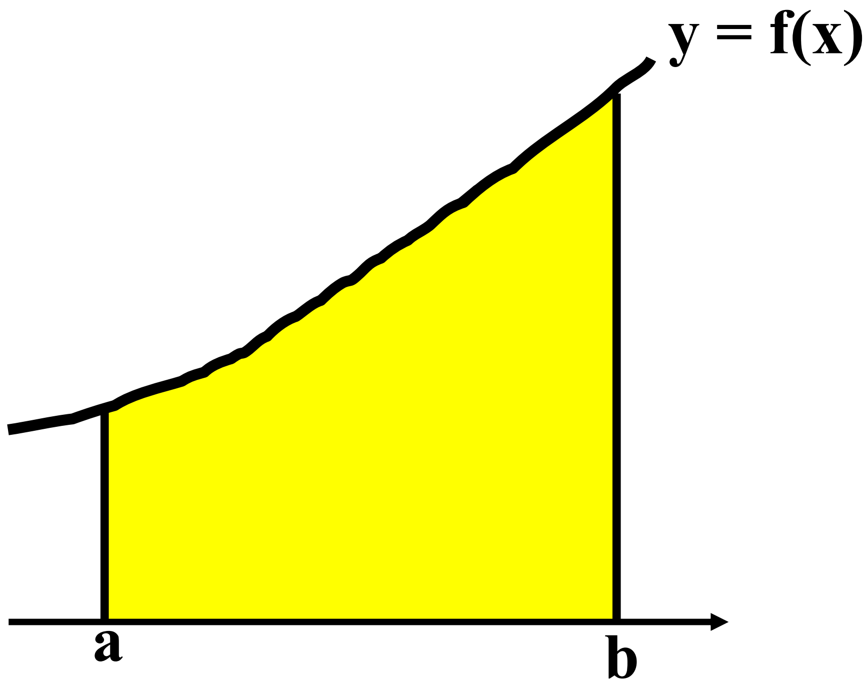
Calculus Lesson #2 Unit 3

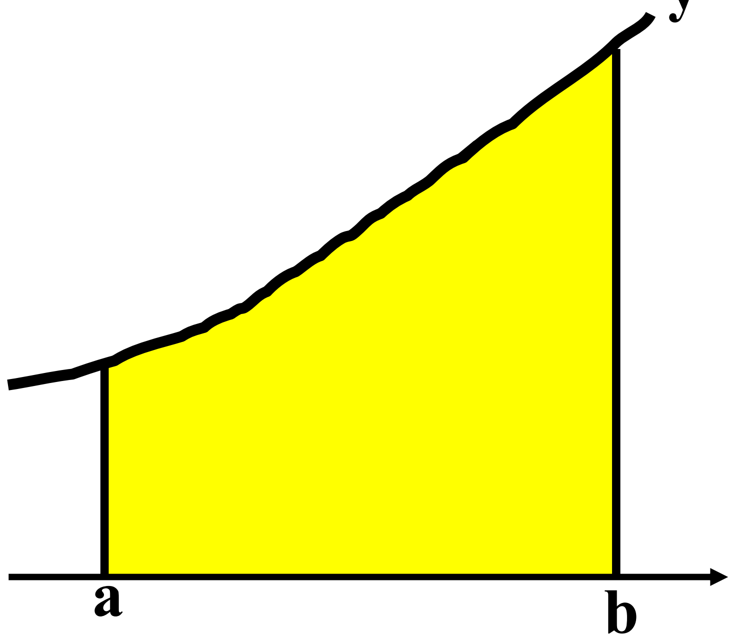
The Fundamental Theorems

Calculus Unit 3 The Fundamental Theorems



Consider the region bounded by the x-axis, the lines x = a and x = b, and the graph of y = f(x).

Calculus Unit 3 The Fundamental Theorems



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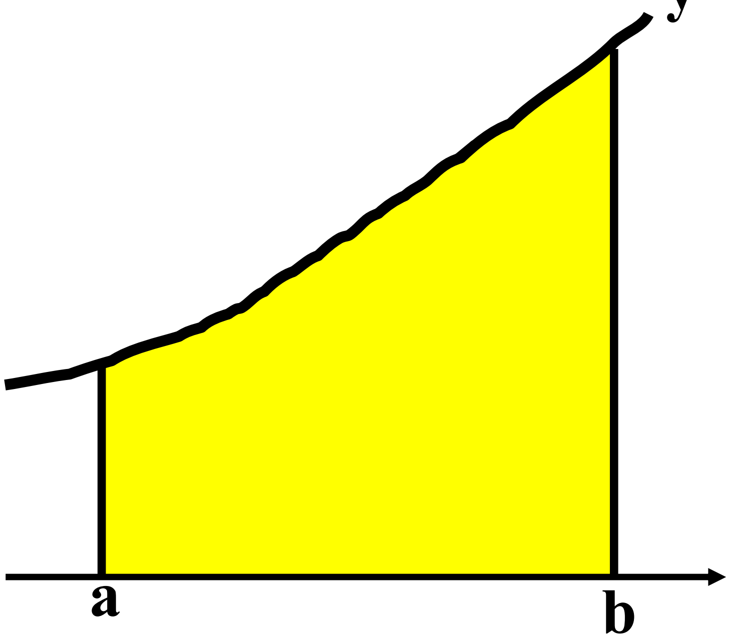

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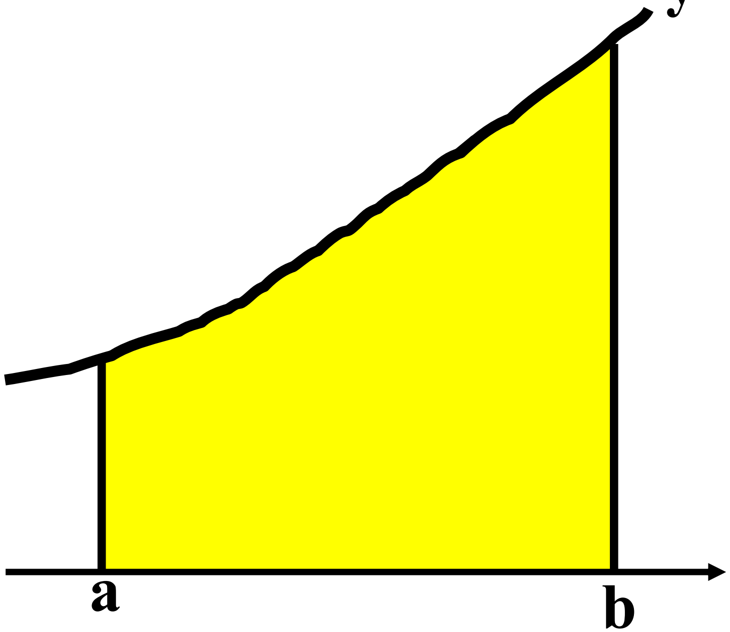

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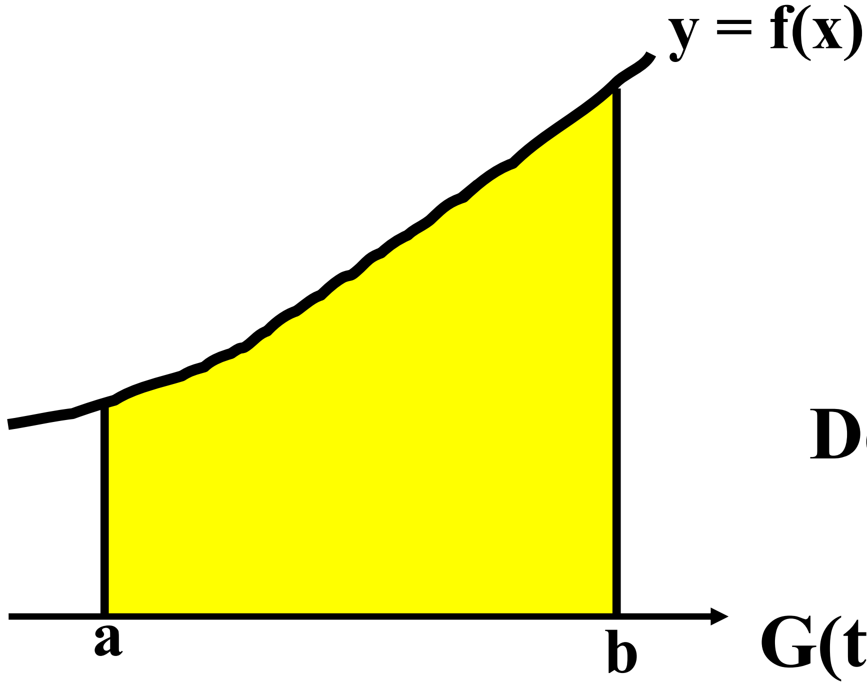

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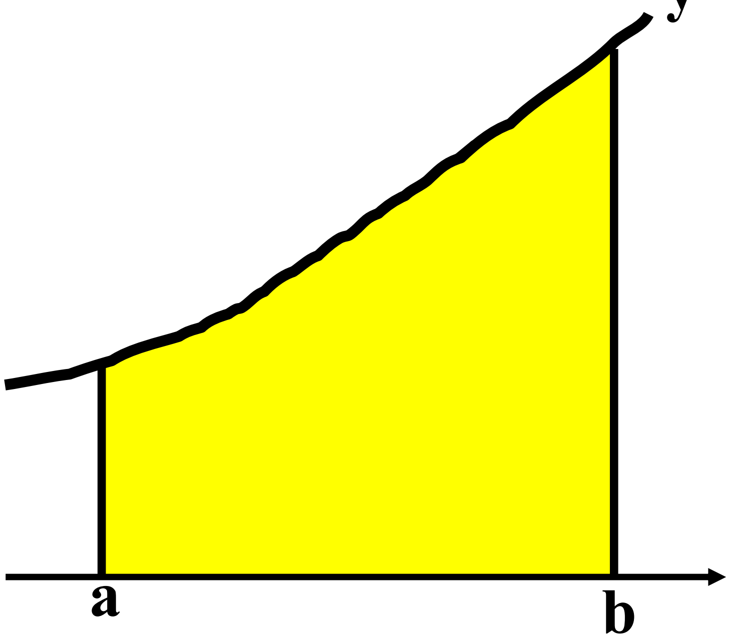

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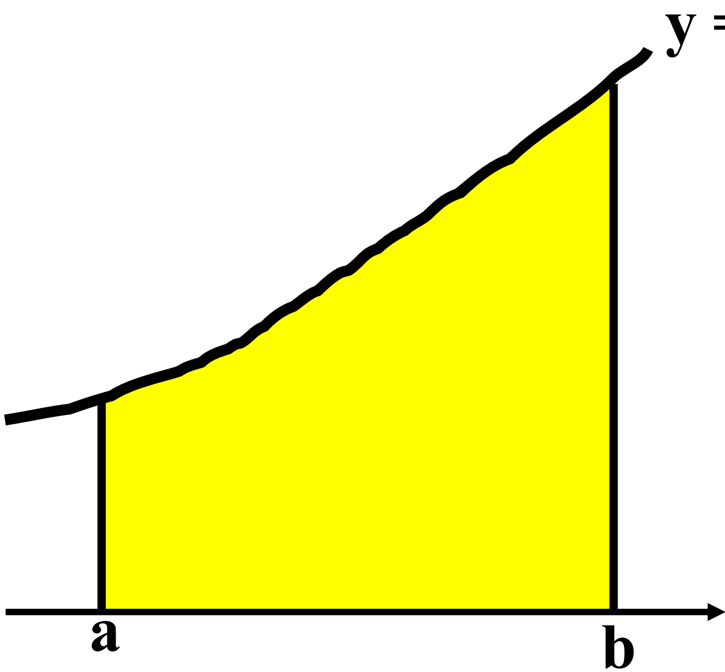

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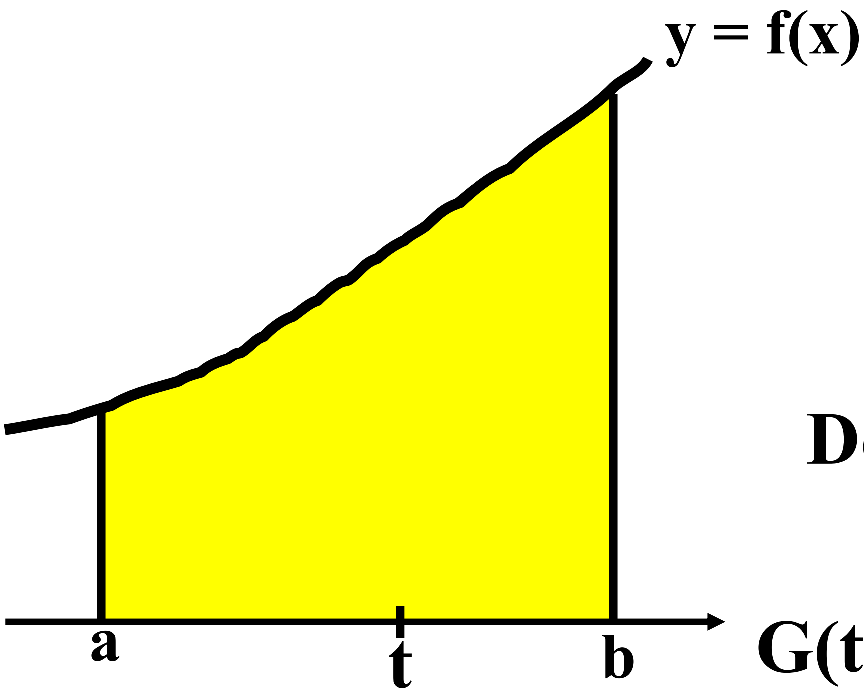

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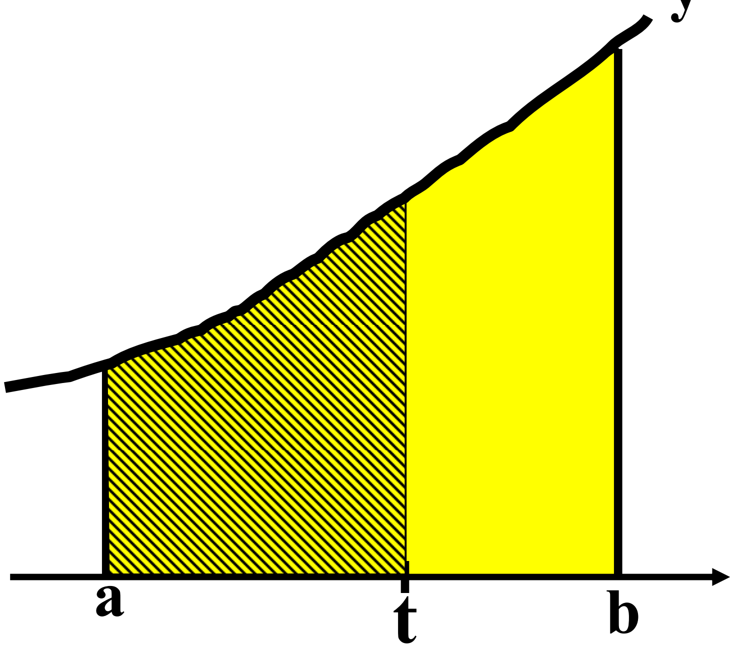

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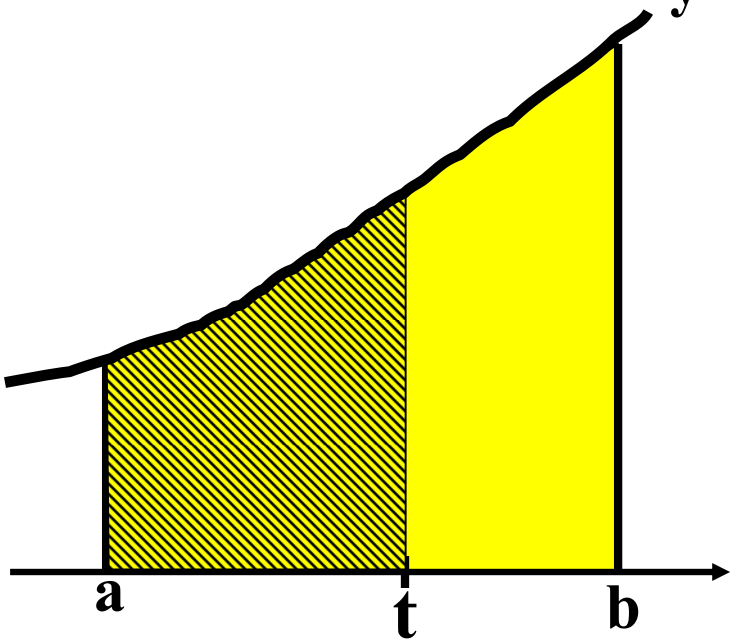

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Our objective is to find $G'(t)$!!

