General Algebra II Unit 5 Introduction to Linear Programming page 1

Consider the following problem.
A farming family wishes to plant some barley and some wheat. They can plant a maximum of 100 acres of barley and a maximum of 80 acres of wheat. However, they only have 120 acres of land available for planting. Barley costs \$20 per acre for seeds, and wheat costs \$30 per acre for seeds. However, they only have \$3000 available for seed costs. They expect a harvest of 1000 pounds per acre of barley and 3000 pounds per acre of wheat. How many acres of each crop should they plant to maximize their total harvest?

Solution: Let x represent the number of acres of barley that they plant.
Let y represent the number of acres of wheat that they plant.
The following inequalities represent the constraints given in the problem concerning the number of acres planted.

$x \leq 100$
$y \leq 80$
$x + y \leq 120$

Since the seed cost is \$20 per acre of barley, the total seed cost for the barley is 20x dollars. Similarly, the total seed cost for the wheat is 30y dollars. Since only \$3000 is available,

$20x + 30y \leq 3000$

Finally, since neither x nor y can be negative,

$x \geq 0$
$y \geq 0$

This set of six inequalities represent the <u>system of constraints</u> for the problem. We can graph this system to find the set of <u>feasible solutions</u> for the problem. To find the number of acres of each which should be planted in order to <u>maximize</u> the total production, we must first represent the total production as a function of x and y. Since 1000 pounds of barley is expected for each acre planted, 1000x (pounds) represents to total harvest of barley expected. Similarly, 3000y (pounds) represents the total harvest of wheat expected. Therefore,

$T = 1000x + 3000y$ (where T is the total harvest)

This relationship is called the <u>objective function</u>.

The principle used to solve problems like this one is stated as follows.

If the set of feasible solutions is represented by a convex polygonal region, and the objective function is a linear expression in the two variables (ax + by) , then its maximum (or minimum) value will occur at a vertex of the region.

Graphing method for solving a linear programming problem.

step 1. Graph the system of constraints, thus showing the set of feasible solutions.

step 2. Find the coordinates of each vertex of the region.

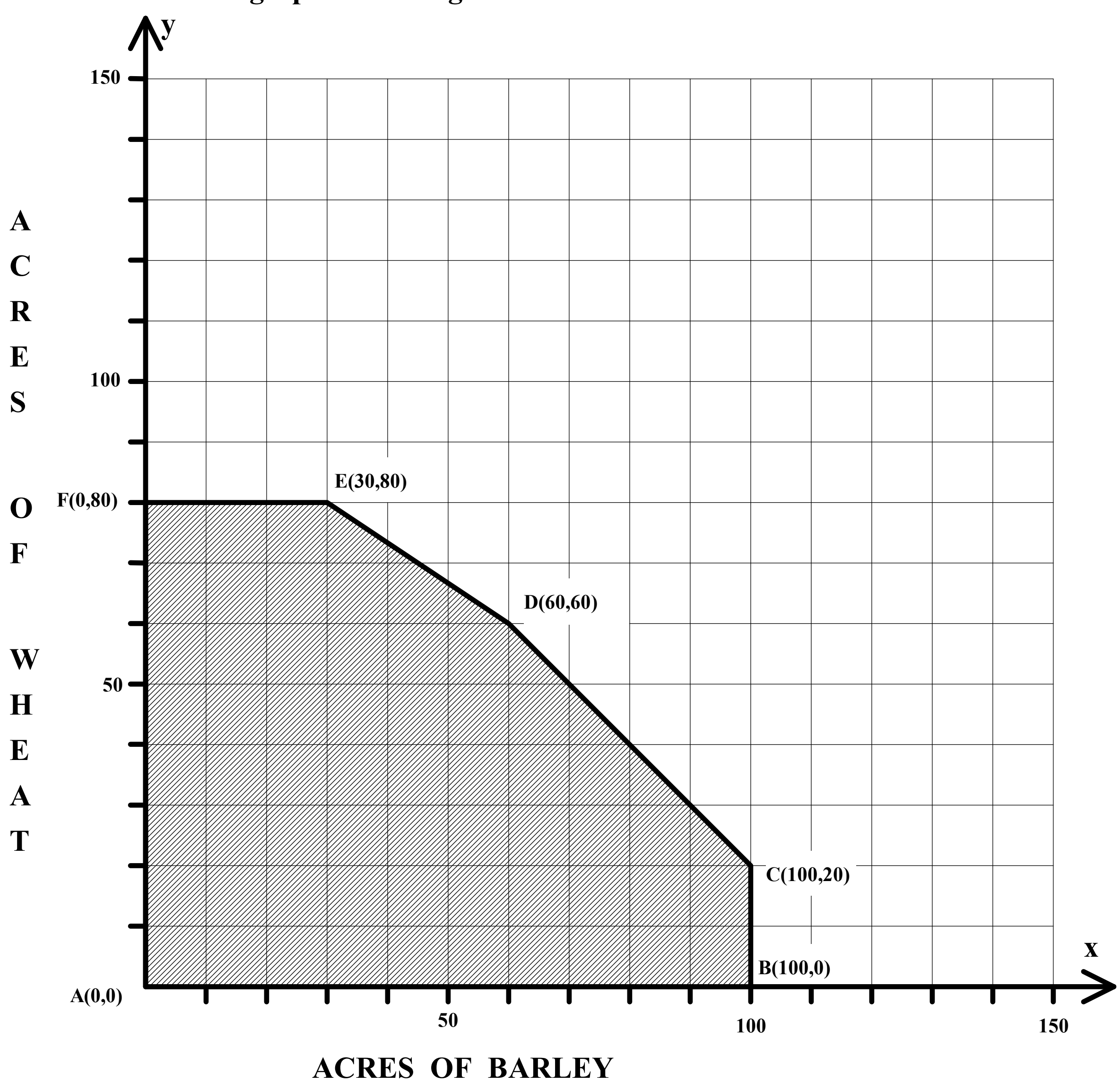
step 3. Evaluate the objective function at each vertex.

step 4. Use the vertex that corresponds to the maximum (or minimum) value to answer the question.

For the problem described for this lesson, the system of constraints was ...

$x \leq 100$	$x \leq 100$
$y \leq 80$	$y \leq 80$
$x + y \leq 120$	$y \leq -x + 120$
$20x + 30y \leq 3000$	$y \leq \frac{-2}{3}x + 100$
$x \geq 0$	$x \geq 0$
$y \geq 0$	$y \geq 0$

The graph of this region is below.



The vertices of the region of feasible solutions are:

A(0 , 0)
B(100 , 0)
C(100 , 20)
D(60 , 60)
E(30 , 80)
F(0 , 80)

The objective function $T = 1000x + 3000y$ should be evaluated at each vertex.

At A, $T = 1000(0) + 3000(0) = 0$
At B, $T = 1000(100) + 3000(0) = 100{,}000$
At C, $T = 1000(100) + 3000(20) = 160{,}000$
At D, $T = 1000(60) + 3000(60) = 240{,}000$
At E, $T = 1000(30) + 3000(80) = 270{,}000$
At F, $T = 1000(0) + 3000(80) = 240{,}000$

Clearly, the maximum total production corresponds to vertex E(30, 80).

Therefore, the farming family should plant 30 acres of barley and 80 acres of wheat.