

Precalculus Worksheet #3 Unit 7 Selected Solutions

Use Gauss-Jordan elimination to solve each of the following systems of equations. In each case you must (a) write the augmented matrix of the system and (b) use elementary row operations to rewrite the augmented matrix in reduced row-echelon form. Show your work neatly organized. (Your process may vary !!)

4. $2x + 3y = 31$
 $3x - 5z = -36$
 $4y - z = 8$

$$\begin{bmatrix} 2 & 3 & 0 & 31 \\ 3 & 0 & -5 & -36 \\ 0 & 4 & -1 & 8 \end{bmatrix} \xrightarrow{-1R_1 + R_2} \begin{bmatrix} 2 & 3 & 0 & 31 \\ 1 & -3 & -5 & -67 \\ 0 & 4 & -1 & 8 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & -5 & -67 \\ 2 & 3 & 0 & 31 \\ 0 & 4 & -1 & 8 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & -3 & -5 & -67 \\ 0 & 9 & 10 & 165 \\ 0 & 4 & -1 & 8 \end{bmatrix} \xrightarrow{-2R_3 + R_2} \begin{bmatrix} 1 & -3 & -5 & -67 \\ 0 & 1 & 12 & 149 \\ 0 & 4 & -1 & 8 \end{bmatrix} \xrightarrow{\substack{3R_2 + R_1 \\ -4R_2 + R_3}}$$

$$\begin{bmatrix} 1 & 0 & 31 & 380 \\ 0 & 1 & 12 & 149 \\ 0 & 0 & -49 & -588 \end{bmatrix} \xrightarrow{R_3 \div -49} \begin{bmatrix} 1 & 0 & 31 & 380 \\ 0 & 1 & 12 & 149 \\ 0 & 0 & 1 & 12 \end{bmatrix} \xrightarrow{\substack{-31R_3 + R_1 \\ -12R_3 + R_2}} \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 12 \end{bmatrix} \rightarrow \begin{matrix} x = 8 \\ y = 5 \\ z = 12 \end{matrix}$$

6. $2x + y + z = -3$
 $x + y - z = -4$
 $5x + 3y + z = 2$

$$\begin{bmatrix} 2 & 1 & 1 & -3 \\ 1 & 1 & -1 & -4 \\ 5 & 3 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & -1 & -4 \\ 2 & 1 & 1 & -3 \\ 5 & 3 & 1 & 2 \end{bmatrix} \xrightarrow{\substack{-2R_1 + R_2 \\ -5R_1 + R_3}} \begin{bmatrix} 1 & 1 & -1 & -4 \\ 0 & -1 & 3 & 5 \\ 0 & -2 & 6 & 22 \end{bmatrix} \xrightarrow{-1R_2}$$

$$\begin{bmatrix} 1 & 1 & -1 & -4 \\ 0 & 1 & -3 & -5 \\ 0 & -2 & 6 & 22 \end{bmatrix} \xrightarrow{\substack{-1R_2 + R_1 \\ 2R_2 + R_3}} \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -5 \\ 0 & 0 & 0 & 12 \end{bmatrix} \rightarrow \text{NO SOLUTION}$$

Clearly this is impossible. $0x + 0y + 0z$ cannot equal 12!