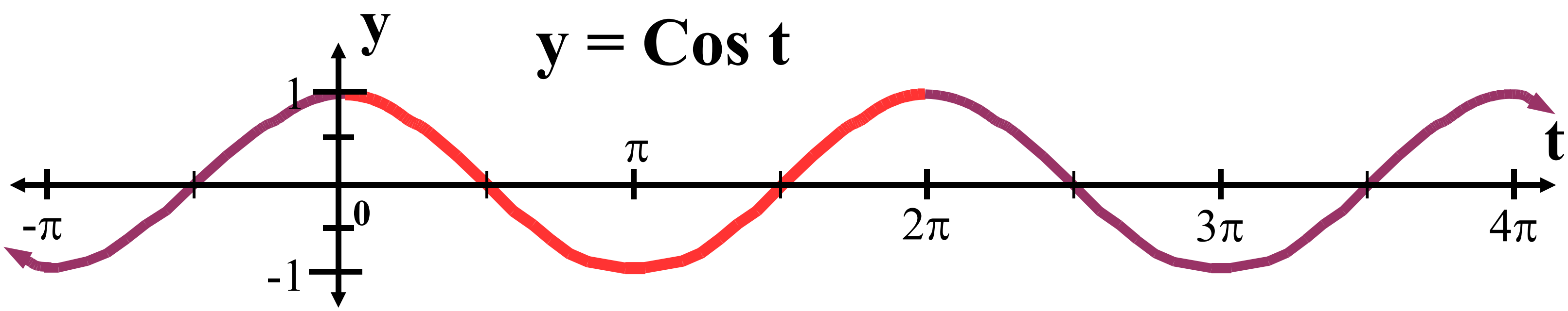
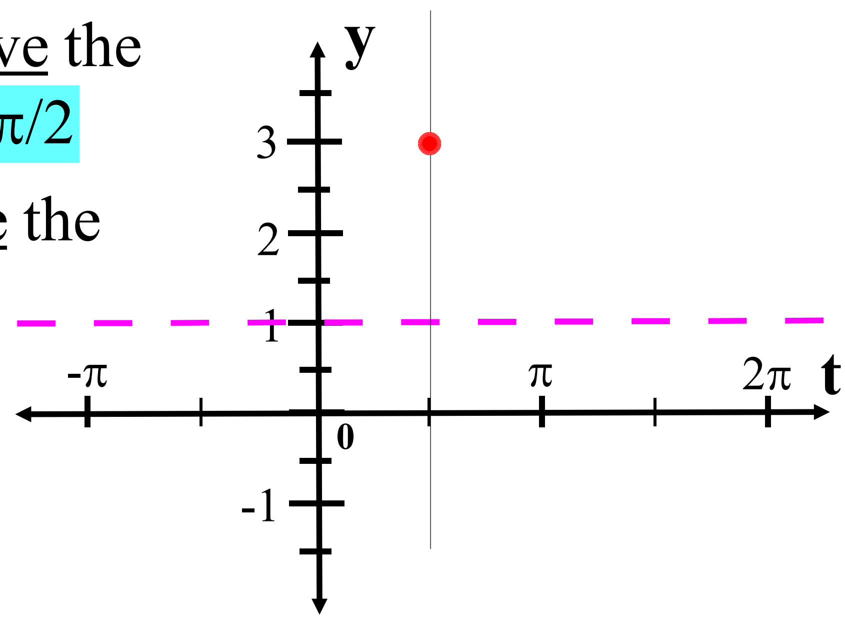


Consider the equation $y = 2\text{Cos}(2t - \pi) + 1$.

Mid-line: $y = 1$

The 'basic cycle' starts <u>2 units above</u> the mid-line when $2t - \pi = 0$. → $t = \pi/2$

The 'basic cycle' ends <u>2 units above</u> the mid-line when $2t - \pi = 2\pi$.



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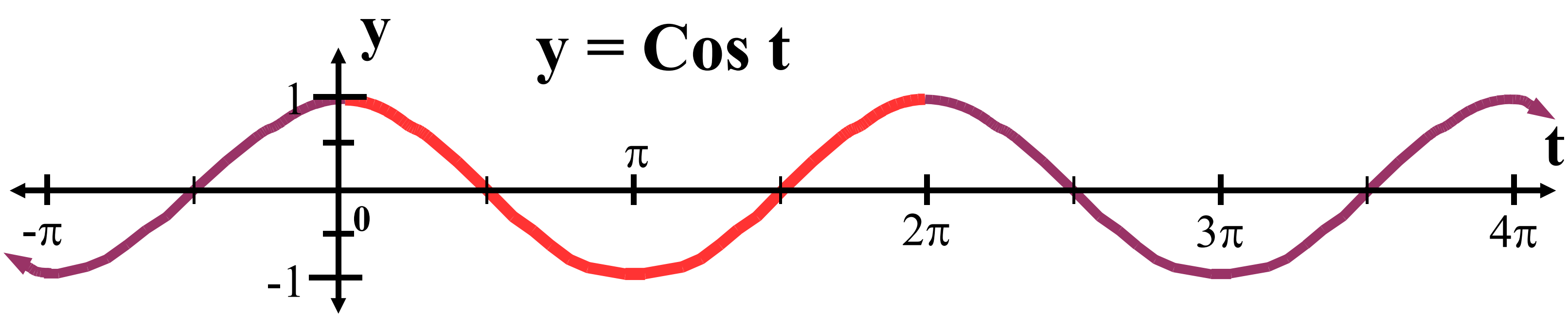
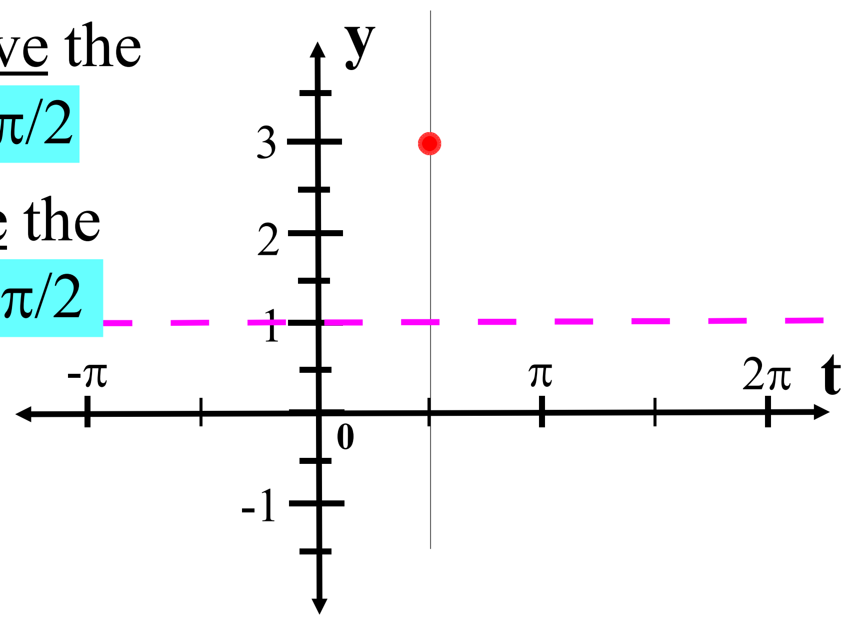


Consider the equation y = **2Cos(2t – π) + 1**.

Mid-line: **y = 1**

The 'basic cycle' starts <u>2 units above</u> the mid-line when 2t – π = 0. ⟶ t = π/2

The 'basic cycle' ends <u>2 units above</u> the mid-line when 2t – π = 2π. ⟶ t = 3π/2



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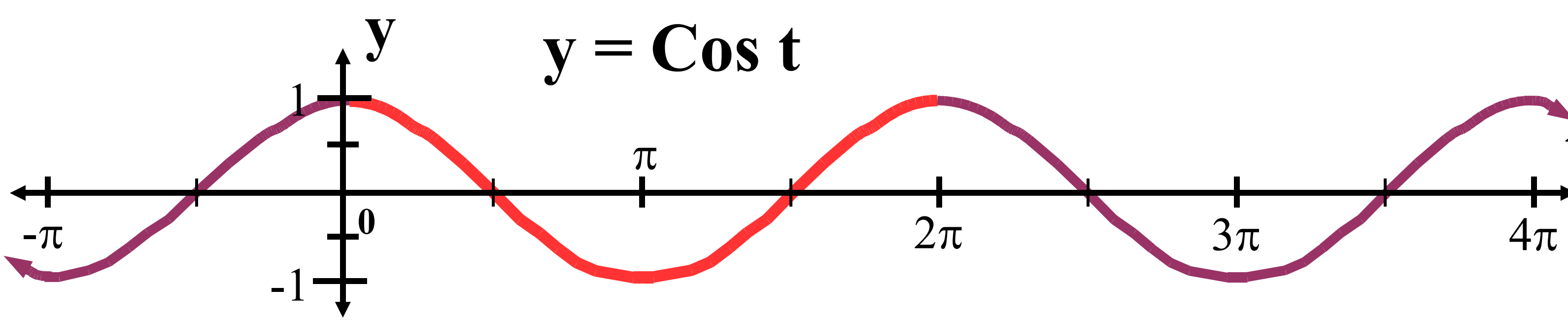

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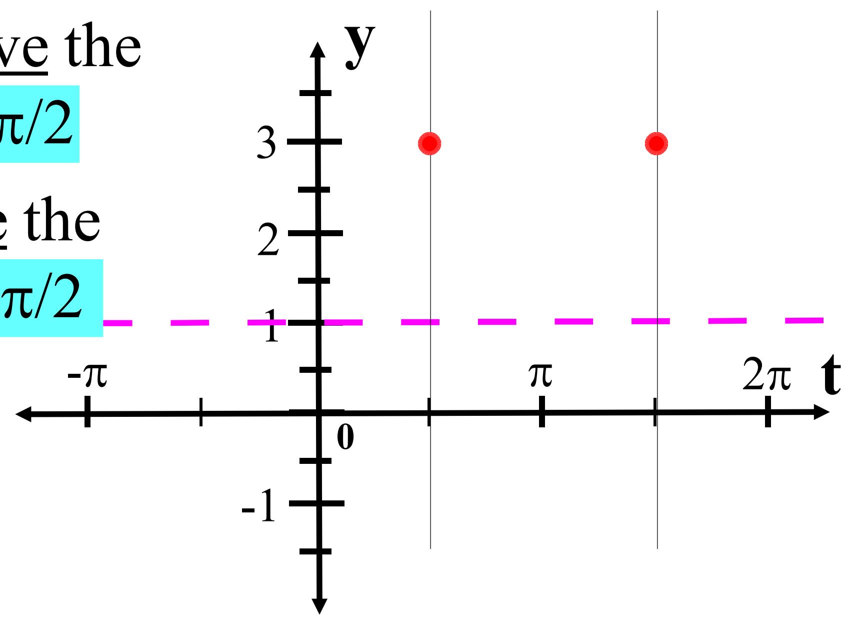

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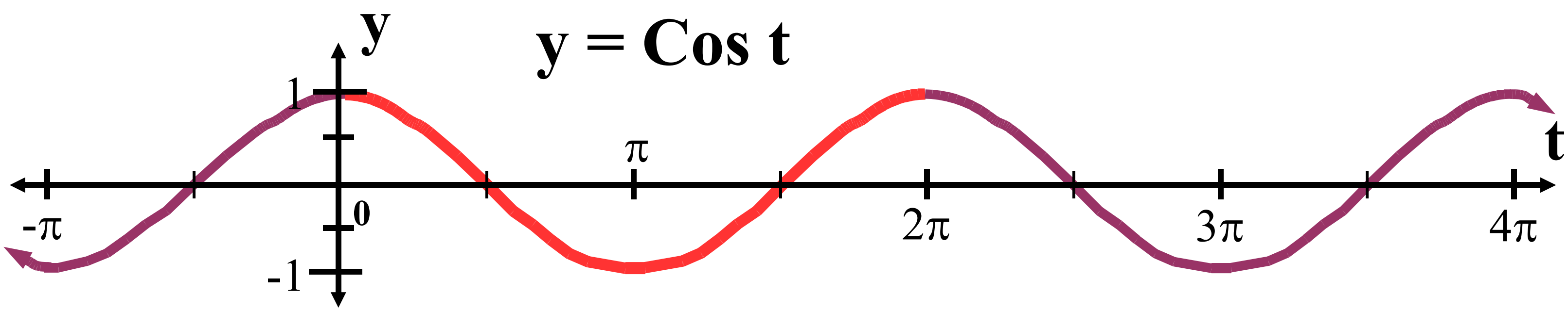

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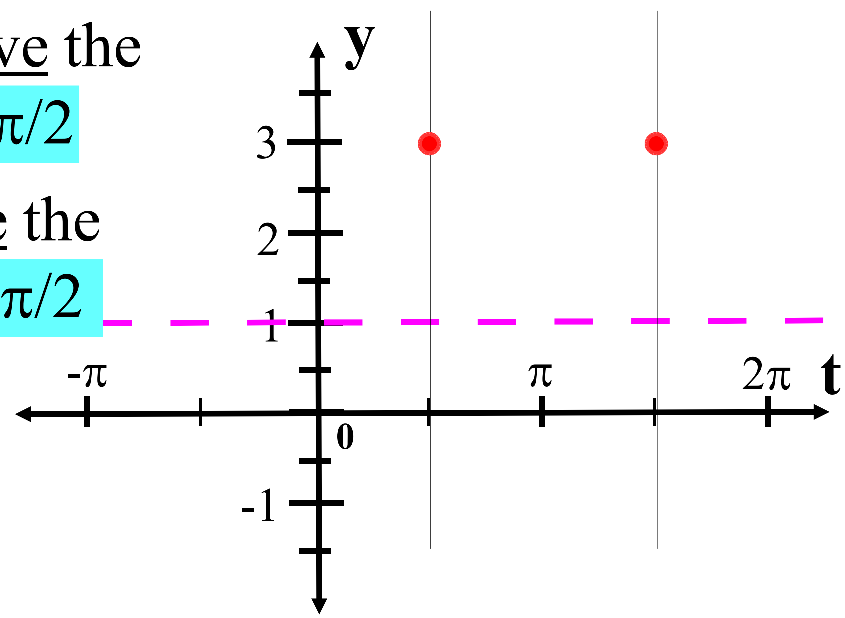
The 'basic cycle' is 2 units below the mid-line when 2t – π = π.



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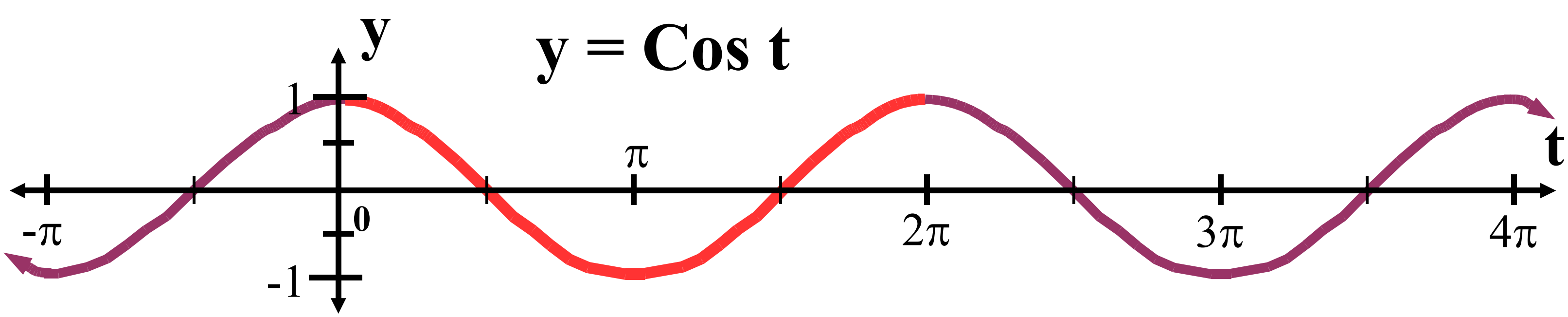

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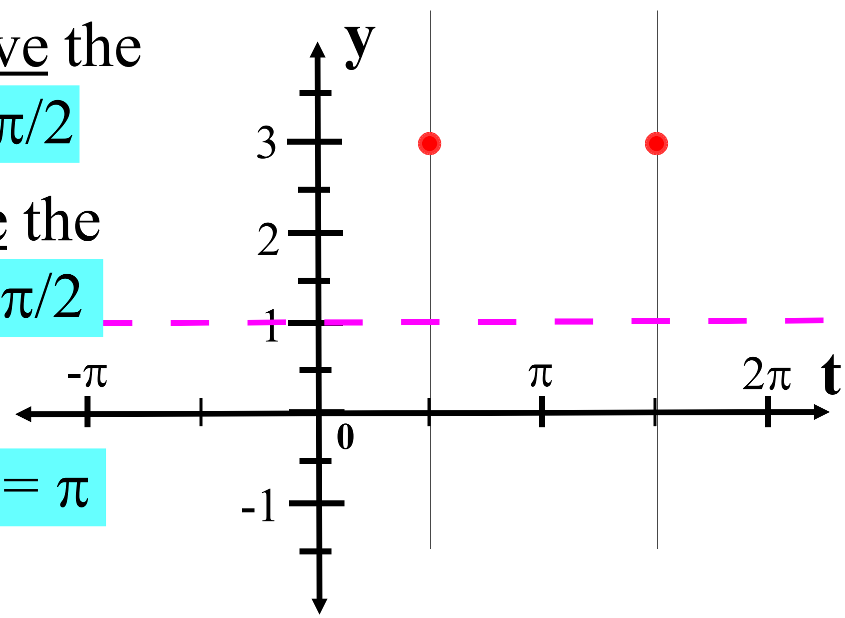
The 'basic cycle' is 2 units below the mid-line when 2t – π = π. → t = π



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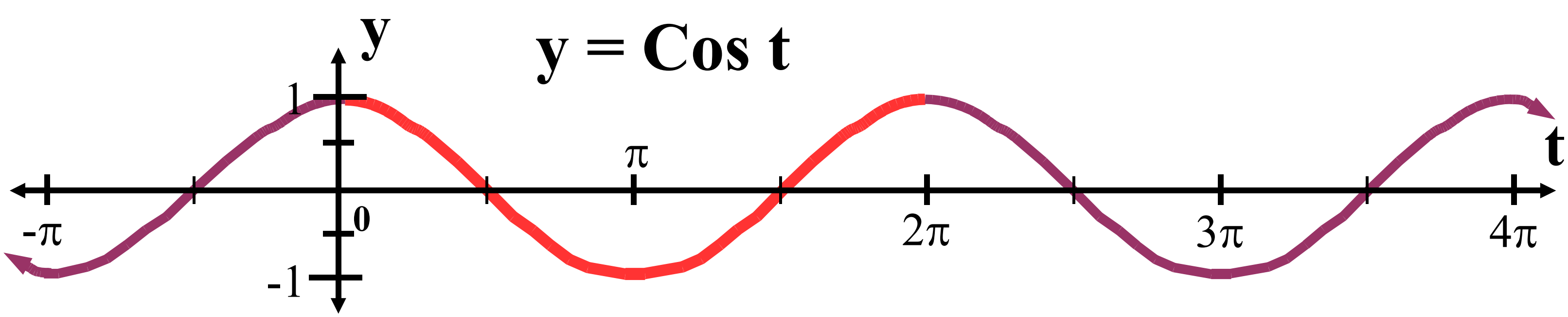


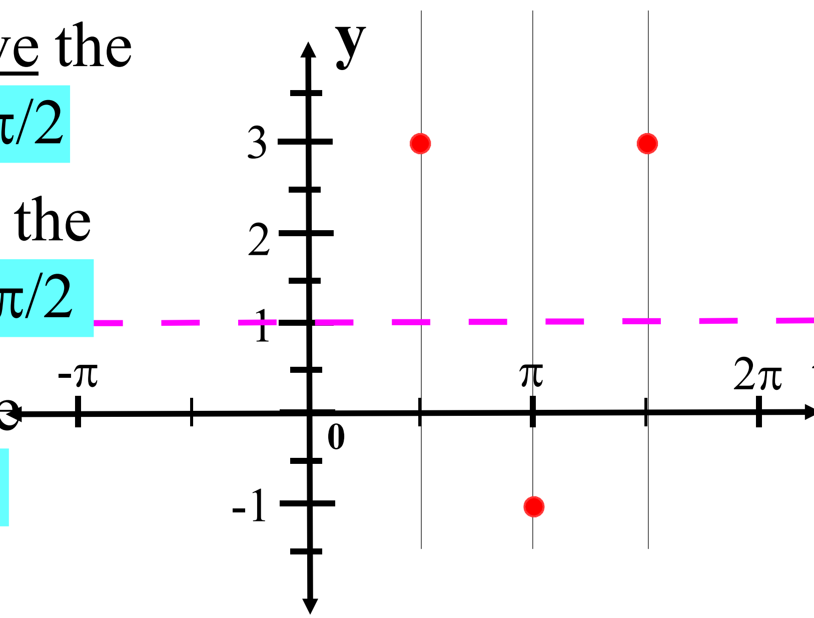
Consider the equation $y = \mathbf{2}\text{Cos}(\mathbf{2t - \pi}) + \mathbf{1}$.

Mid-line: $\mathbf{y = 1}$

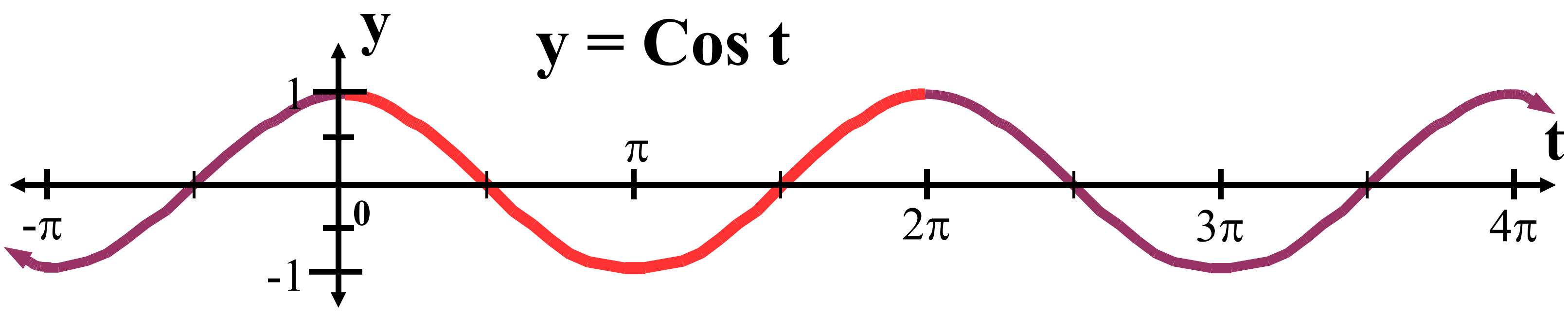
The 'basic cycle' starts 2 units above the mid-line when $2t - \pi = 0$. ⟶ $t = \pi/2$

The 'basic cycle' ends 2 units above the mid-line when $2t - \pi = 2\pi$. ⟶ $t = 3\pi/2$

The 'basic cycle' is 2 units below the mid-line when $2t - \pi = \pi$. ⟶ $t = \pi$



Variations of the Cosine Function



Consider the equation $y = 2\cos(2t - \pi) + 1$.

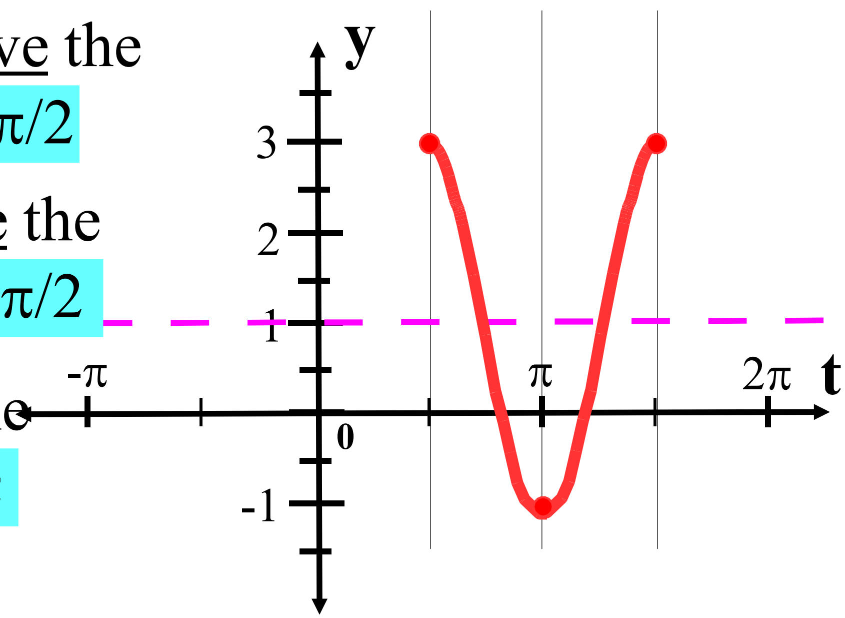
Mid-line: $y = 1$

The 'basic cycle' starts <u>2 units above</u> the mid-line when $2t - \pi = 0$. ⟶ $t = \pi/2$

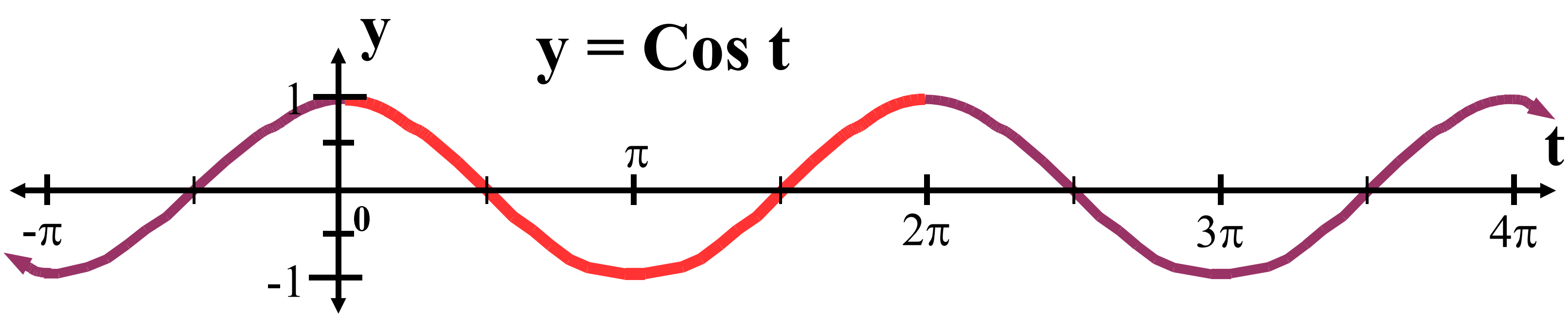
The 'basic cycle' ends <u>2 units above</u> the mid-line when $2t - \pi = 2\pi$. ⟶ $t = 3\pi/2$

The 'basic cycle' is <u>2 units below</u> the mid-line when $2t - \pi = \pi$. ⟶ $t = \pi$

Here is the '**basic cycle**'.



Variations of the Cosine Function



Consider the equation $y = \mathbf{2}\mathrm{\mathbf{Cos}}(\mathbf{2t - \pi}) + \mathbf{1}$.

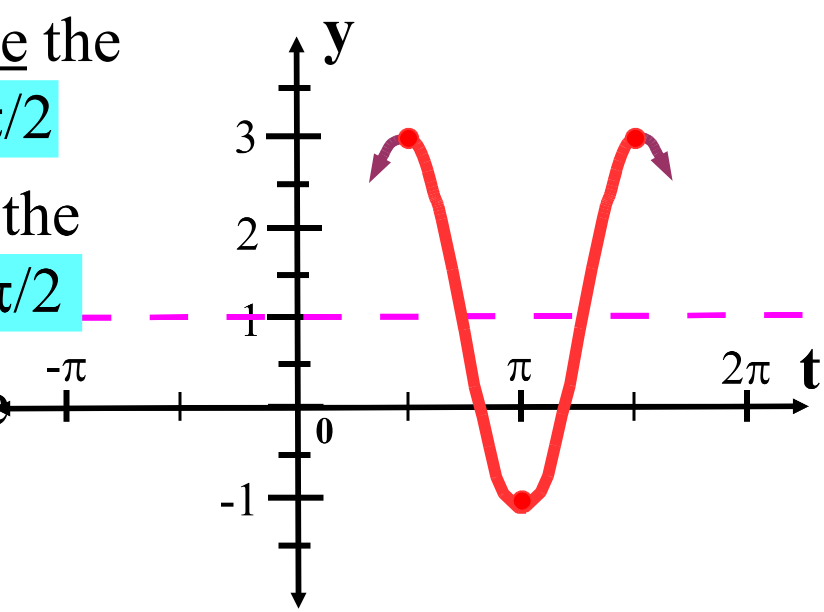
Mid-line: $\mathbf{y = 1}$

The 'basic cycle' starts 2 units above the mid-line when $2t - \pi = 0$. ⟶ $t = \pi/2$

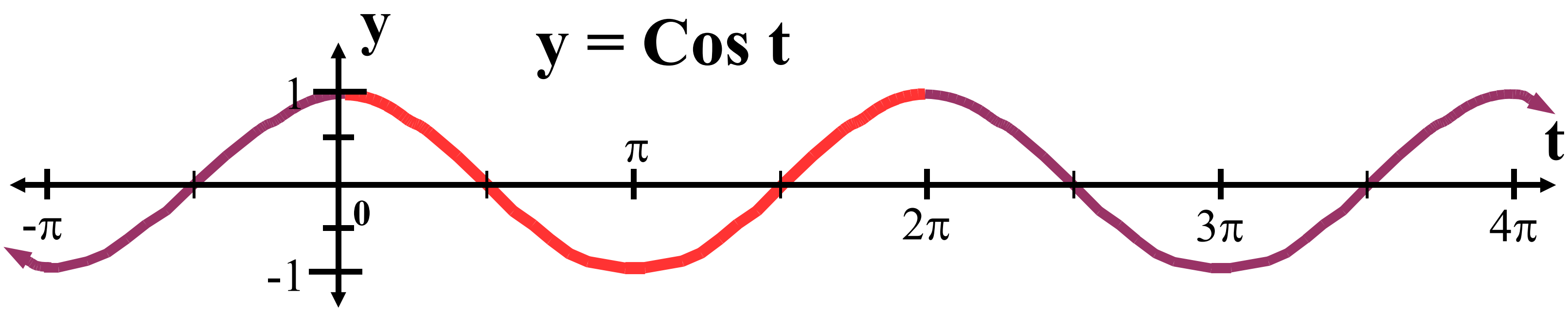
The 'basic cycle' ends 2 units above the mid-line when $2t - \pi = 2\pi$. ⟶ $t = 3\pi/2$

The 'basic cycle' is 2 units below the mid-line when $2t - \pi = \pi$. ⟶ $t = \pi$

Here is the '**basic cycle**'.

