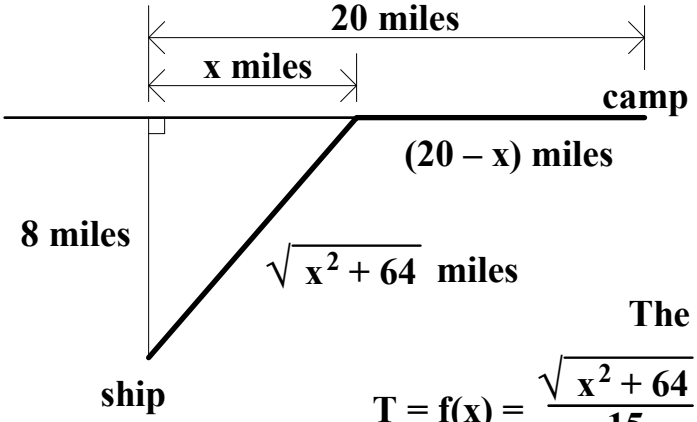
Calculus Worksheet #4 Unit 8 Selected Solutions

2. A messenger is to go ashore from a ship that is 8 miles offshore and deliver a message to a camp that is 20 miles up the beach from the point nearest the ship. He can go 15 mph by boat and will be met by a jeep that can go 25 mph over the beach. Where should he land to complete the trip in as short a time as possible?



Let T_1 represent the time traveling from the ship to the shore, and let T_2 represent the time traveling up the shore to the camp.

$$T_1 = \frac{\sqrt{x^2 + 64}}{15} \quad \text{and} \quad T_2 = \frac{20 - x}{25}$$

The total time, T, is given by the function

$$T = f(x) = \frac{\sqrt{x^2 + 64}}{15} + \frac{20 - x}{25} \quad \text{where } 0 \le x \le 20$$

$$f'(x) = \frac{x}{15\sqrt{x^2 + 64}} - \frac{1}{25}$$

$$f'(x) = 0 \longrightarrow \frac{x}{15\sqrt{x^2 + 64}} = \frac{1}{25} \longrightarrow$$

$$25x = 15\sqrt{x^2 + 64}$$
$$5x = 3\sqrt{x^2 + 64}$$
$$25x^2 = 9(x^2 + 64)$$
$$25x^2 = 9x^2 + 576$$
$$16x^2 = 576$$
$$x^2 = 36$$
$$x = 6$$

x	T
0	1.333 hours
6	1.227 hours
20	1.436 hours

To minimize the total time, he should land at a point that is 14 miles from the camp.