Calculus Notes Unit 7 Related Rates

Notation: If x represents any variable quantity, then dx/dt represents the rate that x is changing.

Examples:

1. Given any distance s, say from a moveable point A on a line to a fixed point B on the line, ds/dt is the rate that s is changing (the velocity of point A).

2. Given any volume V, say of water in a tank, dV/dt is the rate that this volume is changing (the rate that water is being added to the tank)

Solving Related Rate Problems:

Step 1: Analyze the problem stating clearly which rate(s) you are given and which rate(s) you are asked to find.

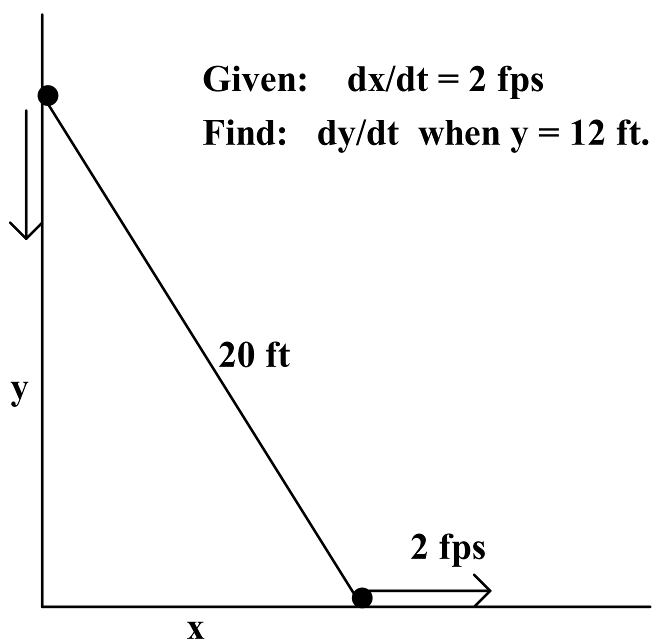
Step 2: Write an equation relating the variables involved (and only those variables).

Step 3: Differentiate each side of the equation with respect to time, thus obtaining an equation relating the rates involved.

Step 4: Solve the equation for the desired rate in terms of the other rates and/or variables.

Step 5. Substitute in the current values of the rates and/or variables to find the desired result.

Example: A 20-foot ladder stands upright against a vertical wall. If the lower end of the ladder is pulled away from the wall (on level ground) at the rate of 2 feet per second (fps), then how fast is the top of the ladder coming down the wall at the instant it is 12 feet above the ground?



Given: dx/dt = 2 fps

Find: dy/dt when y = 12 ft.

$$x^2 + y^2 = 400$$

$$2x(dx/dt) + 2y(dy/dt) = 0$$

$$2y(dy/dt) = -2x(dx/dt)$$

$$dy/dt = \frac{-x(dx/dt)}{y}$$

When y = 12 ft. $x^2 + 144 = 400$

$$x^2 = 256$$

$$x = 16$$

$$dy/dt = \frac{-16(2)}{12}$$

$$dy/dt = -8/3$$

Answer: The ladder is coming down the wall at 8/3 fps (2 ft. 8 in. per second).