Calculus Worksheet #1 Unit 8 Selected Solutions

5. $y = \tan(5x)$

$dy = f'(x)\,dx$

$dy = 5\sec^2(5x)\,dx$

7. $\sqrt{50}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$x = 49$ $\quad \Delta x = 1$

$\sqrt{50} \approx \sqrt{49} + \frac{1}{2\sqrt{49}}(1)$

$\sqrt{50} \approx 7 + \frac{1}{14}$

$\sqrt{50} \approx \frac{99}{14}$

10. $\sqrt[3]{7.9}$

$f(x + \triangle x) \approx f(x) + f'(x)\,dx$

$f(x) = \sqrt[3]{x}$ $\quad f'(x) = \frac{1}{3}x^{\frac{-2}{3}}$

$x = 8$ $\quad \Delta x = -0.1$

$\sqrt[3]{7.9} \approx \sqrt[3]{8} + \frac{1}{3}(8)^{\frac{-2}{3}}(-0.1)$

$\sqrt[3]{7.9} \approx 2 + (\frac{1}{3})(\frac{1}{4})(\frac{-1}{10})$

$\sqrt[3]{7.9} \approx 2 + \frac{-1}{120}$

$\sqrt[3]{7.9} \approx \frac{239}{120}$

11. A brass sphere with a diameter of 1 inch is given a gold plating which is .005 inches thick. What is the approximate volume of gold used?
(For a sphere, $V = (4/3)\pi r^3$.)

Clearly, the volume of gold needed is the increase in the volume of the sphere caused by the gold plating. Since, $V = f(r) = (4/3)\pi r^3$, the increase in the volume, $\triangle V$, can be approximated using the differential $dV = f'(r)\,dr$. $f'(r) = 4\pi r^2$. Therefore, $\triangle V \approx 4\pi r^2 dr$. Since the diameter is 1 inch, $r = 0.5$ in. Since the gold plating is to be 0.005 inches thick, $dr = \triangle r = 0.005$ inches. Therefore, $\triangle V \approx 4\pi(.5)^2(0.005) = .005\pi \approx 0.0157$.

The volume of the gold is about 0.0157 cubic inches.