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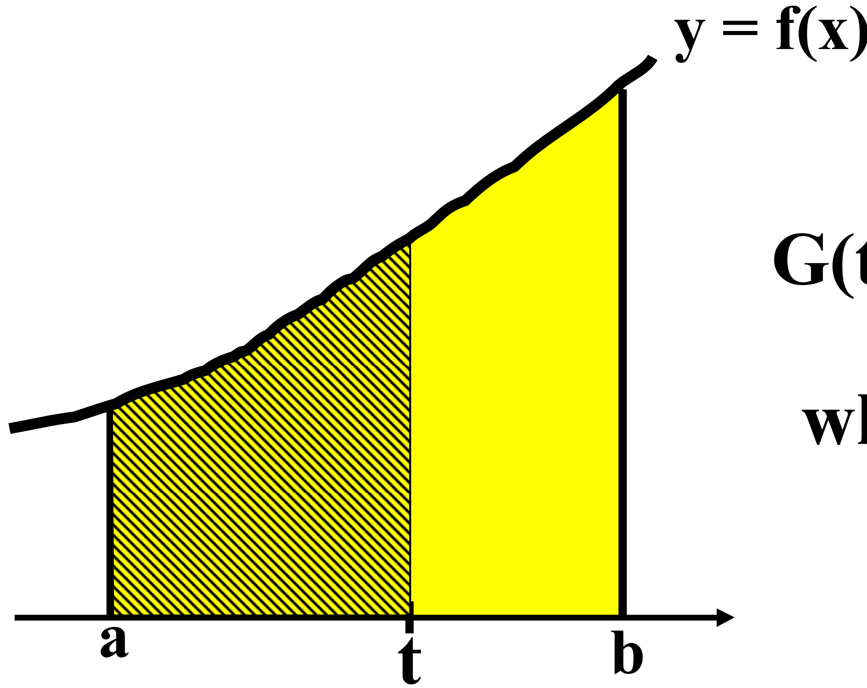

$$G(t) = \int_a^t f(x)\,dx$$

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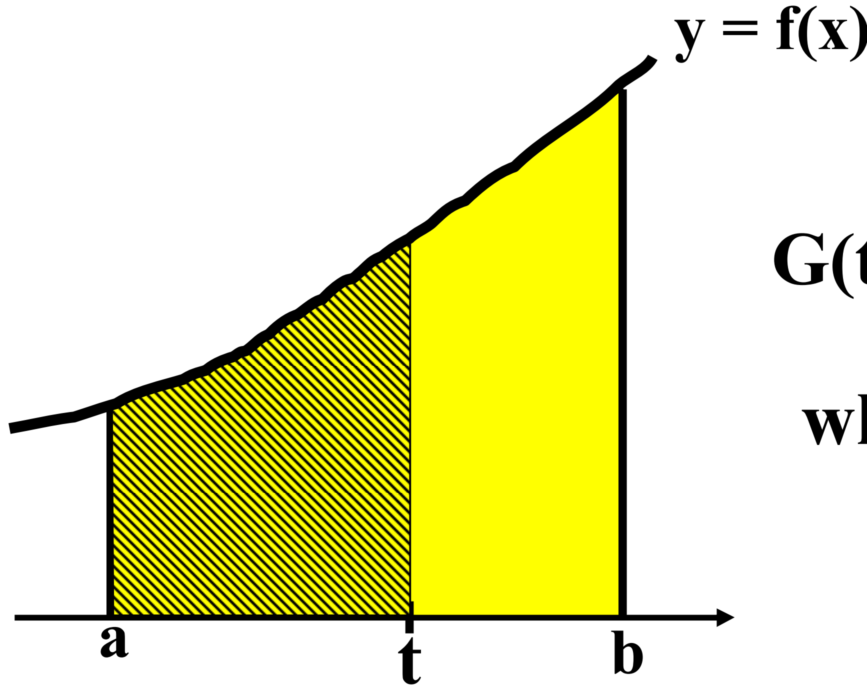

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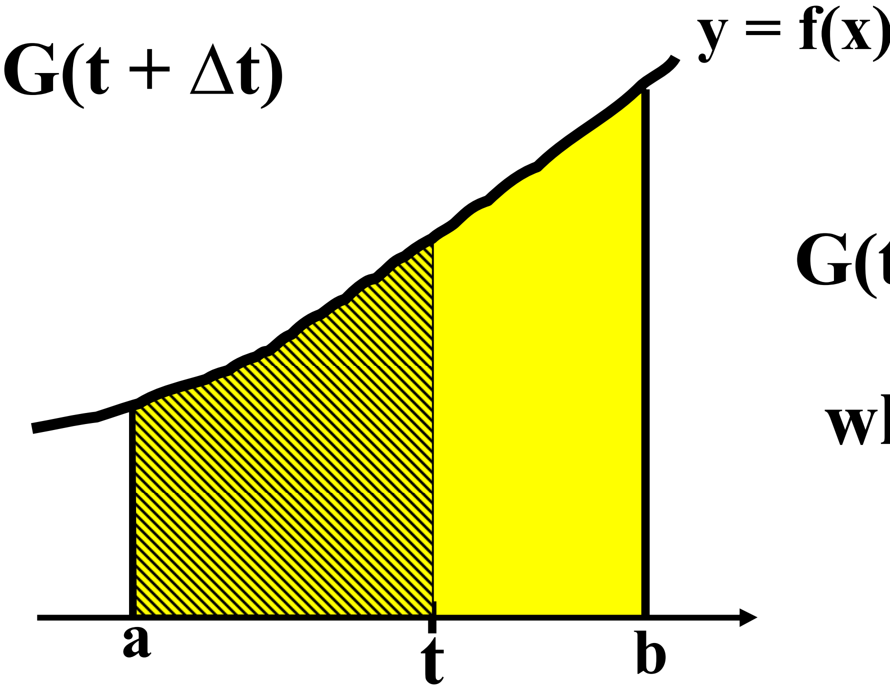
Step 1: Find $G(t + \Delta t)$



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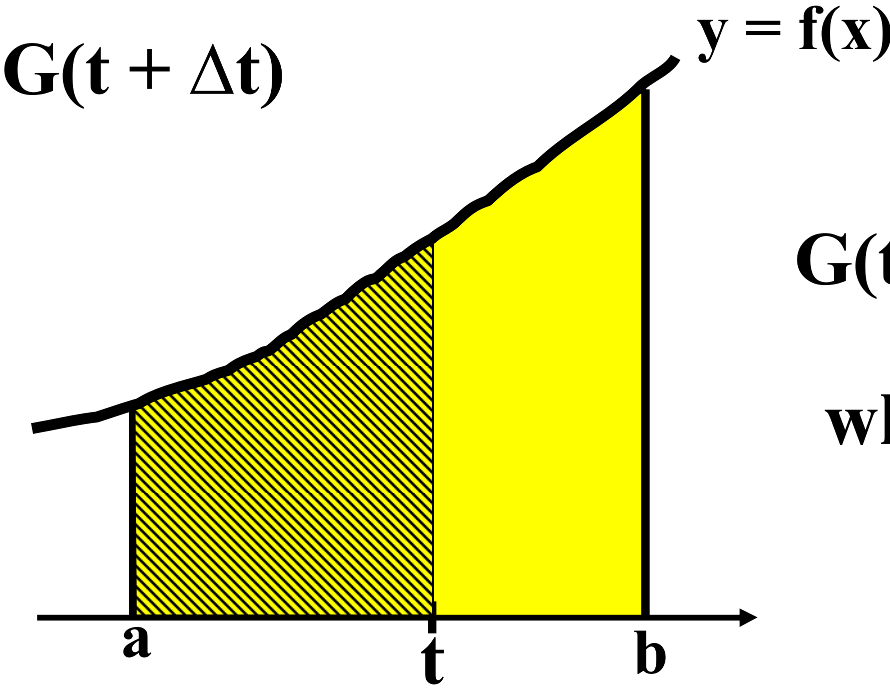

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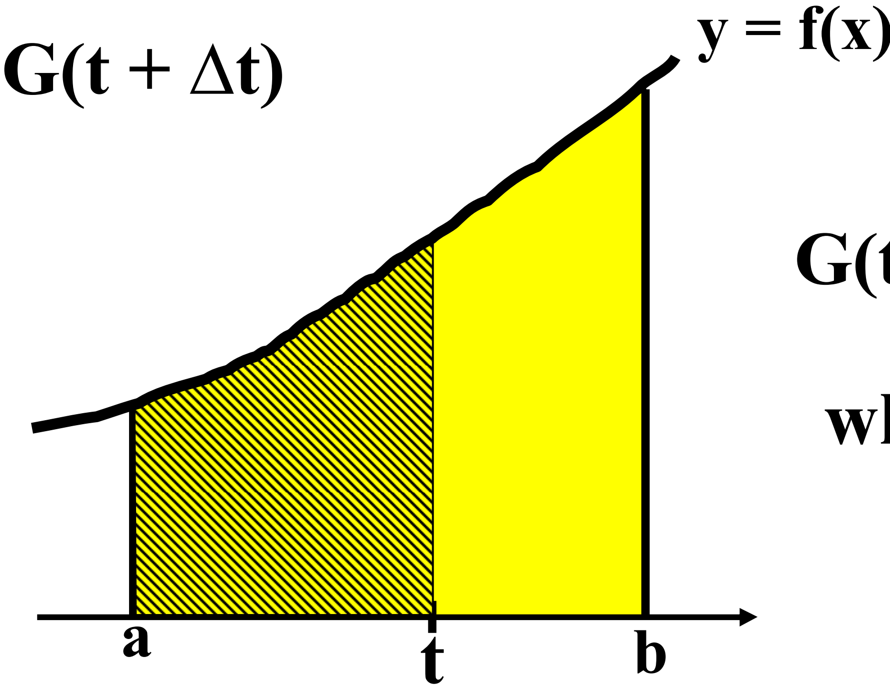

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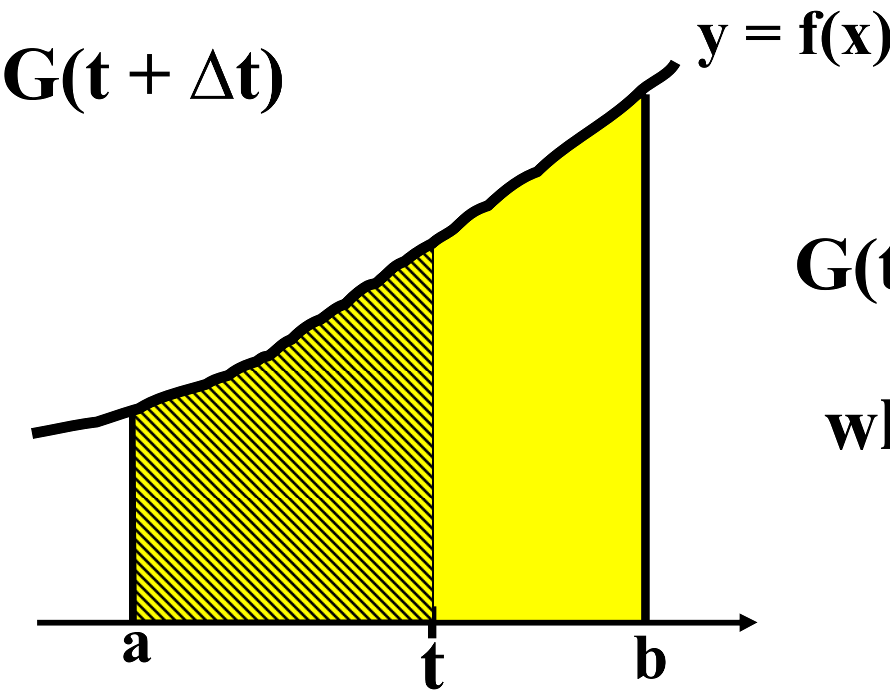

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Calculus   Unit 3   The Fundamental Theorems

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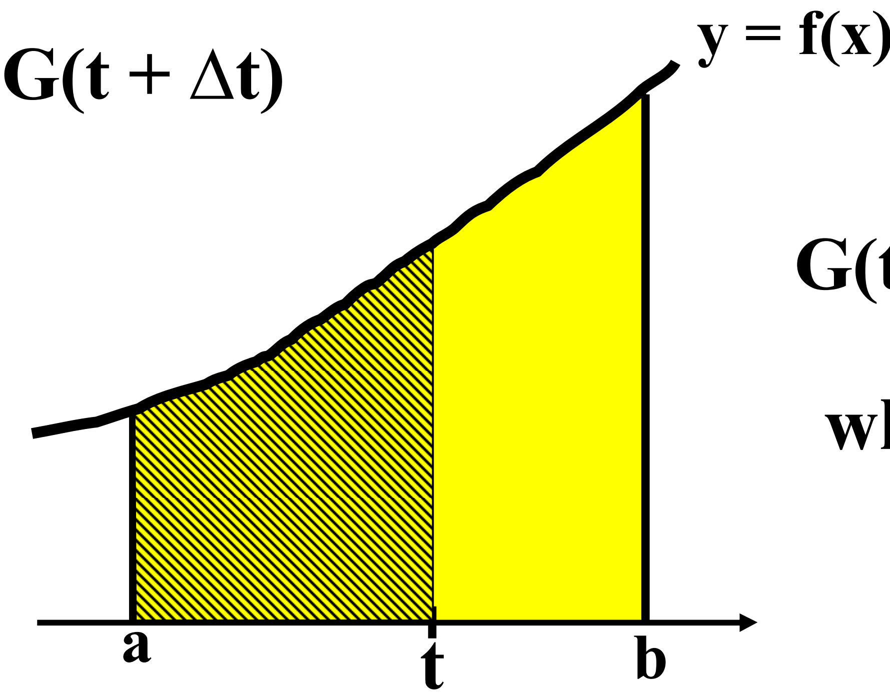

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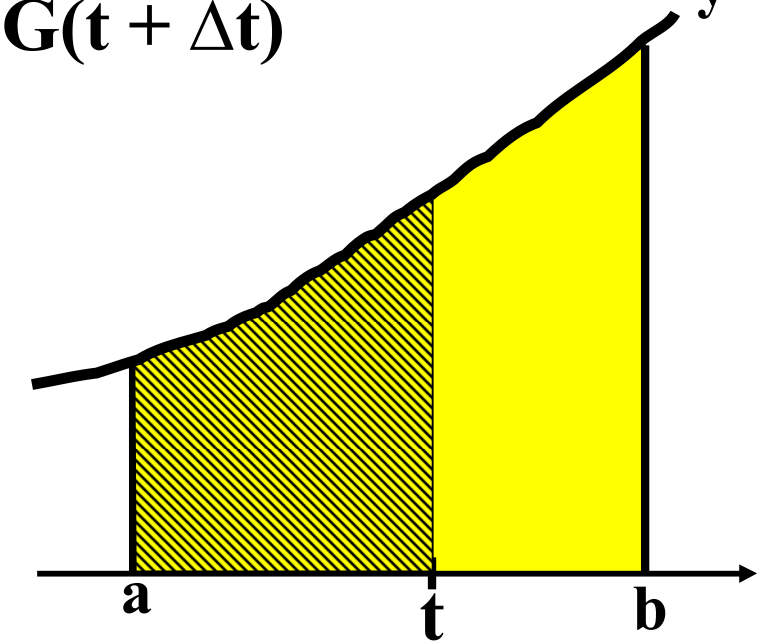

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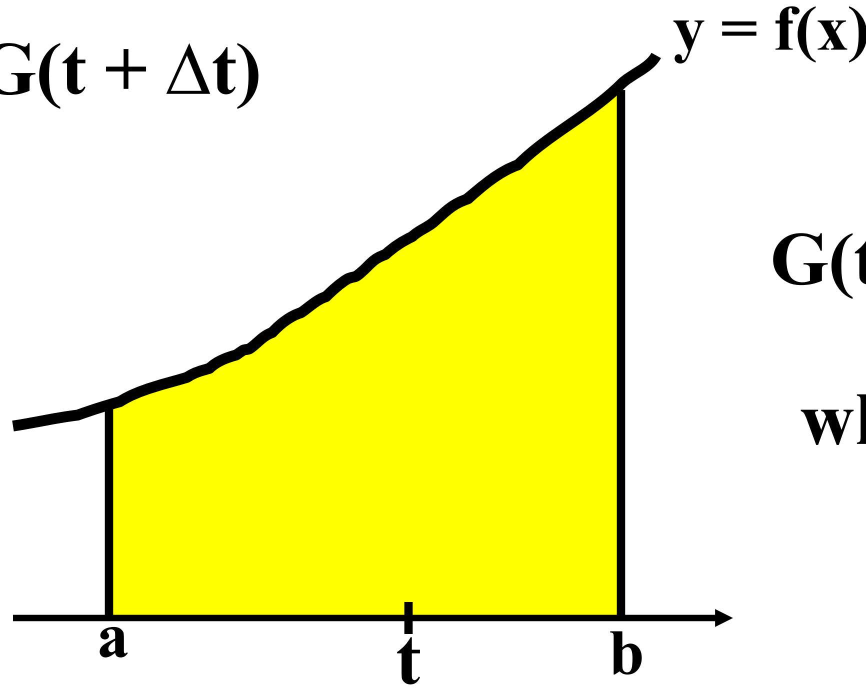

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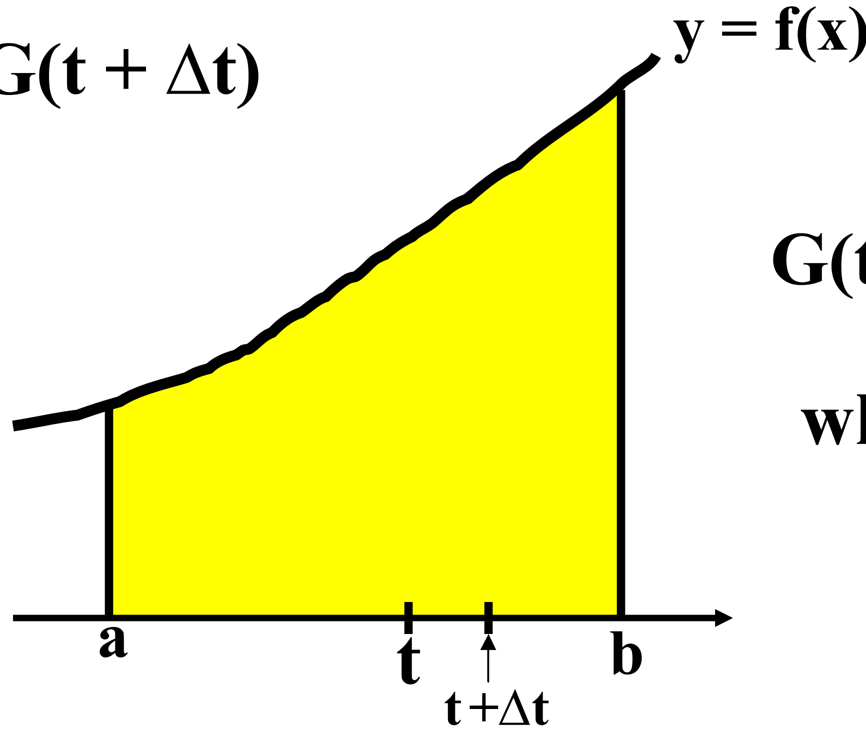