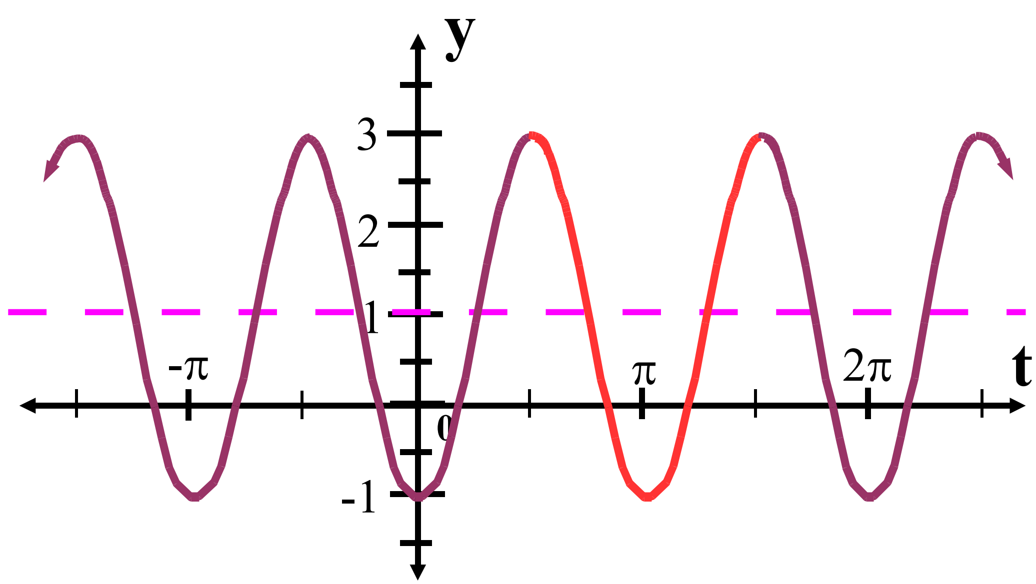


Variations of the Cosine Function

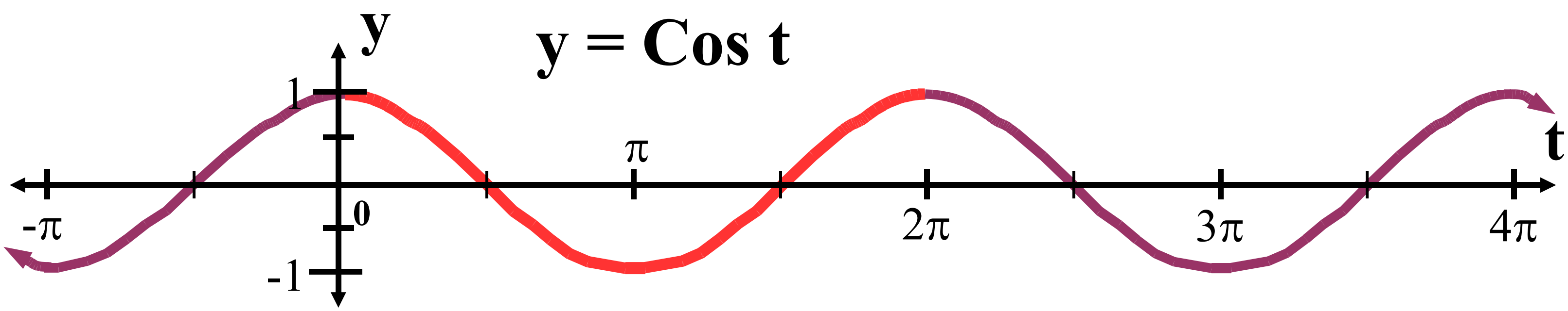


Here is a more complete graph.

$y = 2\text{Cos}(2t - \pi) + 1$

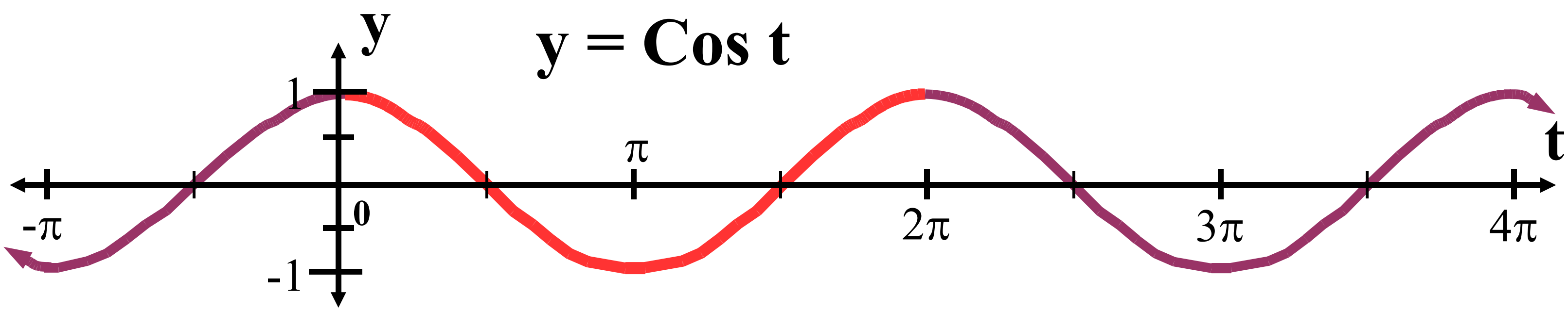


Variations of the Cosine Function



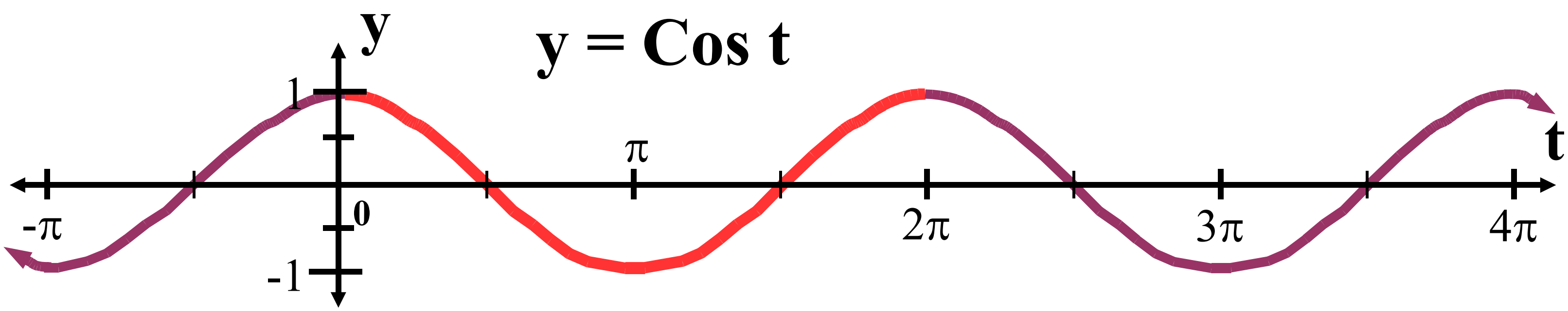
Consider the equation y = **-0.5Cos(t + π/2) – 1**.

Variations of the Cosine Function



Consider the equation y = -0.5**Cos(t** + π/2**) – 1**.

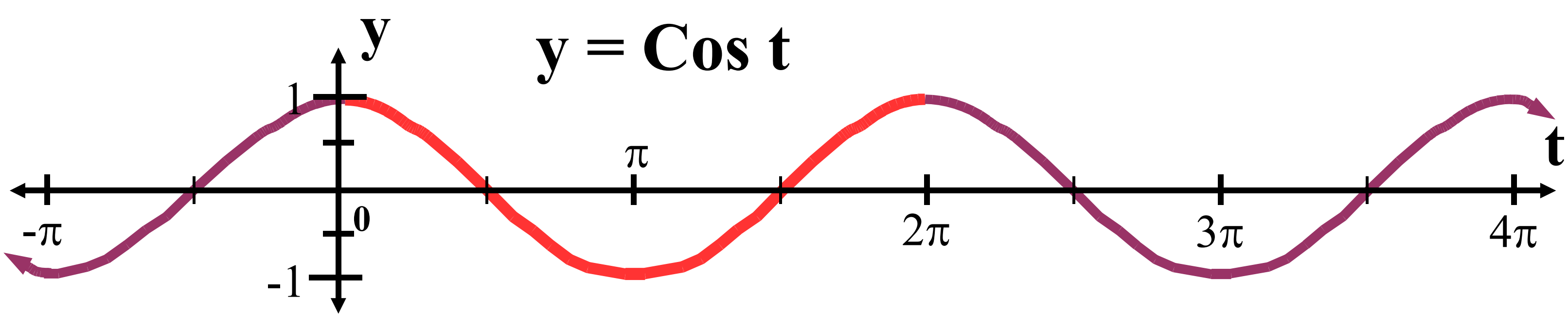
Variations of the Cosine Function



Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line:

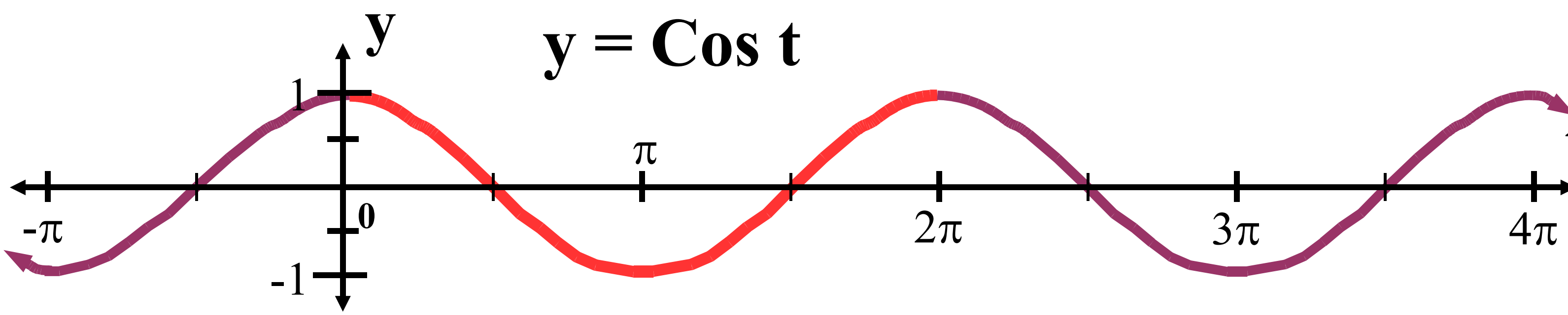
Variations of the Cosine Function



Consider the equation y = -0.5**Cos(t + π/2) – 1**.

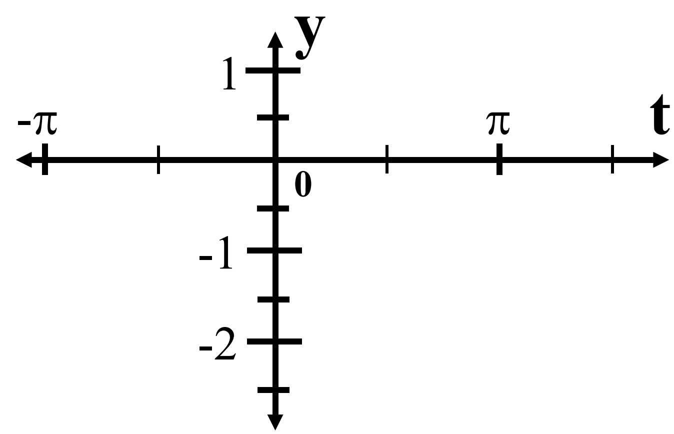
Mid-line: **y = -1**

Variations of the Cosine Function

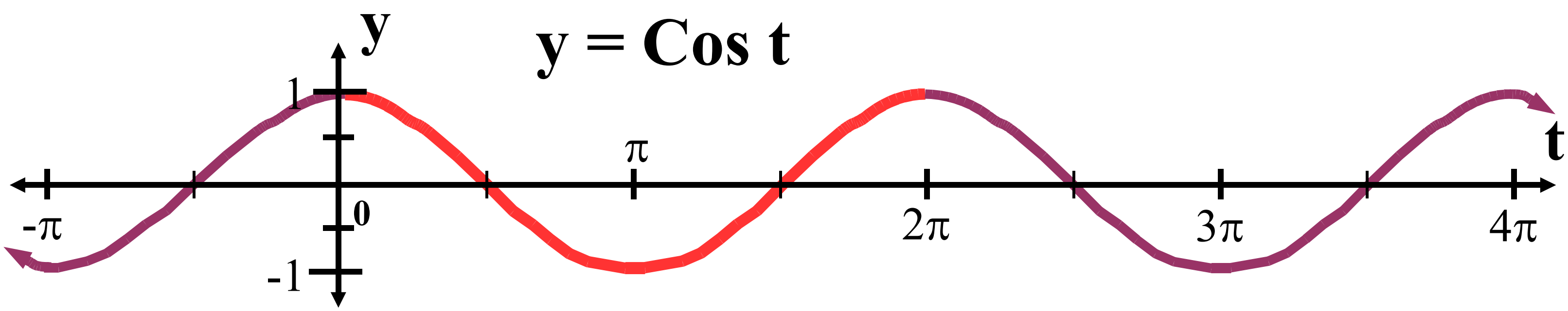


Consider the equation y = -0.5**Cos(t + π/2)** **– 1**.

Mid-line: **y = -1**



Variations of the Cosine Function



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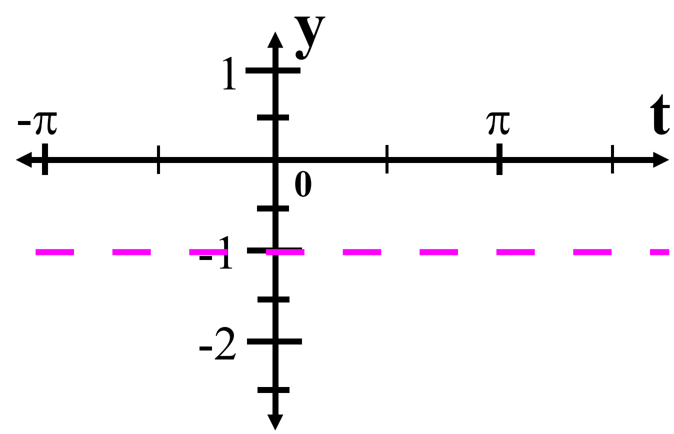

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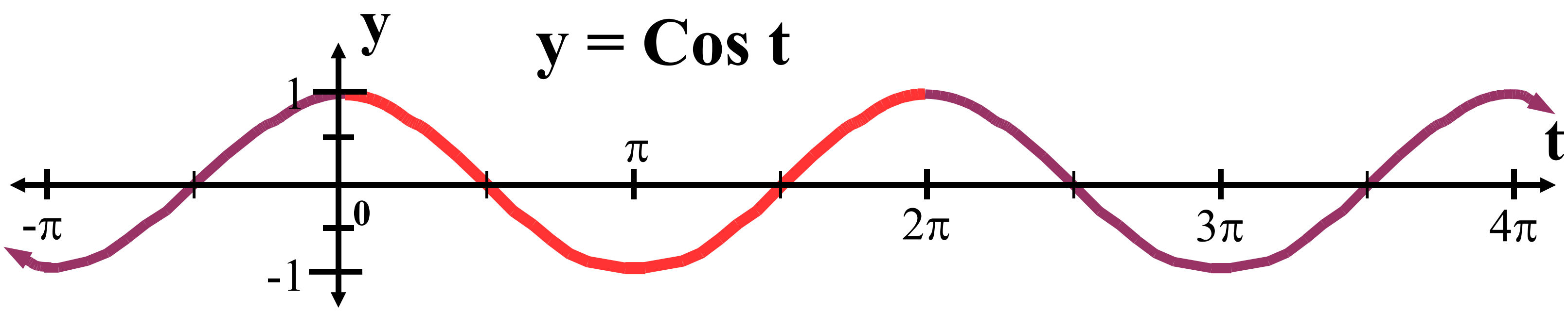

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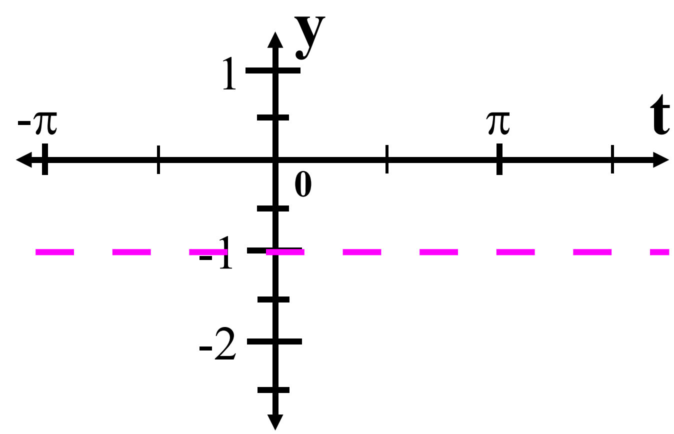

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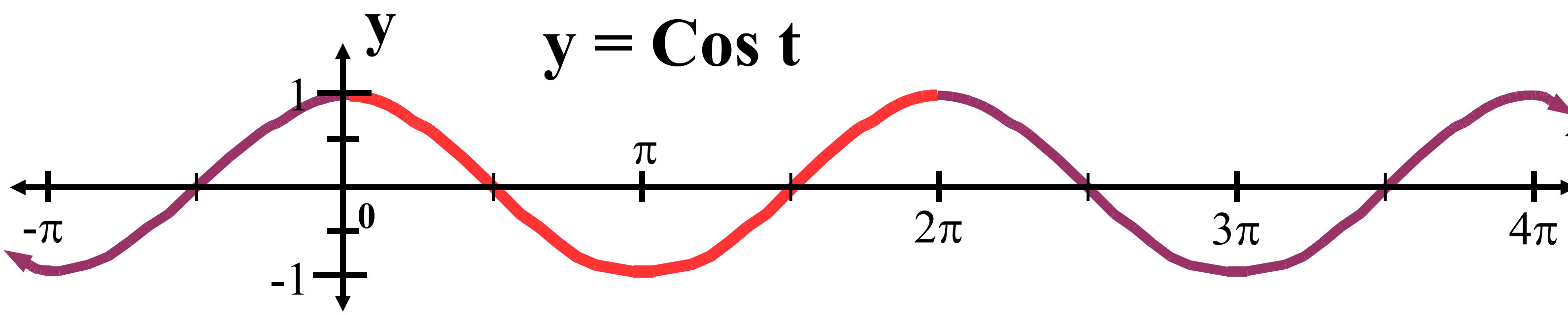
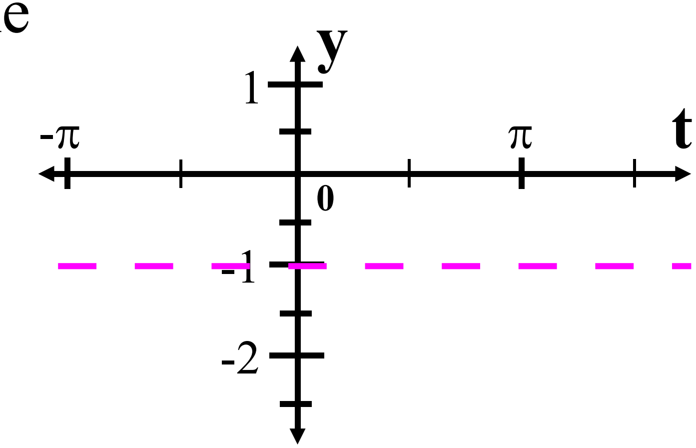


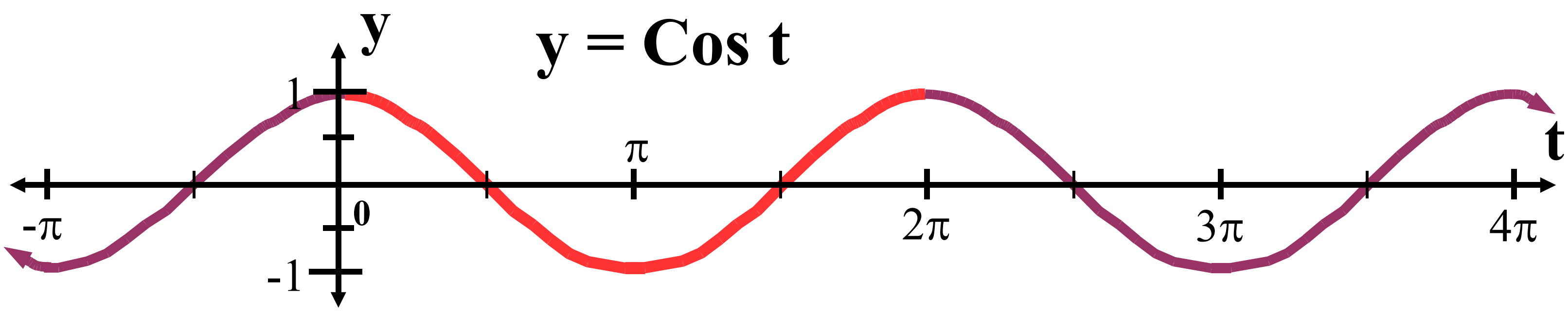
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line



Variations of the Cosine Function



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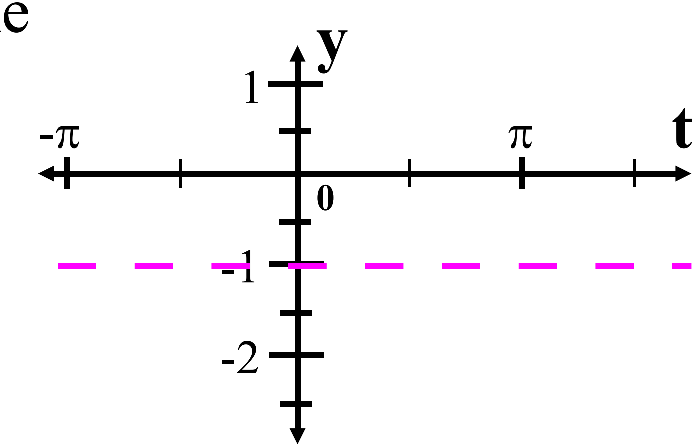
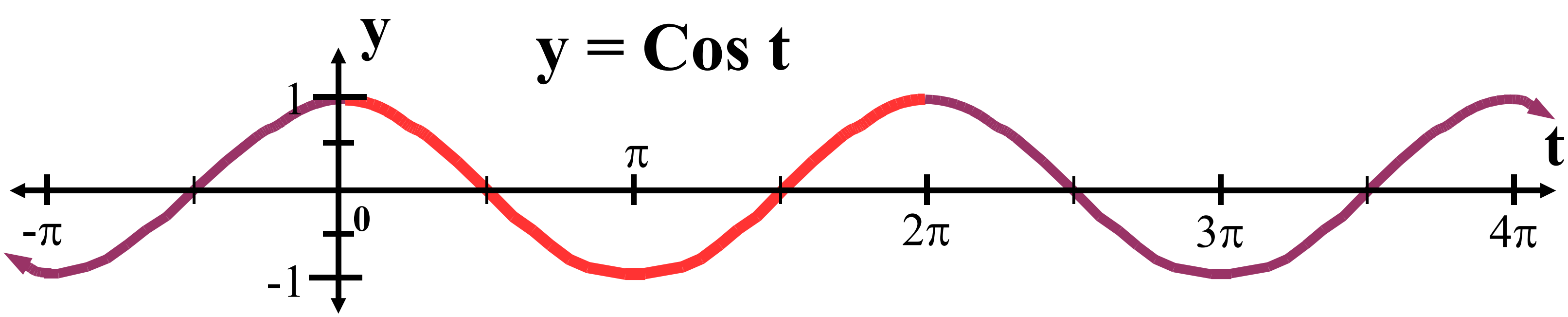


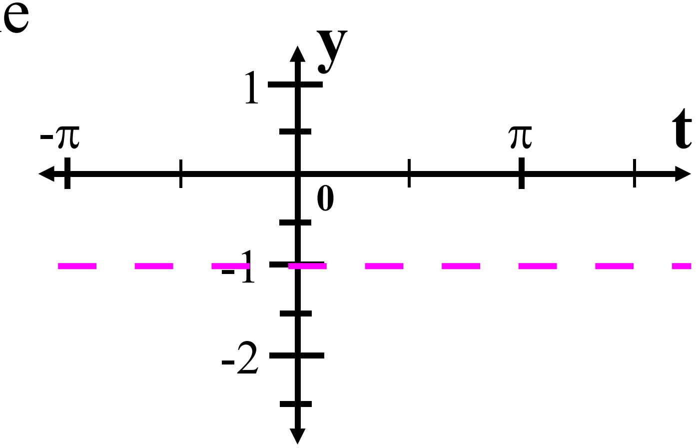
Variations of the Cosine Function



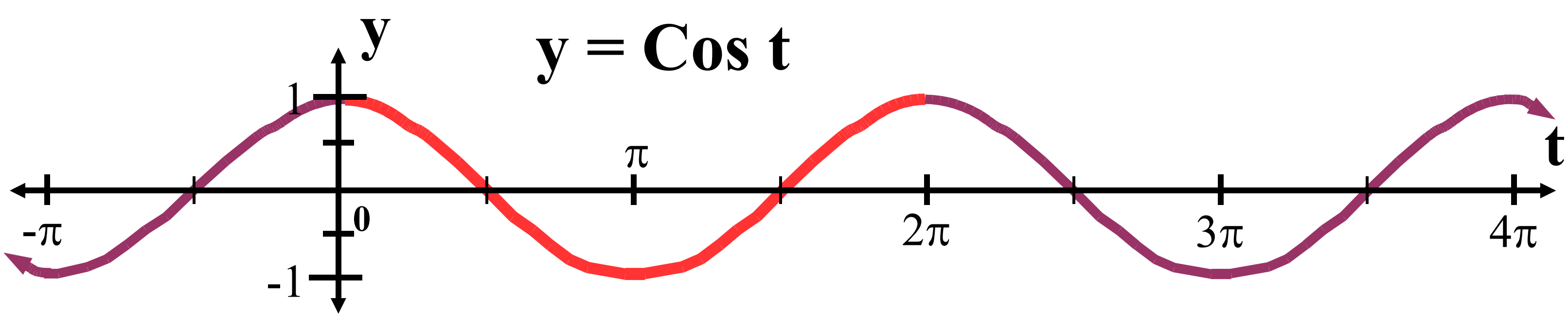
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0.



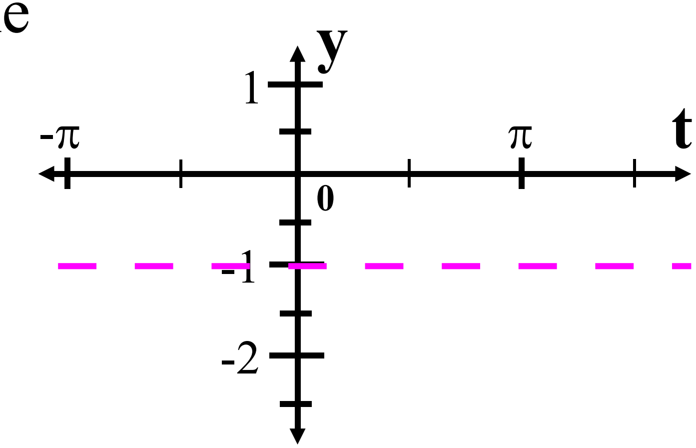
Variations of the Cosine Function



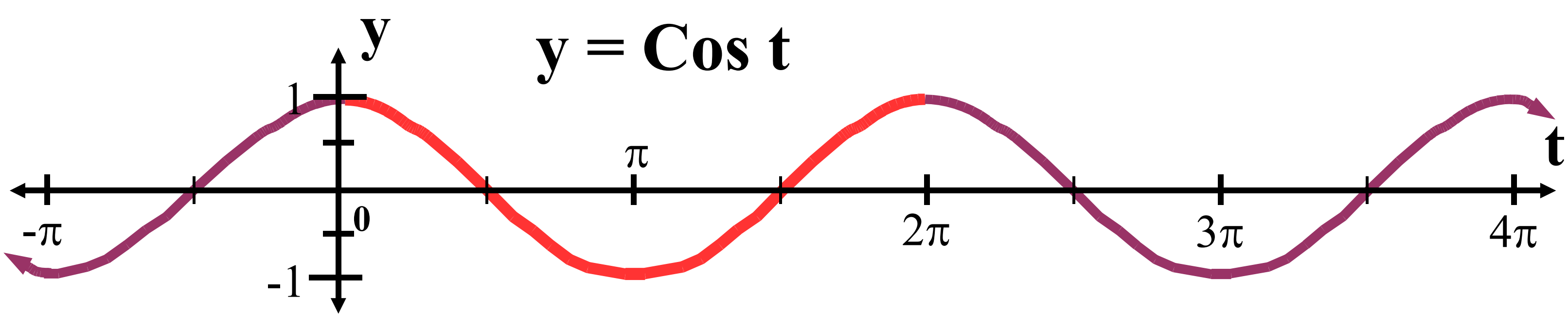
Consider the equation y = $-0.5\mathbf{Cos(t + \pi/2) - 1}$.

