Calculus Worksheet #5 Unit 4 Selected Solutions

This is the key relationship between the three functions, where s represents position, v represents velocity, and a represents acceleration. If s = f(t) , then v = f '(t) and a = f ''(t).

2. A stone is propelled downward from a point that is 200 feet above the ground. If the initial velocity of the stone is 40 fps, then a) how long will it take for the stone to hit the ground, and b) what will be its speed as it hits the ground?

Let s represent the distance the stone is above the ground. This establishes the positive direction as upward. Since the acceleration due to gravity near the earth's surface is about 32 feet per second per second downward, a = f ''(t) = -32.

$$v = f'(t) = \int -32\,dt = -32t + C$$

Clearly, the constant here is the value of v when t = 0, the initial velocity. $v = -32t + v_i$. Since the stone was propelled downward at 40 fps, $v_i = -40$ feet per second.

Therefore, v = f '(t) = -32t – 40.

$$s = f(t) = \int (-32t - 40)dt = -16t^2 - 40t + C$$

The constant here is the initial value of s. $s = -16t^2 - 40t + s_i$. Since the stone was propelled downward from a point that was 200 feet above the ground, $s_i = 200$ feet.

Therefore, $s = f(t) = -16t^2 - 40t + 200$.

a) How long will it take for the stone to hit the ground?

Find the value of t when s = 0.

$$-16t^2 - 40t + 200 = 0$$
$$2t^2 + 5t - 25 = 0$$
$$(2t - 5)(t + 5) = 0$$
$$2t - 5 = 0 \text{ or } t + 5 = 0$$
$$t = 2.5 \text{ of } t = -5$$

The stone will hit the ground in 2.5 seconds.

b) What will be its speed as it hits the ground?

Find the value of v when t = 2.5 seconds.

$$v = f'(t)\ -32t - 40$$

When t = 2.5 seconds, v = f '(2.5) = -32(2.5) – 40 = -120 fps

The stone is moving downward at 120 feet per second as it hits the ground.