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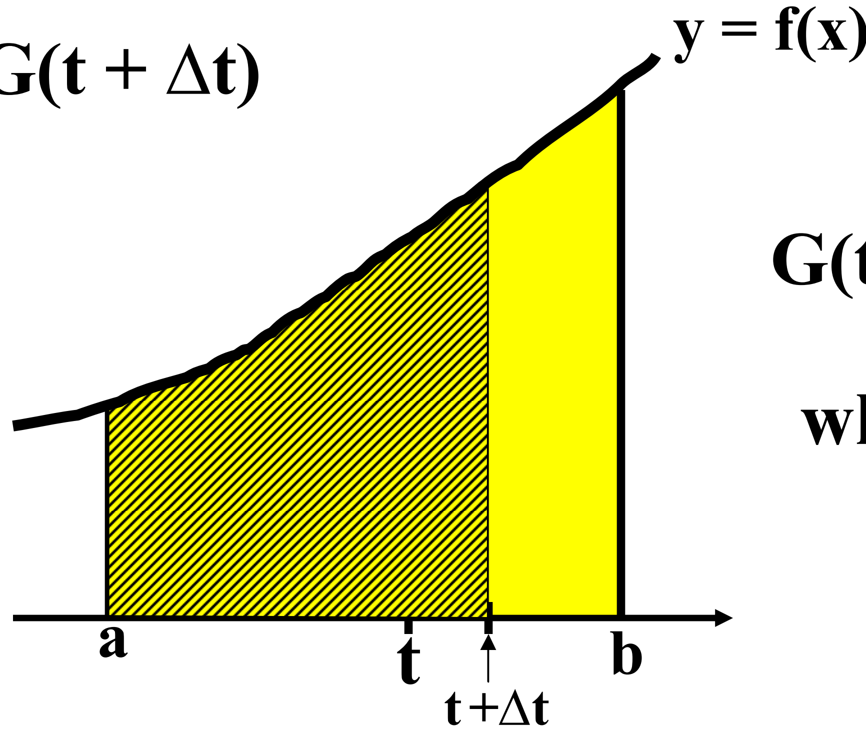

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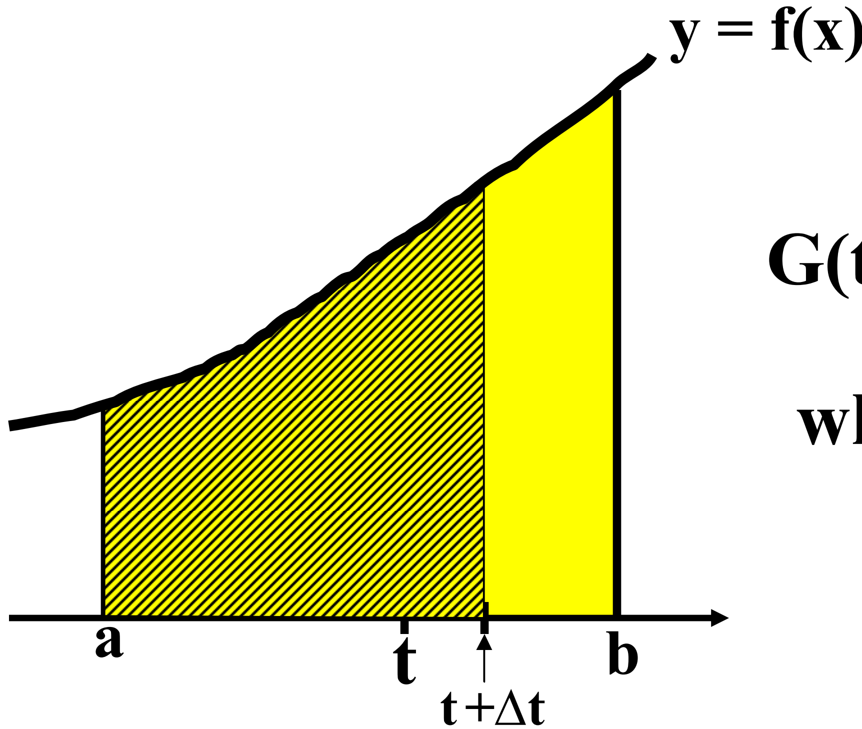

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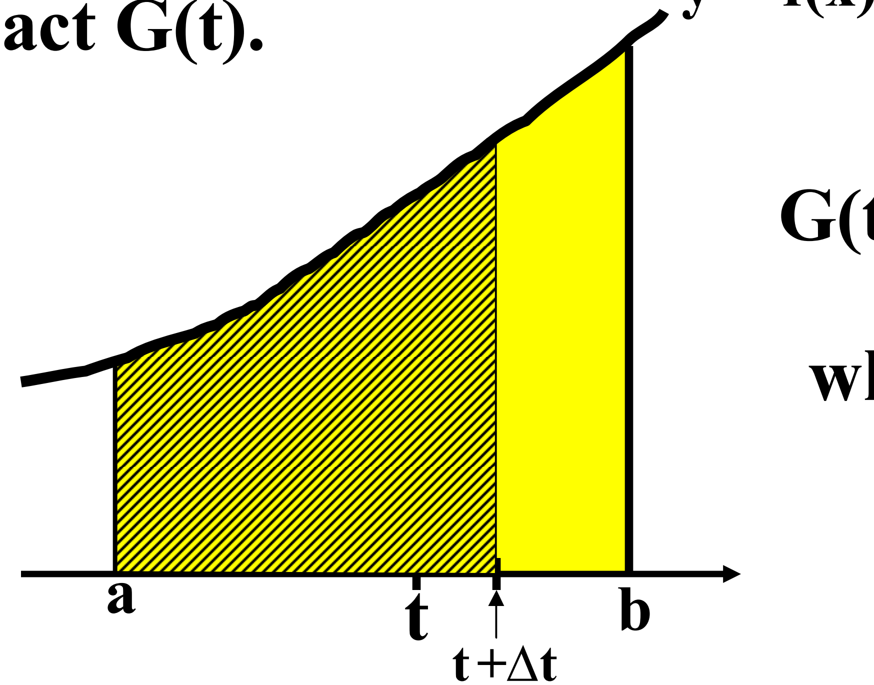

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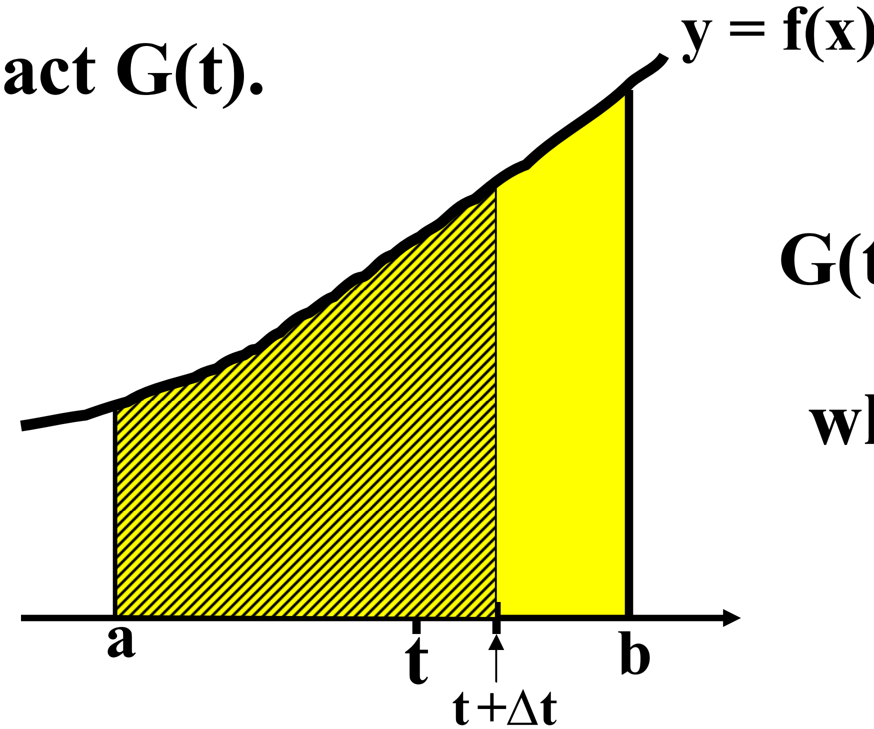

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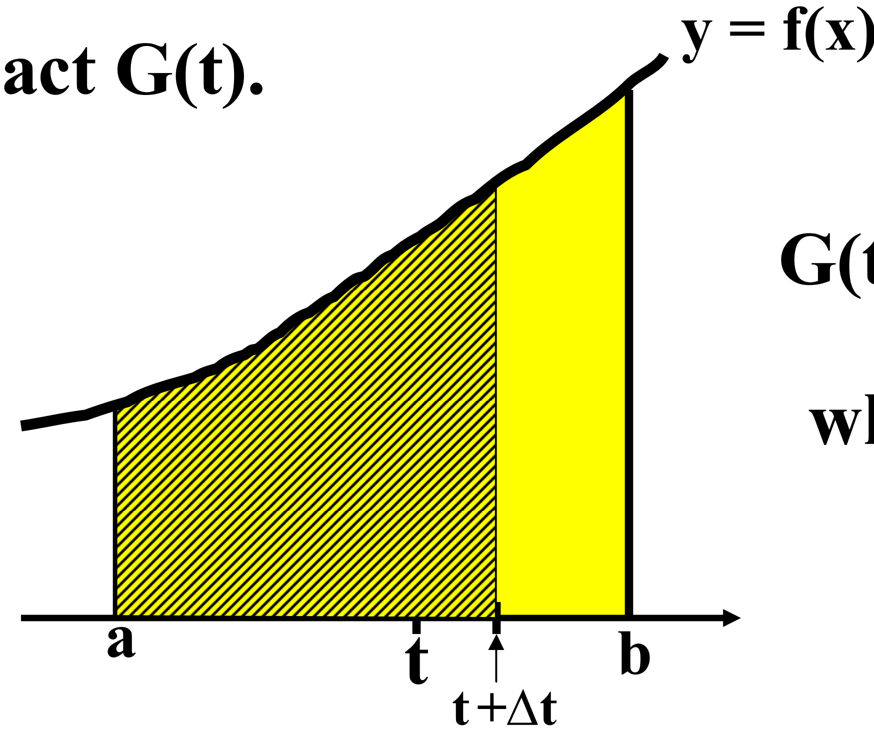

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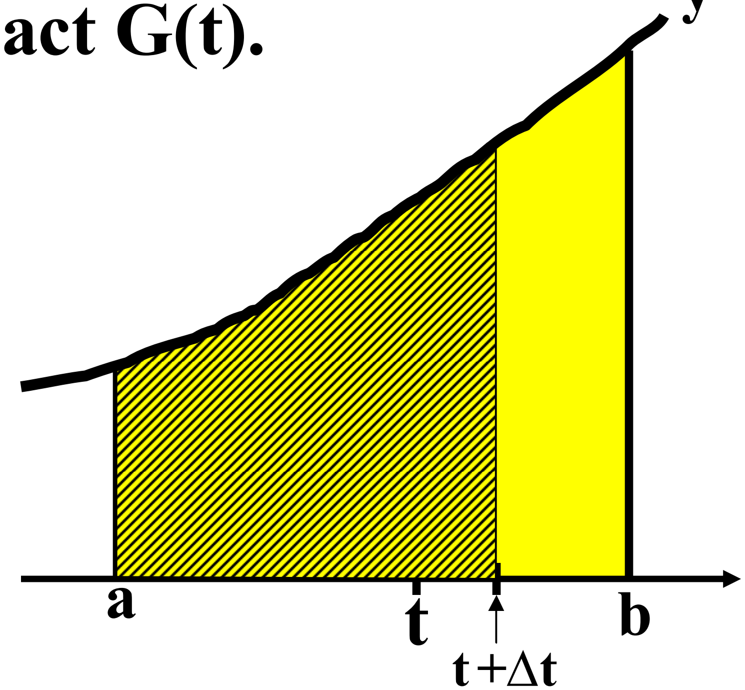

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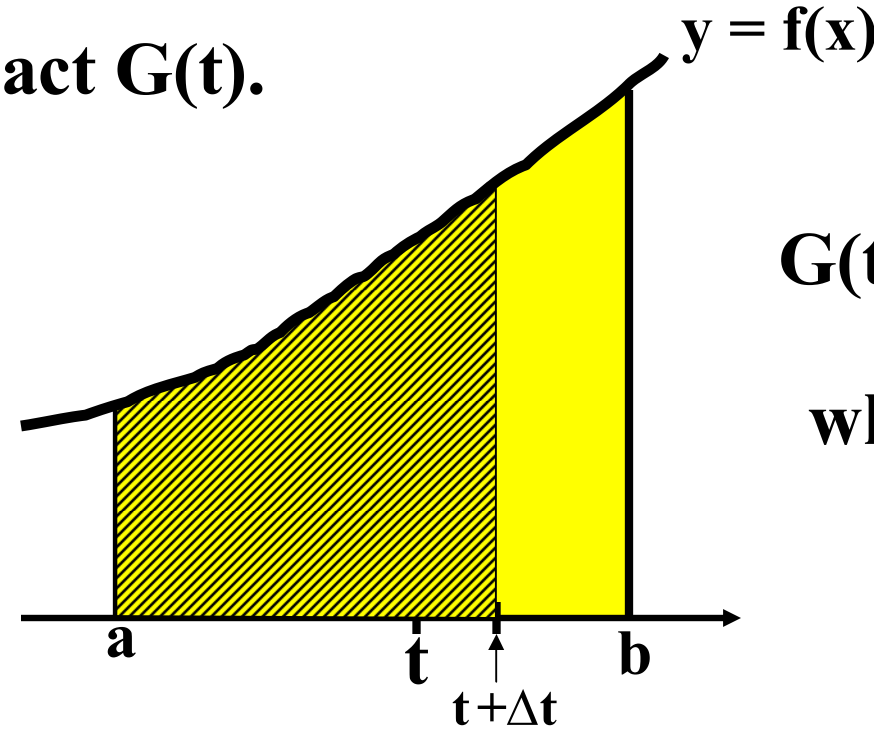

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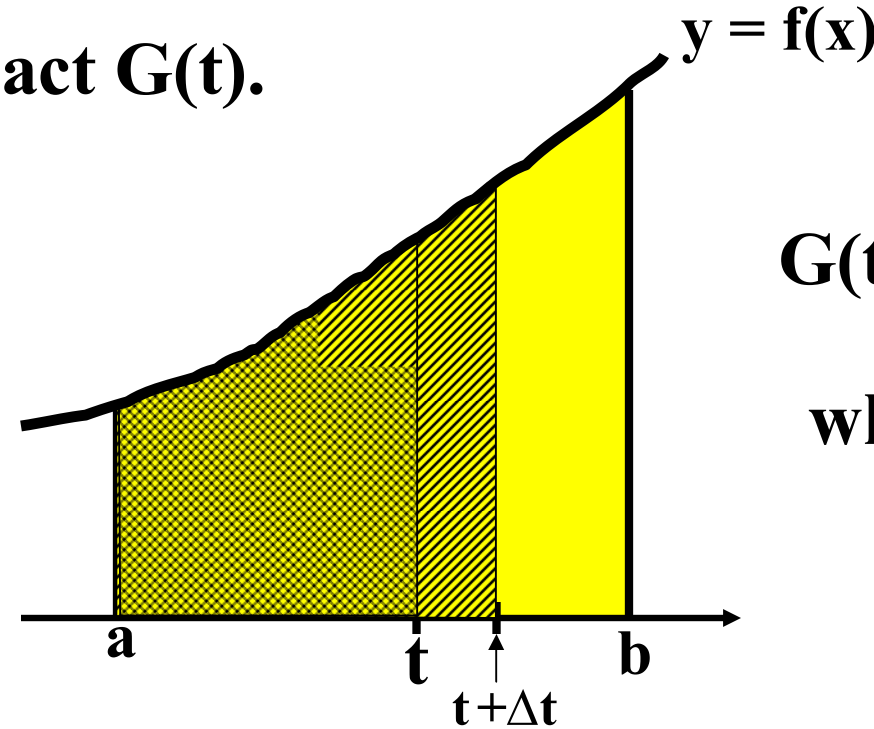

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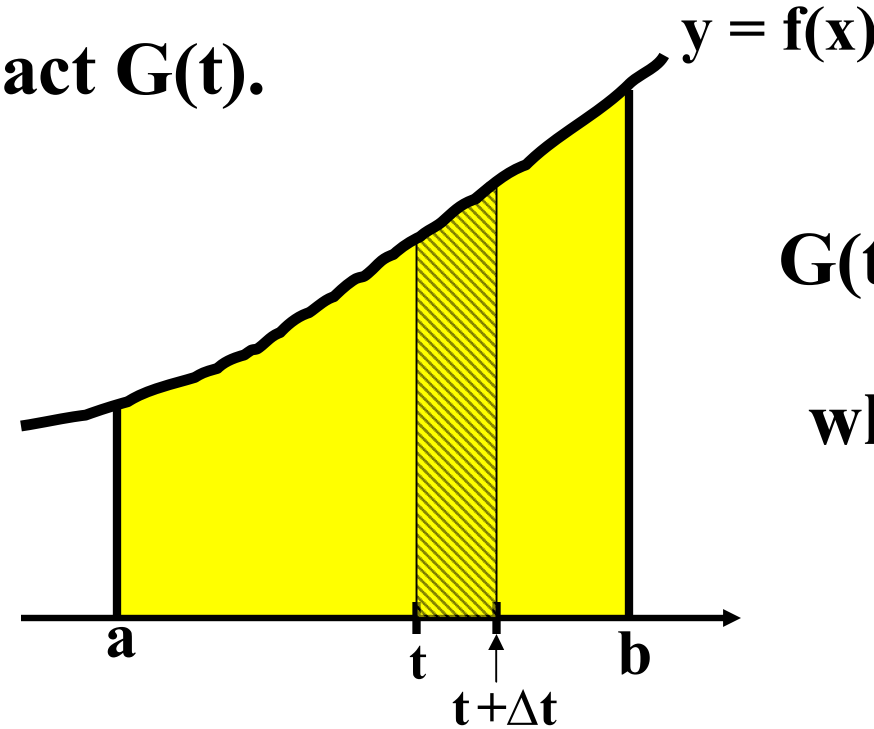