Mid-line: **y = -1**

The 'basic cycle' starts 0.5 units below the mid-line when t + π/2 = 0. → t = -π/2



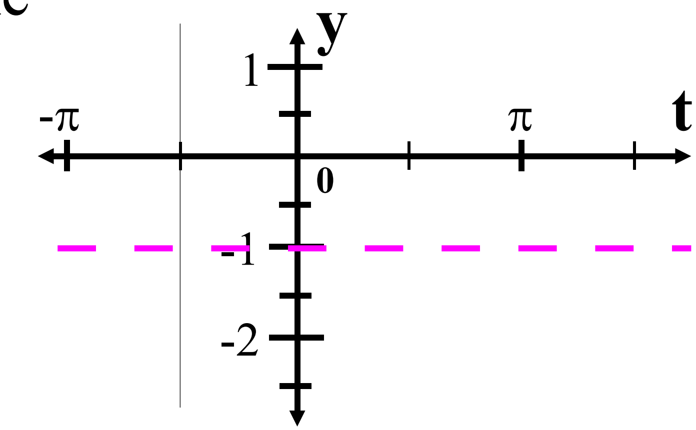
Variations of the Cosine Function



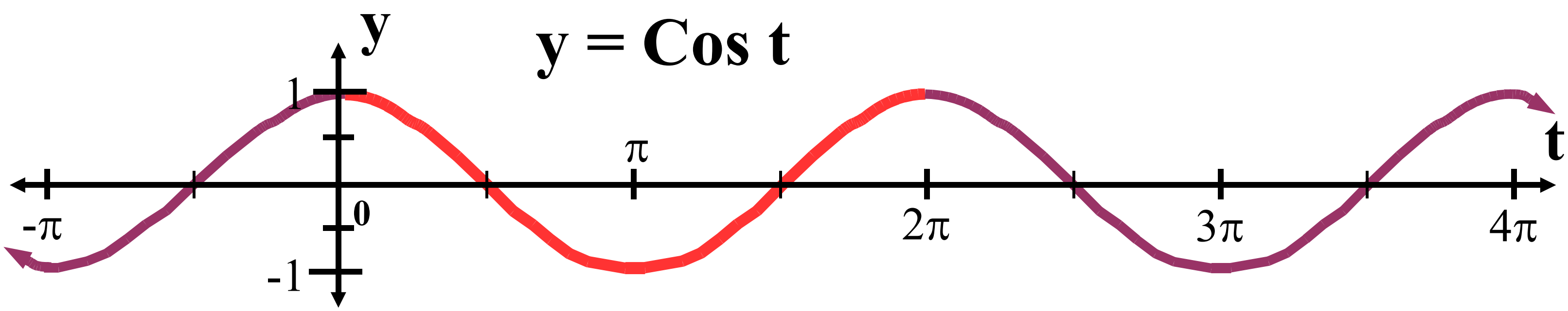
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. ⟶ t = -π/2



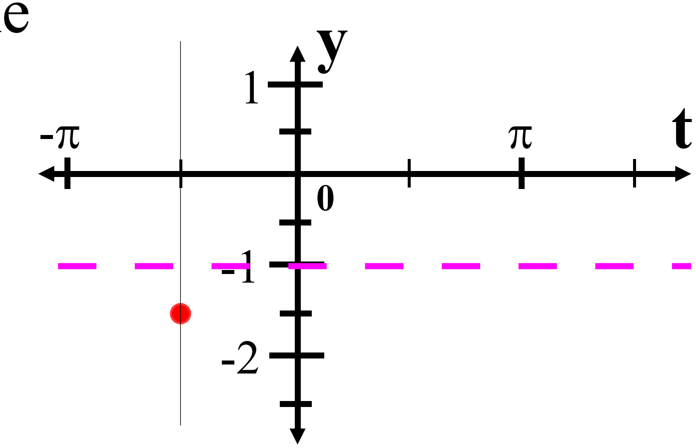
Variations of the Cosine Function



Consider the equation $y = -0.5\mathbf{Cos}(\mathbf{t} + \pi/2) - \mathbf{1}$.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when $t + \pi/2 = 0$. → $t = -\pi/2$



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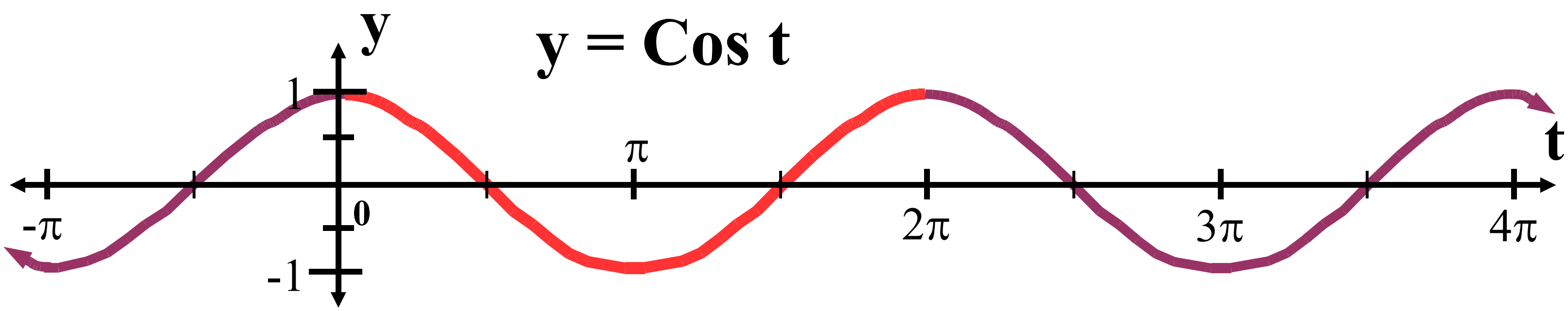
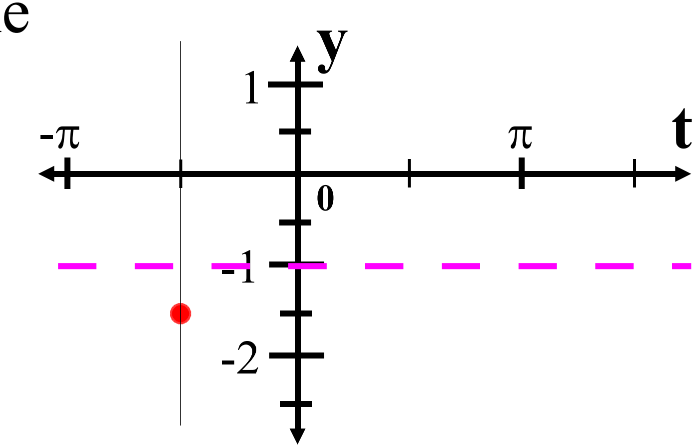


Consider the equation $y = -0.5\mathbf{Cos}(\mathbf{t} + \pi/2) - \mathbf{1}$.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when $t + \pi/2 = 0$. → $t = -\pi/2$

The 'basic cycle' ends <u>0.5 units below</u> the mid-line



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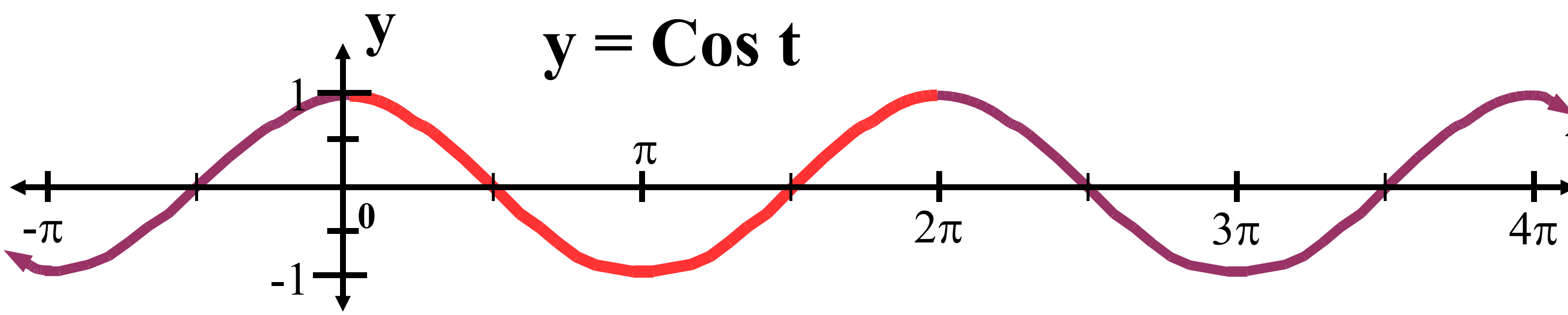
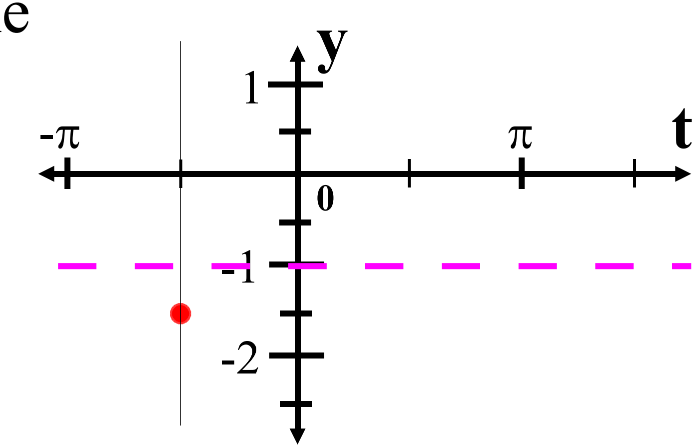


Consider the equation y = -0.5**Cos(t + π/2) – 1**.

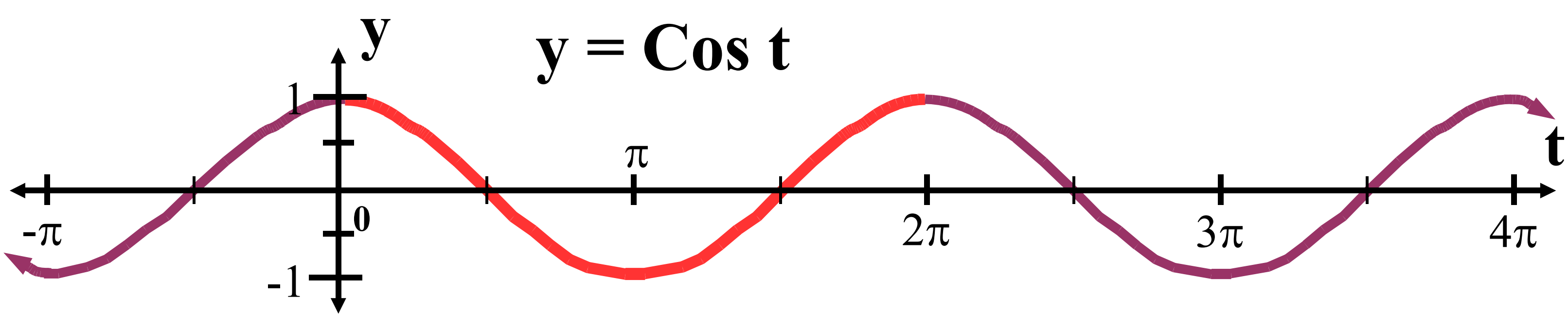
Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when t + π/2 = 2π.



Variations of the Cosine Function

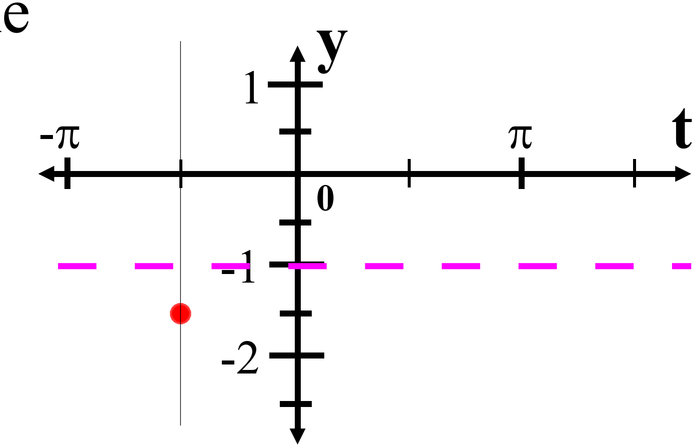


Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when t + π/2 = 2π. → t = 3π/2



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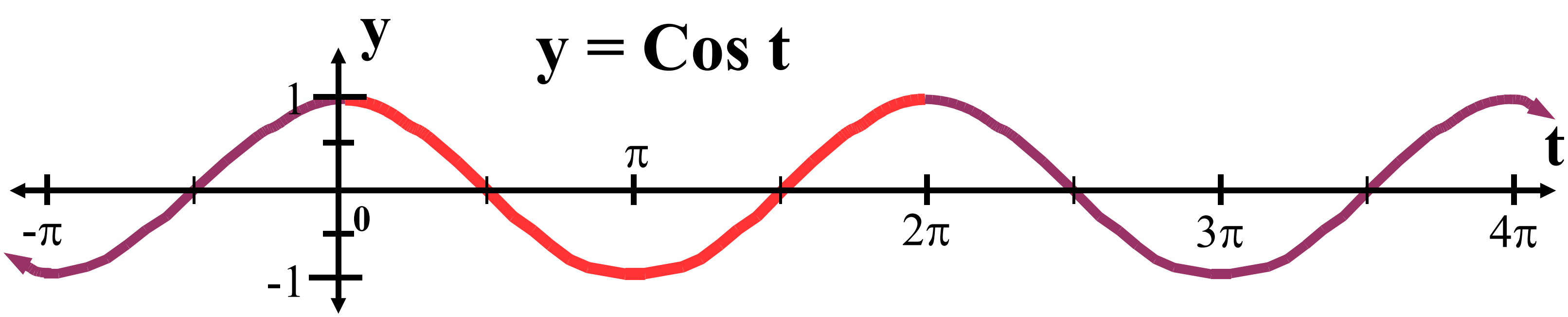
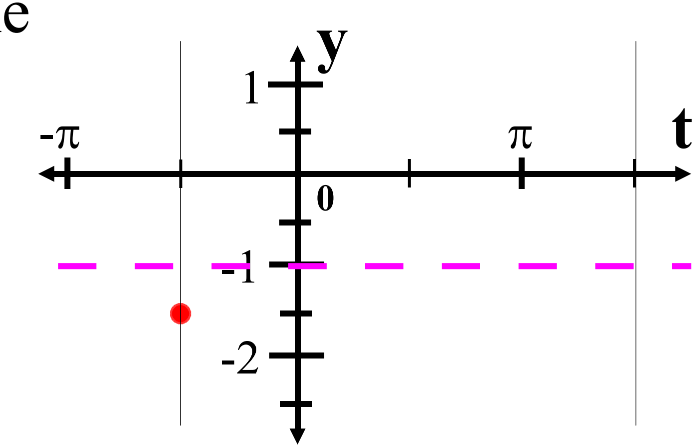


Consider the equation $y = -0.5\mathbf{Cos(t + \pi/2) - 1}$.

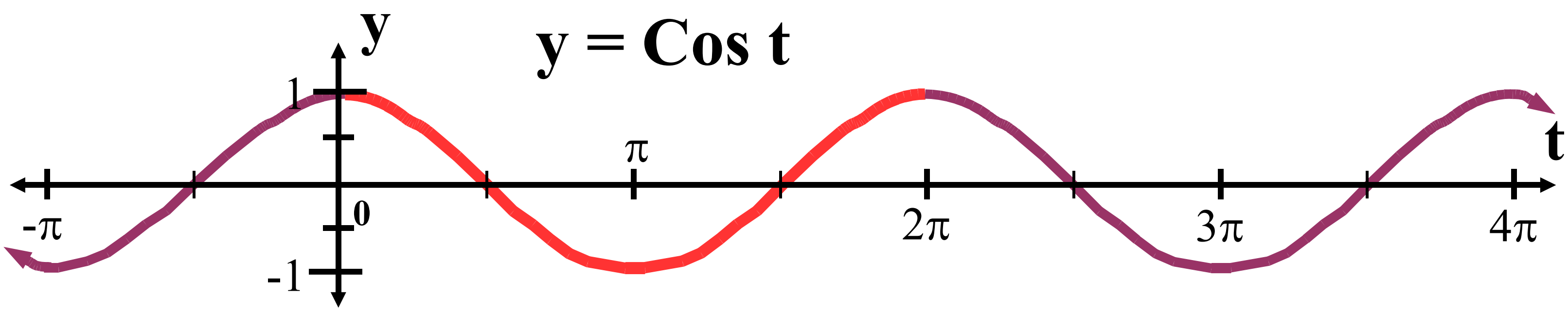
Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when $t + \pi/2 = 0$. → $t = -\pi/2$

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when $t + \pi/2 = 2\pi$. → $t = 3\pi/2$



Variations of the Cosine Function

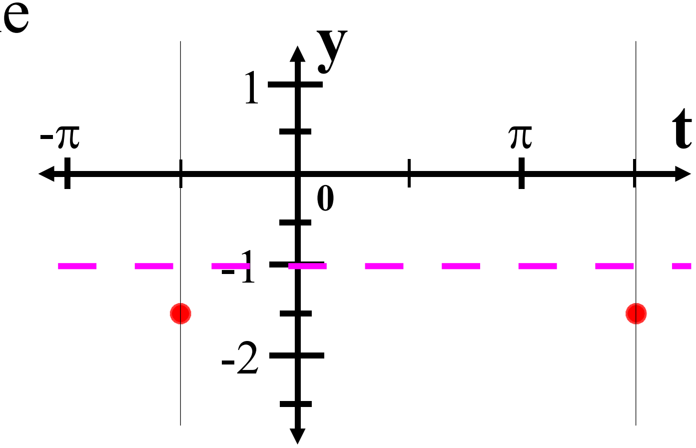


Consider the equation y = **-0.5Cos(t + π/2) – 1**.

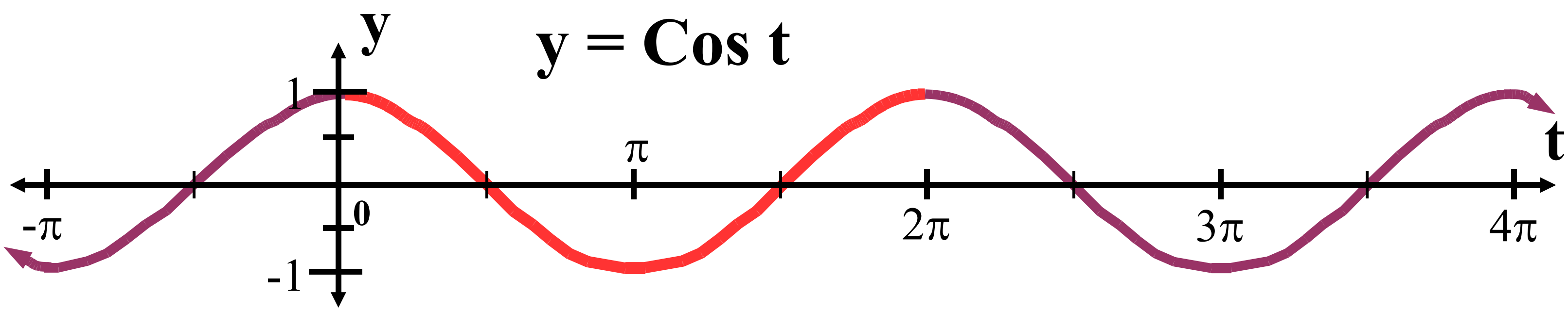
Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when t + π/2 = 2π. → t = 3π/2



Variations of the Cosine Function



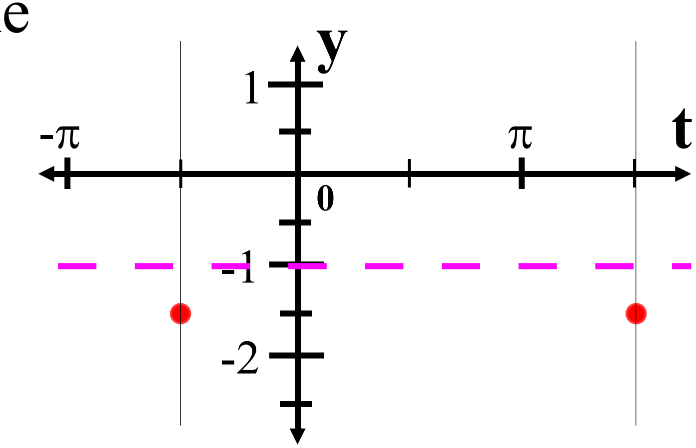
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts 0.5 units below the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends 0.5 units below the mid-line when t + π/2 = 2π. → t = 3π/2

The 'basic cycle' is 0.5 units above the mid-line



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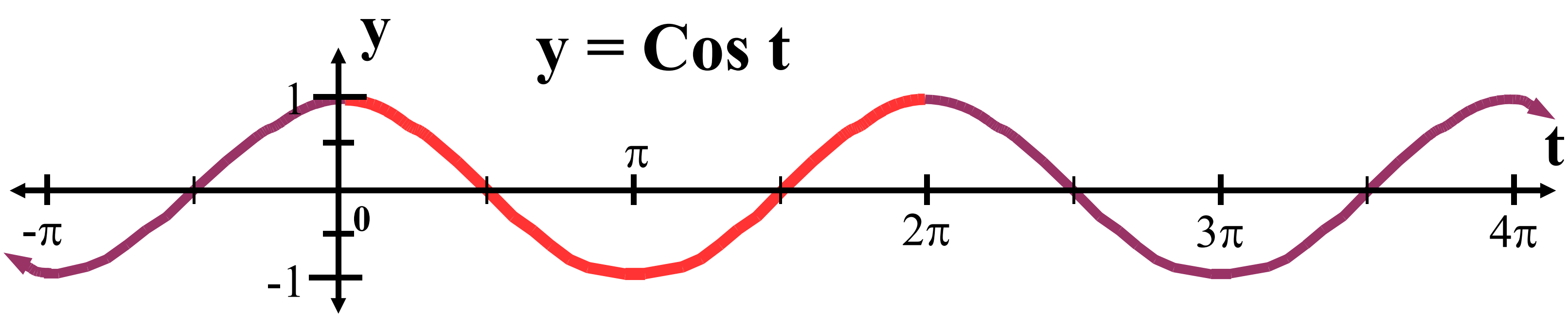


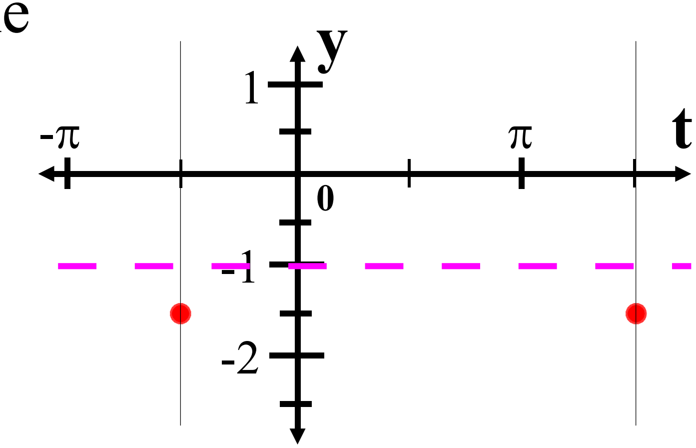
Consider the equation $y = -0.5\mathbf{Cos(t + \pi/2) - 1}$.

Mid-line: $\mathbf{y = -1}$

The 'basic cycle' starts 0.5 units below the mid-line when $t + \pi/2 = 0$. → $t = -\pi/2$

The 'basic cycle' ends 0.5 units below the mid-line when $t + \pi/2 = 2\pi$. → $t = 3\pi/2$

The 'basic cycle' is 0.5 units above the mid-line when $t + \pi/2 = \pi$.



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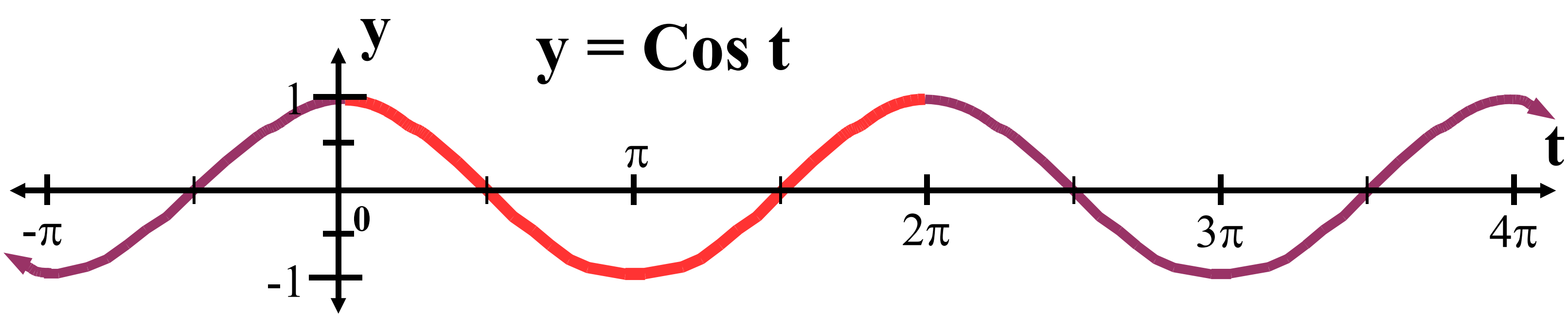


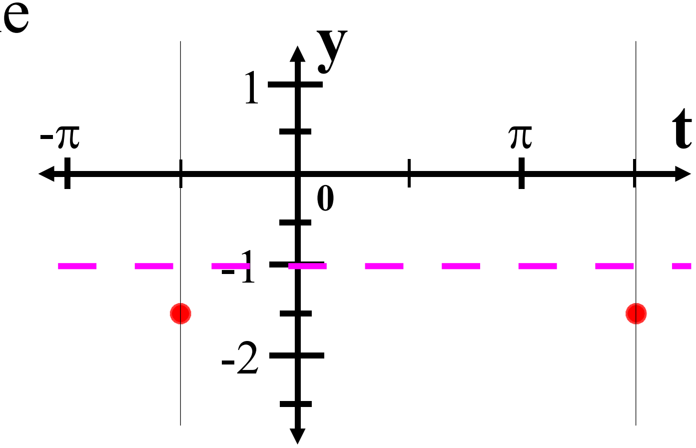
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when t + π/2 = 2π. → t = 3π/2

The 'basic cycle' is <u>0.5 units above</u> the mid-line when t + π/2 = π. → t = π/2



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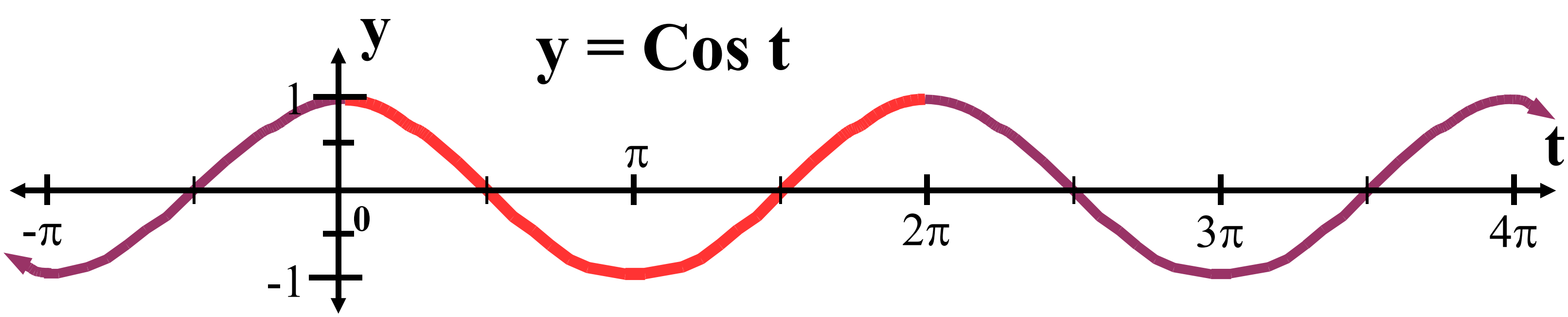

Consider the equation $y = -0.5\mathbf{Cos(t + \pi/2) - 1}$.

