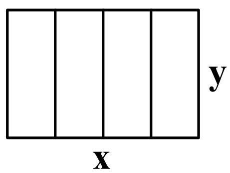
General Algebra 2 Worksheet #6 Unit 9 Selected Solutions

6. Sam wants to fence in a rectangular plot of land and to divide it into four equal areas using three lengths of fencing parallel to two opposite sides. If he has a total of 800 feet of fencing to work with, then find the dimensions that will maximize the total area enclosed. What is the maximum area?



Consider the diagram shown. Let x represent the length of the rectangular plot of land. Let y represent its width. Clearly, the total amount of fencing required is 2x + 5y.

Once again, to maximize the area, we must represent the area as a function of one variable.

A = xy where $2x + 5y = 800$

$$5y = -2x + 800$$

$$y = -0.4x + 160$$

Therefore, $A = f(x) = x(-0.4x + 160)$

$$A = f(x) = -0.4x^2 + 160x$$

The maximum area corresponds to the vertex of this function. There are two common methods used to find the vertex.

At the vertex, x = -B/2A

$$x = -160/(-0.8) = 200$$

The maximum area is

$$f(200) = -0.4(200)^2 + 160(200) = 16{,}000$$

$$A = -0.4(x^2 - 400x)$$

$$A - 16{,}000 = -0.4(x^2 - 400x + 40{,}000)$$

$$A - 16{,}000 = -0.4(x - 200)^2$$

The vertex is (200, <u>16,000</u>).

For maximum area, x = 200.

$$y = -0.4(200) + 160 = -80 + 160 = 80.$$

The plot with maximum area is 200 feet long and 80 feet wide.
The plot will have a maximum area of 16,000 square feet.