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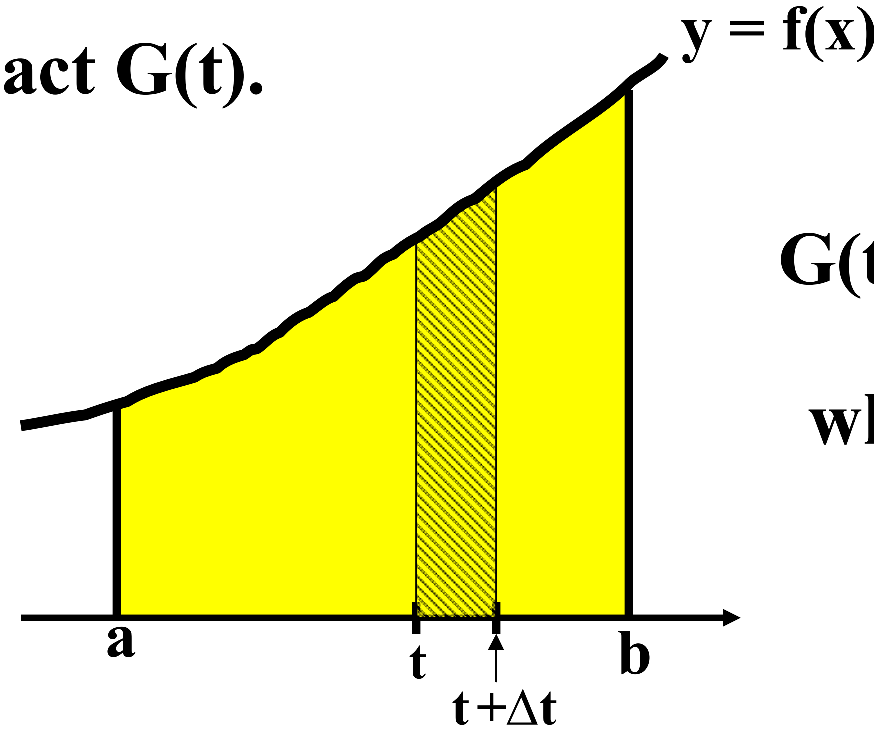

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$$G(t + \Delta t) - G(t)$$

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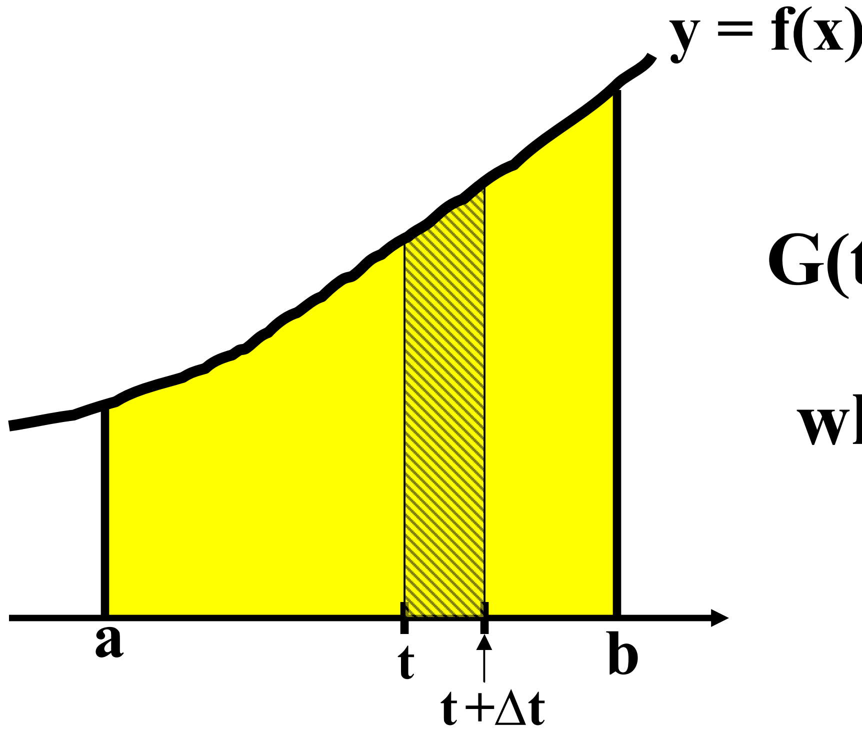

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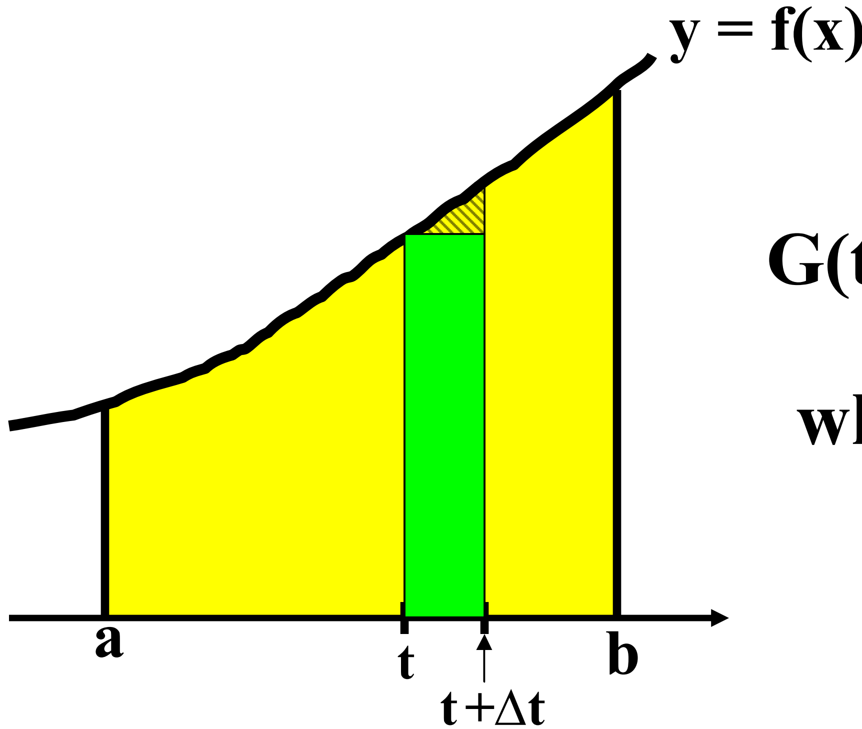

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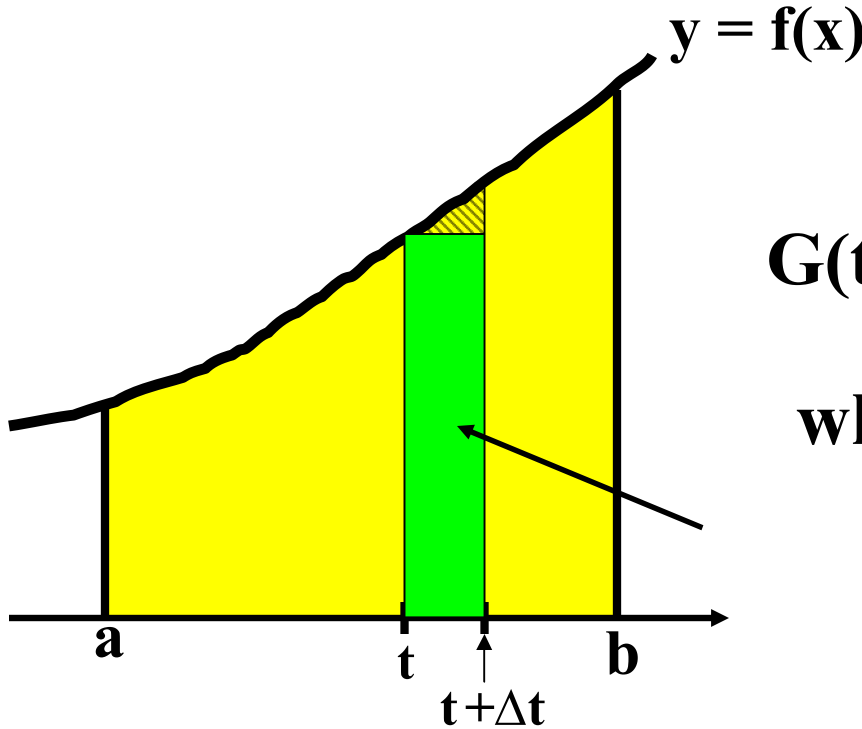

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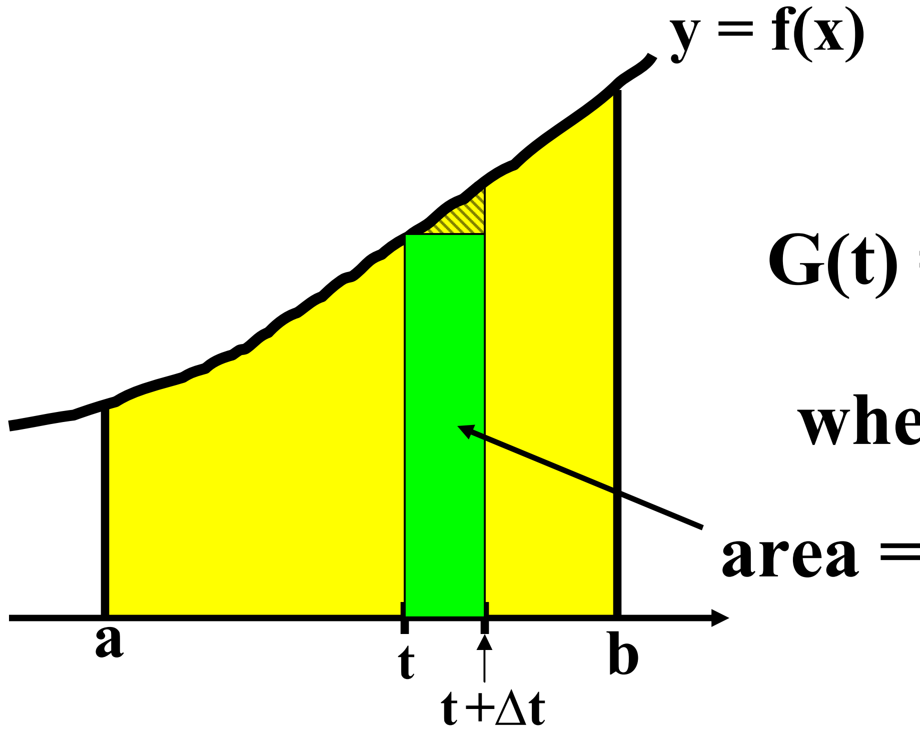

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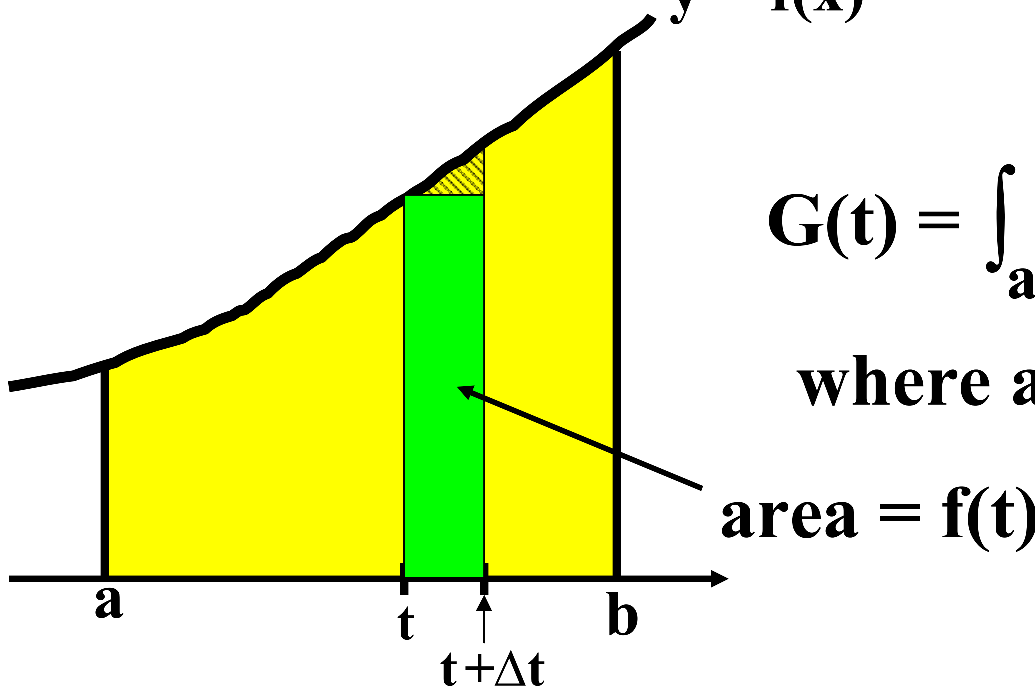

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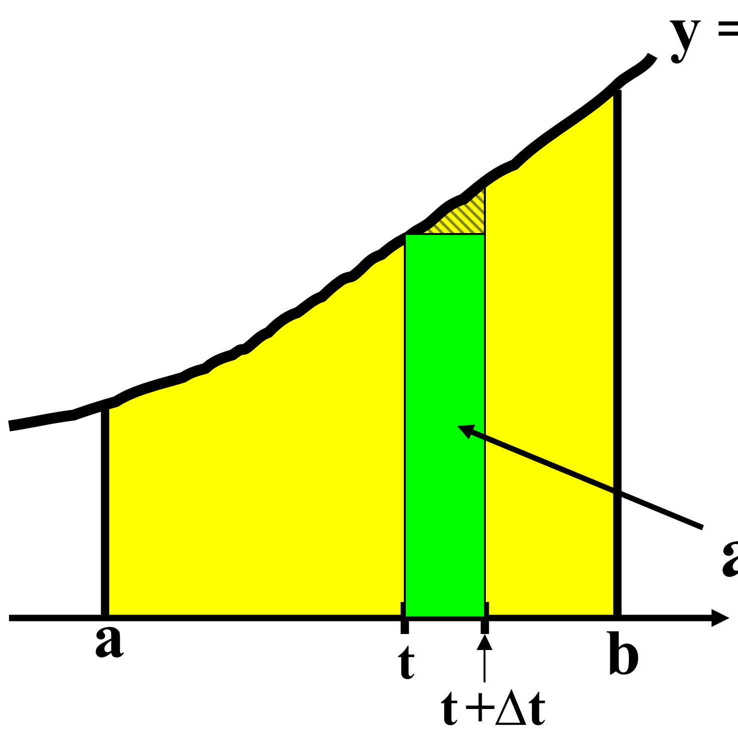

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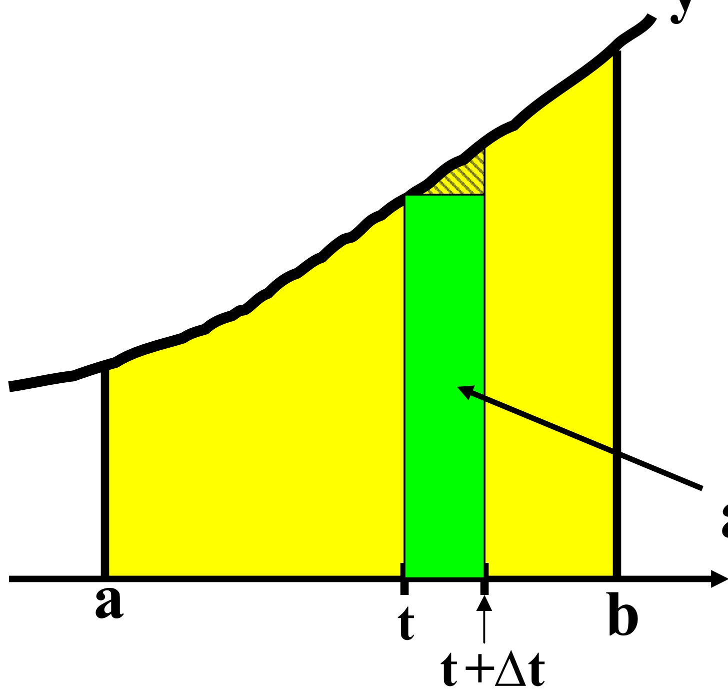