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The 'basic cycle' ends 0.5 units below the mid-line when $t + \pi/2 = 2\pi$. → $t = 3\pi/2$

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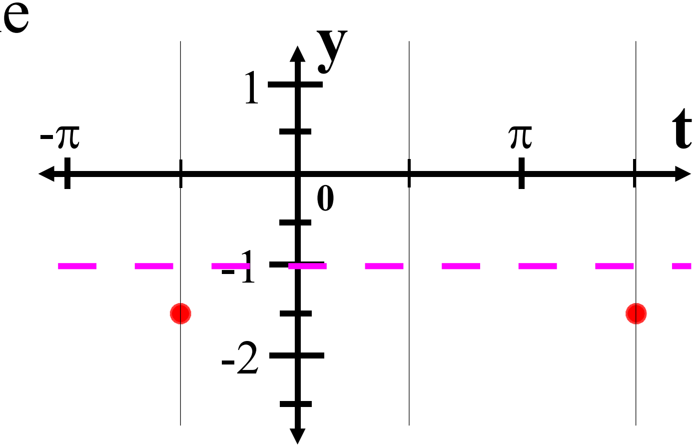

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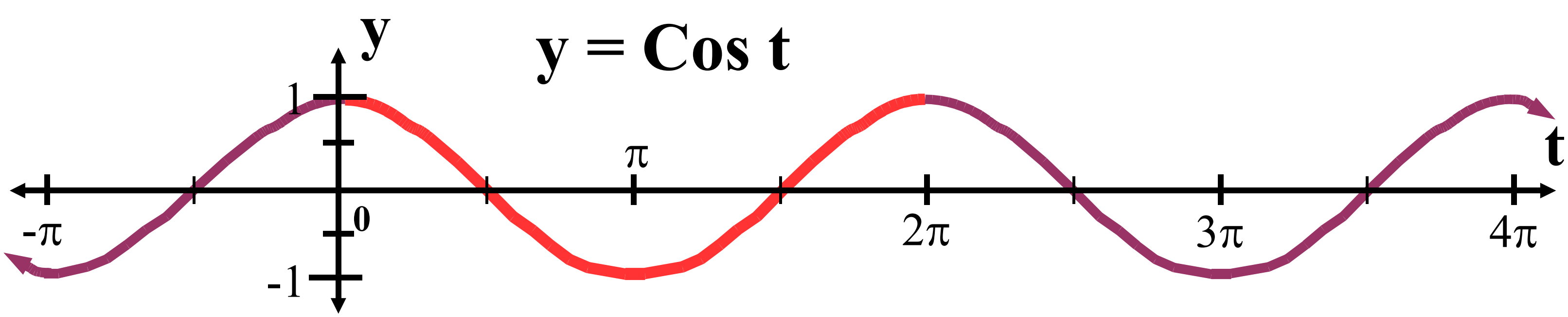


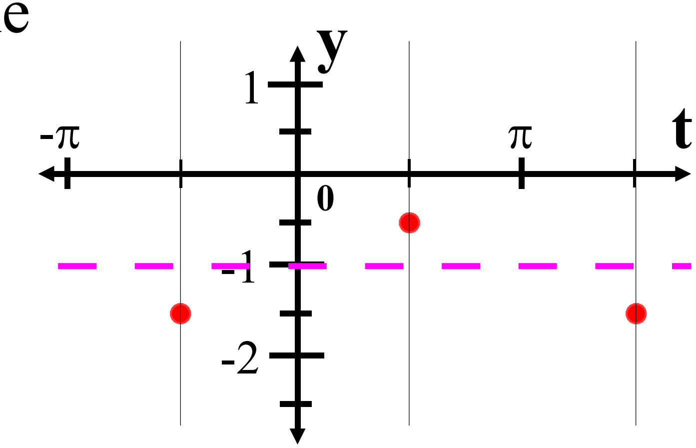
Consider the equation y = **-0.5Cos(t + π/2) – 1**.

Mid-line: **y = -1**

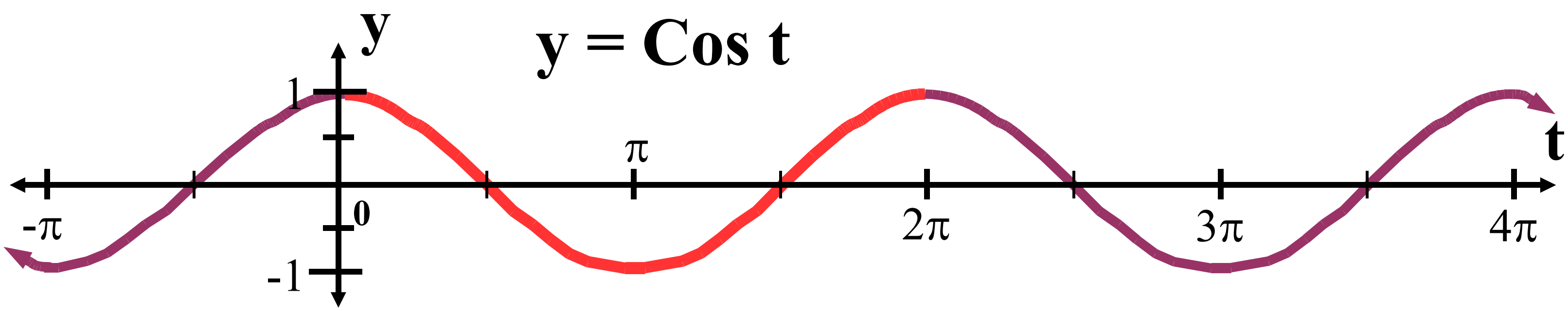
The 'basic cycle' starts <u>0.5 units below</u> the mid-line when t + π/2 = 0. → t = -π/2

The 'basic cycle' ends <u>0.5 units below</u> the mid-line when t + π/2 = 2π. → t = 3π/2

The 'basic cycle' is <u>0.5 units above</u> the mid-line when t + π/2 = π. → t = π/2



Variations of the Cosine Function



Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts 0.5 units below the mid-line when t + π/2 = 0. → t = -π/2

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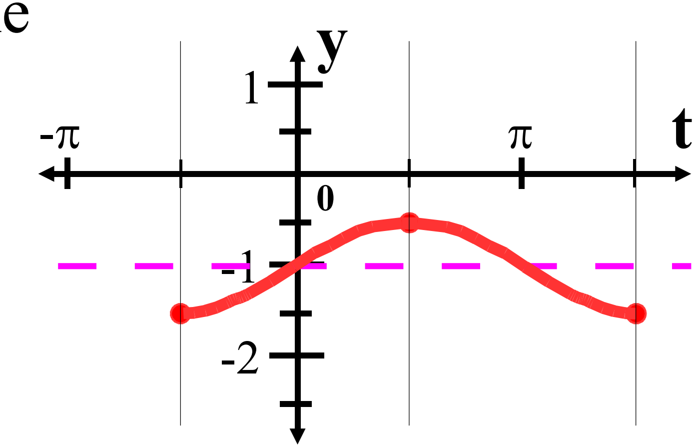
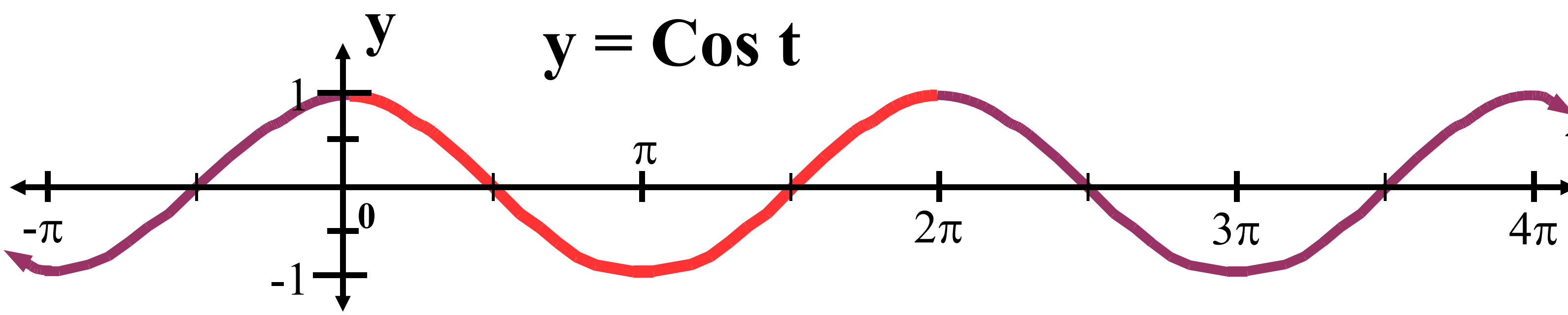
The 'basic cycle' is 0.5 units above the mid-line when t + π/2 = π. → t = π/2

Here is the '**basic cycle**'.



Variations of the Cosine Function



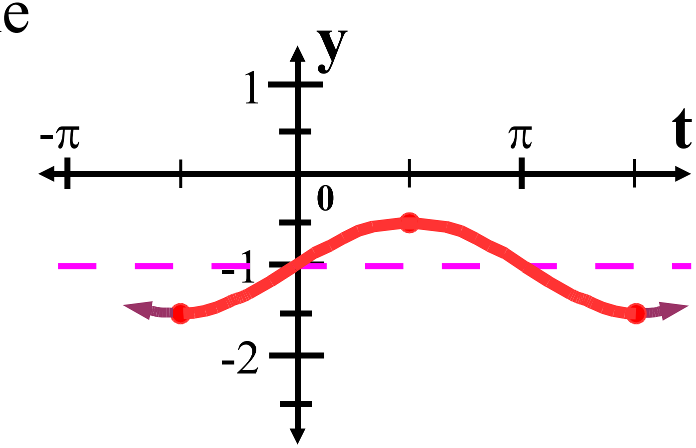
Consider the equation y = -0.5**Cos(t + π/2) – 1**.

Mid-line: **y = -1**

The 'basic cycle' starts 0.5 units below the mid-line when t + π/2 = 0. → t = -π/2

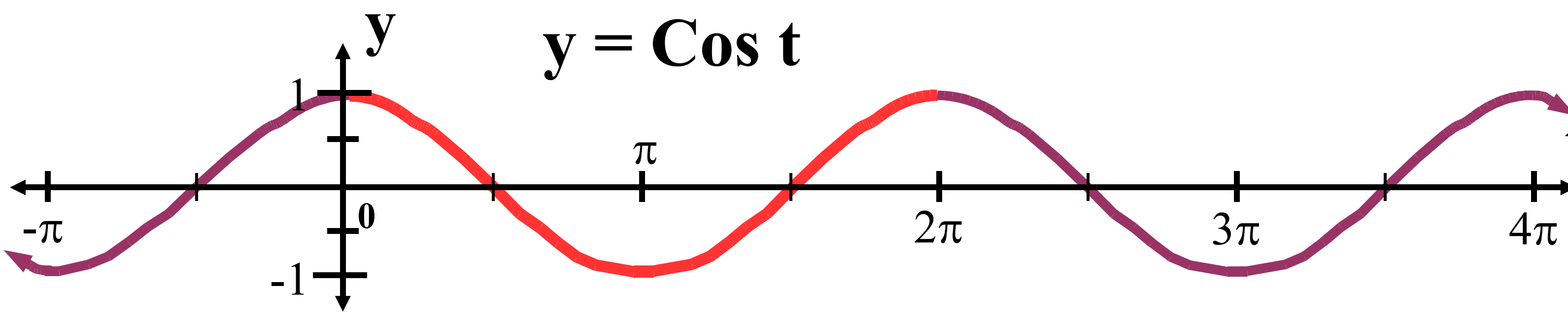
The 'basic cycle' ends 0.5 units below the mid-line when t + π/2 = 2π. → t = 3π/2

The 'basic cycle' is 0.5 units above the mid-line when t + π/2 = π. → t = π/2



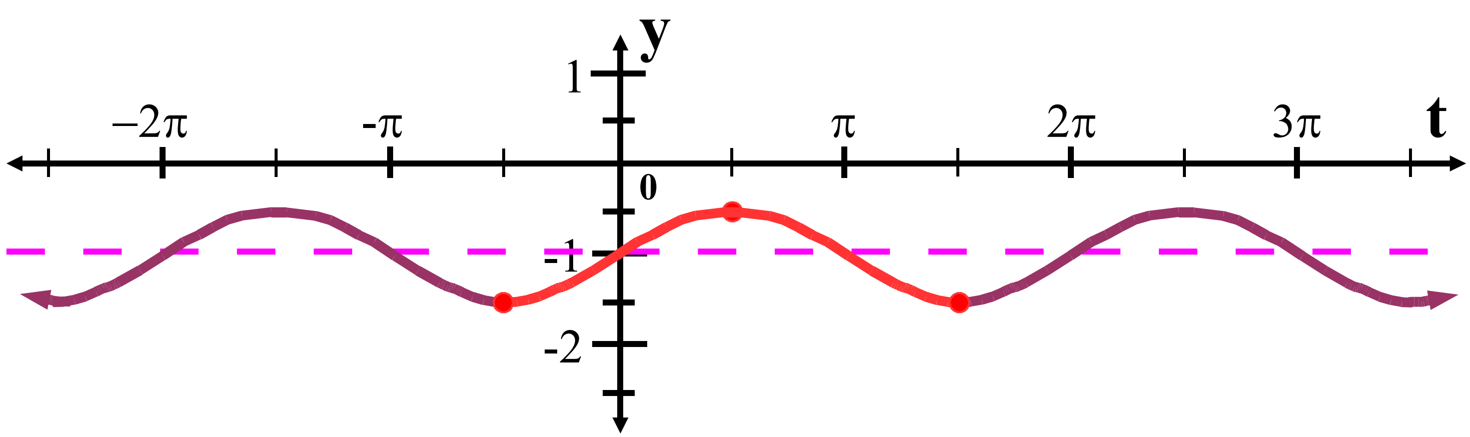
Here is the '**basic cycle**'.

Variations of the Cosine Function

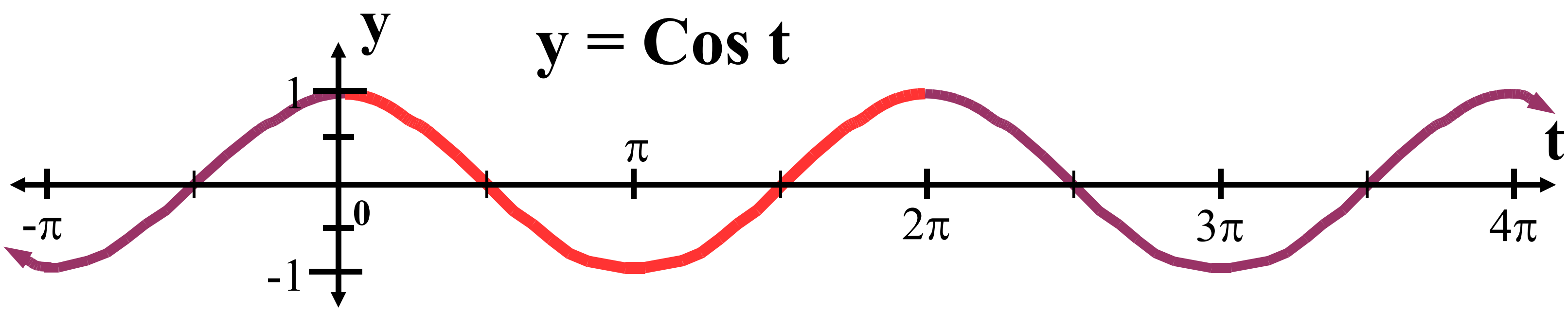


Here is a more complete graph.

y = -0.5Cos(t + π/2) – 1

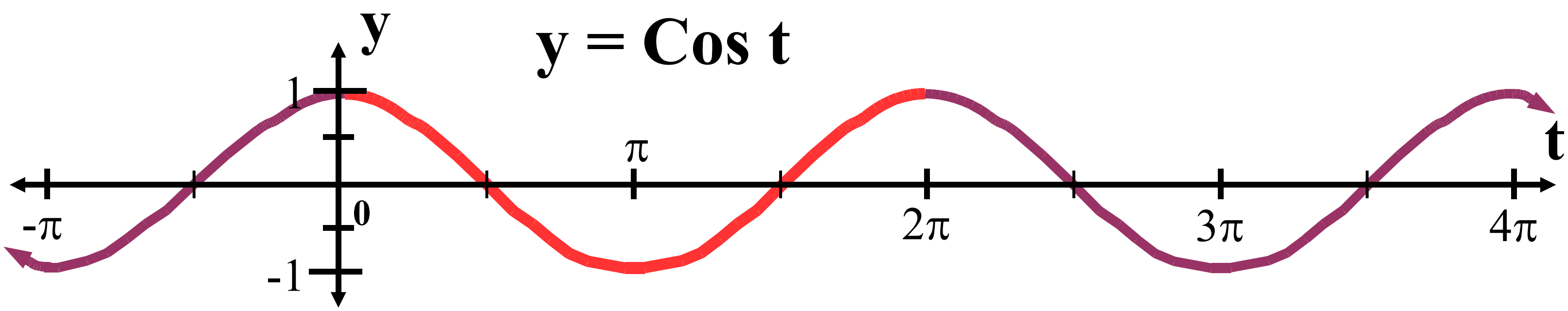


Variations of the Cosine Function



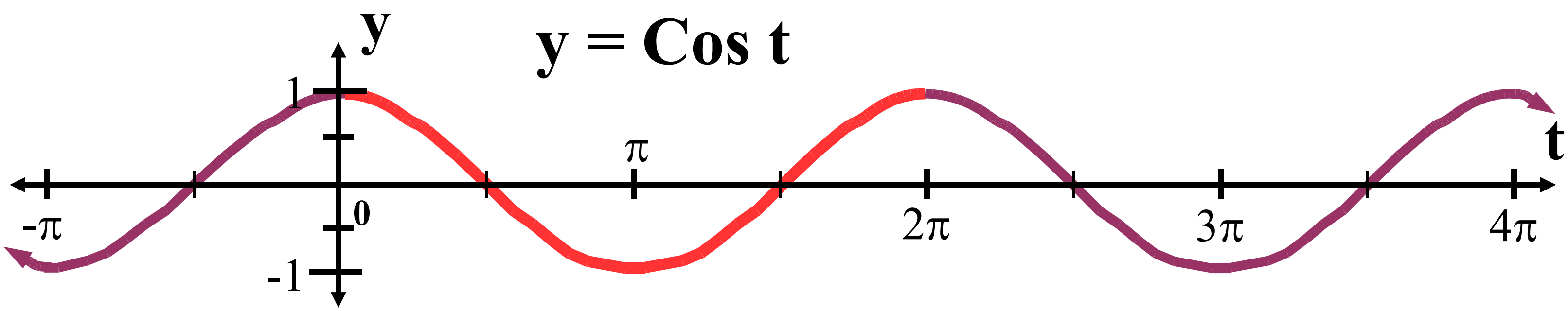
Consider the equation y = **-Cos(0.5t +** π**/3) + 2**.

Variations of the Cosine Function



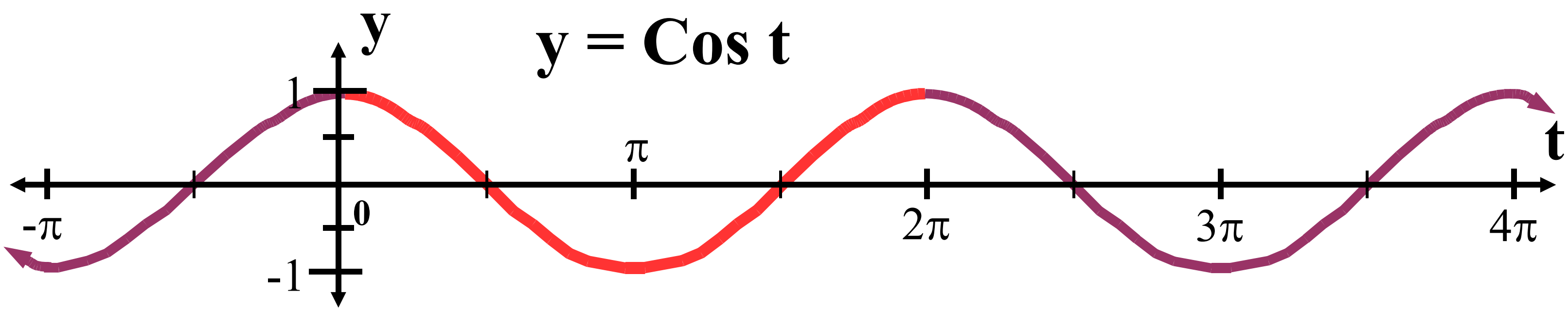
Consider the equation y = **-Cos(0.5t** + **π/3)** + **2**.

Variations of the Cosine Function

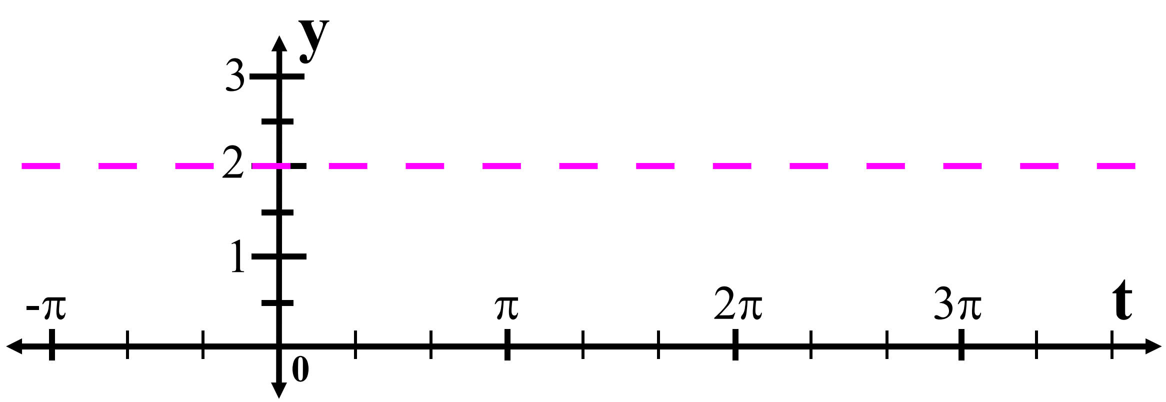


Consider the equation y = **-Cos(0.5t +** π/3**) + 2**. → Mid-line: **y = 2**

Variations of the Cosine Function



Consider the equation y = **-Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**



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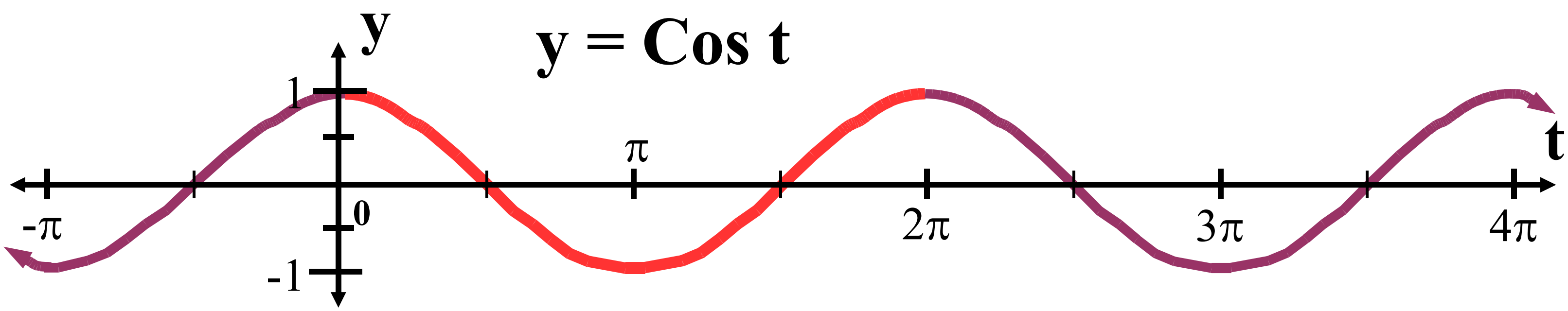
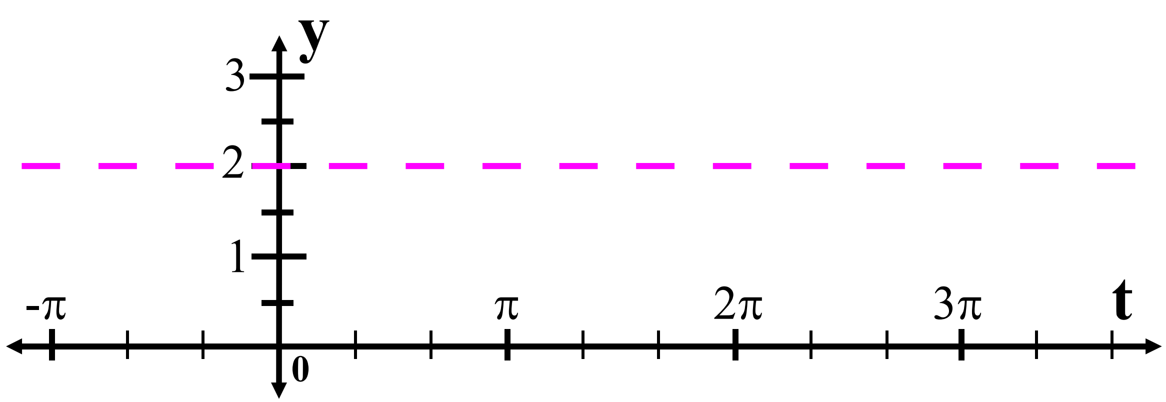
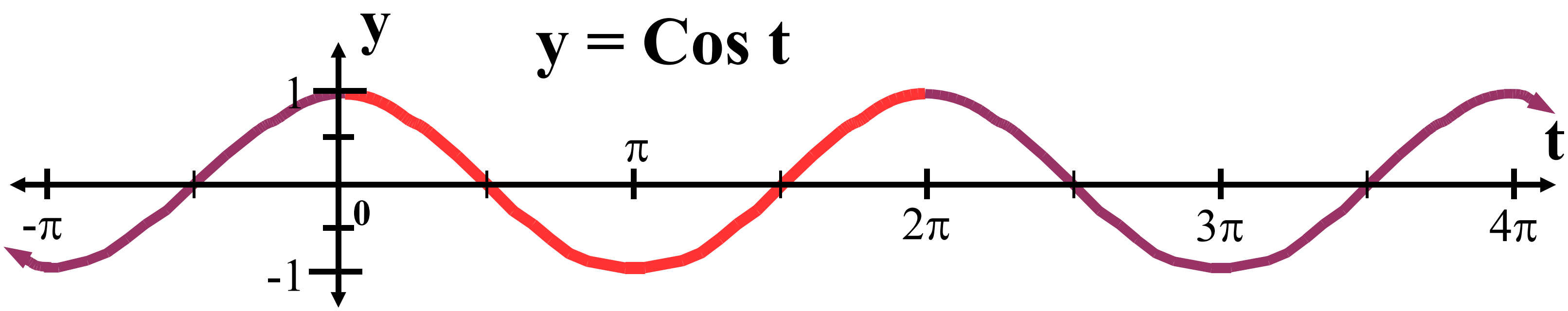


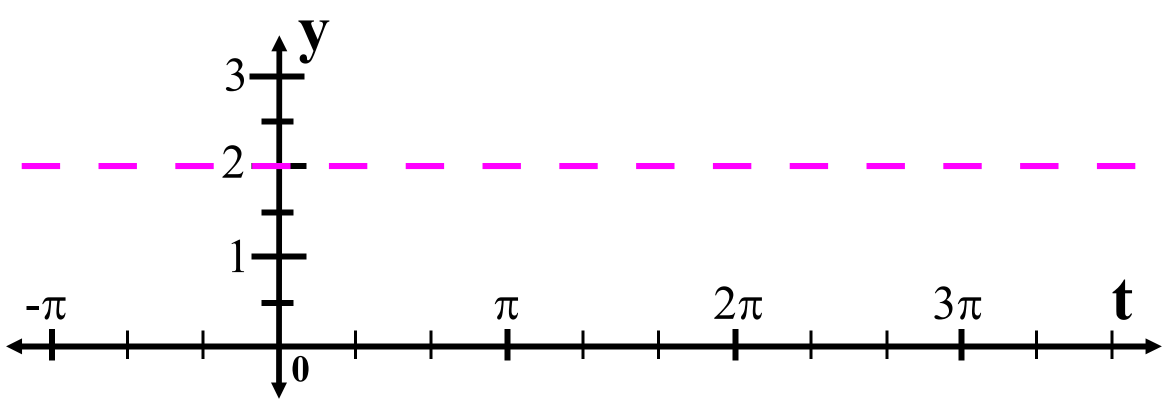
Consider the equation y = **-Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**



