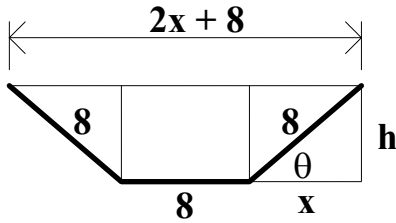


Calculus Worksheet #5 Unit 8 Selected Solutions

4. A strip of metal 24 inches wide is to be made into a gutter by bending up an 8 inch strip along each side. What should the angle between the sides and the bottom be in order to maximize the volume the gutter can hold?



To maximize the volume of the gutter, the cross-sectional area must be maximum. The area, A , of a trapezoid is found using the formula below.

$$A = \frac{(b_1 + b_2)h}{2}$$

$$b_1 = 8 \quad \longrightarrow \quad A = (x + 8)h$$

$$b_2 = 2x + 8$$

$$\text{Since } x = 8\cos \theta \text{ and } h = 8\sin \theta ,$$

$$A = (8\cos \theta + 8)(8\sin \theta)$$

$$A = 64(\cos \theta \sin \theta + \sin \theta) \quad \text{where } 0 < \theta \leq 90^\circ$$

$$A' = 64[\cos^2 \theta - \sin^2 \theta + \cos \theta]$$

$$A' = 0 \quad \longrightarrow \quad \cos^2 \theta - 1 + \cos^2 \theta + \cos \theta = 0$$

$$2\cos^2 \theta + \cos \theta - 1 = 0$$

$$(2\cos \theta - 1)(\cos \theta + 1) = 0$$

$$\cos \theta = 1/2 \quad \text{or} \quad \cos \theta = -1$$

$$\theta = 60^\circ \quad \text{or} \quad \theta = 180^\circ$$

The volume will be a maximum if the angle is 120° .