Calculus   Worksheet #5   Unit 5   Selected Solutions

Find $\dfrac{d^2y}{dx^2}$ for each of the following. Simplify your answer as much as possible.

2. $x^2 - 2y^2 = 7$

$2x - (4y)dy/dx = 0$

$(-4y)dy/dx = -2x$

$dy/dx = \dfrac{x}{2y}$

$\dfrac{d^2y}{dx^2} = \dfrac{2y(1) - x(2)dy/dx}{4y^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{2y - 2x\left(\frac{x}{2y}\right)}{4y^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{2y - \frac{x^2}{y}}{4y^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{2y^2 - x^2}{4y^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{-7}{4y^3}$

4. $y^2 - 2x - 2y = 1$

$(2y)dy/dx - 2 - (2)dy/dx = 0$

$(2y - 2)dy/dx = 2$

$dy/dx = \dfrac{1}{y-1} = (y-1)^{-1}$

$\dfrac{d^2y}{dx^2} = -1(y-1)^{-2}(dy/dx)$

$\dfrac{d^2y}{dx^2} = -1(y-1)^{-2}(y-1)^{-1}$

$\dfrac{d^2y}{dx^2} = -1(y-1)^{-3} = \dfrac{-1}{(y-1)^3}$

6. $x^2 + xy + y^2 = 10$

$2x + (x)dy/dx + y(1) + (2y)dy/dx = 0$

$(x + 2y)dy/dx = -2x - y$

$dy/dx = \dfrac{-2x - y}{x + 2y}$

$\dfrac{d^2y}{dx^2} = \dfrac{(x + 2y)(-2 - dy/dx) - (-2x - y)(1 + 2(dy/dx))}{(x + 2y)^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{(x + 2y)(-2 - \left(\frac{-2x-y}{x+2y}\right)) - (-2x - y)(1 + 2\left(\frac{-2x-y}{x+2y}\right))}{(x + 2y)^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{(x + 2y)\left(\frac{-3y}{x + 2y}\right) - (-2x - y)\left(\frac{-3x}{x + 2y}\right)}{(x + 2y)^2}$

$\dfrac{d^2y}{dx^2} = \dfrac{-3xy - 6y^2 - 6x^2 - 3xy}{(x + 2y)^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{-6x^2 - 6xy - 6y^2}{(x + 2y)^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{-6(x^2 + xy + y^2)}{(x + 2y)^3}$

$\dfrac{d^2y}{dx^2} = \dfrac{-60}{(x + 2y)^3}$