General Algebra 2 Worksheet #5 Unit 9 Selected Solutions

5. The owner of a large apartment building with fifty units has found that if the rent for each unit is \$360 per month, then all of the units will be rented. But one unit will become vacant for each increase of \$10 per month. What rate should be charged per month per unit in order to maximize the total monthly income? What is the maximum monthly income?

The challenge is to represent the total monthly income as a function of one variable. The total income can be determined using the equation

total income = (number of apartments rented)(price per apartment).

Consider the following table showing the relationship between number of apartments rented and the price per apartment. This table uses the given fact that 'one apartment will become vacant for each \$10 increase in the price.

number of apartments rented: ...	**50**	**49**	**48**	**47**	**in general 50 – x**
cost per apartment	**\$360**	**\$370**	**\$380**	**\$390**	**in general 360 + 10x**

The function for total income, T, is

$$T = (50 - x)(360 + 10x)$$
$$T = 18{,}000 + 500x - 360x - 10x^2$$
$$T = -10x^2 + 140x + 18{,}000$$

This is clearly a parabola, opening downward. Therefore, the maximum total income will correspond to the vertex.

At the vertex, $x = -B/2A$.

$x = -140/-20 = 7$

The maximum value of

$T = -10(7)^2 + 140(7) + 1800 = 18{,}490$

$$T - 18{,}000 = -10(x^2 - 14x)$$
$$T - 18{,}000 - 490 = -10(x^2 - 14x + 49)$$
$$T - 18{,}490 = -10(x - 7)^2$$

The vertex is (7, <u>18,490</u>).

Therefore, the maximum total income of \$18,490 is achieved when x = 7. The corresponding price per apartment is 360 + 10x = 360 + 10(7) = \$430.