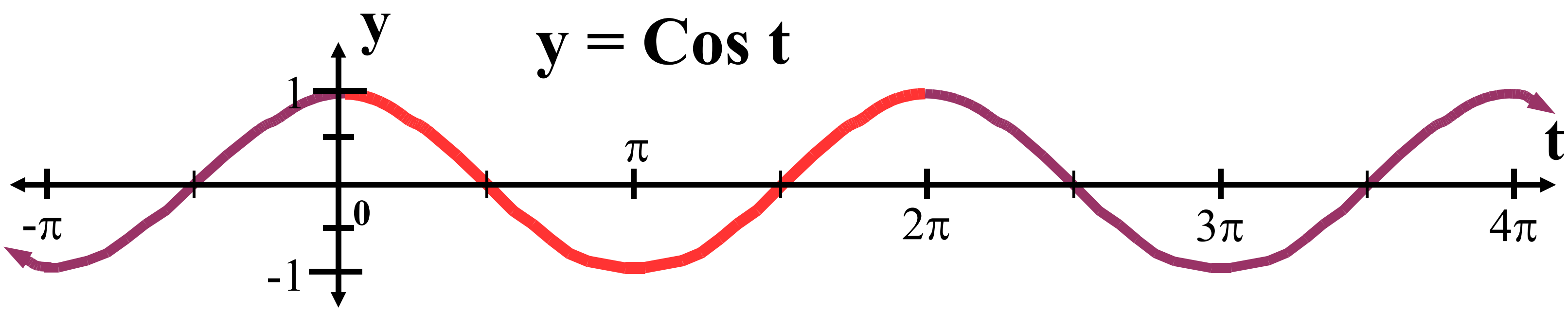
Variations of the Cosine Function



Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

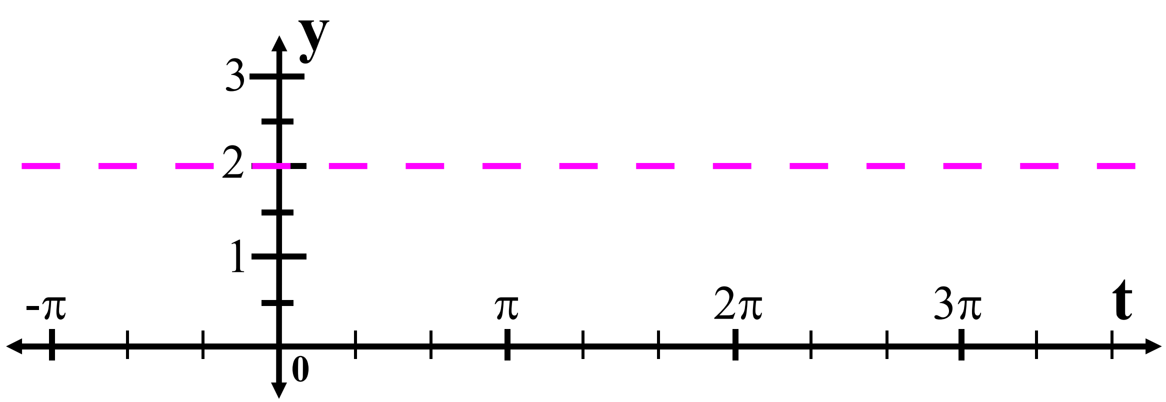


Variations of the Cosine Function

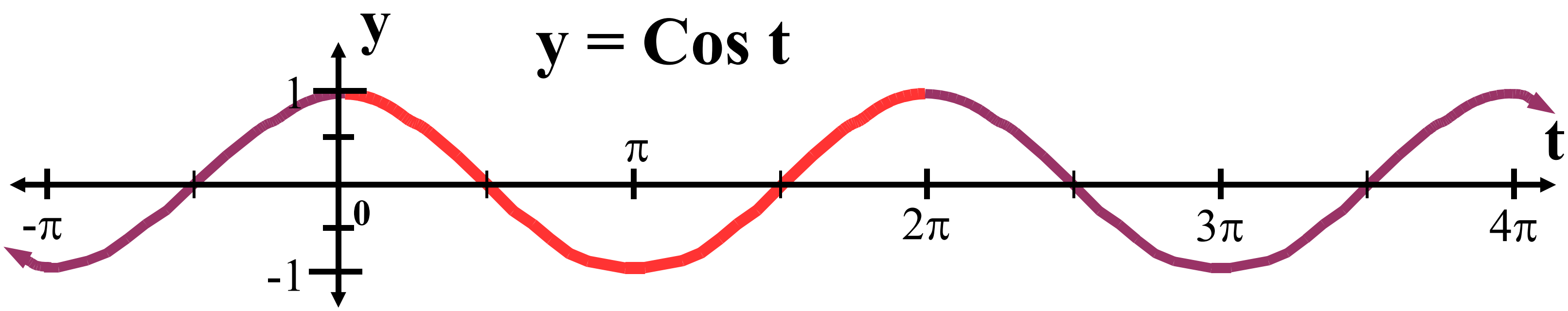


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

The 'basic cycle' starts <u>1 unit below</u> the mid-line



Variations of the Cosine Function



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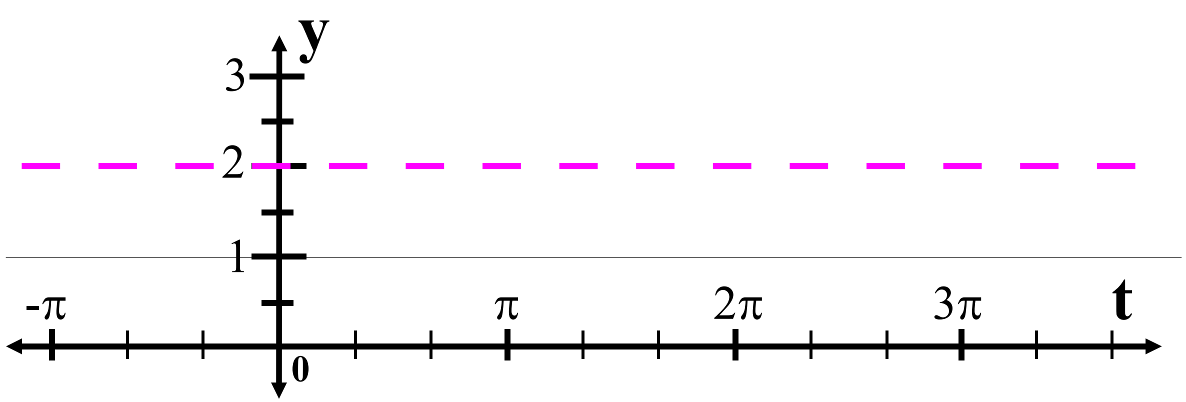
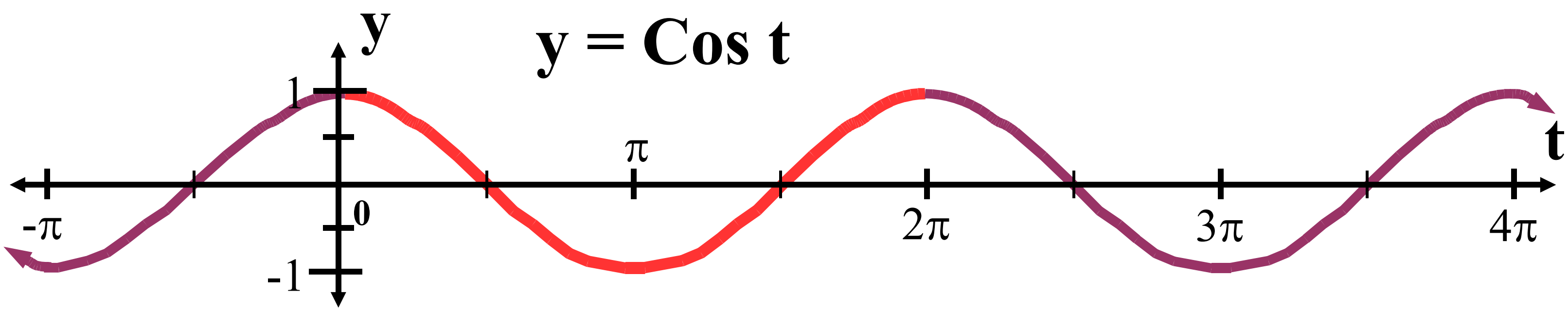
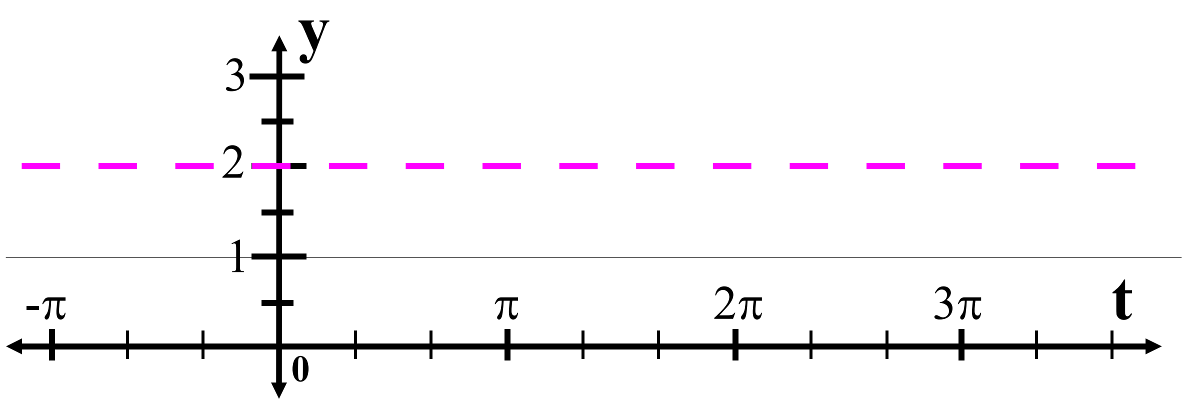


Variations of the Cosine Function

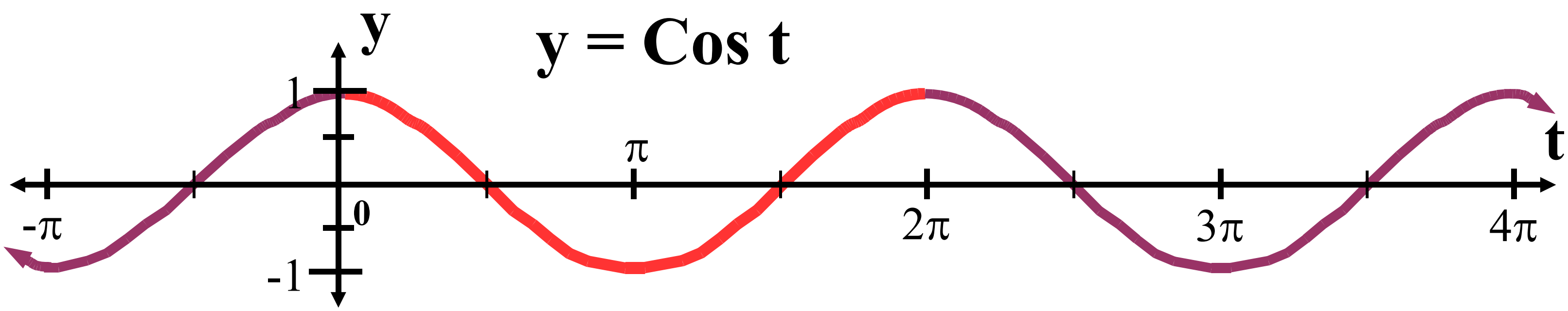


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

The 'basic cycle' starts 1 unit below the mid-line when 0.5t + π/3 = 0.

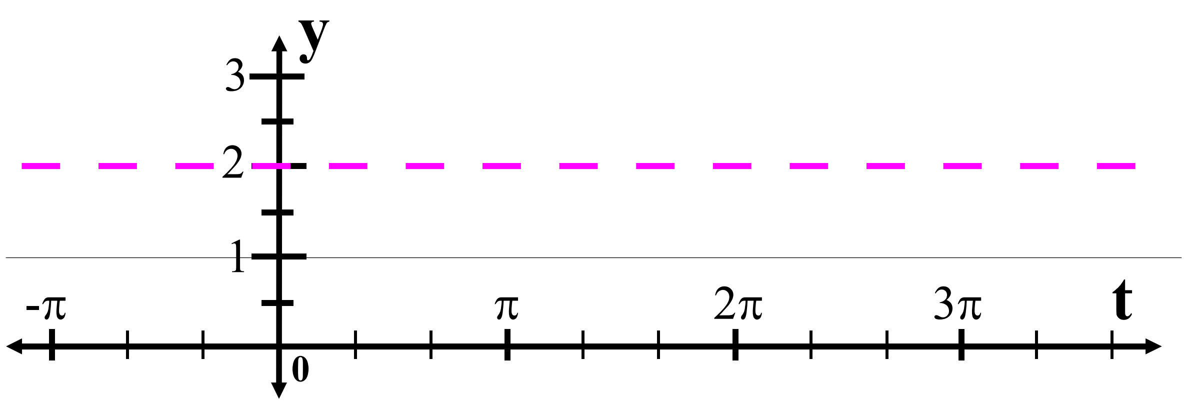


Variations of the Cosine Function

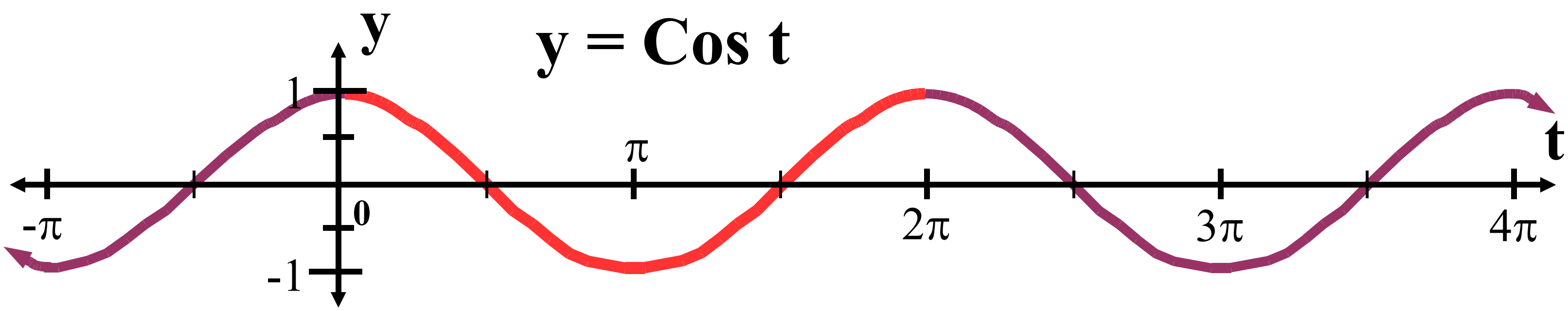


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

The 'basic cycle' starts <u>1 unit below</u> the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**



Variations of the Cosine Function



Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

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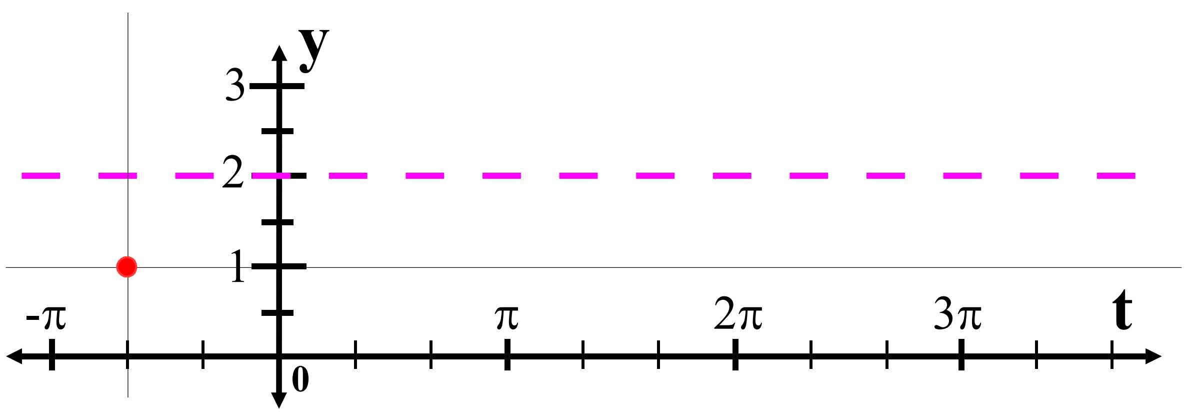
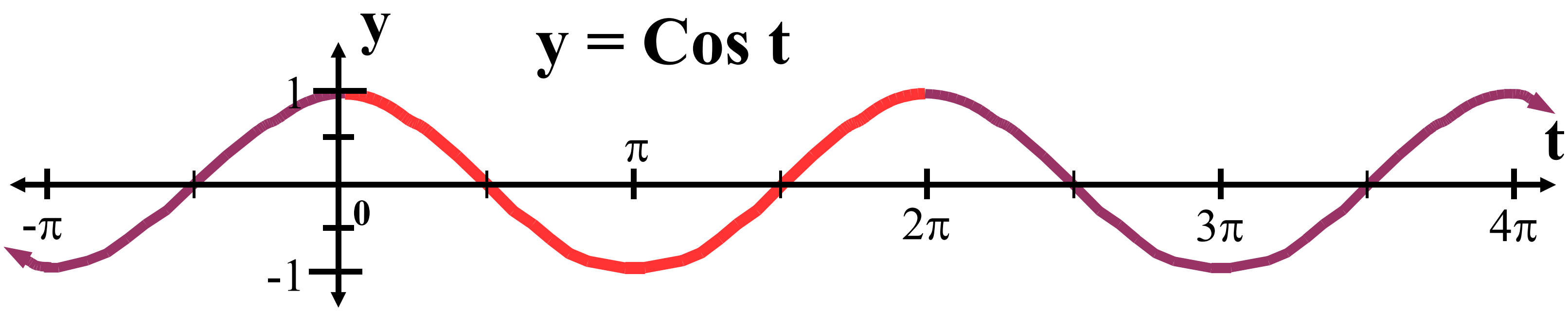


Variations of the Cosine Function



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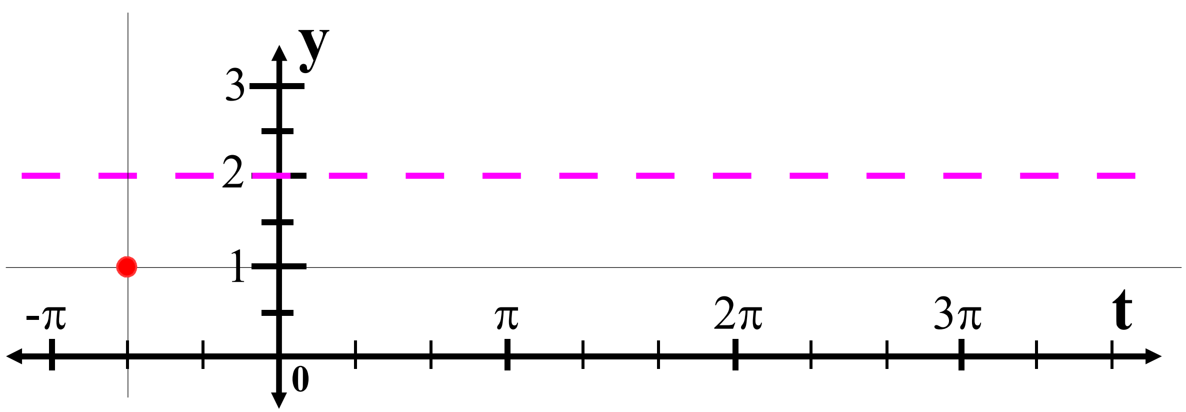
The 'basic cycle' starts 1 unit below the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends 1 units below the mid-line



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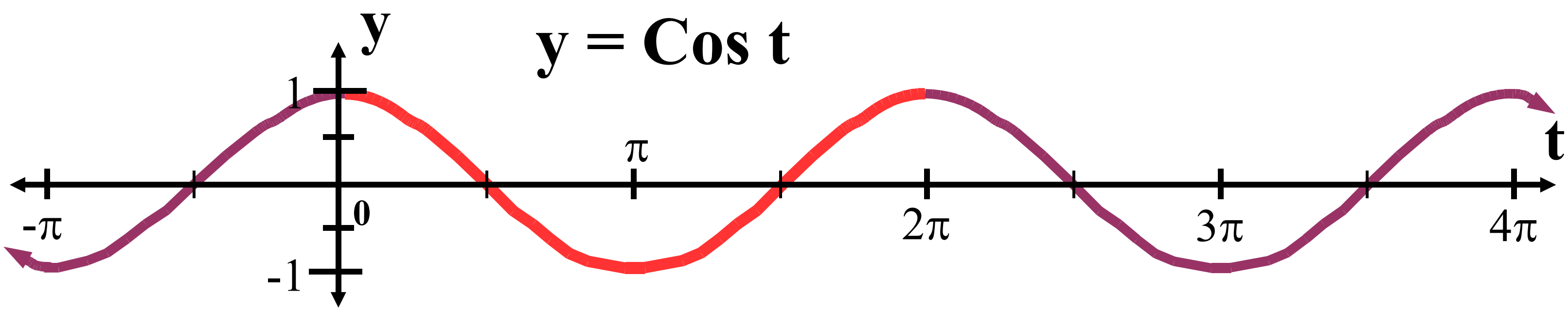
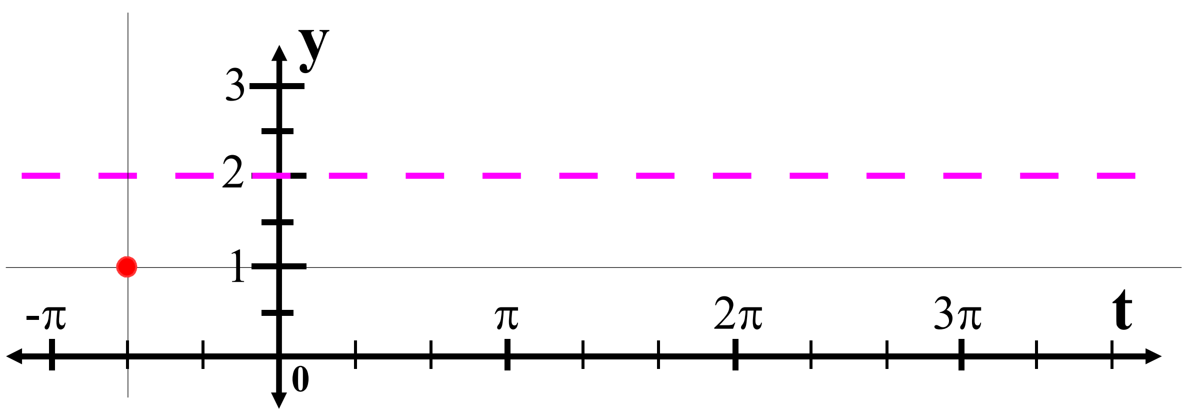


Consider the equation y = -1**Cos(0.5t + π/3)** + **2**. → Mid-line: **y = 2**

The 'basic cycle' starts 1 unit below the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends 1 units below the mid-line when 0.5t + π/3 = 2π.



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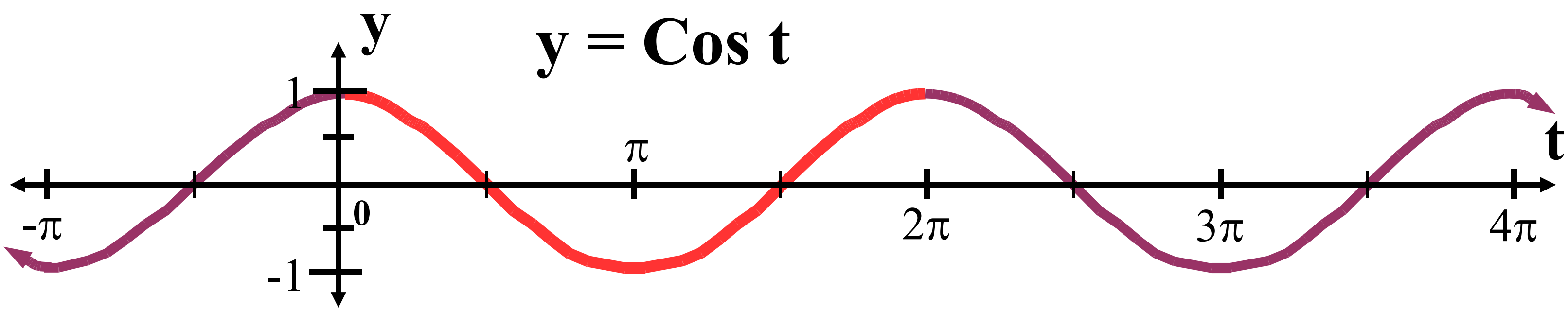
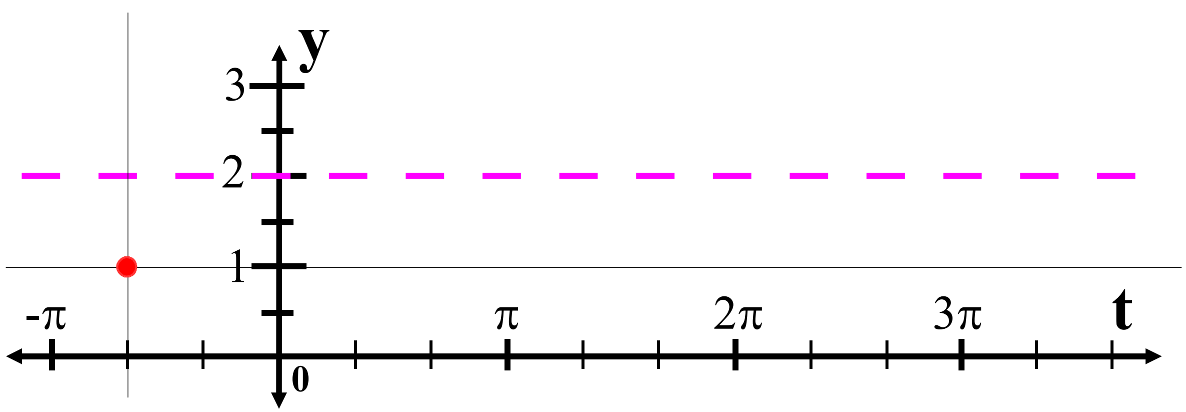


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

The 'basic cycle' starts <u>1 unit below</u> the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends <u>1 units below</u> the mid-line when 0.5t + π/3 = 2π. → **t = 10π/3**



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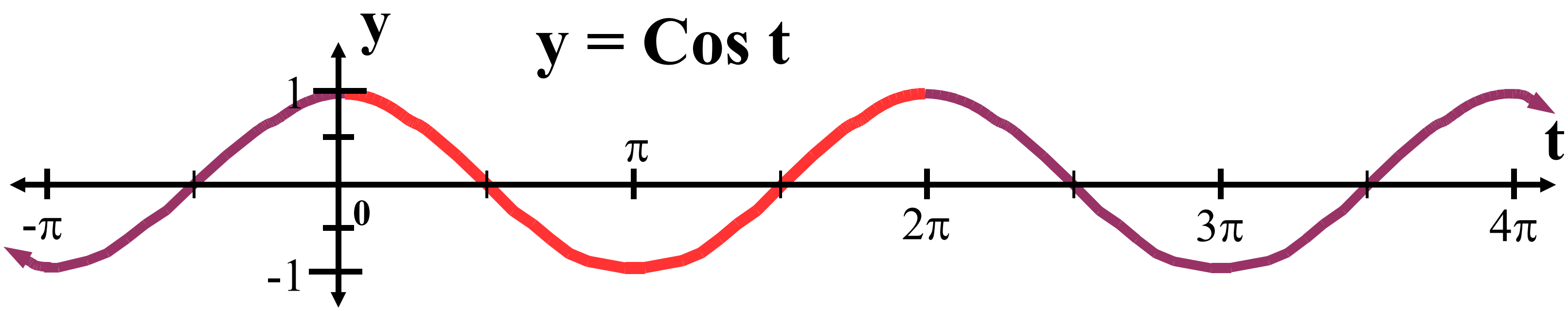
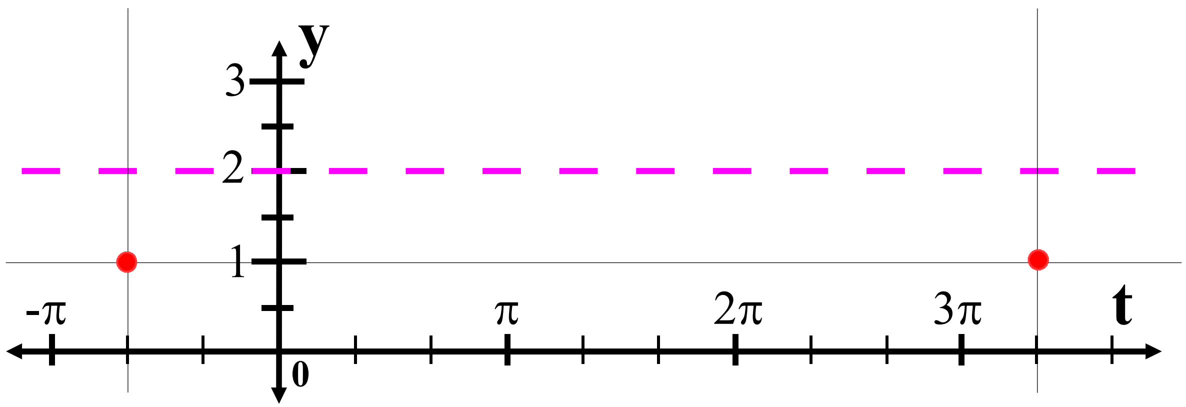


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

The 'basic cycle' starts <u>1 unit below</u> the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends <u>1 units below</u> the mid-line when 0.5t + π/3 = 2π. → **t = 10π/3**