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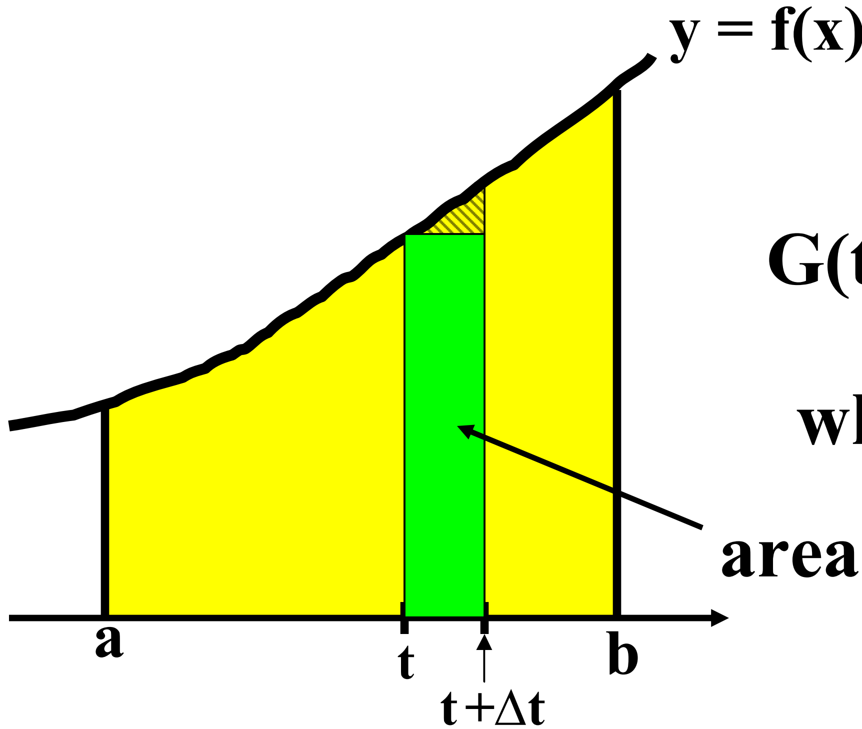

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area $= f(t) \cdot \Delta t$

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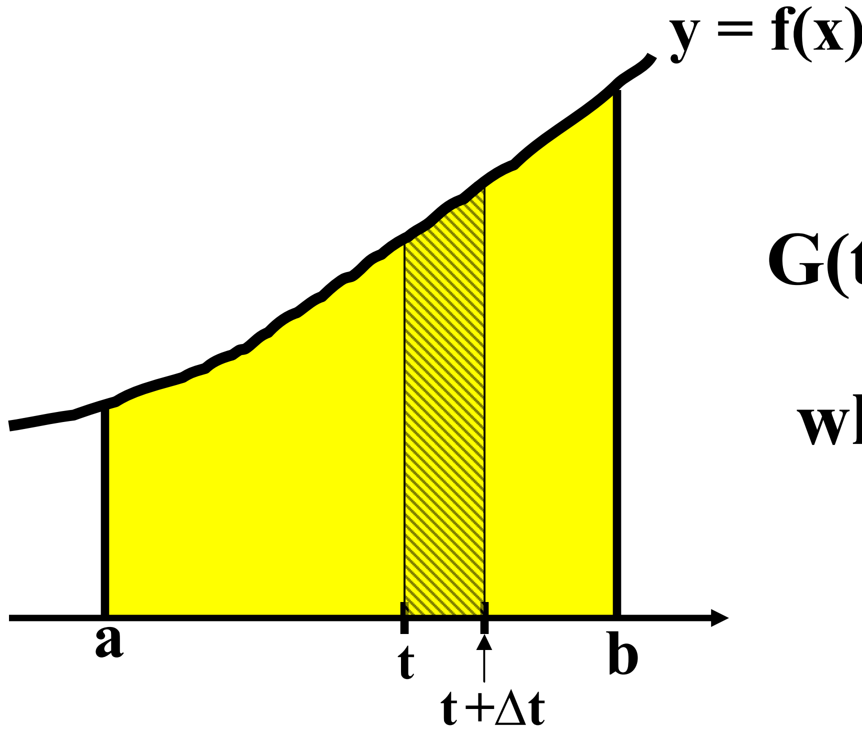

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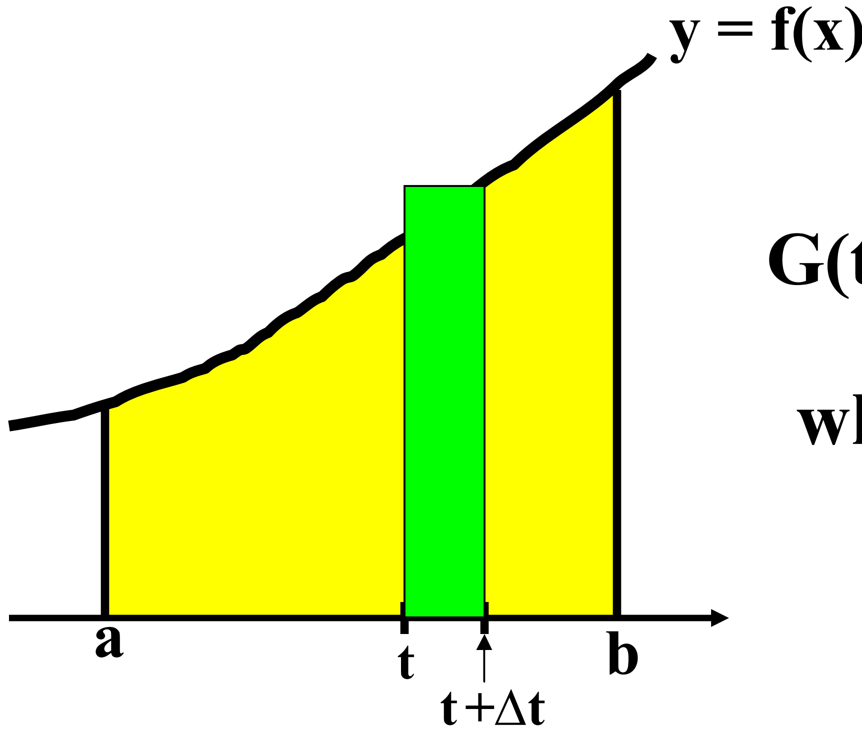

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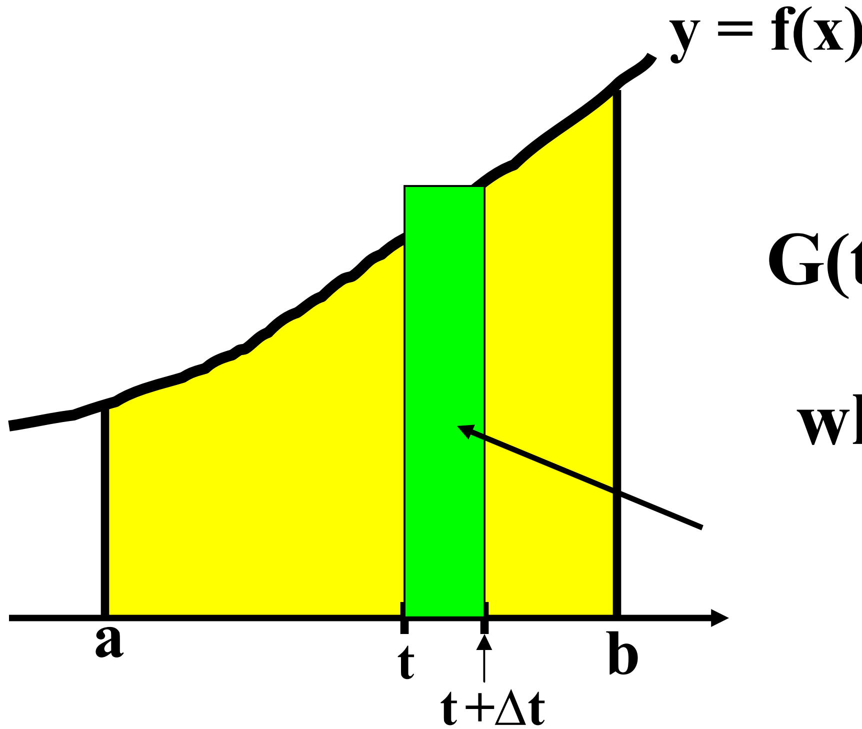

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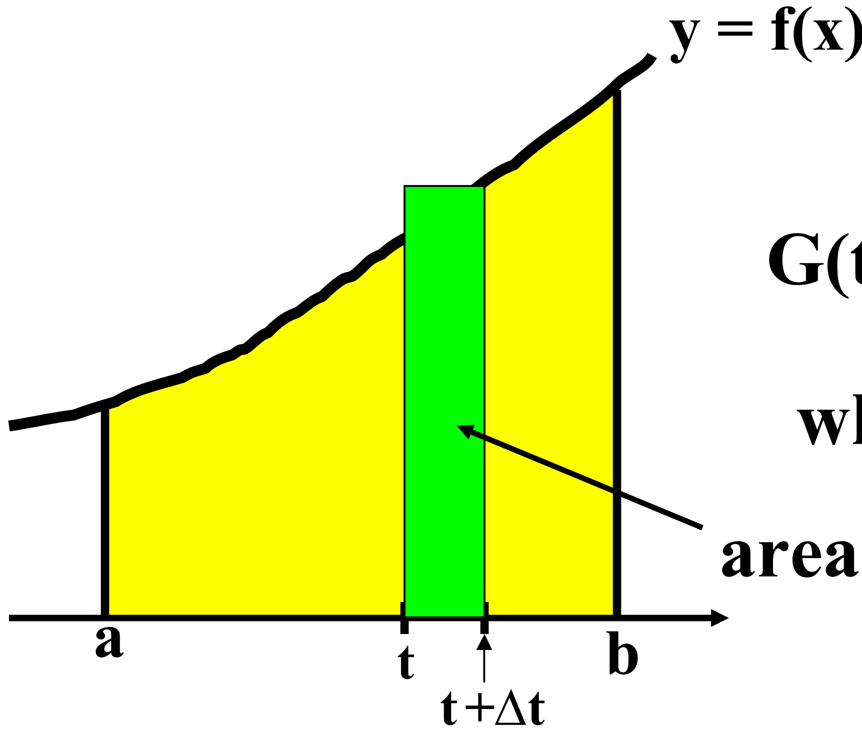

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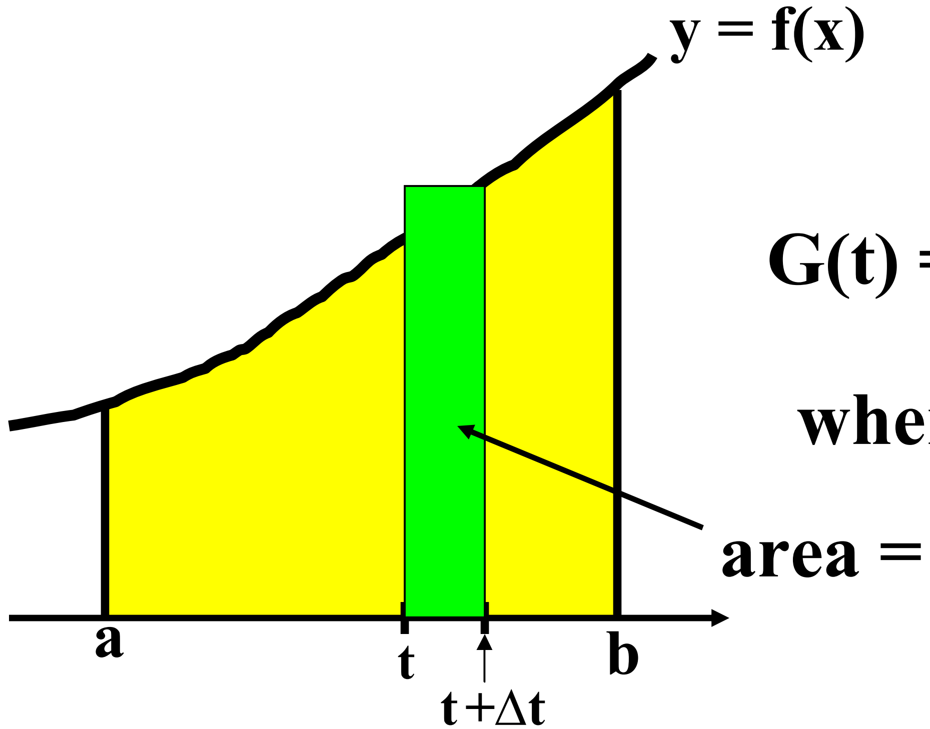

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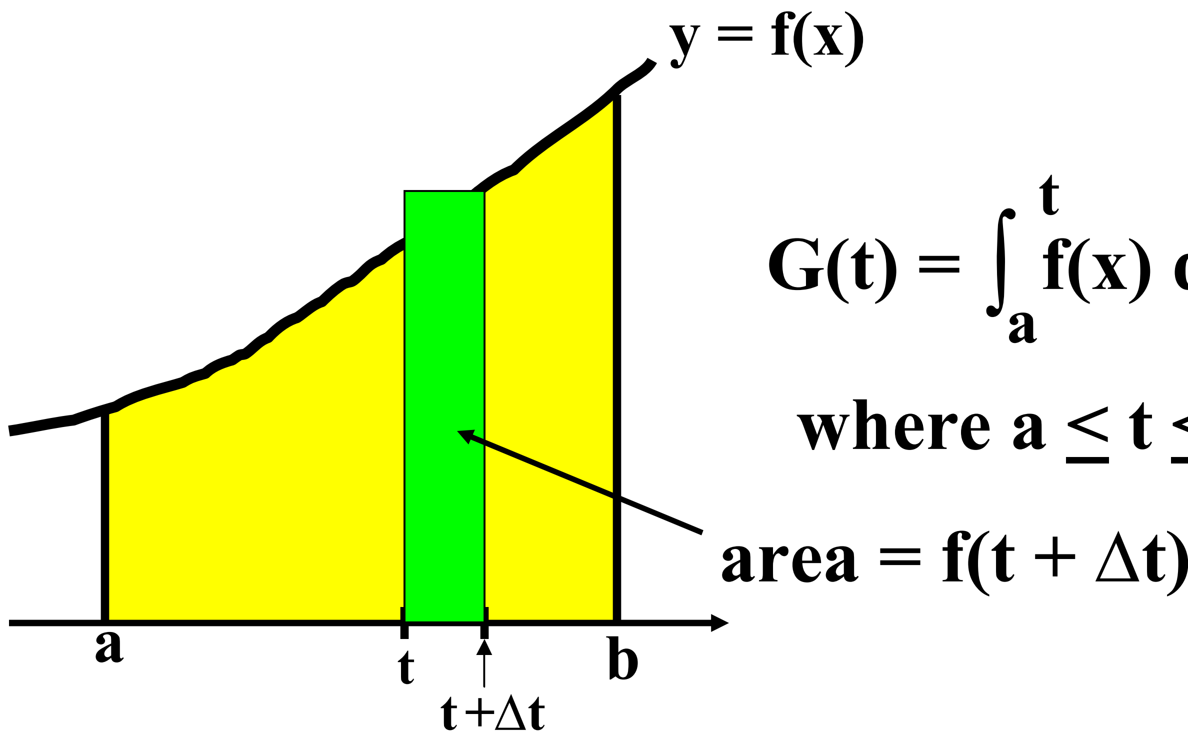

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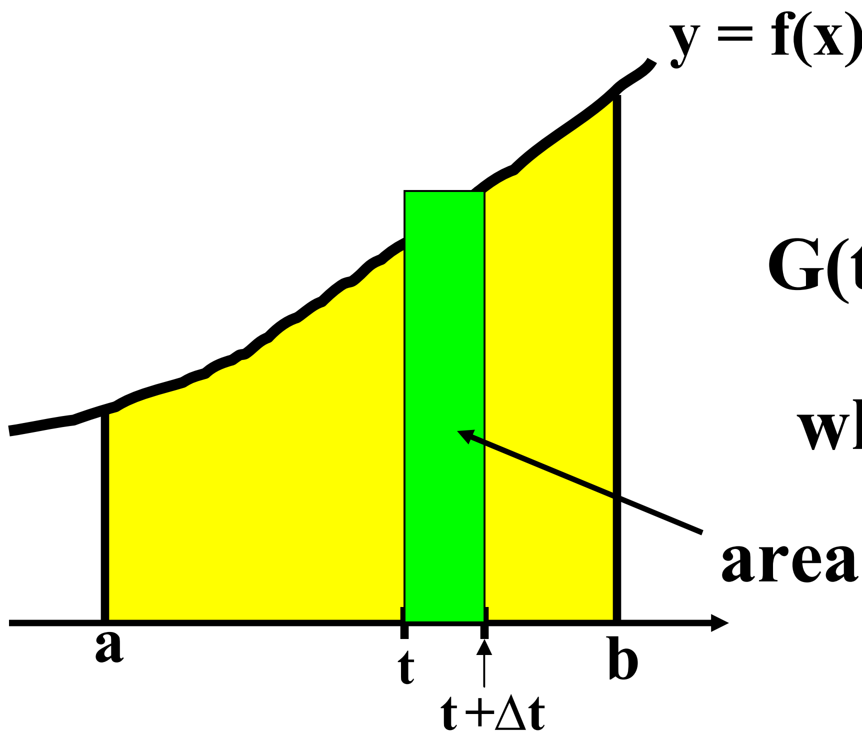

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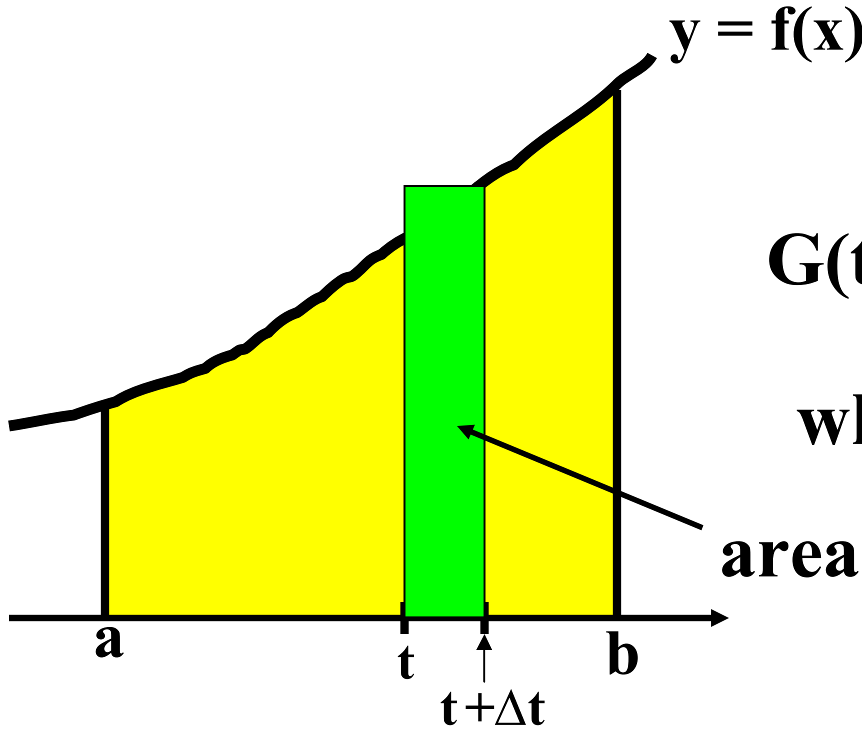

