Calculus Worksheet #2 Unit 8 Selected Solutions

4. $\sqrt{25.1}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt{x} \qquad f'(x) = \frac{1}{2\sqrt{x}}$

$x = 25 \qquad \Delta x = 0.1$

$$\sqrt{25.1} \approx \sqrt{25} + \left(\frac{1}{2\sqrt{25}}\right)\left(\frac{1}{10}\right)$$

$$\sqrt{25.1} \approx 5 + \frac{1}{100}$$

$$\sqrt{25.1} \approx 5.01$$

6. $\sqrt[3]{62}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt[3]{x} \qquad f'(x) = \frac{1}{3}x^{\frac{-2}{3}}$

$x = 64 \qquad \Delta x = -2$

$$\sqrt[3]{62} \approx \sqrt[3]{64} + \frac{1}{3}(64)^{\frac{-2}{3}}(-2)$$

$$\sqrt[3]{62} \approx 4 + \left(\frac{1}{3}\right)\left(\frac{1}{16}\right)(-2)$$

$$\sqrt[3]{62} \approx 4 + \frac{-1}{24}$$

$$\sqrt[3]{62} \approx \frac{95}{24}$$

8. Find the approximate change in sin x per 1 degree change in x for each of the following values of x.

a) $x = 0$ b) $x = \pi/6$ c) $x = \pi/3$ d) $x = \pi/2$

If $y = \sin x$, then the 'change in sin x' can be represented by $\triangle y$. This can be approximated using $dy = f'(x)\,dx$. Clearly, $f'(x) = \cos x$. Therefore, $\triangle y \approx \cos x\ dx = (\cos x)(\triangle x)$. Since the problem asks for the change in the sin x 'per 1 degree change in x', $\triangle x = 1° = \pi/180$.
For each given value of x, the value of $\triangle y$ can be approximated using the equation $\triangle y \approx (\cos x)(\pi/180)$.

a) If $x = 0$, $\triangle y \approx (\cos 0)(\pi/180) = \pi/180 \approx .0175$
c) If $x = \pi/3$, $\triangle y \approx (\cos \pi/3)(\pi/180) = (1/2)(\pi/180) = \pi/360 \approx .00873$