General Algebra 2 Worksheet #8 Unit 10 Selected Solutions

3. Find the sum of the first 50 terms of the sequence defined by $a_n = 4n - 1$.

$a_1 = 4(1) - 1 = 3$ This is an arithmetic series. $S_n = \frac{n}{2}(a_1 + a_n)$

$a_2 = 4(2) - 1 = 7$ $a_{50} = 4(50) - 1 = 199$ $S_{50} = \frac{50}{2}(a_1 + a_{50})$

$a_3 = 4(3) - 1 = 11$ $S_{50} = 25(3 + 199) = 25(202) = 5{,}050$

5. Find the sum of the first 10 terms of the sequence defined by $a_{n+1} = -2a_n$ where $a_1 = -1$.

$a_1 = -1$ This is a geometric series. $S_n = \frac{a_1(1 - r^n)}{1 - r}$

$a_2 = (-2)(-1) = 2$ $a_1 = -1$ $r = -2$ $n = 10$

$a_3 = (-2)(2) = -4$ $S_{10} = \frac{-1[1 - (-2)^{10}]}{1 - (-2)} = 341$

9. Evaluate the series $5 + 8 + 11 + 14 + ... + 701$.

This is an arithmetic series.

$a_1 = 5$ $d = 3$ $a_n = 701$ $S_n = \frac{n}{2}(a_1 + a_n)$

$a_n = a_1 + (n - 1)d$ $3n + 2 = 701$ $S_{233} = \frac{233}{2}(5 + 701)$

$a_n = 5 + (n - 1)3$ $3n = 699$

$a_n = 5 + 3n - 3 = 3n + 2$ $n = 233$ $S_{233} = (116.5)(706) = 82{,}249$

12. $\sum_{k=1}^{5} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 1 + 4 + 9 + 16 + 25 = 55$

15. $\sum_{i=1}^{\infty} (2)\left(\frac{2}{3}\right)^{(i-1)} = (2)\left(\frac{2}{3}\right)^0 + (2)\left(\frac{2}{3}\right)^1 + (2)\left(\frac{2}{3}\right)^2 + ...$

infinite geometric series $S = \frac{a_1}{1 - r}$

$a_1 = 2$ $r = \frac{2}{3}$

$S = \frac{2}{1 - \frac{2}{3}} = \frac{2}{\frac{1}{3}} = (2)(3) = 6$

18. A job has a starting salary of \$14,000 with a guaranteed increase of 3% per year. Find the total salary for the first sixteen years.

geometric series

$a_1 = 14{,}000$ $r = 1.03$ $n = 16$

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

$S_{16} = \frac{14{,}000(1 - 1.03^{16})}{1 - 1.03} \approx 282{,}196.34$

The total salary is about \$282,196.