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The Equation of a Line

In this part of the unit, a line will be described. Your job will be to write its equation. We will start with a review of the three types of lines and their most common equations.

Type 1: Horizontal Line	**Equation: $y = k$**
Type 2: Vertical Line	**Equation: $x = k$**
Type 3: Oblique Line	**Equation: $y = mx + b$ (slope-intercept equation)**

When writing the equation of a line, first determine the type of line, then find its equation. Horizontal and vertical lines should be fairly routine (especially once you have done a few practice problems). Oblique lines are the most challenging. Of course if both the slope and the y-intercept are known, then the slope-intercept equation can be written easily. In the event that the y-intercept is not given, however, there is another equation that proves to be very useful. It is called the point-slope equation. If you know a point on the line, represented by (x_1, y_1), and the slope of the line, represented by m, then an equation of the line is $y - y_1 = m(x - x_1)$. This equation is called a point-slope equation of the line. The slope-intercept equation can easily be derived from this equation.

Consider the following examples.

Find the equation of each line described. If the line is oblique, then write the slope-intercept equation.

1. The line through (3, -1) and (-2, -1)
Note that the two points have the same y-coordinates.
This tells us that the line is horizontal. **$y = -1$**

2. The line through (3, -1) and (3, 5)
Note that the two points have the same x-coordinates.
This tells us that the line is vertical. **$x = 3$**

3. The line through (-2, 5) with slope 0
The fact that the slope is 0 tells us that the line is horizontal.
The line contains the point (-2, 5). **$y = 5$**

4. The line through (-2, 5) with "no slope"
The phrase "no slope" is commonly used to describe the slope of a vertical line. The slope of a vertical line is undefined.
The line contains the point (-2, 5). **$x = -2$**

5. The line with slope 3 and y-intercept -1
This line is oblique. We are given that $m = 3$ and $b = -1$. **$y = 3x - 1$**

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Find the equation of each line described. If the line is oblique, then write the slope-intercept equation.

6. The line with slope -2 through the point (0, 5)
This line is oblique. We are given that m = -2.
Since the line contains the point (0, 5), b = 5. **$y = -2x + 5$**
Note: The y-intercept, b, is the value of y when x = 0.

7. The line through (0, 1) and (2, 5)
This line is oblique. We must find the slope first.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{2 - 0} = \frac{4}{2} = 2$$

Since the line contains the point (0, 1), b = 1. **$y = 2x + 1$**

8. The line with slope 4 through the point (3, 1)
The line is oblique. We are given that m = 4.
We are not given the y-intercept. We will use the point-slope equation. $y - y_1 = m(x - x_1)$
Since the line contains the point (3, 1), $x_1 = 3$ **and** $y_1 = 1$.
The point-slope equation becomes $y - 1 = 4(x - 3)$.
Solving for y, we get $y - 1 = 4x - 12$
$y = 4x - 11$ **$y = 4x - 11$**

9. The line through (-2, 3) and (2, -5)
This line is oblique. We must find the slope first.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{2 - -2} = \frac{-8}{4} = -2$$

We are not given the y-intercept. We will use the point-slope equation. $y - y_1 = m(x - x_1)$
Since the line contains the point (2, -5), $x_1 = 2$ **and** $y_1 = -5$.
The point-slope equation becomes $y - -5 = -2(x - 2)$.
Solving for y, we get $y + 5 = -2x + 4$
$y = -2x - 1$ **$y = -2x - 1$**
Note: The point (-2, 3) would have worked as well.
Using this point, $x_1 = -2$ **and** $y_1 = 3$.
The point-slope equation becomes $y - 3 = -2(x - -2)$.
Solving for y, we get $y - 3 = -2(x + 2)$
$y - 3 = -2x - 4$
$y = -2x - 1$ **!!!**