Calculus Worksheet #2 Unit 3 Selected Solutions

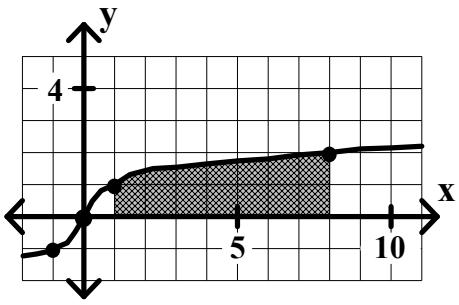
2. $\int_1^8 \sqrt[3]{x}\,dx$

$a = 1$
$b = 8$
$n = 5$

$$\Delta x = \frac{b-a}{n} = \frac{8-1}{5} = 1.4$$

$a = x_0 = 1$
$x_1 = 2.4$
$x_2 = 3.8$
$x_3 = 5.2$
$x_4 = 6.6$
$b = x_5 = 8$

$x_1^* = 1.7$
$x_2^* = 3.1$
$x_3^* = 4.5$
$x_4^* = 5.9$
$x_5^* = 7.3$



$$S_L = \sum_{i=1}^{n} f(x_{i-1})\Delta x = \sum_{i=1}^{5} \sqrt[3]{x_{i-1}}\,\Delta x$$

$$S_U = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{5} \sqrt[3]{x_i}\,\Delta x$$

$$S_M = \sum_{i=1}^{n} f(x_i^*)\Delta x = \sum_{i=1}^{5} \sqrt[3]{x_i^*}\,\Delta x$$

$$S_L = \sqrt[3]{x_0}\Delta x + \sqrt[3]{x_1}\Delta x + \sqrt[3]{x_2}\Delta x + \sqrt[3]{x_3}\Delta x + \sqrt[3]{x_4}\Delta x =$$
$$= [\sqrt[3]{x_0} + \sqrt[3]{x_1} + \sqrt[3]{x_2} + \sqrt[3]{x_3} + \sqrt[3]{x_4}\,]\Delta x =$$
$$= [\sqrt[3]{1} + \sqrt[3]{2.4} + \sqrt[3]{3.8} + \sqrt[3]{5.2} + \sqrt[3]{6.6}\,]\ (1.4) \approx (7.51)(1.4) = 10.51$$

$$S_U = \sqrt[3]{x_1}\Delta x + \sqrt[3]{x_2}\Delta x + \sqrt[3]{x_3}\Delta x + \sqrt[3]{x_4}\Delta x + \sqrt[3]{x_5}\Delta x =$$
$$= [\sqrt[3]{x_1} + \sqrt[3]{x_2} + \sqrt[3]{x_3} + \sqrt[3]{x_4} + \sqrt[3]{x_5}\,]\Delta x =$$
$$= [\sqrt[3]{2.4} + \sqrt[3]{3.8} + \sqrt[3]{5.2} + \sqrt[3]{6.6} + \sqrt[3]{8}\,]\ (1.4) \approx (8.51)(1.4) = 11.91$$

$$S_M = \sqrt[3]{x_1^*}\Delta x + \sqrt[3]{x_2^*}\Delta x + \sqrt[3]{x_3^*}\Delta x + \sqrt[3]{x_4^*}\Delta x + \sqrt[3]{x_5^*}\Delta x =$$
$$= [\sqrt[3]{x_1^*} + \sqrt[3]{x_2^*} + \sqrt[3]{x_3^*} + \sqrt[3]{x_4^*} + \sqrt[3]{x_5^*}\,]\Delta x =$$
$$= [\sqrt[3]{1.7} + \sqrt[3]{3.1} + \sqrt[3]{4.5} + \sqrt[3]{5.9} + \sqrt[3]{7.3}\,]\ (1.4) \approx (8.05)(1.4) = 11.27$$