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Calculus Unit 3 The Fundamental Theorems

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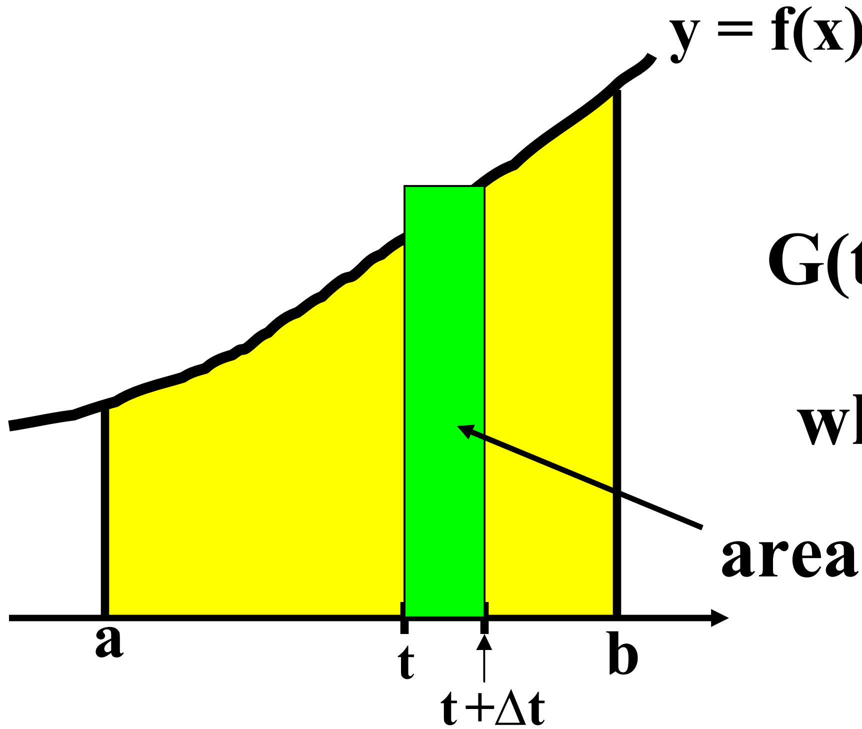

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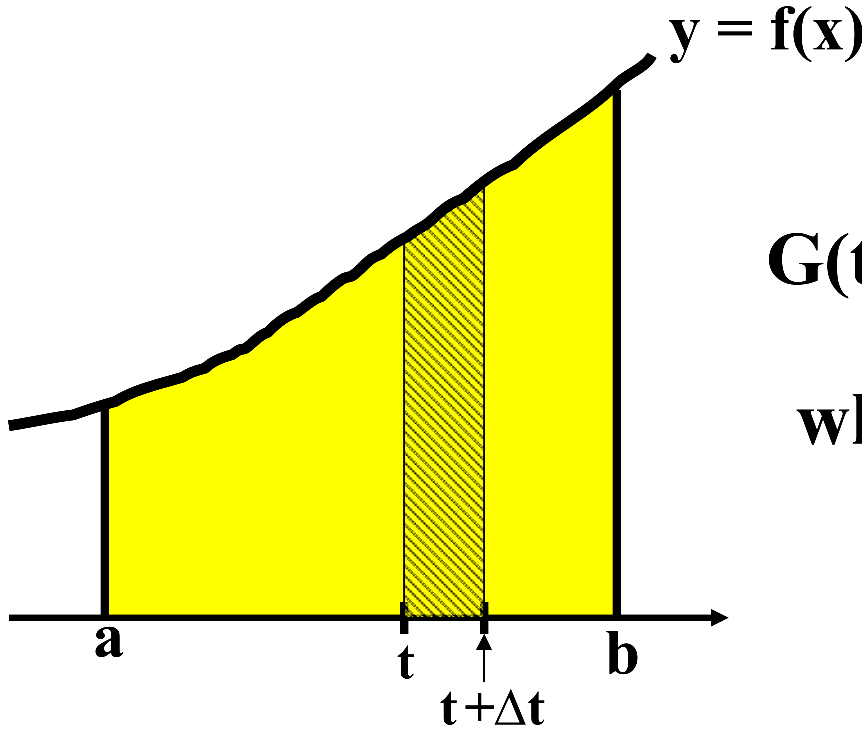

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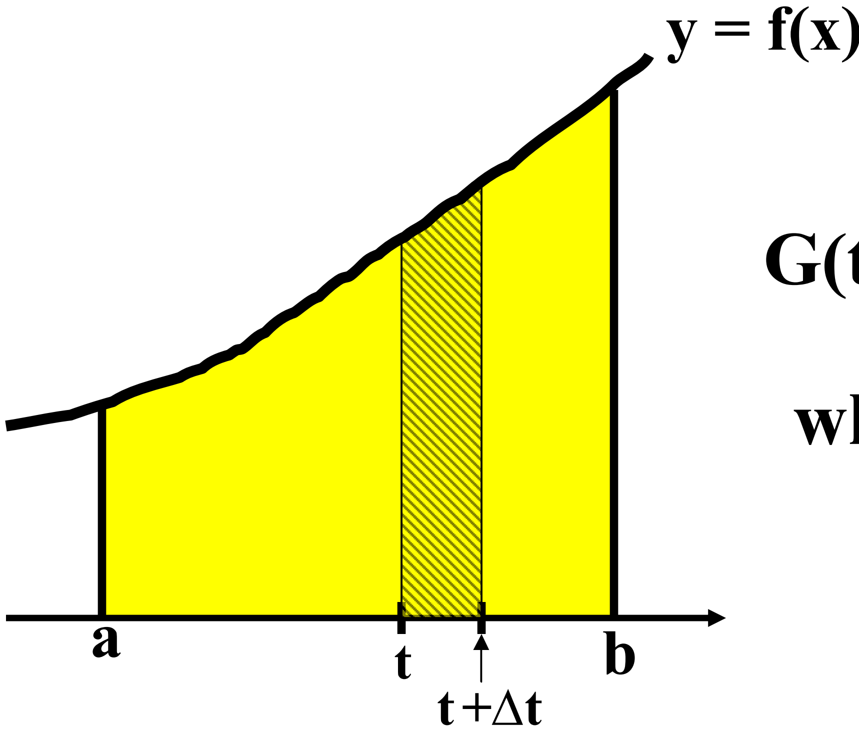

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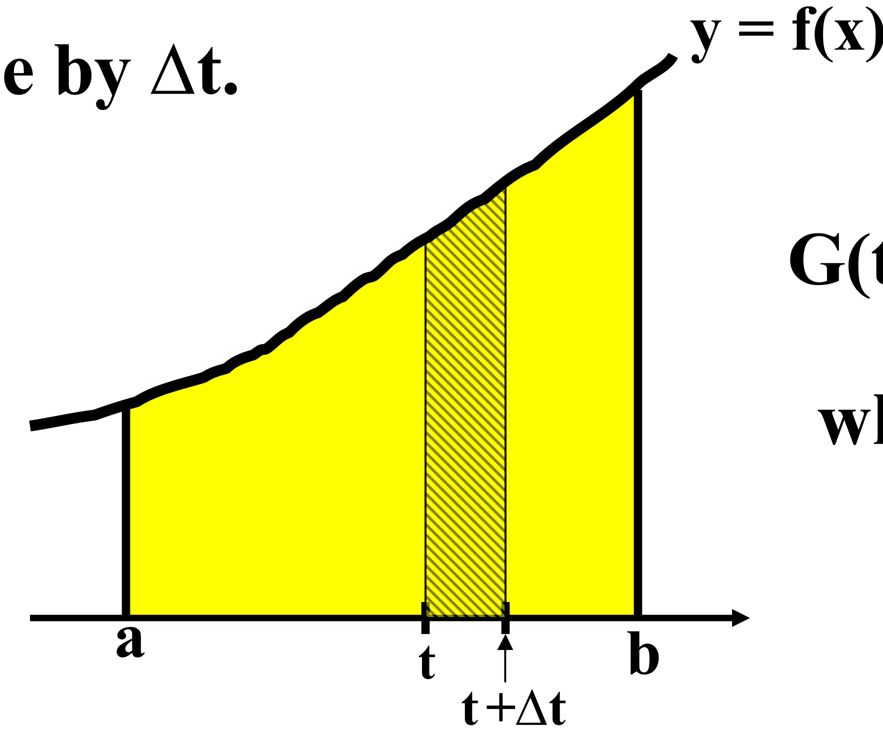

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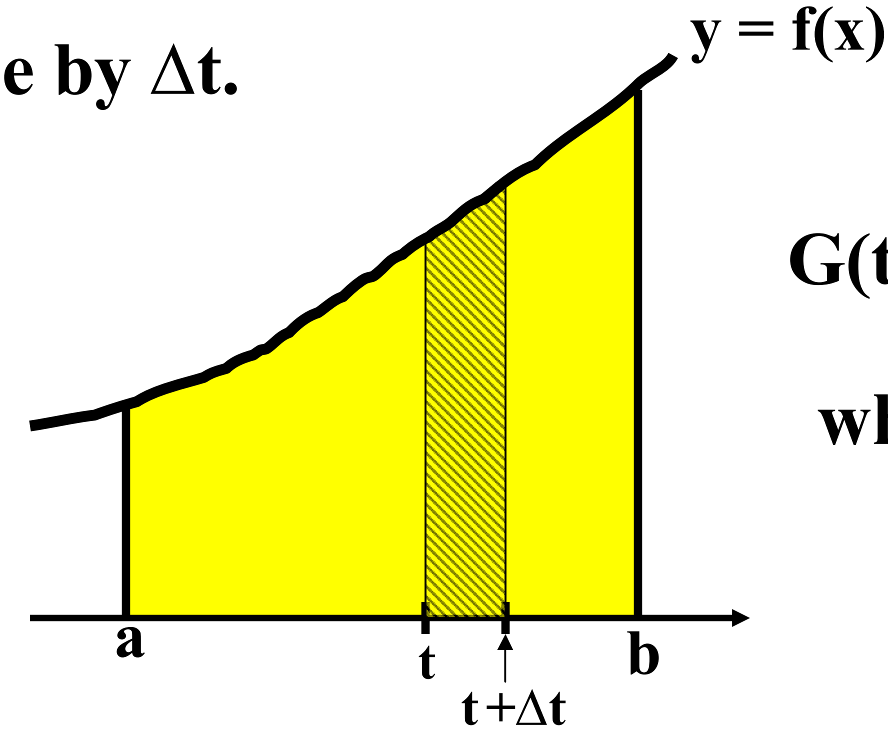

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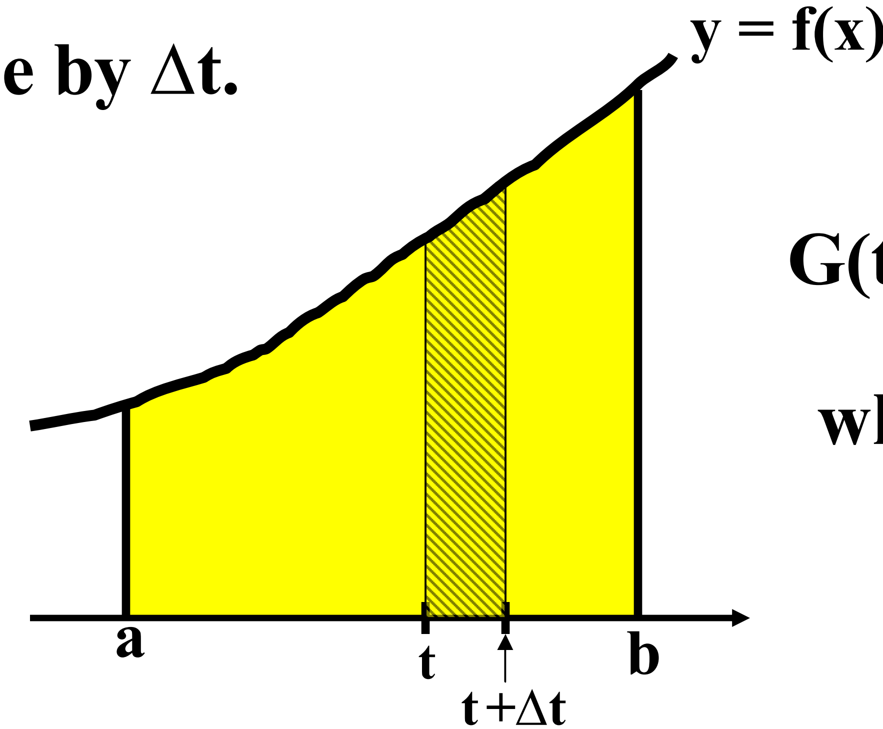

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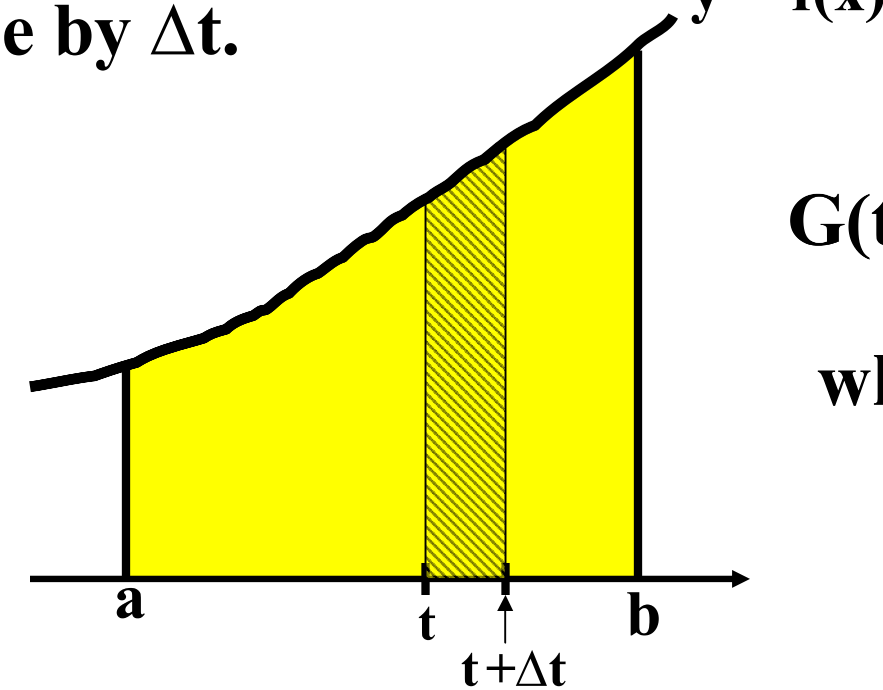

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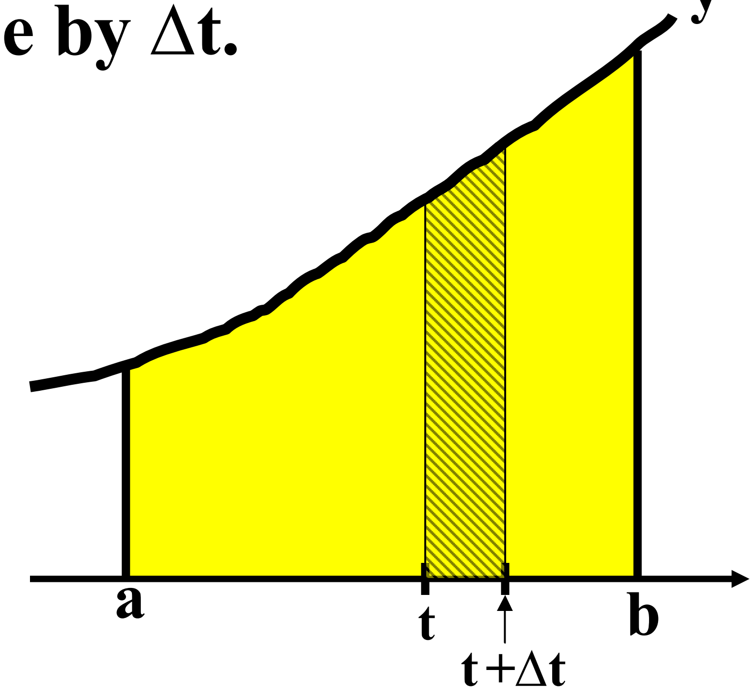

