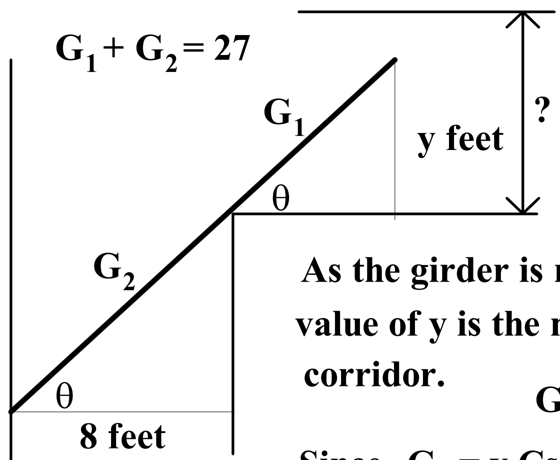
Calculus Worksheet #6 Unit 8 Selected Solutions

4. A 27 foot girder has to be moved down a corridor that is 8 feet wide and around a right angle turn into another corridor. What is the minimum width of the second corridor? (Ignore the width of the girder.)



As the girder is moved around the corner, the maximum value of y is the minimum width necessary for the second corridor.

$$G_1 + G_2 = 27$$

Since $G_1 = y \operatorname{Csc}\theta$ **and** $G_2 = 8\operatorname{Sec}\theta$

$$y \operatorname{Csc}\theta + 8\operatorname{Sec}\theta = 27$$

$$y = 27\operatorname{Sin}\theta - 8\operatorname{Tan}\theta$$

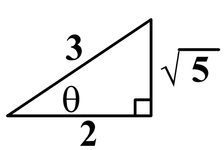
$$y' = 27\operatorname{Cos}\theta - 8\operatorname{Sec}^2\theta$$

$y' = 0 \longrightarrow 27\operatorname{Cos}\theta - 8\operatorname{Sec}^2\theta = 0$

$$\operatorname{Cos}^3\theta = \frac{8}{27}$$

$$\operatorname{Cos}\theta = \frac{2}{3}$$

Note: $y'' = -27\operatorname{Sin}\theta - 16\operatorname{Sec}^2\theta\operatorname{Tan}\theta$
Clearly $y'' < 0$. Therefore, the stationary point corresponds to a maximum value of y.



$\operatorname{Sec}\theta = \frac{3}{2}$ **and** $\operatorname{Sin}\theta = \frac{\sqrt{5}}{3}$

$G_2 = 8\operatorname{Sec}\theta = 12$ **feet** $\longrightarrow$ $G_1 = 15$ **feet**

$y = G_1\operatorname{Sin}\theta = 5\sqrt{5} \approx 11.2$ **feet**

The second corridor must be at least 11.2 feet wide.