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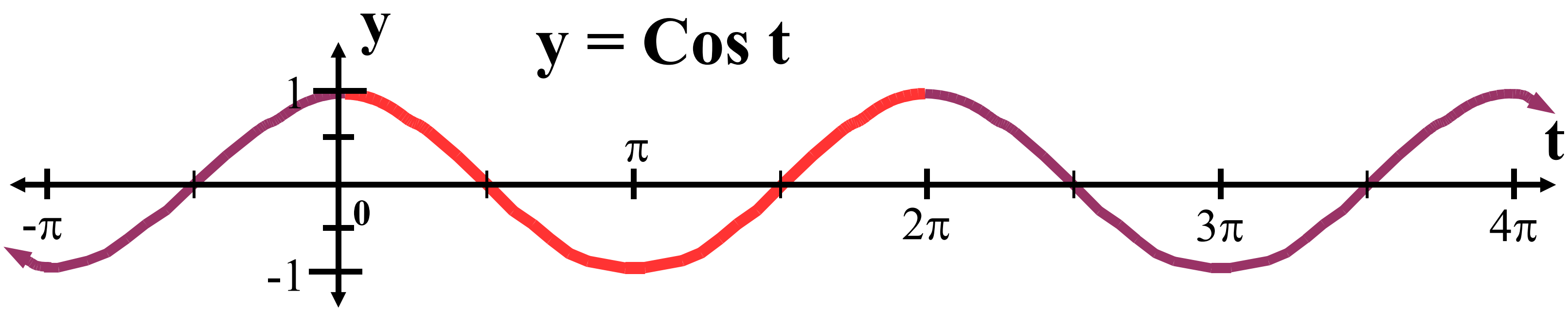
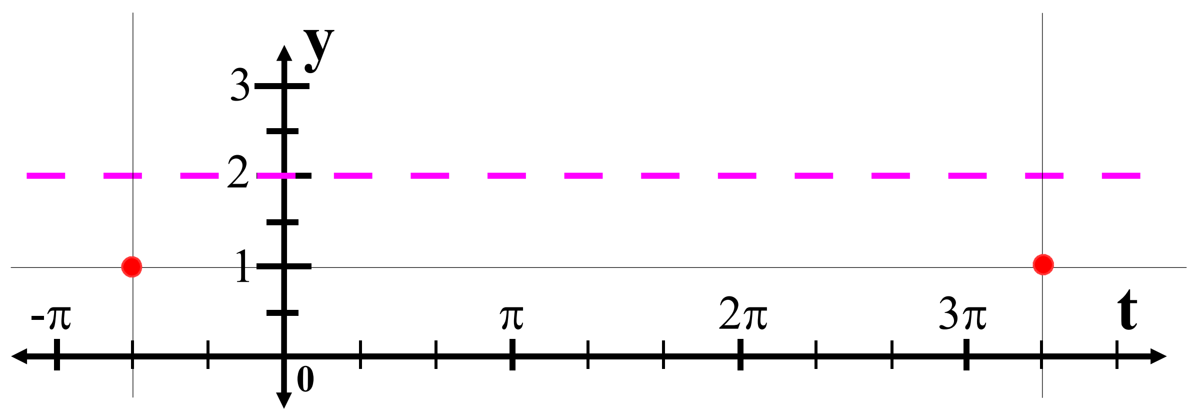


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

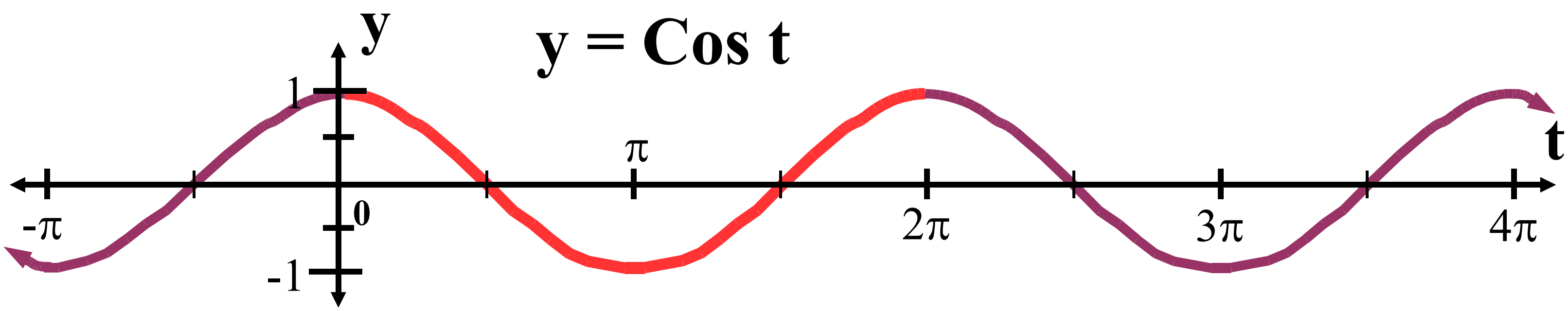
The 'basic cycle' starts <u>1 unit below</u> the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends <u>1 units below</u> the mid-line when 0.5t + π/3 = 2π. → **t = 10π/3**

The 'basic cycle' is <u>1 unit above</u> the mid-line



Variations of the Cosine Function

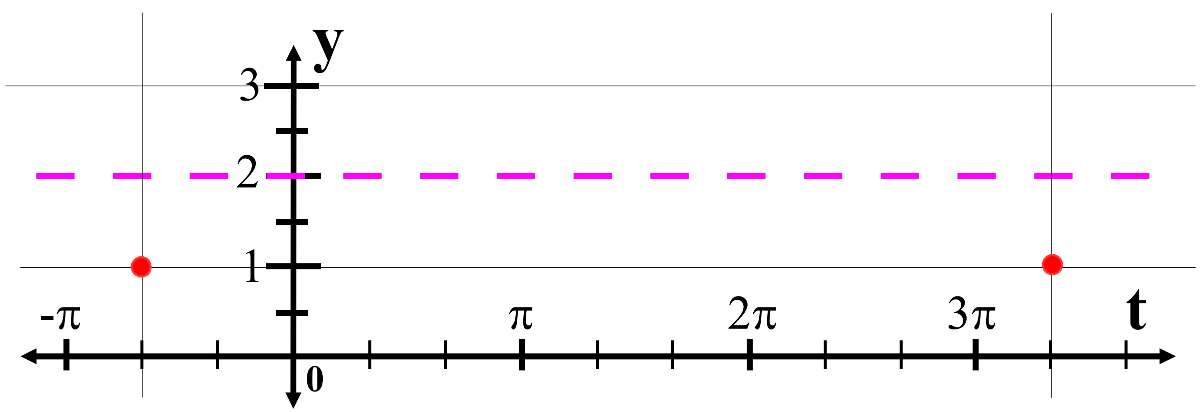


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

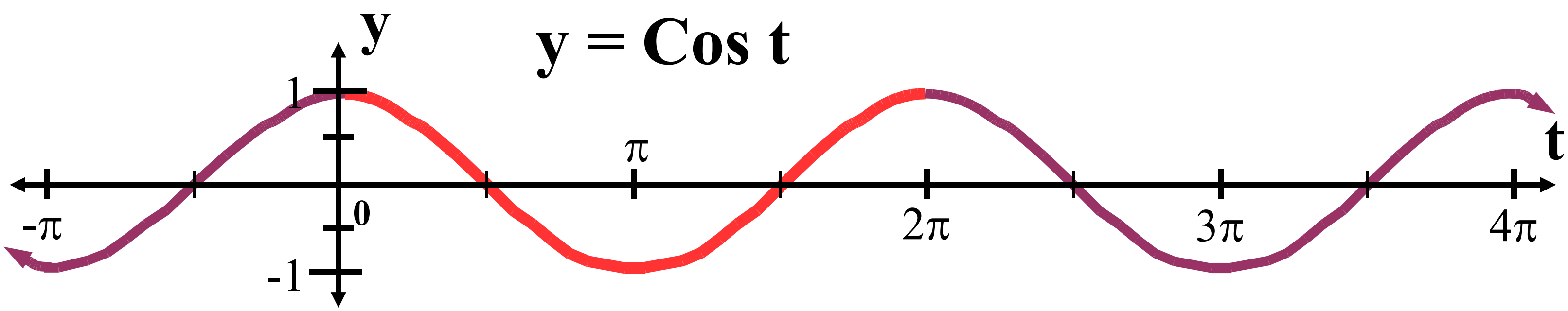
The 'basic cycle' starts <u>1 unit below</u> the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends <u>1 units below</u> the mid-line when 0.5t + π/3 = 2π. → **t = 10π/3**

The 'basic cycle' is <u>1 unit above</u> the mid-line



Variations of the Cosine Function

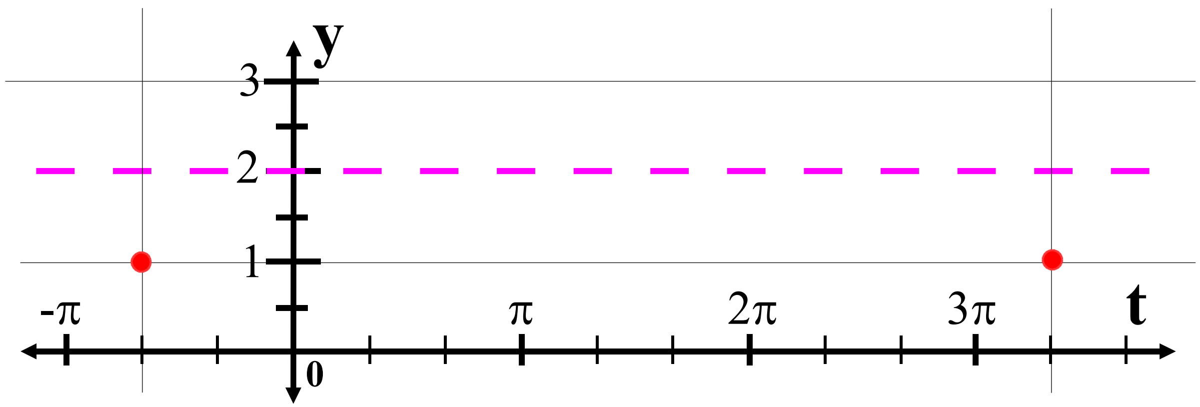


Consider the equation $y = -1\text{Cos}(0.5t + \pi/3) + 2$. → Mid-line: **y = 2**

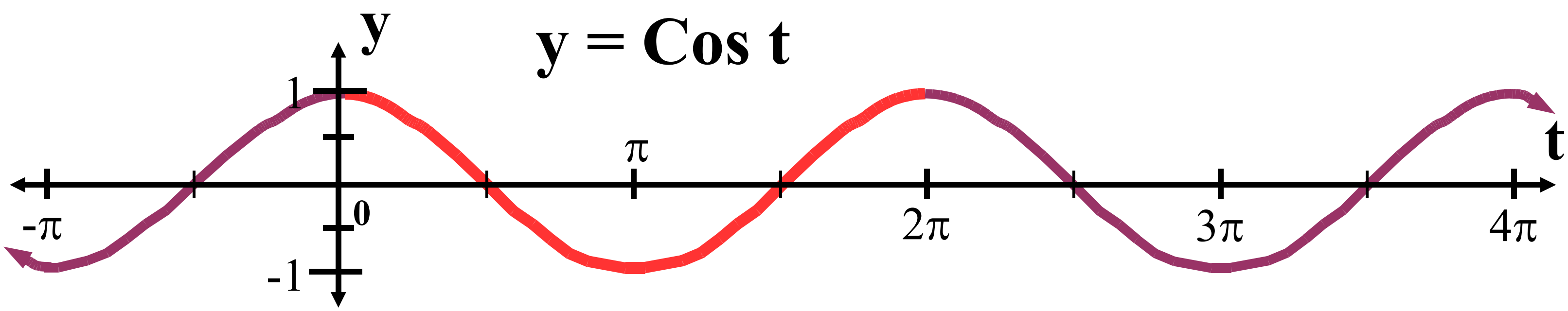
The 'basic cycle' starts 1 unit below the mid-line when $0.5t + \pi/3 = 0$. → **$t = -2\pi/3$**

The 'basic cycle' ends 1 units below the mid-line when $0.5t + \pi/3 = 2\pi$. → **$t = 10\pi/3$**

The 'basic cycle' is 1 unit above the mid-line when $0.5t + \pi/3 = \pi$.



Variations of the Cosine Function

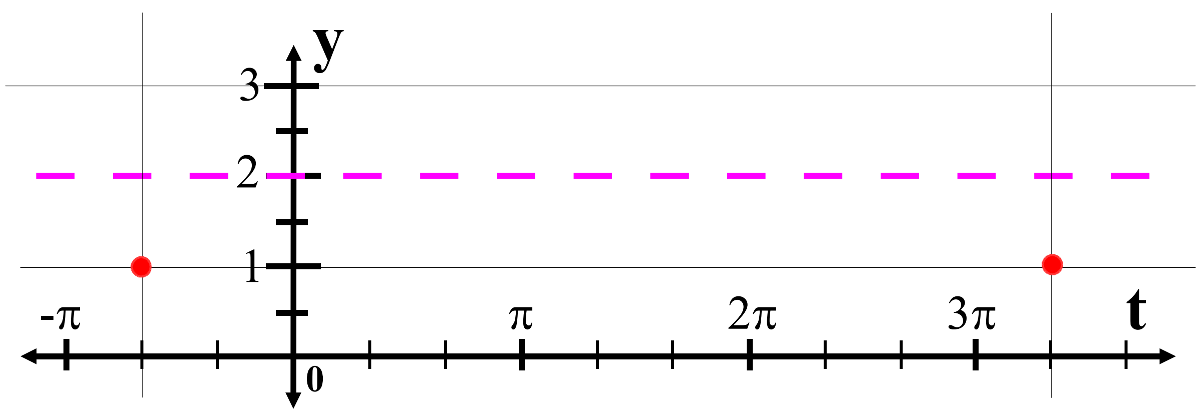


Consider the equation $y = -1\text{Cos}(0.5t + \pi/3) + 2$. → Mid-line: **y = 2**

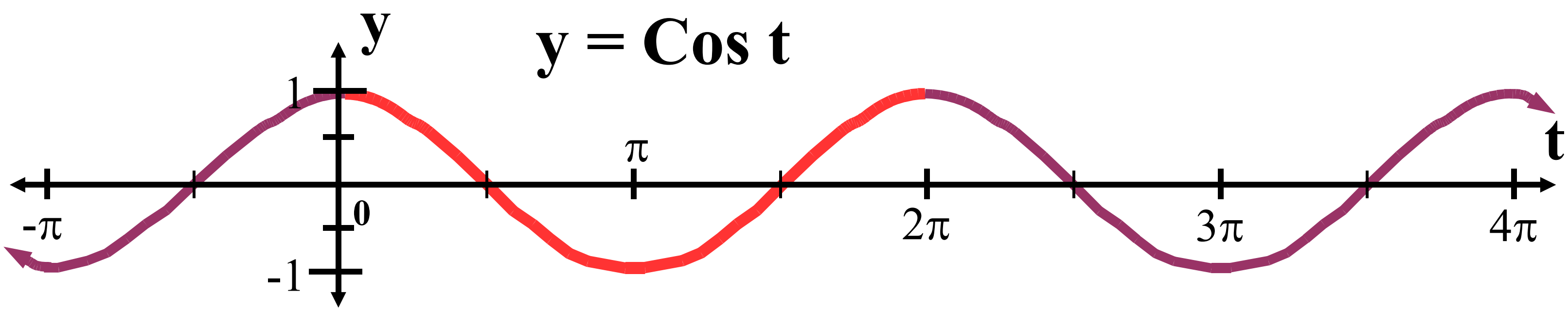
The 'basic cycle' starts 1 unit below the mid-line when $0.5t + \pi/3 = 0$. → **$t = -2\pi/3$**

The 'basic cycle' ends 1 units below the mid-line when $0.5t + \pi/3 = 2\pi$. → **$t = 10\pi/3$**

The 'basic cycle' is 1 unit above the mid-line when $0.5t + \pi/3 = \pi$. → **$t = 4\pi/3$**



Variations of the Cosine Function

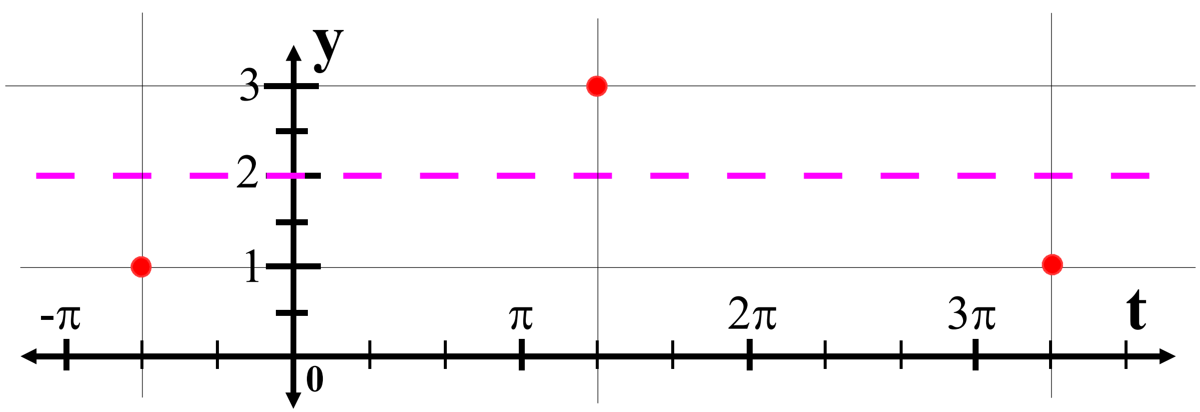


Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

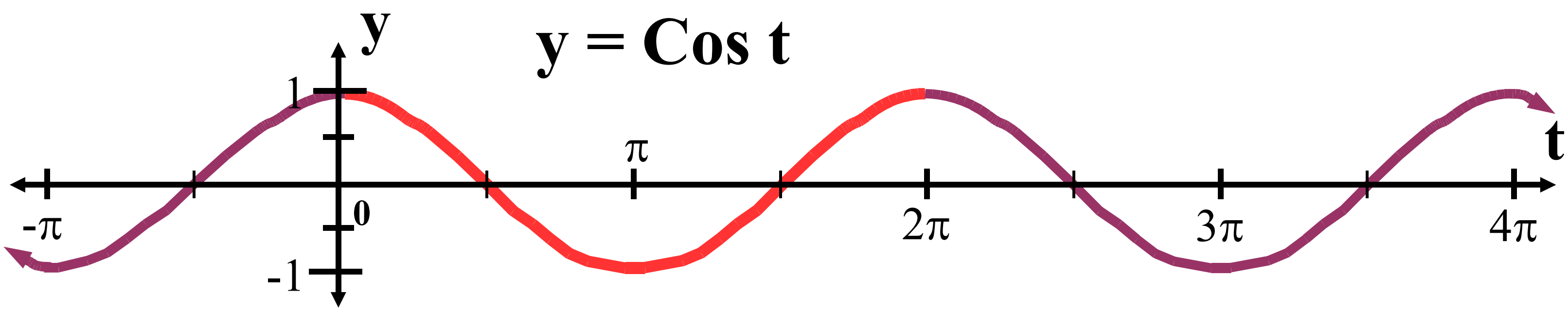
The 'basic cycle' starts 1 unit below the mid-line when 0.5t + π/3 = 0. → **t = -2π/3**

The 'basic cycle' ends 1 units below the mid-line when 0.5t + π/3 = 2π. → **t = 10π/3**

The 'basic cycle' is 1 unit above the mid-line when 0.5t + π/3 = π. → **t = 4π/3**



Variations of the Cosine Function



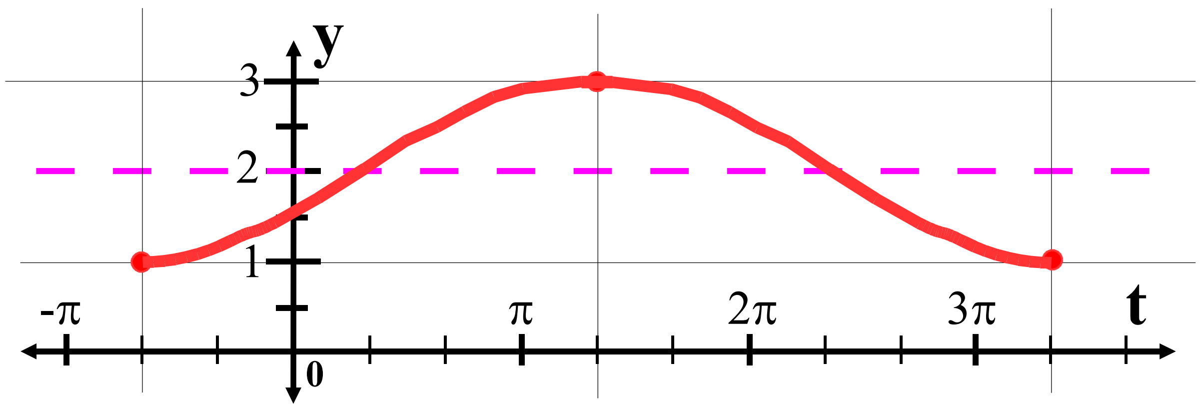
Consider the equation y = -1Cos(0.5t + π/3) + 2. ⟶ Mid-line: **y = 2**

The 'basic cycle' starts 1 unit below the mid-line when 0.5t + π/3 = 0. ⟶ **t = -2π/3**

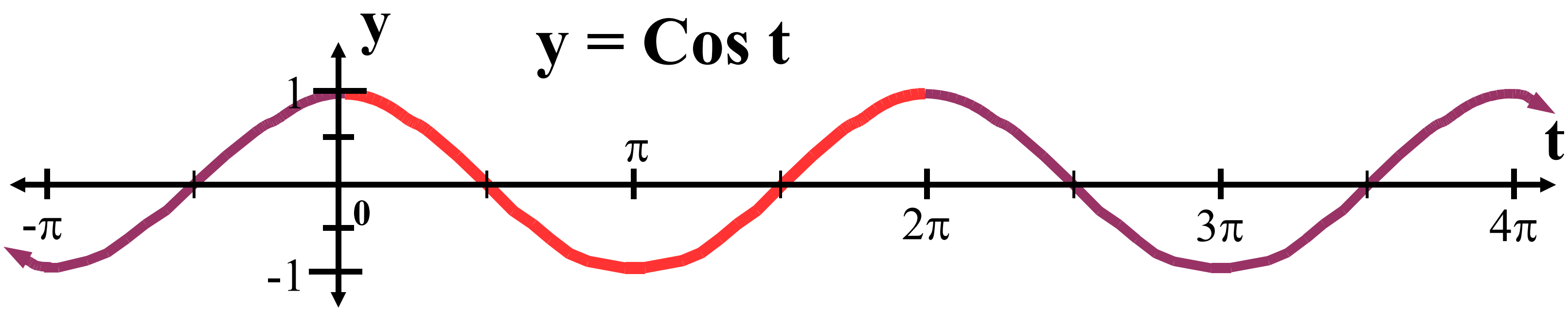
The 'basic cycle' ends 1 units below the mid-line when 0.5t + π/3 = 2π. ⟶ **t = 10π/3**

The 'basic cycle' is 1 unit above the mid-line when 0.5t + π/3 = π. ⟶ **t = 4π/3**

Here is the '**basic cycle**'.



Variations of the Cosine Function



Consider the equation y = -1**Cos(0.5t + π/3) + 2**. → Mid-line: **y = 2**

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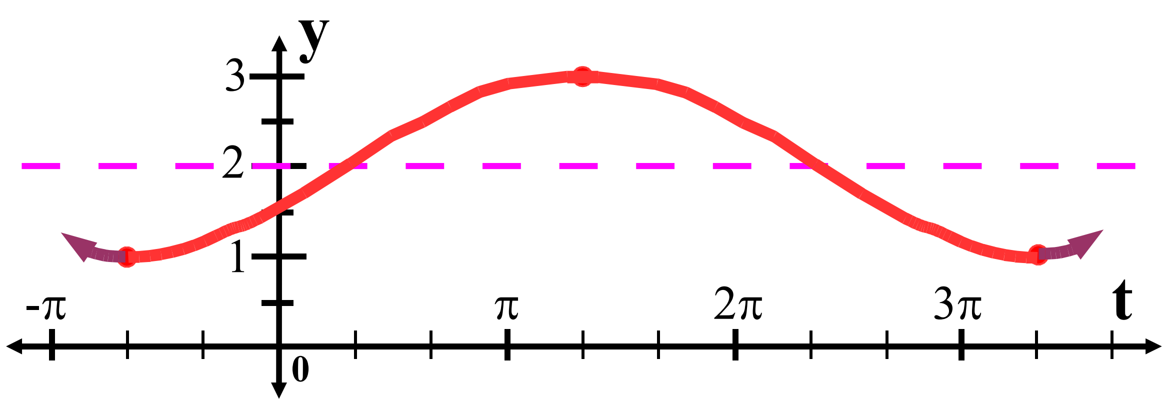
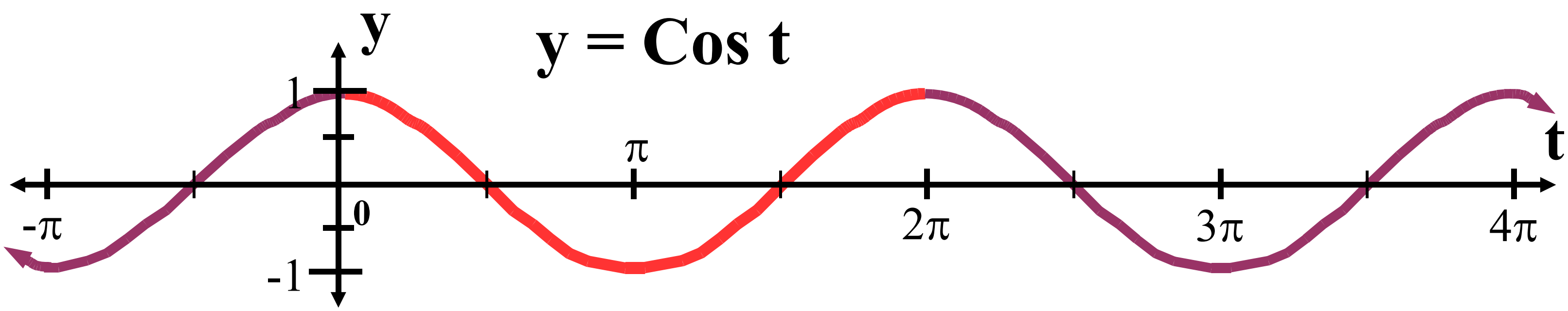
Here is the '**basic cycle**'.



