Calculus Worksheet #4 Unit 4 Selected Solutions

This is the key relationship between the three functions, where s represents position, v represents velocity, and a represents acceleration. If s = f(t) , then v = f '(t) and a = f ''(t).

3. The velocity at time t of a particle moving on a straight line is $v = 3t^2 - 9t + 6$ (fps) where $t \geq 0$.

a. Express the acceleration, a, of the particle as a function of t. $a = 6t - 9$

Given: $v = f'(t) = 3t^2 - 9t + 6$.

$$a = f''(t) = 6t - 9$$

b. Find the acceleration of the particle when t = 3s.

When t = 3 s, $a = f''(3) = 6(3) - 9 = 9$

When t = 3 s, the acceleration is 9 feet per second per second.

c. If s is the distance that the particle is from its starting point, then express s as a function of t.

Given: $v = f(t) = 3t^2 - 9t + 6$

$$s = f(t) = \int (3t^2 - 9t + 6)dt$$

$$s = f(t) = t^3 - \frac{9}{2}t^2 + 6t + C$$

Since s is the distance from starting position, f(0) = 0! Therefore C = 0.

$$s = f(t) = t^3 - \frac{9}{2}t^2 + 6t$$

d. When will the particle be at rest? What is its position when it is at rest relative to its starting point?

The particle is at rest when $v = 3t^2 - 9t + 6 = 0$.

$$t^2 - 3t + 2 = 0$$
$$(t - 1)(t - 2) = 0$$
$$t = 1 \text{ or } t = 2$$

When t = 1, s = f(1) = 2.5. When t = 2, s = f(2) = 2.

The particle is at rest after 1 second 2.5 feet from its starting position. It is at rest again after 2 seconds 2 feet (in the same direction) from its starting position.

e. How far does the particle move from t = 3s to t = 5s?

When t = 3s, s = f(3) = 4.5 feet. When t = 5s, s = f(5) = 42.5 feet. Since v > 0 during the entire time interval, the particle is moving in the same direction for the entire interval. Therefore the distance moved is f(5) – f(3) = 42.5 – 4.5.

The particle moved 38 feet during the time interval.