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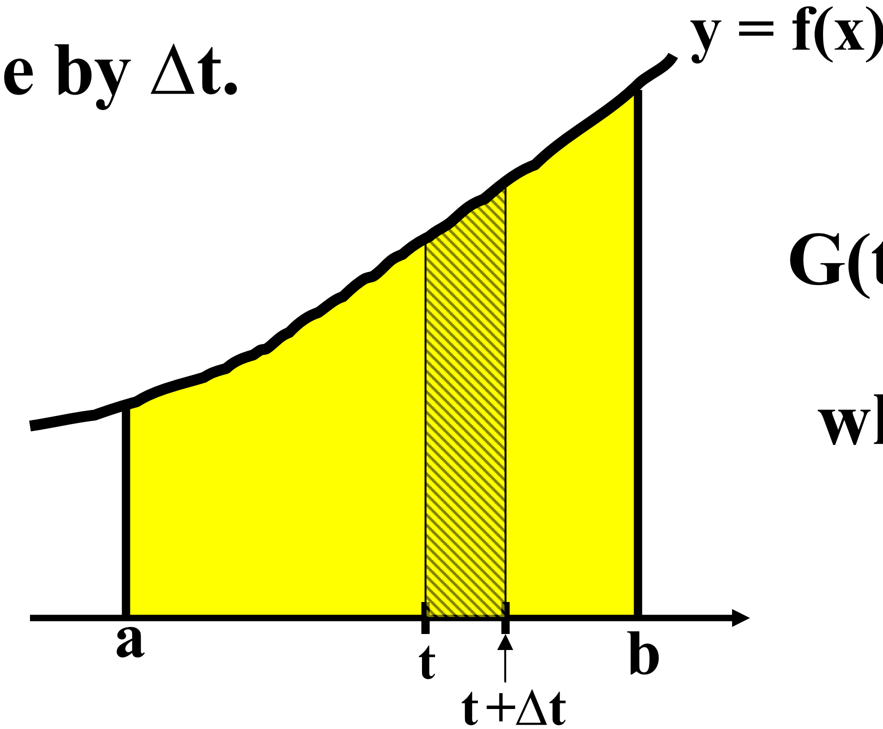

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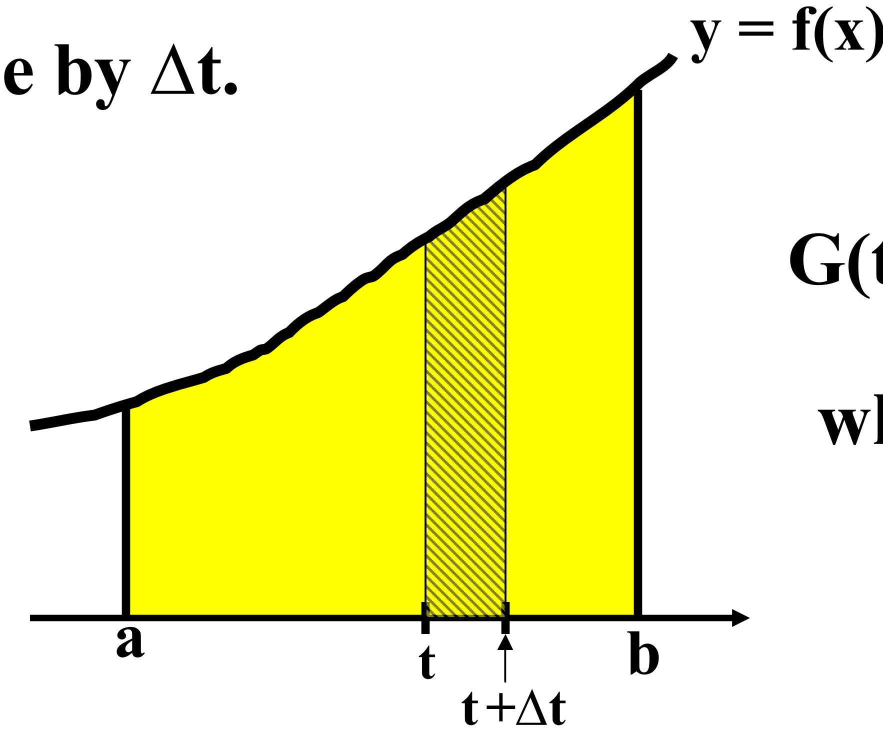

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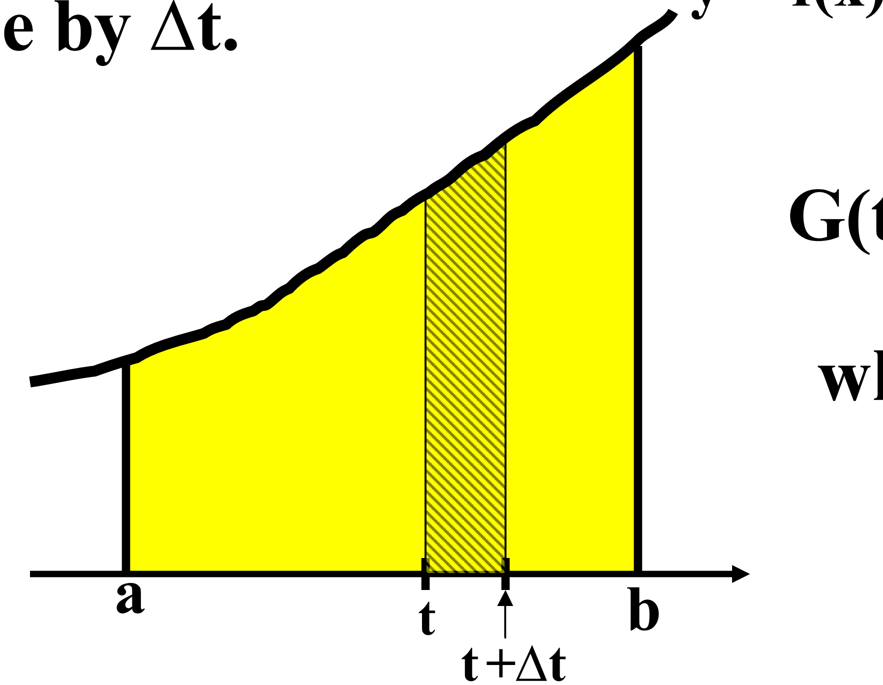

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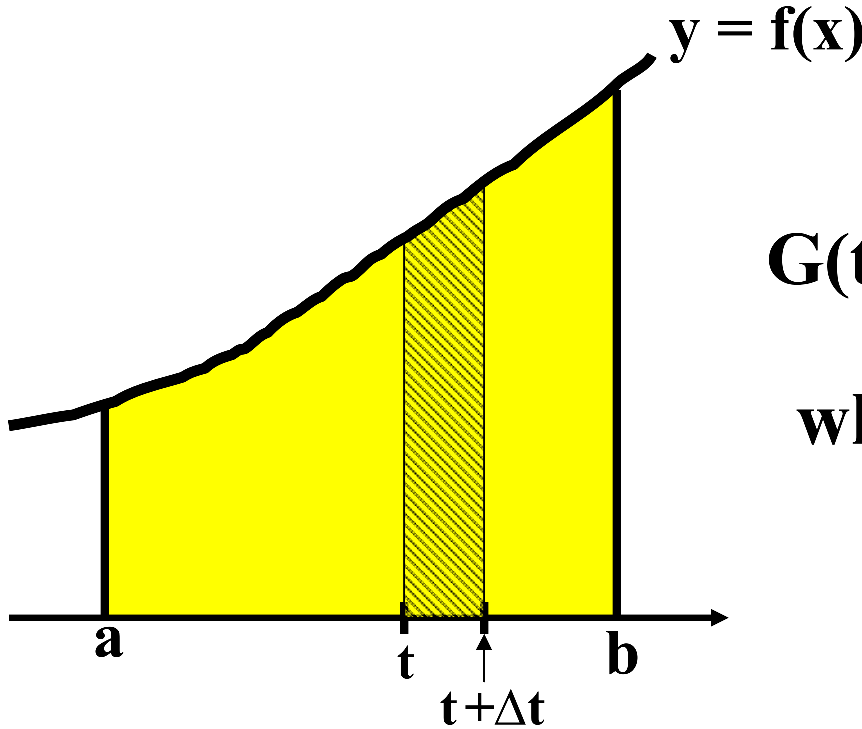

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Calculus Unit 3 The Fundamental Theorems

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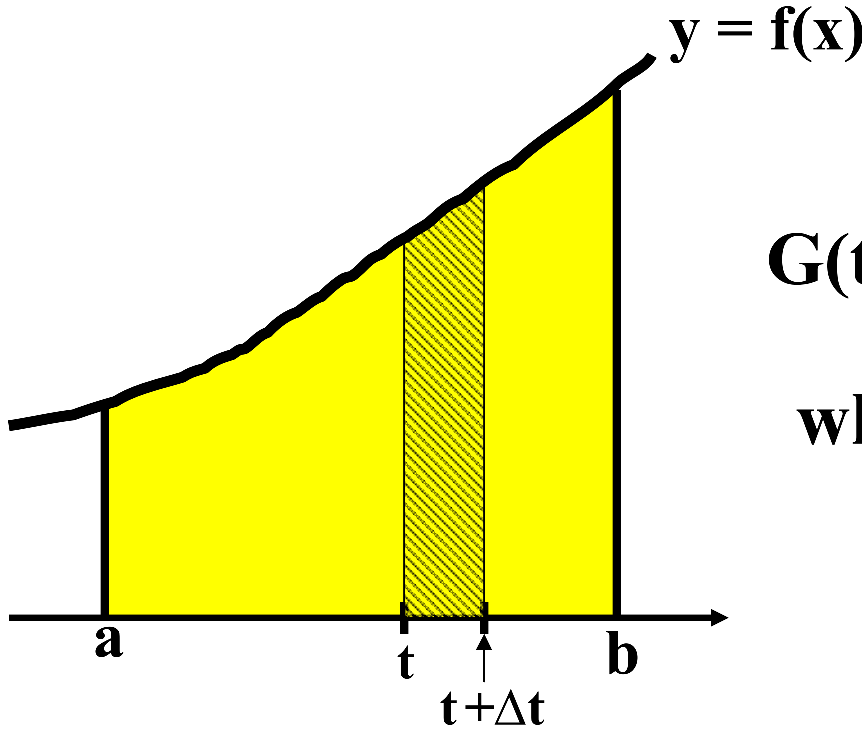

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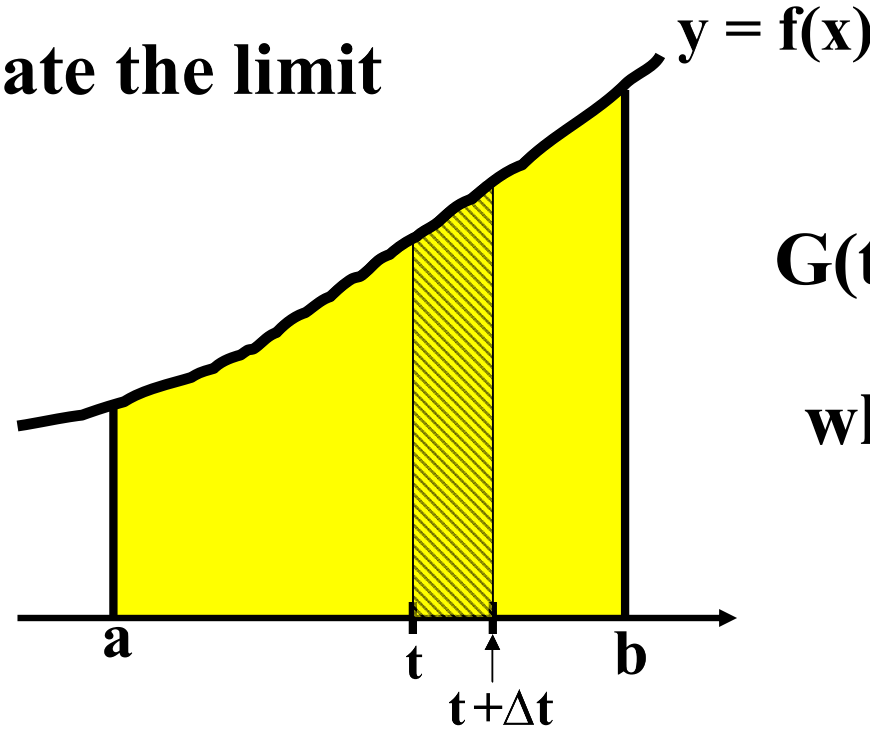

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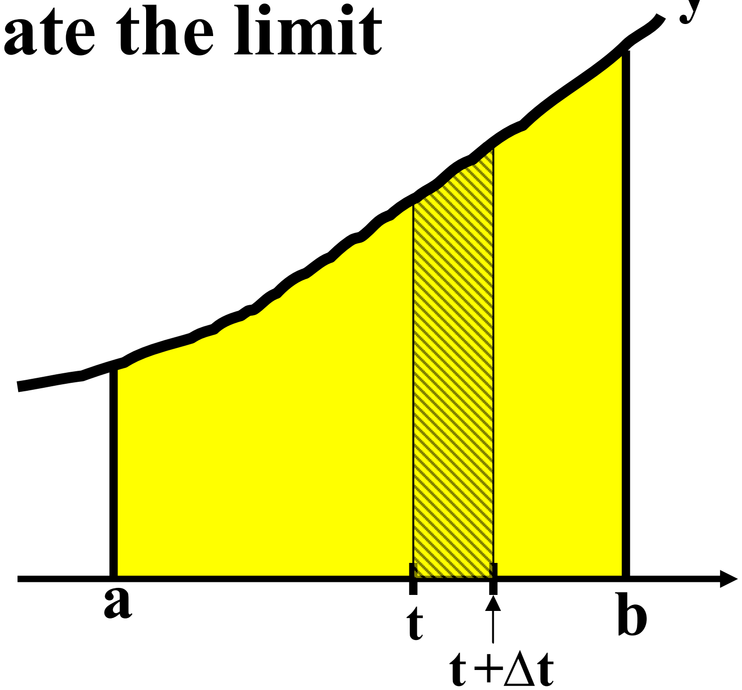

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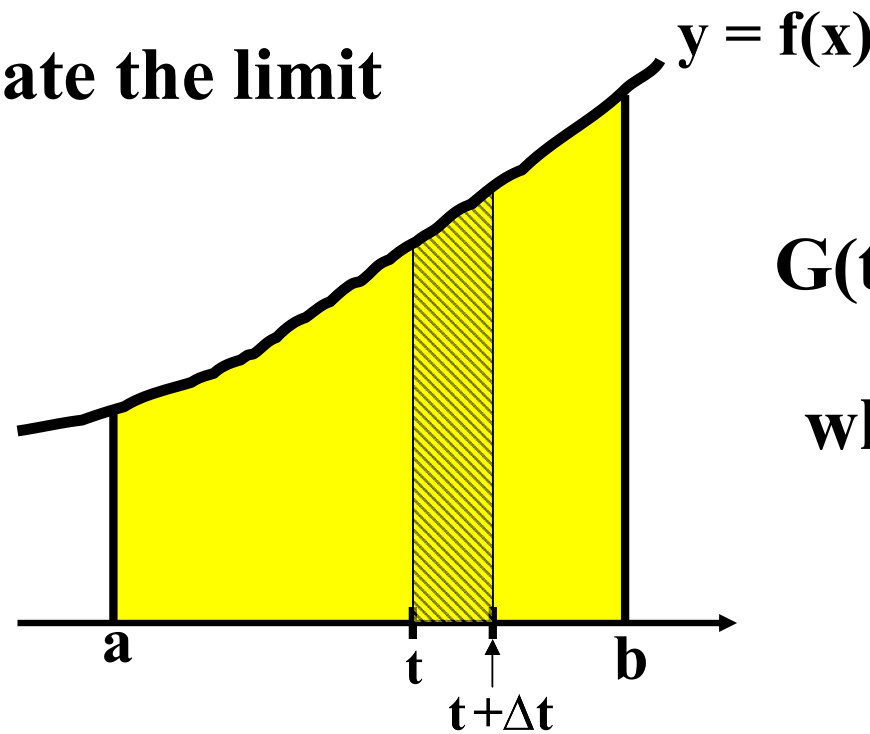

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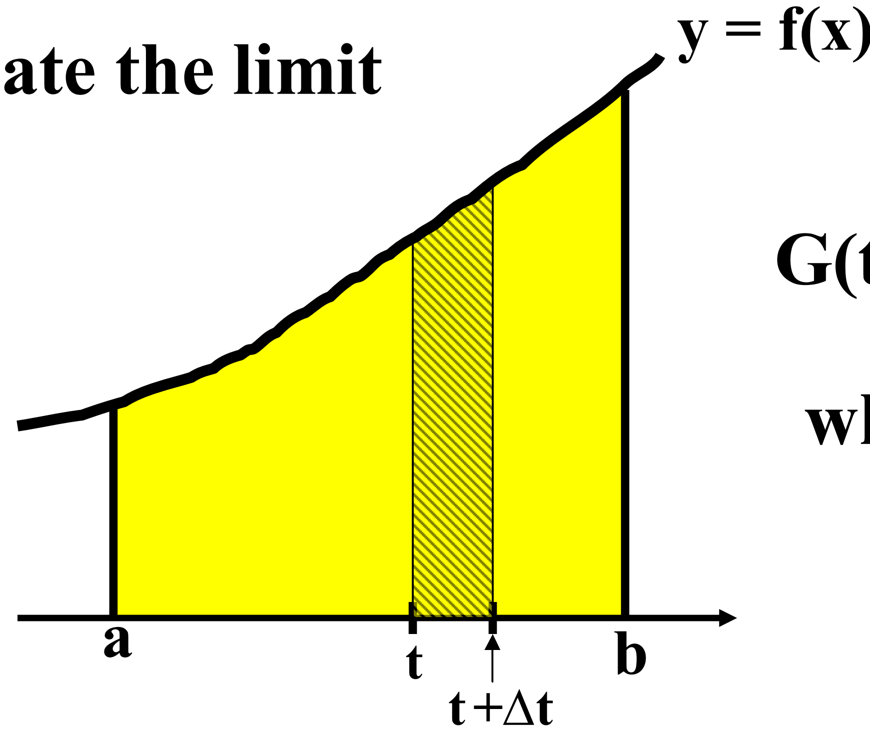

