

Calculus Worksheet #6 Unit 9 Selected Solutions

Integrate each of the following.

$$1. \int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2\cos \theta d\theta}{2\cos \theta} = \int d\theta = \theta + C$$

$$\text{let } \theta = \sin^{-1}\left(\frac{x}{2}\right) \qquad = \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$x = 2\sin \theta$$

$$\sqrt{4-x^2} = 2\cos \theta$$

$$dx = 2\cos \theta d\theta$$

$$4. \int \sqrt{4-x^2} dx = \int (2\cos \theta)(2\cos \theta d\theta) = 4 \int \cos^2 \theta d\theta$$

$$\text{let } \theta = \sin^{-1}\left(\frac{x}{2}\right) \qquad = 2 \int [1 + \cos(2\theta)] d\theta = 2 \int d\theta + 2 \int \cos(2\theta) d\theta$$

$$x = 2\sin \theta$$

$$\sqrt{4-x^2} = 2\cos \theta$$

$$dx = 2\cos \theta d\theta$$

$$= 2\theta + \sin(2\theta) + C = 2\theta + 2\sin \theta \cos \theta + C$$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x}{2} \sqrt{4-x^2} + C$$

$$5. \int \frac{dx}{x^2+9} = \int \frac{3\sec^2 \theta d\theta}{9\sec^2 \theta} = \frac{1}{3} \int d\theta = \frac{1}{3} \theta + C$$

$$\text{let } \theta = \tan^{-1}\left(\frac{x}{3}\right) \qquad = \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$x = 3\tan \theta$$

$$x^2+9 = 9\sec^2 \theta$$

$$dx = 3\sec^2 \theta d\theta$$