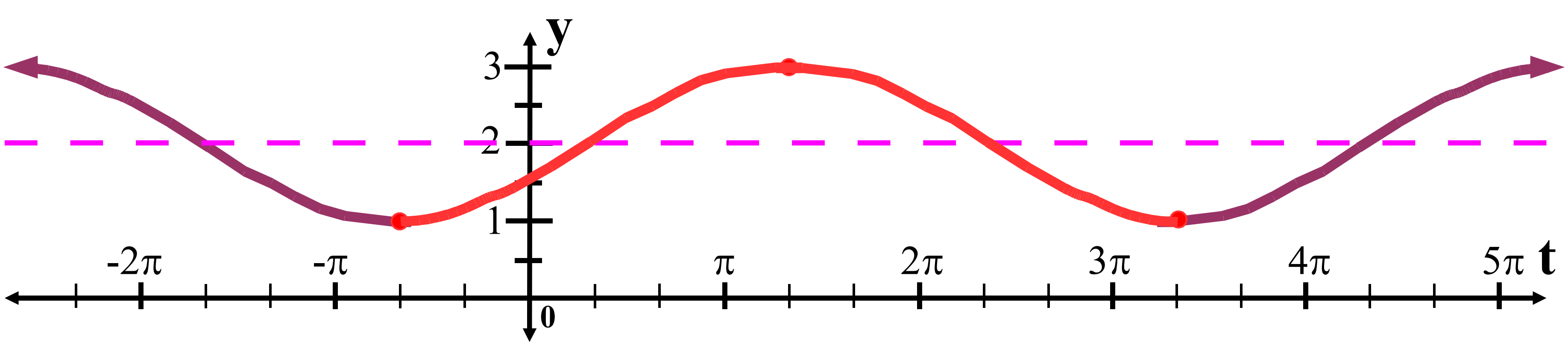
Variations of the Cosine Function

y = Cos t

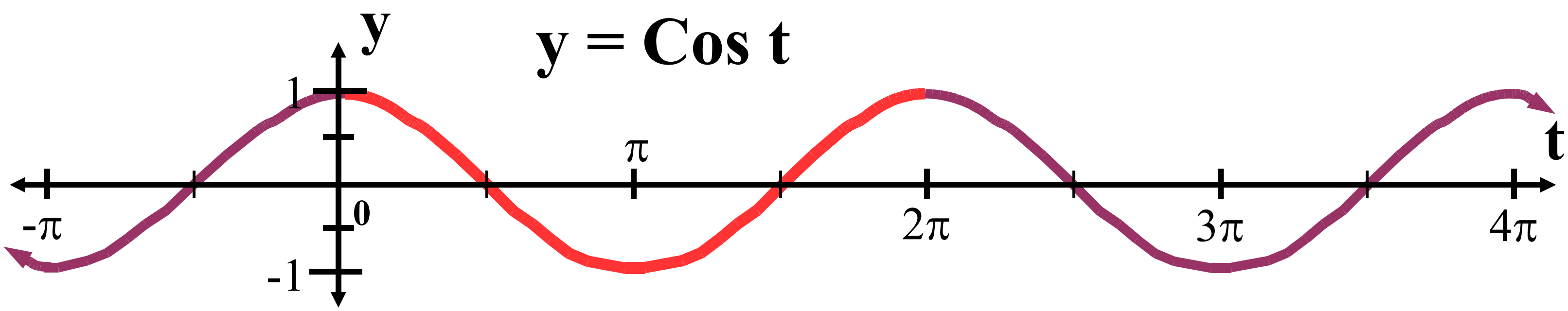


Here is a more complete graph.

y = -Cos(0.5t + π/3) + 2

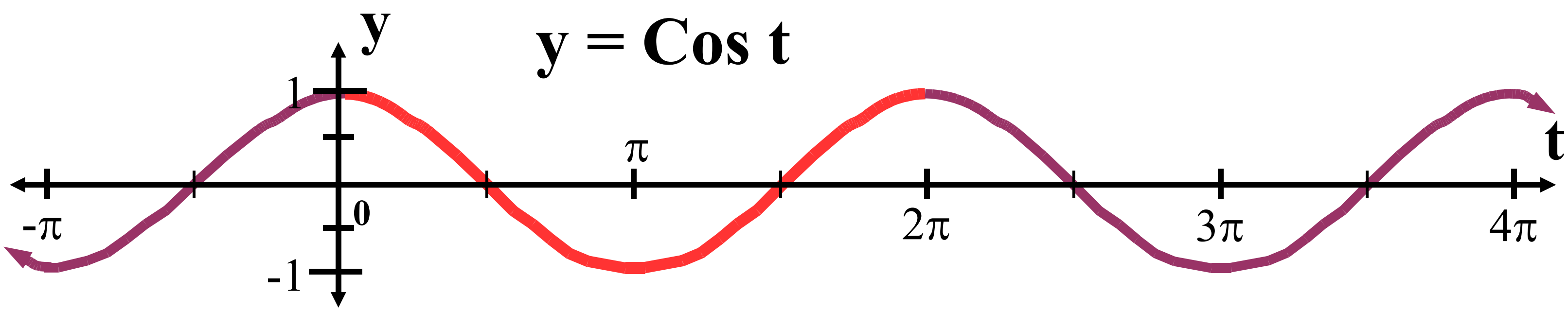


Variations of the Cosine Function



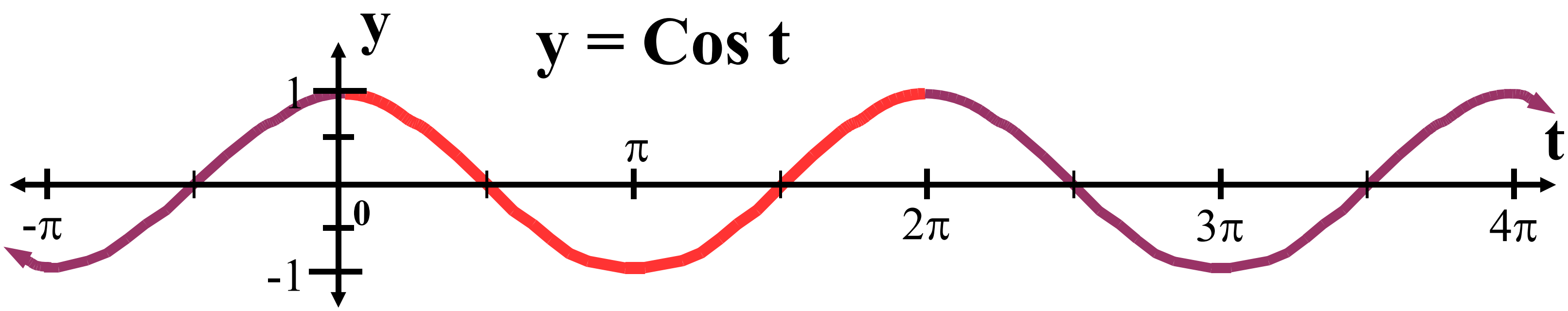
Consider the equation y = 0.5**Cos(1.5t – π/4) – 0.5**.

Variations of the Cosine Function



Consider the equation $y = 0.5\mathbf{Cos(1.5t - \pi/4) - 0.5}$.

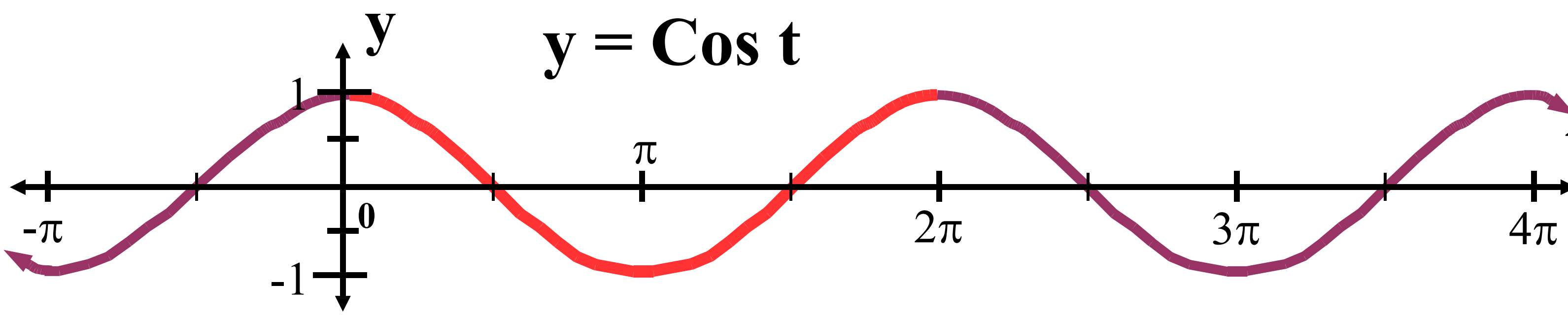
Variations of the Cosine Function



Consider the equation $y = 0.5\mathbf{Cos(1.5t - \pi/4) - 0.5}$.

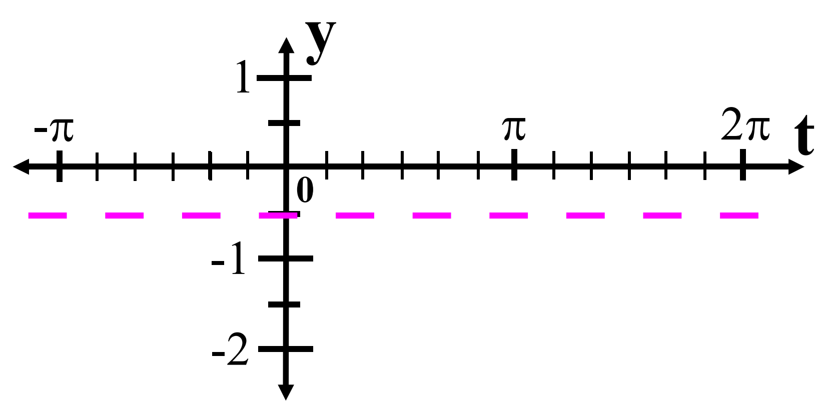
Mid-line: $\mathbf{y = -0.5}$

Variations of the Cosine Function

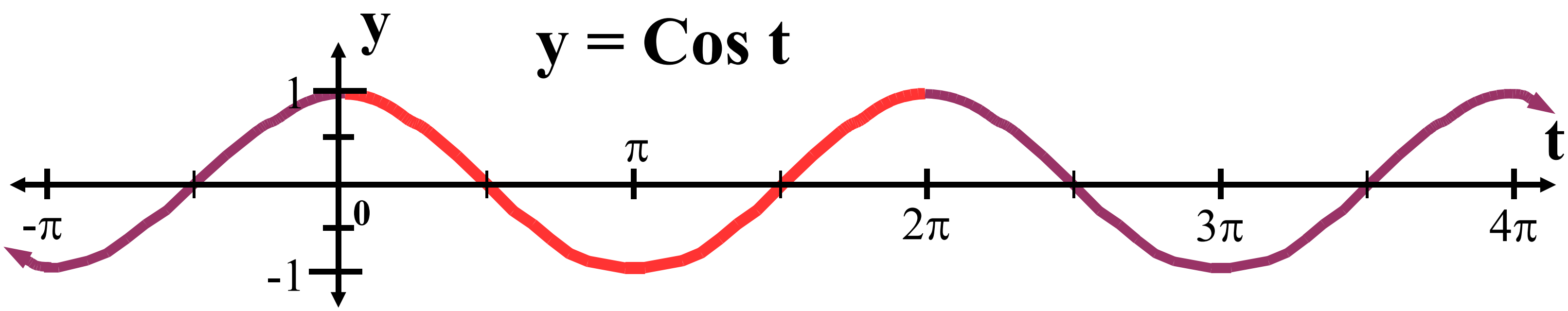


Consider the equation y = 0.5**Cos(1.5t – π/4) – 0.5**.

Mid-line: **y = -0.5**

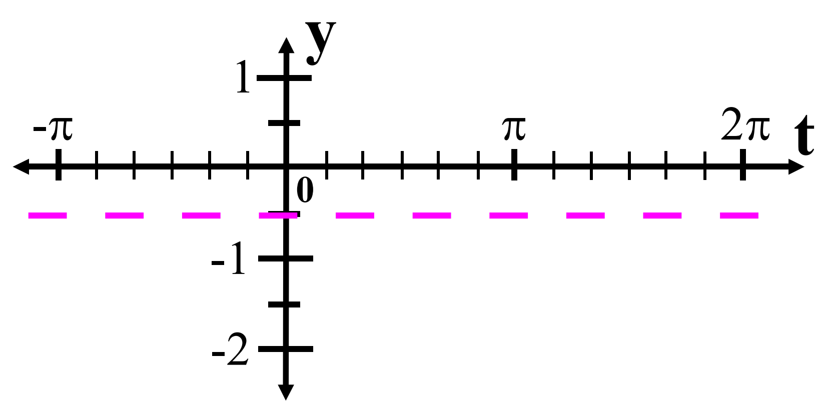


Variations of the Cosine Function



Consider the equation y = 0.5**Cos(1.5t – π/4) – 0.5**.

Mid-line: **y = -0.5**



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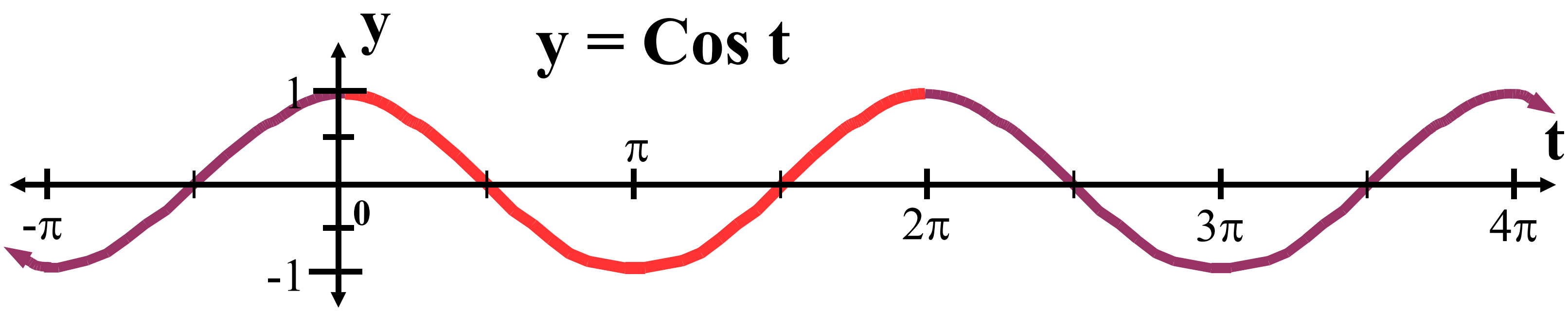

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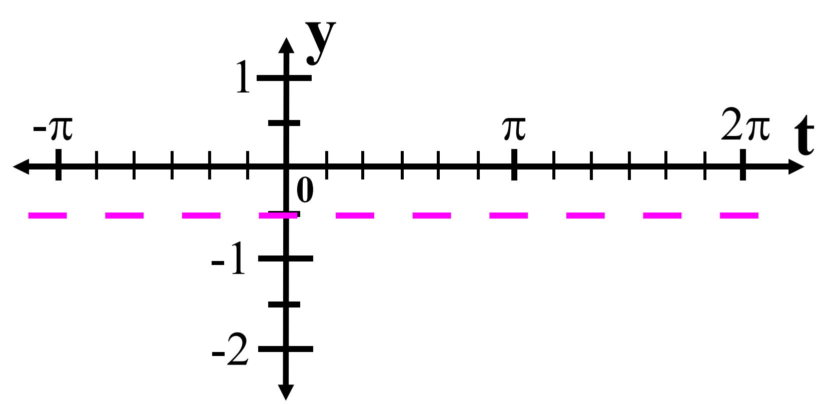

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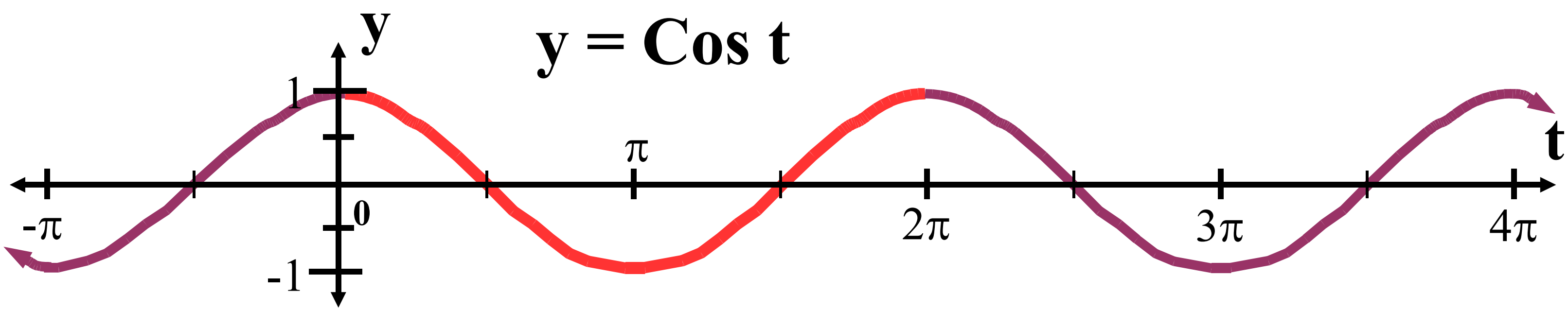

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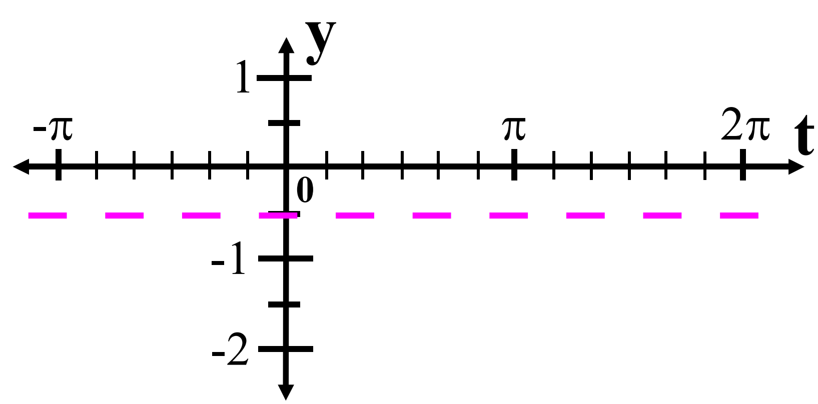

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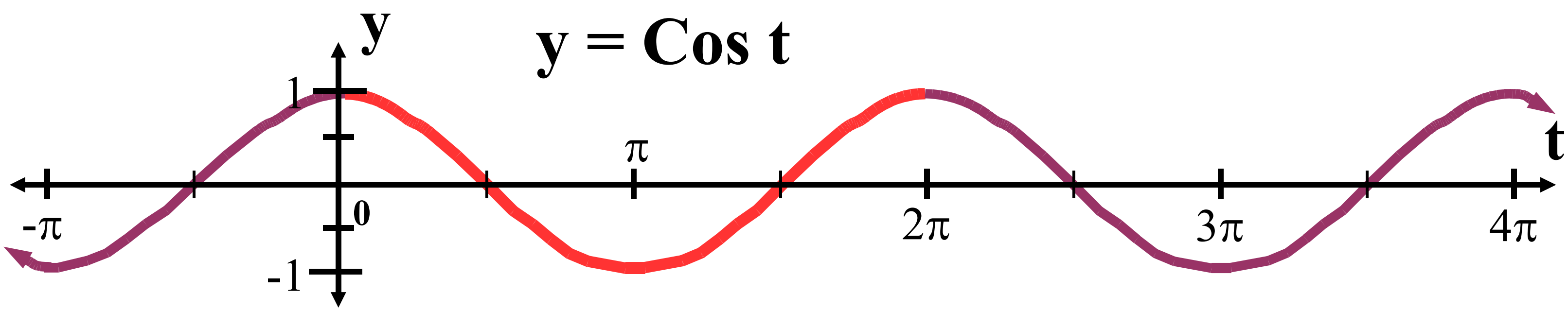

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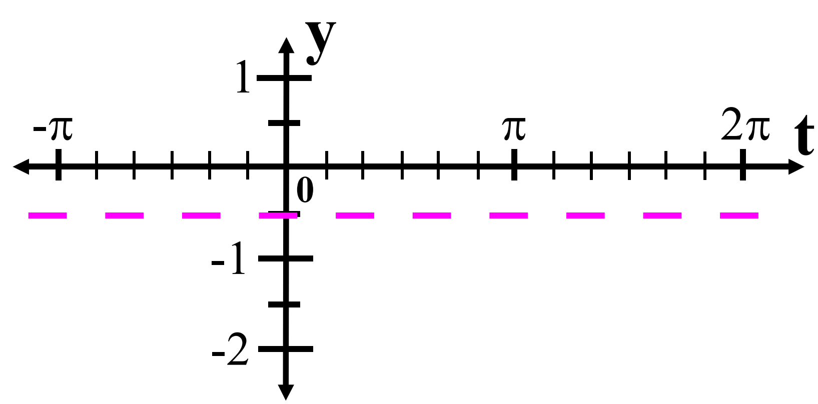

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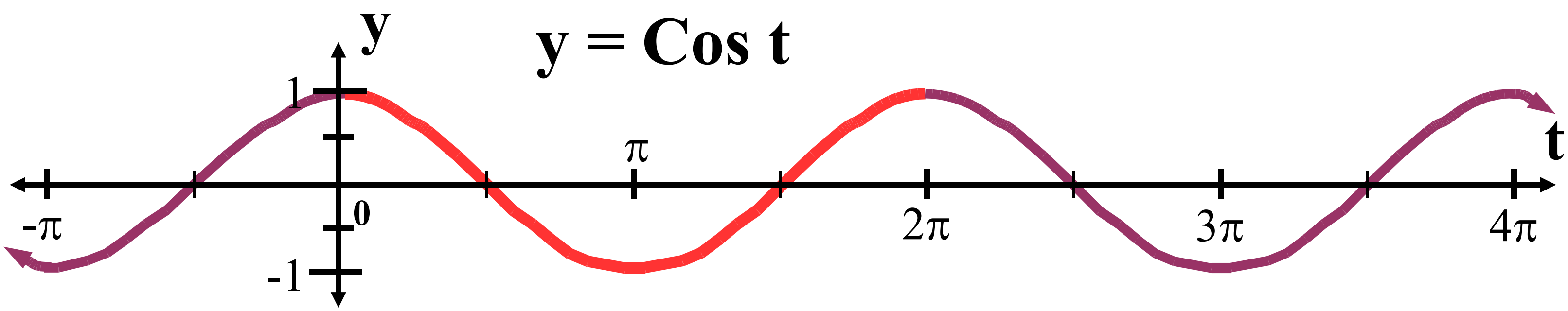

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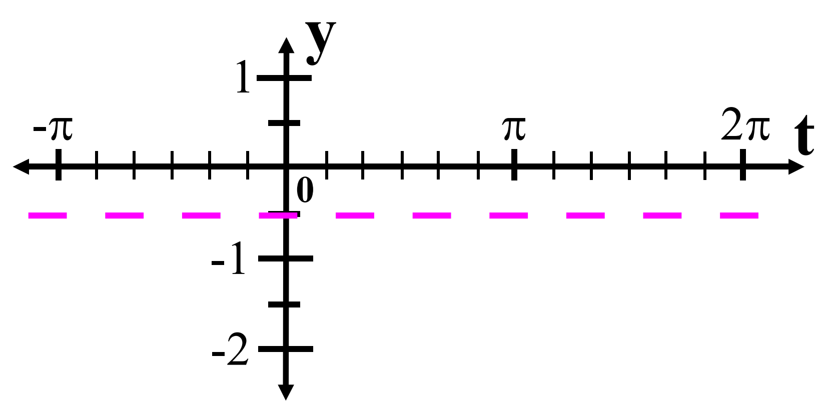
Mid-line: **y = -0.5**

The 'basic cycle' starts 0.5 units above the mid-line when $1.5t - \pi/4 = 0$. → **$t = \pi/6$**



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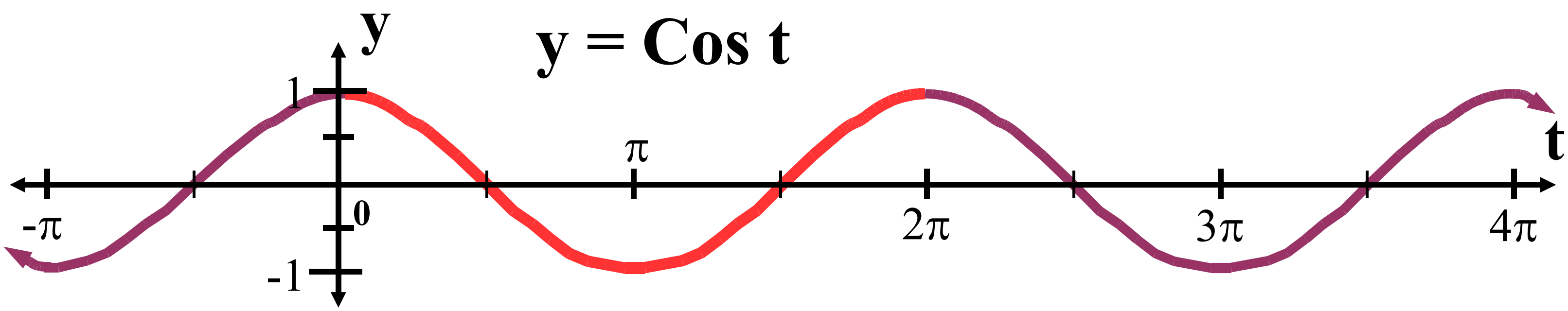

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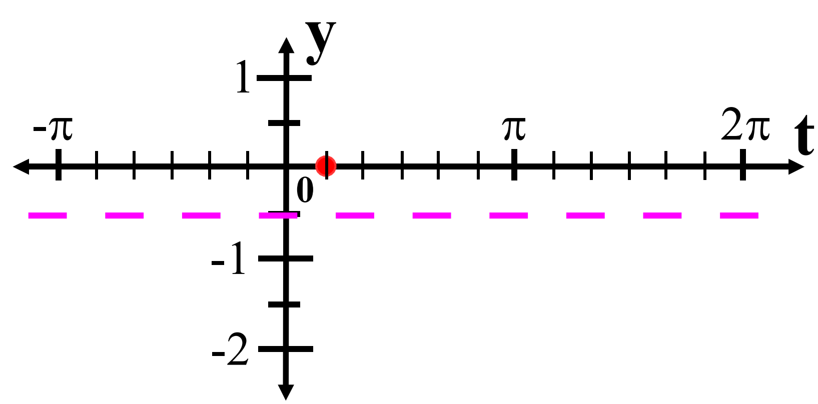

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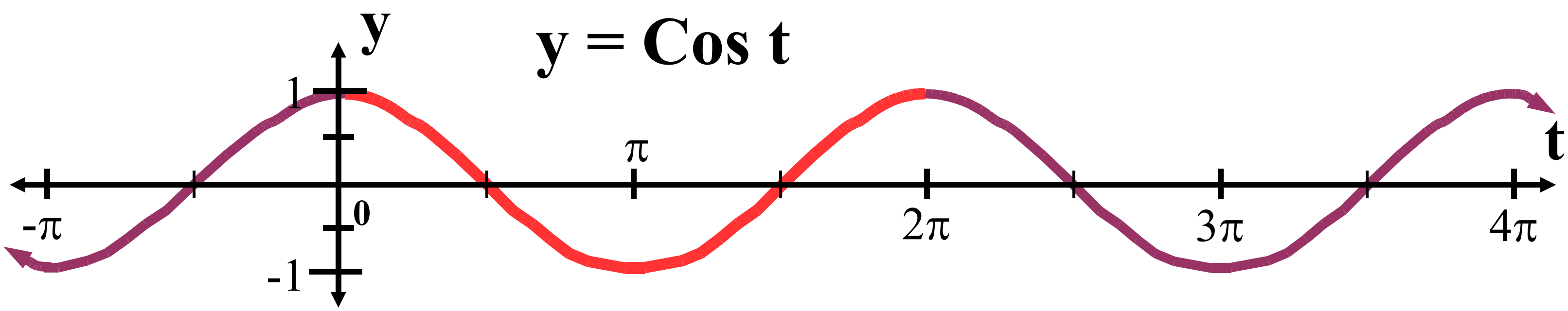

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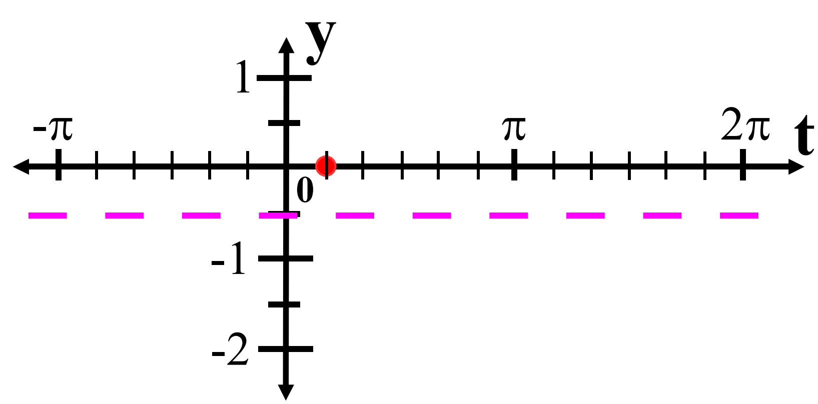

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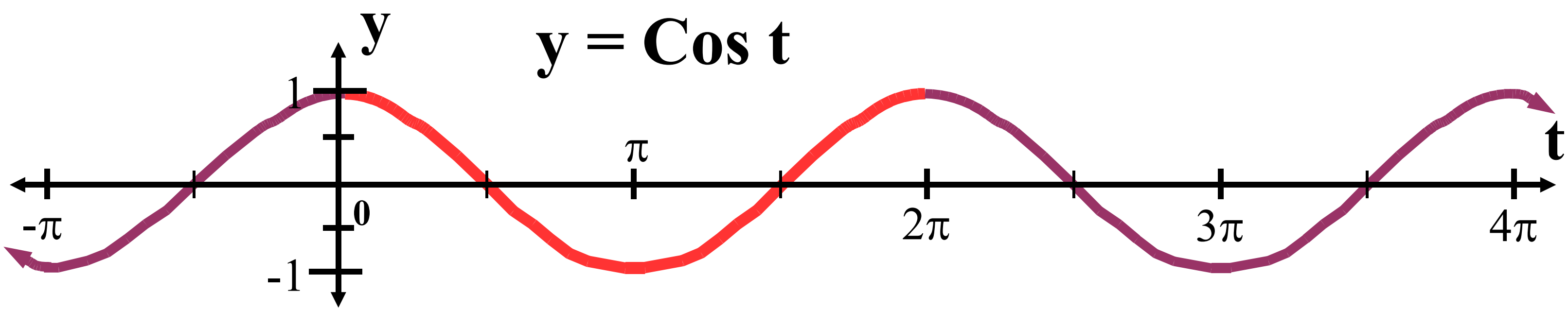

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Mid-line: **y = -0.5**

The 'basic cycle' starts 0.5 units above the mid-line when $1.5t - \pi/4 = 0$. → **$t = \pi/6$**

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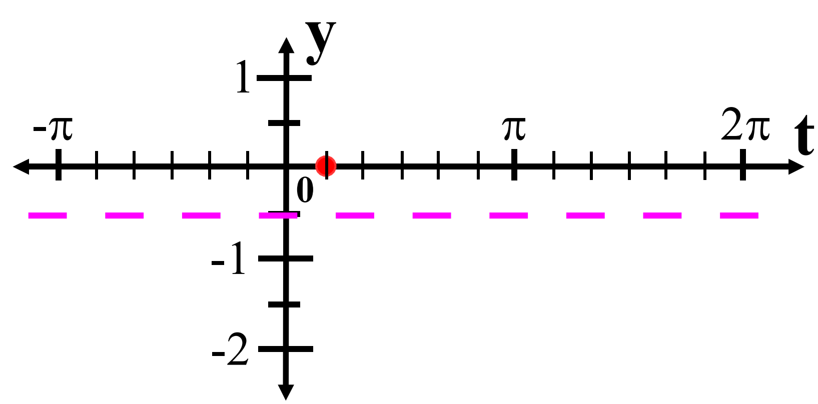

