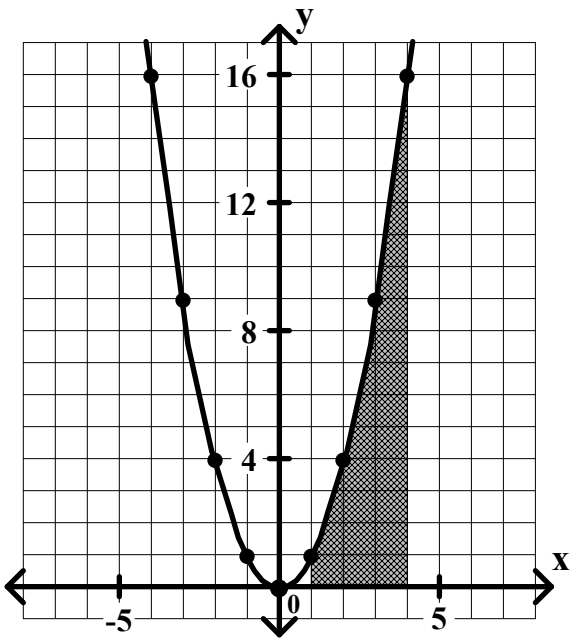
Calculus Worksheet #1 Unit 3 Selected Solutions

1. $\int_1^4 x^2 dx$

$a = 1$
$b = 4$
$n = 5$

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{5} = 0.6$$



$a = x_0 = 1$
$x_1 = 1.6$ $\quad x_1^* = 1.3$
$x_2 = 2.2$ $\quad x_2^* = 1.9$
$x_3 = 2.8$ $\quad x_3^* = 2.5$
$x_4 = 3.4$ $\quad x_4^* = 3.1$
$b = x_5 = 4$ $\quad x_5^* = 3.7$

$$S_L = \sum_{i=1}^{n} f(x_{i-1})\Delta x = \sum_{i=1}^{5} (x_{i-1})^2 \Delta x$$

$$S_U = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{5} (x_i)^2 \Delta x$$

$$S_M = \sum_{i=1}^{n} f(x_i^*)\Delta x = \sum_{i=1}^{5} (x_i^*)^2 \Delta x$$

$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x =$
$= [f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4)]\Delta x =$
$= [(x_0)^2 + (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2]\Delta x =$
$= [(1)^2 + (1.6)^2 + (2.2)^2 + (2.8)^2 + (3.4)^2](0.6) = [27.8](0.6) = 16.68$

$S_U = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x =$
$= [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]\Delta x =$
$= [(x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 + (x_5)^2]\Delta x =$
$= [(1.6)^2 + (2.2)^2 + (2.8)^2 + (3.4)^2 + (4)^2](0.6) = [42.8](0.6) = 25.68$

$S_M = f(x^*_1)\Delta x + f(x^*_2)\Delta x + f(x^*_3)\Delta x + f(x^*_4)\Delta x + f(x^*_5)\Delta x =$
$= [f(x^*_1) + f(x^*_2) + f(x^*_3) + f(x^*_4) + f(x^*_5)]\Delta x =$
$= [(x^*_1)^2 + (x^*_2)^2 + (x^*_3)^2 + (x^*_4)^2 + (x^*_5)^2]\Delta x =$
$= [(1.3)^2 + (1.9^2 + (2.5)^2 + (3.1)^2 + (3.7)^2](0.6) = [34.85](0.6) = 20.91$