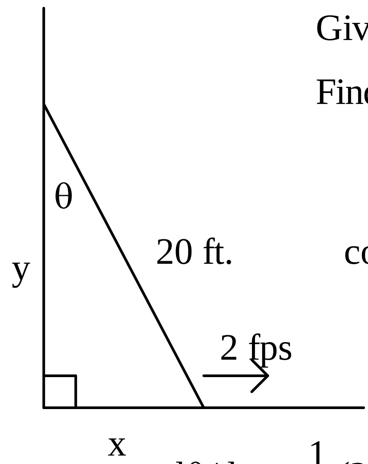
Calculus Worksheet #3 Unit 7 Selected Solutions

1. A 20-foot ladder stands upright against a vertical wall. If the lower end of the ladder is pulled away from the wall (on level ground) at the rate of 2 feet per second (fps), then how fast is the angle between the ladder and the wall increasing at the instant the top of the ladder is 16 feet above the ground? (Express your answer in degrees per second rounded to 2 significant figures.)



Given: $dx/dt = 2$ fps

Find: $d\theta/dt$ when $y = 16$ feet

$$\sin\theta = \frac{x}{20}$$

$$\cos\theta\,(d\theta/dt) = \frac{1}{20}(dx/dt)$$

$$d\theta/dt = \frac{1}{20}(dx/dt)\sec\theta$$

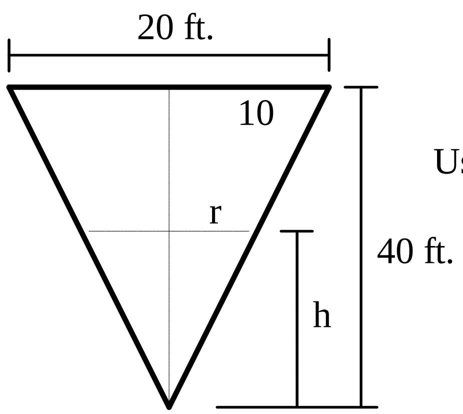
When $y = 16$ ft., $\sec\theta = 1.25$

$$d\theta/dt = \frac{1}{20}(2)(1.25) = .125 \text{ radians per second} \approx \mathbf{7.2^\circ\ per\ second}$$

The angle is increasing at about 7.2 degrees per second.

4. A conical reservoir with its axis vertical is 40 feet deep and 20 feet across the top. If water is being added at the rate of 20 cubic feet per minute, then how fast is the water rising the instant it is 5 feet deep? (Express your final answer in inches per second rounded to 2 significant digits.)

Not drawn to scale.



Given: $dV/dt = 20$ cu. ft. per minute

Find: dh/dt when $h = 5$ ft.

For a cone: $V = \frac{1}{3}\pi r^2 h$

Using similar triangles, $\frac{r}{h} = \frac{1}{4} \Rightarrow r = \frac{1}{4}h$

$$V = \frac{1}{48}\pi h^3$$

$$dV/dt = \frac{1}{16}\pi h^2(dh/dt)$$

$$dh/dt = \frac{16(dV/dt)}{\pi h^2}$$

When $h = 5$ ft., $dh/dt = \frac{(16)(20)}{25\pi} = \frac{64}{5\pi}$ ft. per min. $\approx$ **0.8 inches per second**

The water is rising at about 0.8 inches per second.