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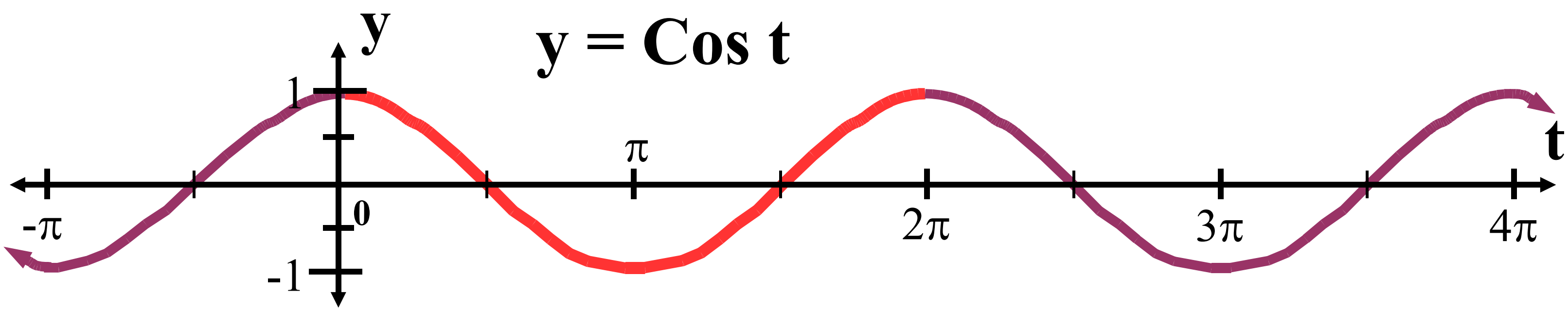

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Mid-line: $\mathbf{y = -0.5}$

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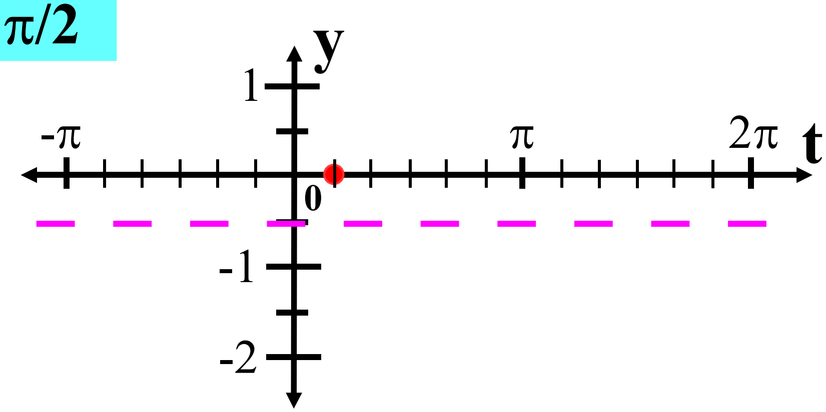
The 'basic cycle' ends 0.5 units above the mid-line when $1.5t - \pi/4 = 2\pi$. → $\mathbf{t = 3\pi/2}$



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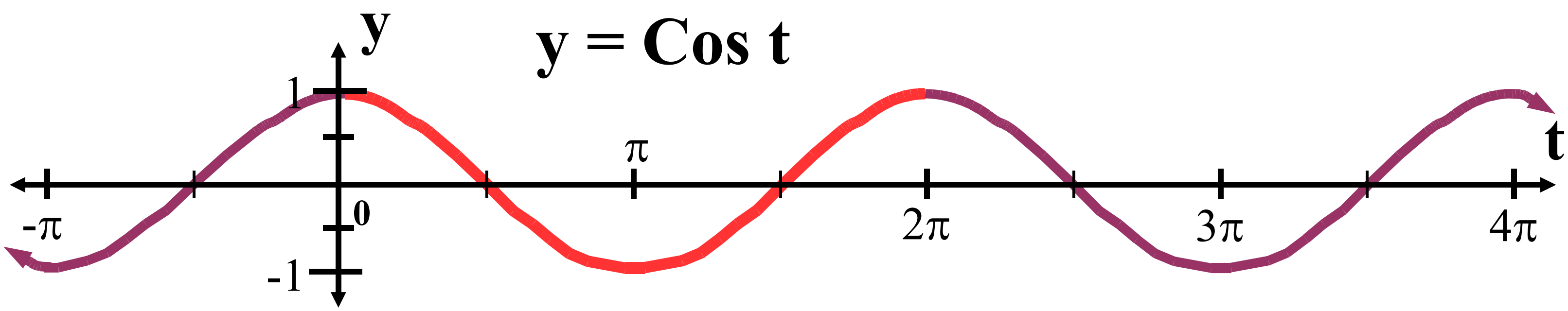

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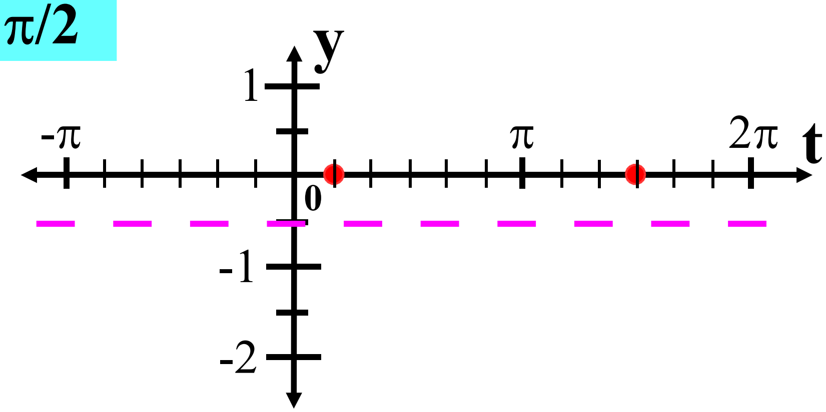

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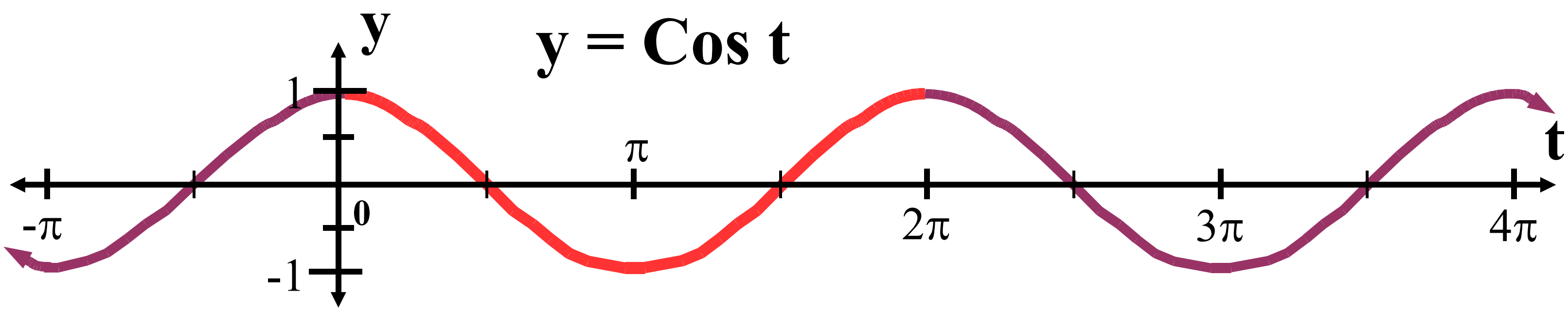

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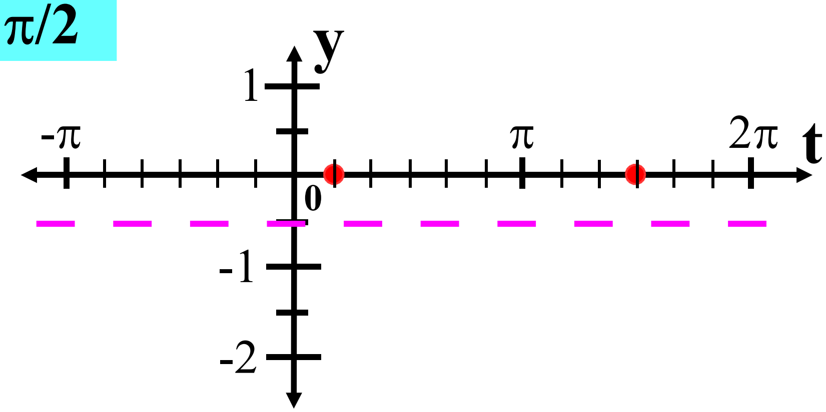
The 'basic cycle' ends <u>0.5 units above</u> the mid-line when $1.5t - \pi/4 = 2\pi$. → **t = 3π/2**

The 'basic cycle' is <u>0.5 units below</u> the mid-line



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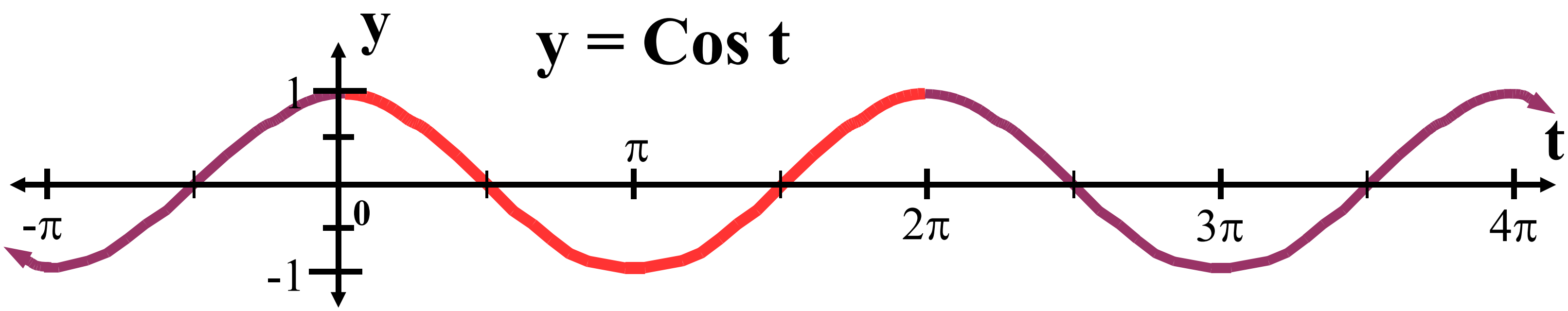

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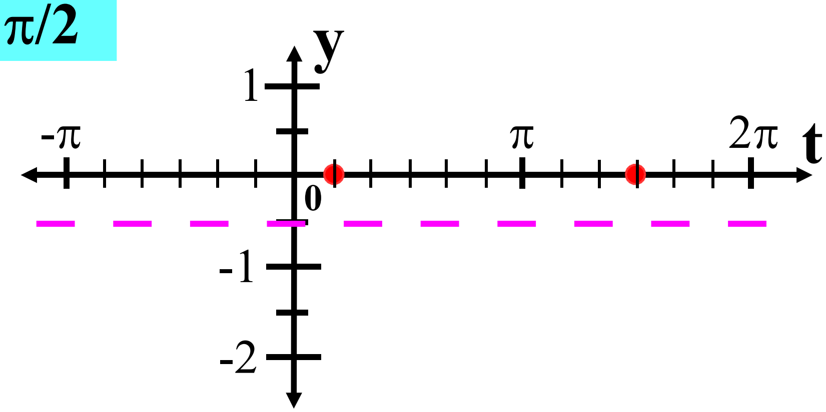

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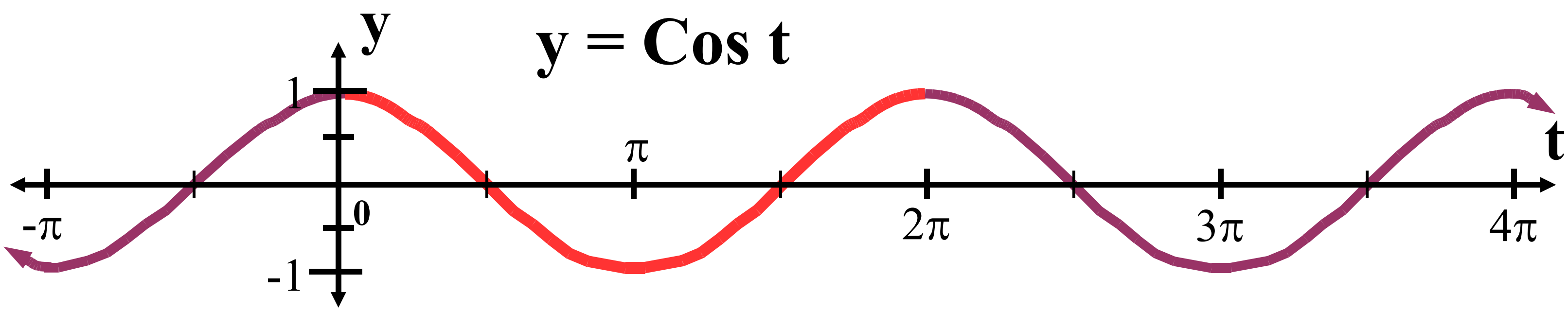

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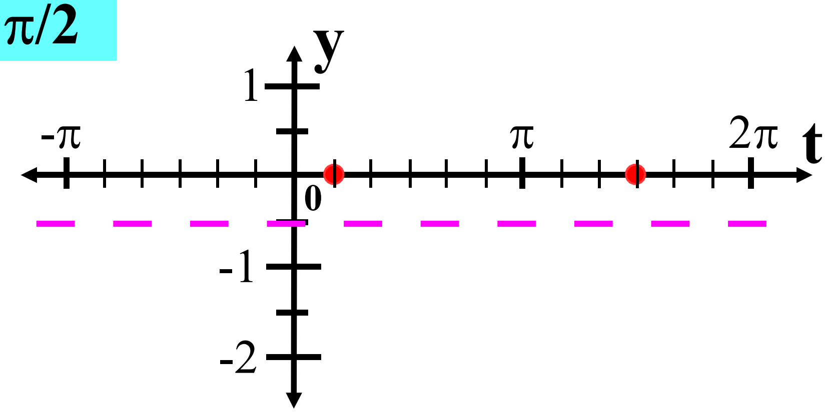
→ $\mathbf{t = 5\pi/6}$



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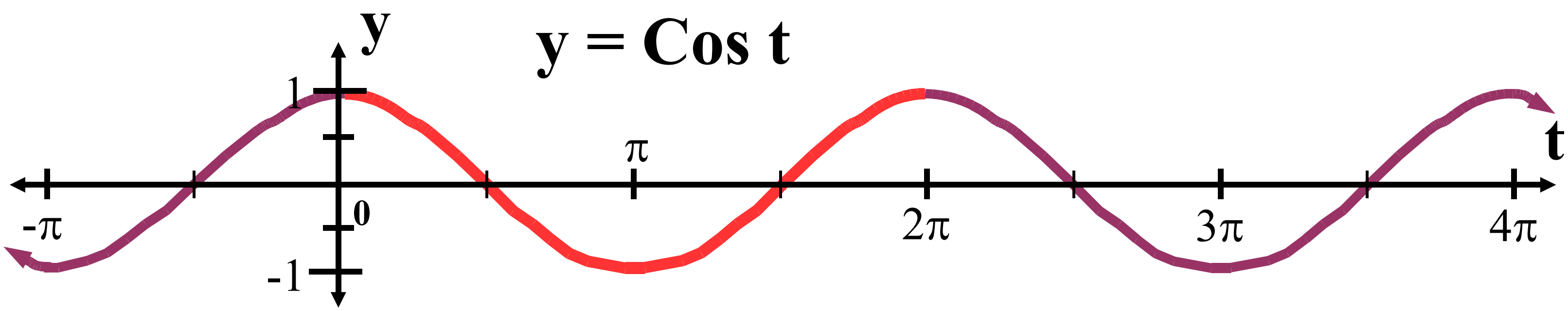

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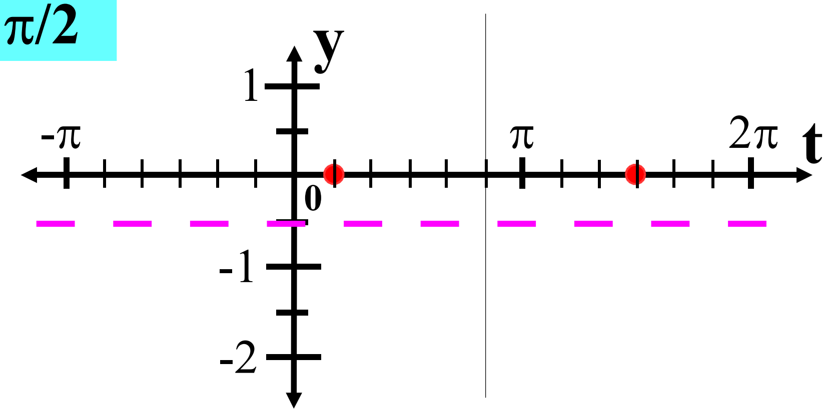

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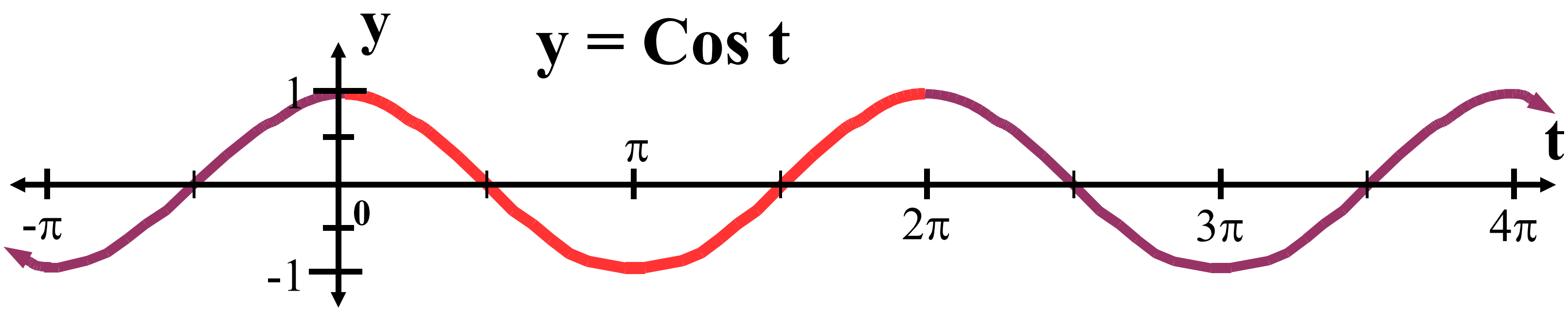

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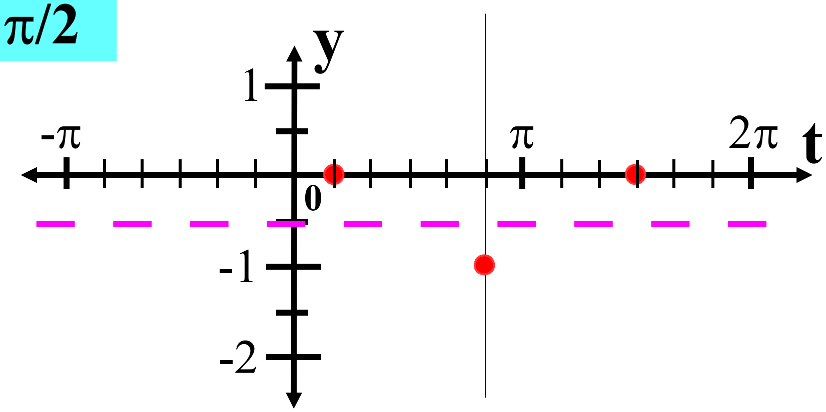

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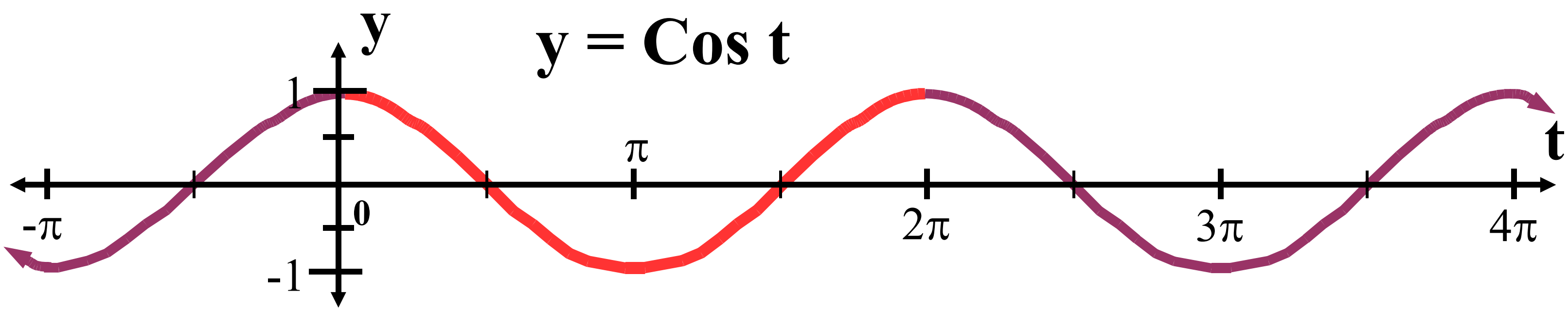

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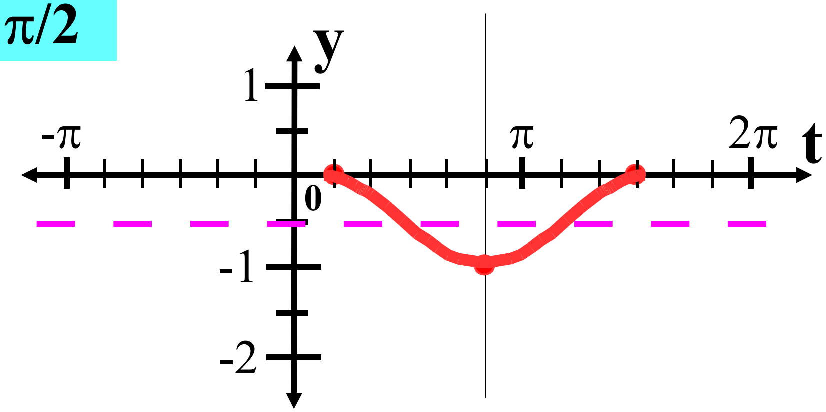
Here is the basic cycle.



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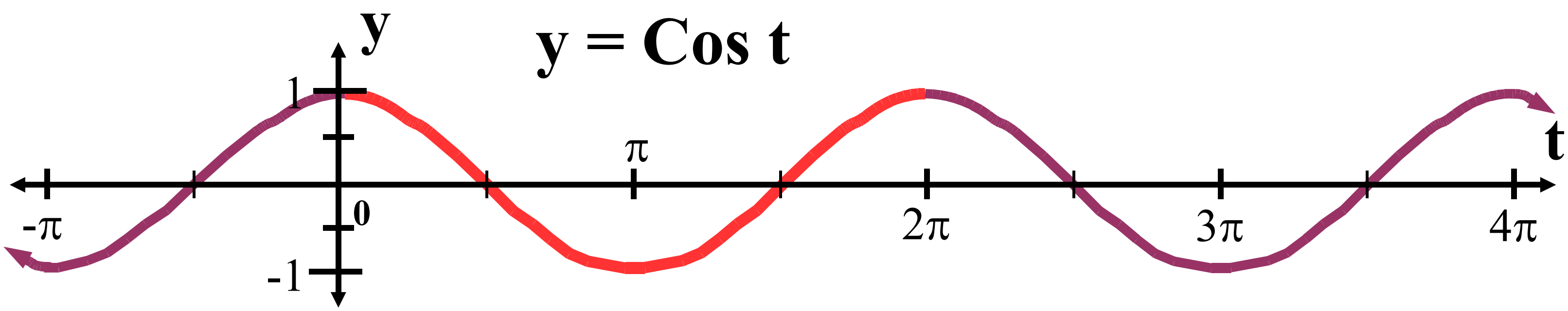

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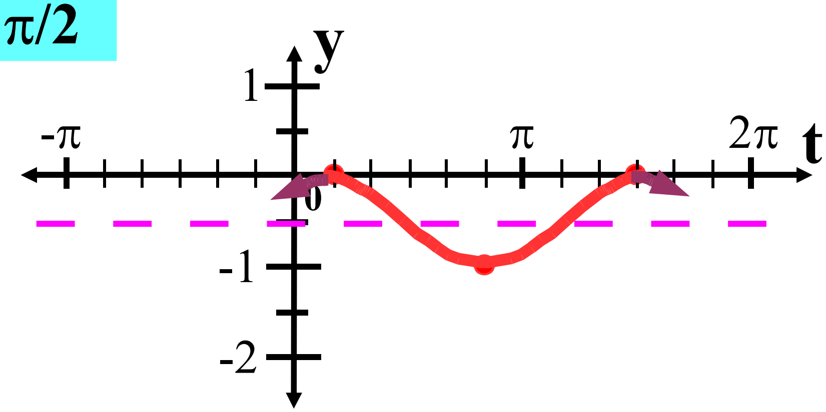
Here is the basic cycle.



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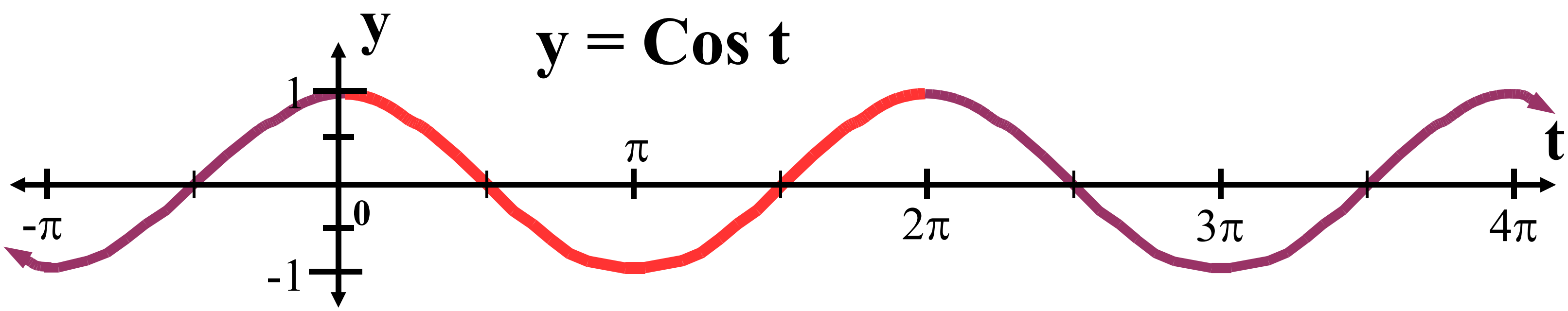
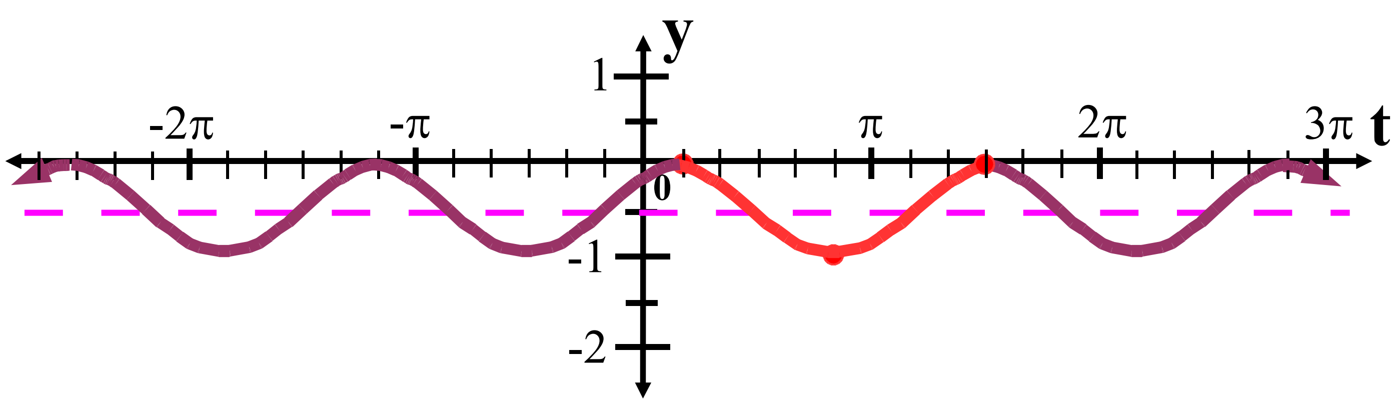


Here is a more complete graph.

$y = 0.5\cos(1.5t - \pi/4) - 0.5$



Variations of the Cosine Function

Consider the equation $y = A\cos(Bt + C) + D$.

(1) The amplitude of the 'cosine wave' is the absolute value of A.

(2) If $A > 0$, then the basic cycle starts at its maximum value and ends at its maximum value. It is at its minimum value “half-way' through the cycle. It crosses the mid-line ¼ way through the cycle and again ¾ way through the cycle.

(3) If $A < 0$, then the basic cycle starts at its minimum value and ends at its minimum value. It is at its maximum value “half-way' through the cycle. It crosses the mid-line ¼ way through the cycle and again ¾ way through the cycle.

(4) The equation of the mid-line is $y = D$.

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(5) The basic cycle starts when $Bt + C = 0$.

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(6) The period of the 'cosine wave' $= 2\pi/|B|$.

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at its minimum valu… way' through the
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The End

(4) The equation of the mid-line is y = D.

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