Calculus Worksheet #3 Unit 8 Selected Solutions

4. $\sqrt{15.8}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$x = 16$ $\quad \triangle x = -0.2$

$\sqrt{15.8} \approx \sqrt{16} + \left(\frac{1}{2\sqrt{16}}\right)\left(\frac{-1}{5}\right)$

$\sqrt{15.8} \approx 4 + \frac{-1}{40}$

$\sqrt{15.8} \approx \frac{159}{40}$

7. $\sqrt[3]{64.2}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt[3]{x}$ $\quad f'(x) = \frac{1}{3}x^{\frac{-2}{3}}$

$x = 64$ $\quad \triangle x = 0.2$

$\sqrt[3]{64.2} \approx \sqrt[3]{64} + \frac{1}{3}(64)^{\frac{-2}{3}}(0.2)$

$\sqrt[3]{64.2} \approx 4 + \left(\frac{1}{3}\right)\left(\frac{1}{16}\right)\left(\frac{1}{5}\right)$

$\sqrt[3]{64.2} \approx 4 + \frac{1}{240}$

$\sqrt[3]{64.2} \approx \frac{961}{240}$

10. A steel cabinet is to be in the shape of a cube, measuring 20 inches on each side, with a greatest possible error allowed of 0.1 inches. (Measurements like this can be written as 20 ± 0.1 inches.) What is the greatest possible error that can result in the volume of the cabinet?

If x represents the length of each side of the cabinet, then any error in this distance can be represented as $\triangle x$. The resulting error in the volume of the cabinet can be represented as $\triangle V$.
Clearly, for this cabinet, $V = f(x) = x^3$. $\triangle V$ can be approximated using the differential $dv = f'(x)\,dx$. $f'(r) = 3x^2$. Therefore, $\triangle V \approx 3x^2\,dx = 3x^2\,\triangle x$. In this problem $x = 20$ inches and $\triangle x = 0.1$ inches. Therefore, $\triangle V \approx 3(20)^2(0.1) = 120$.

The greatest possible error in the volume is about 120 cubic inches.