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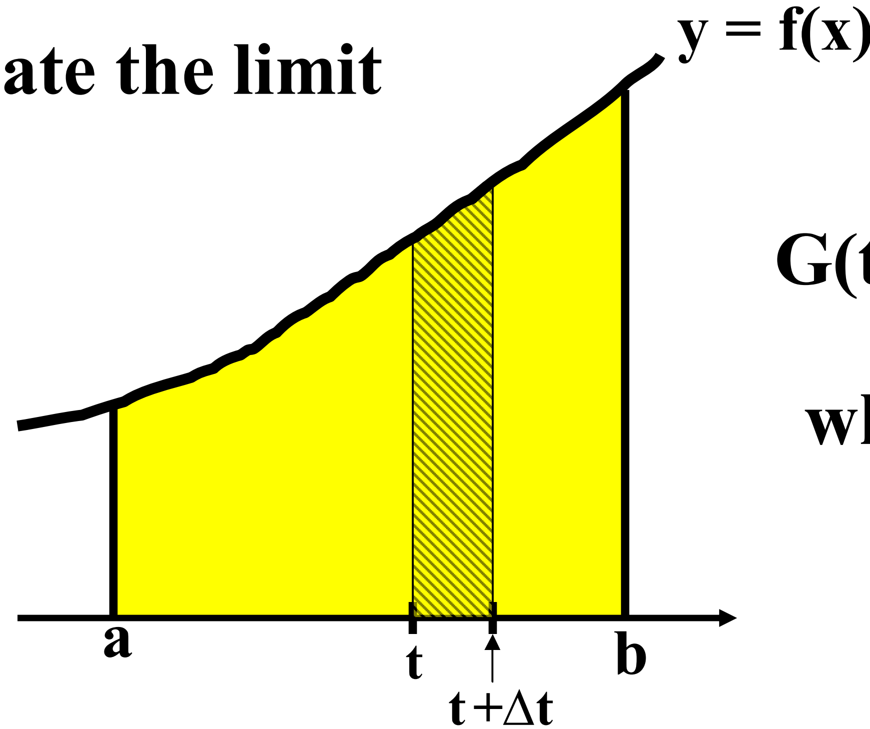

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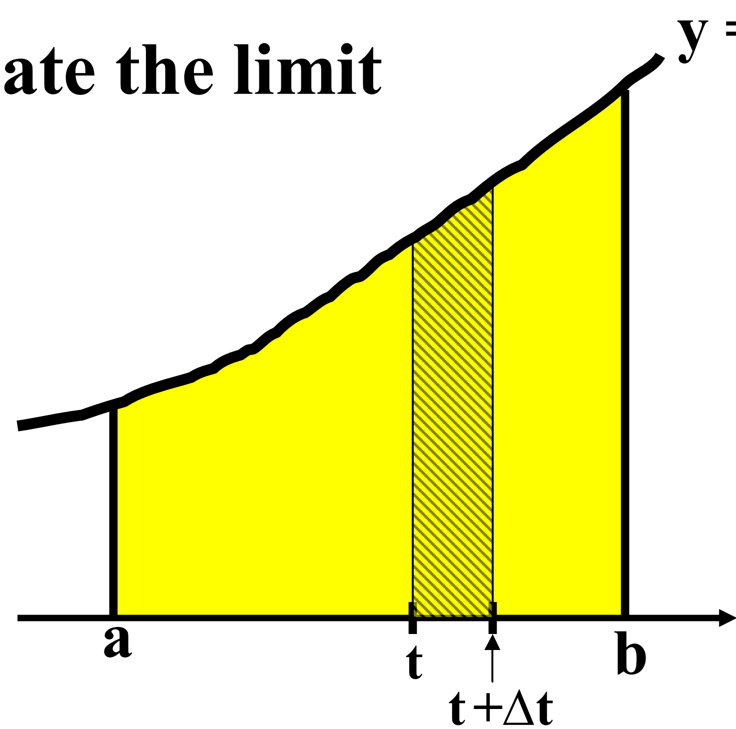

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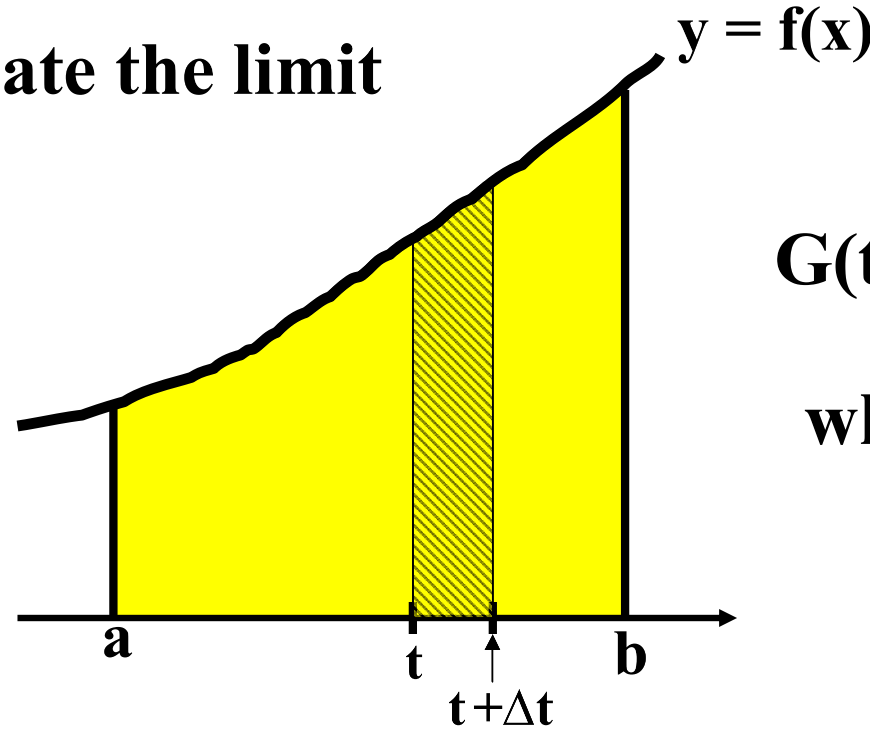

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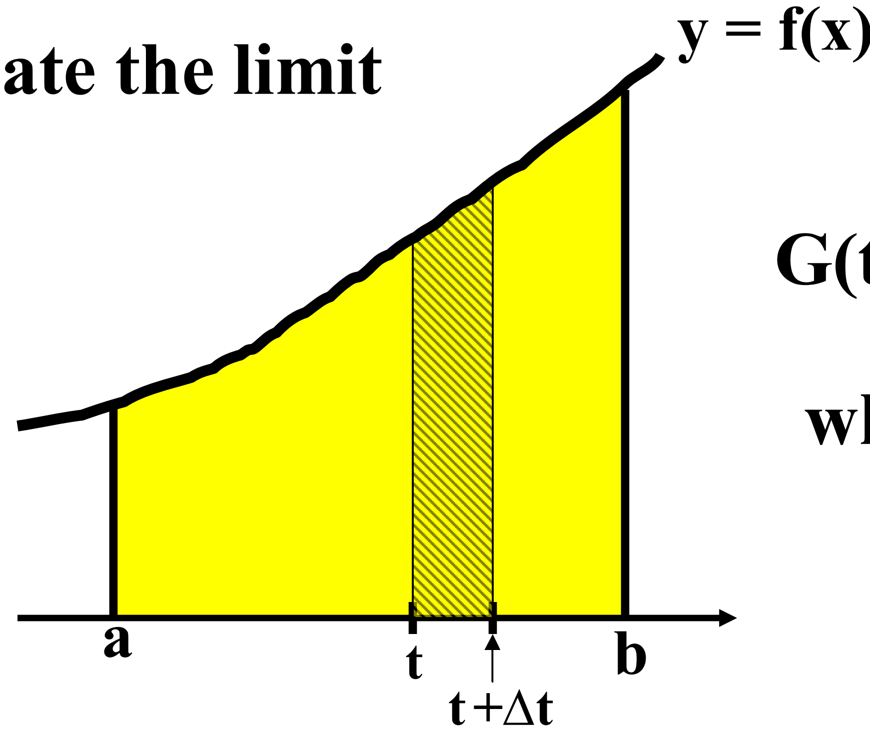

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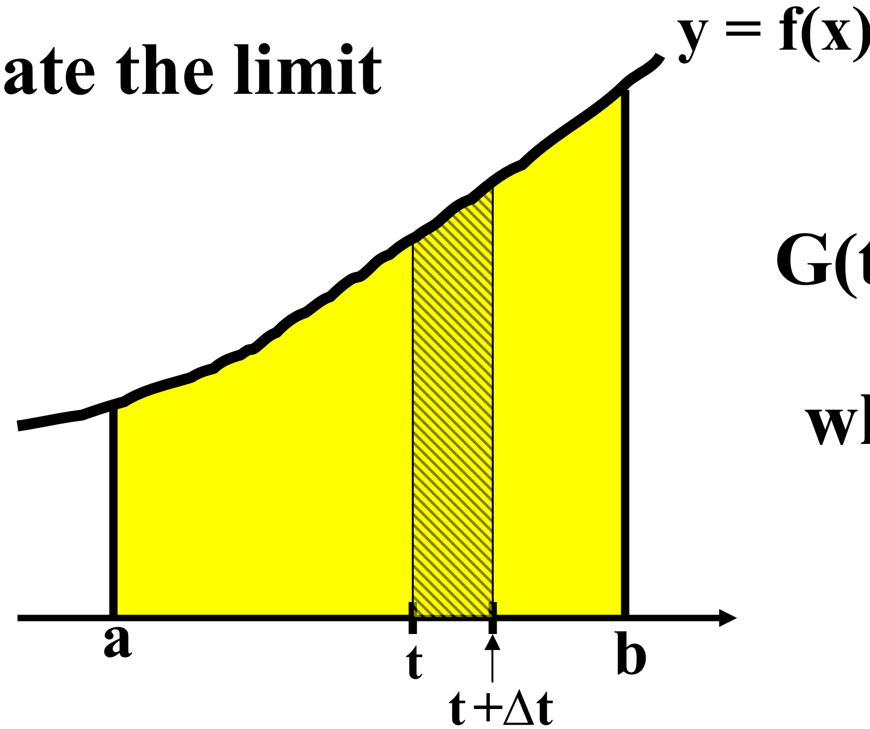

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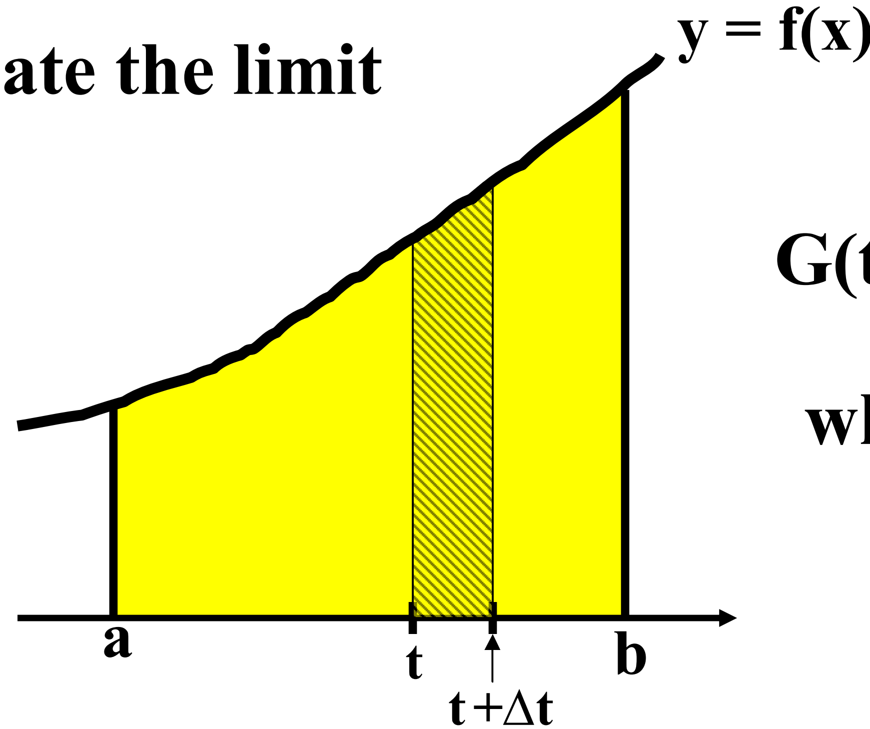

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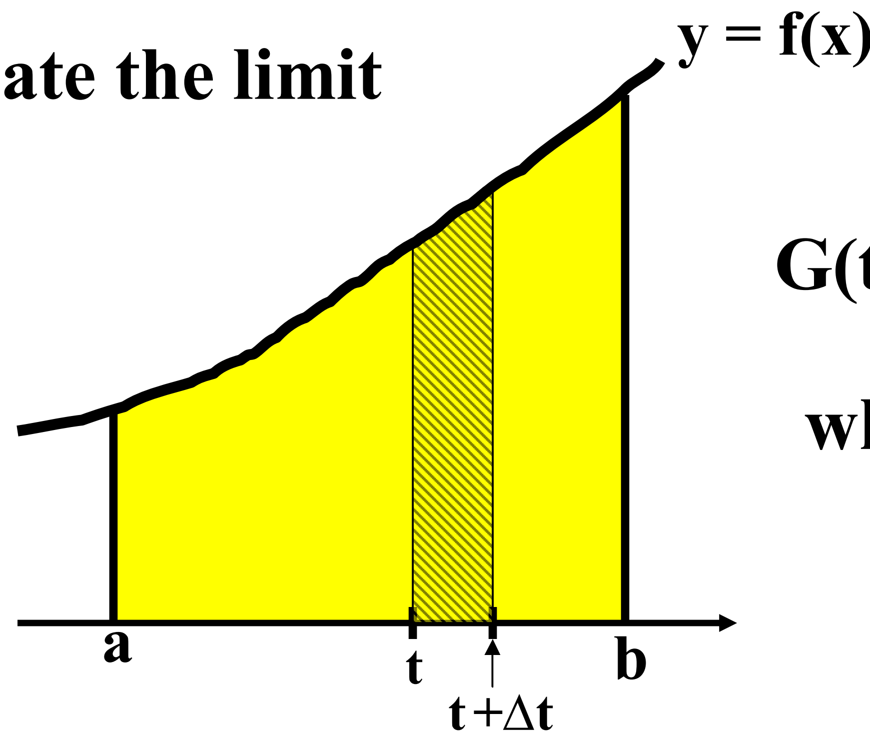

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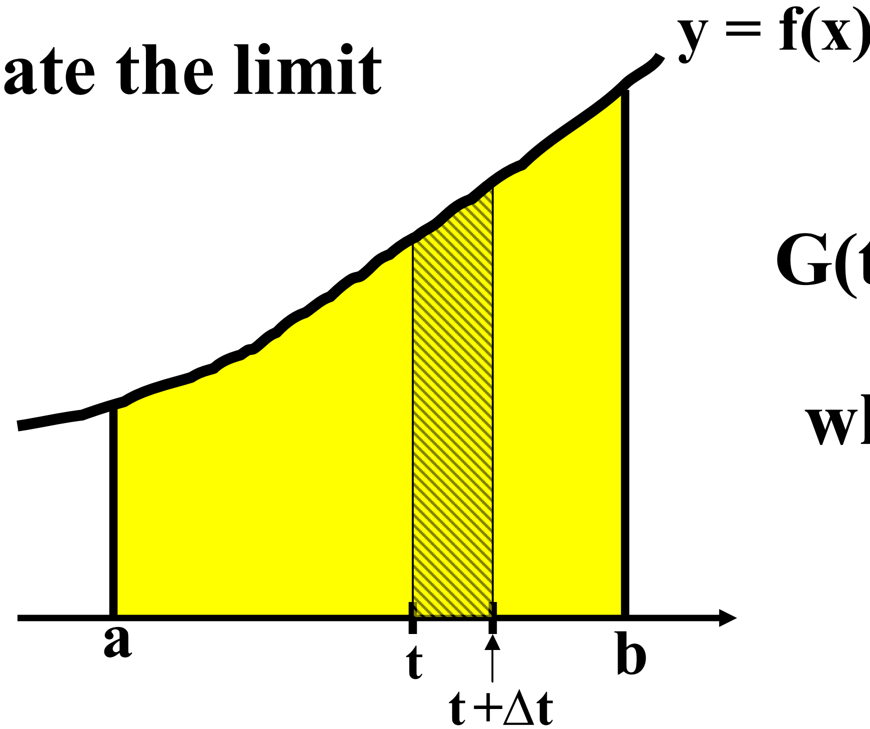

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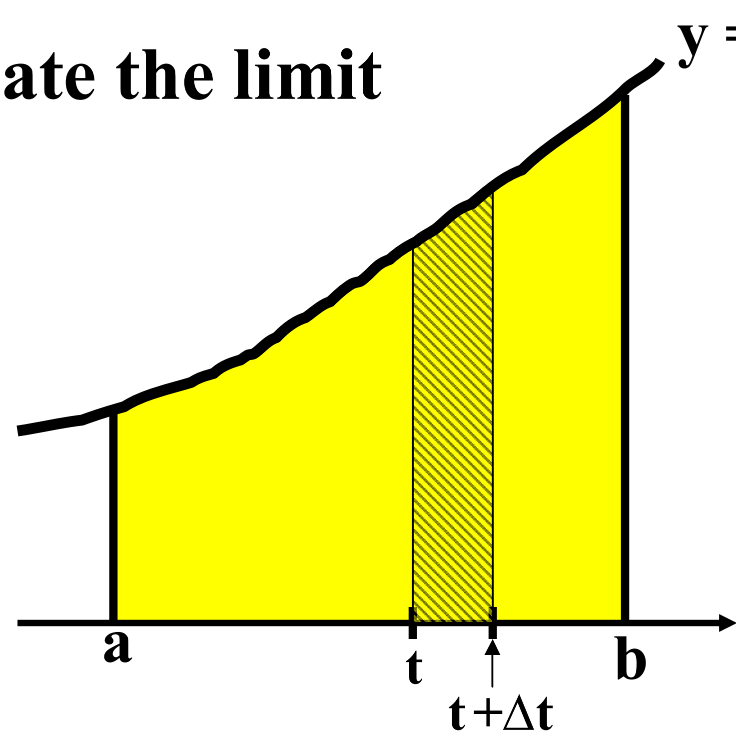

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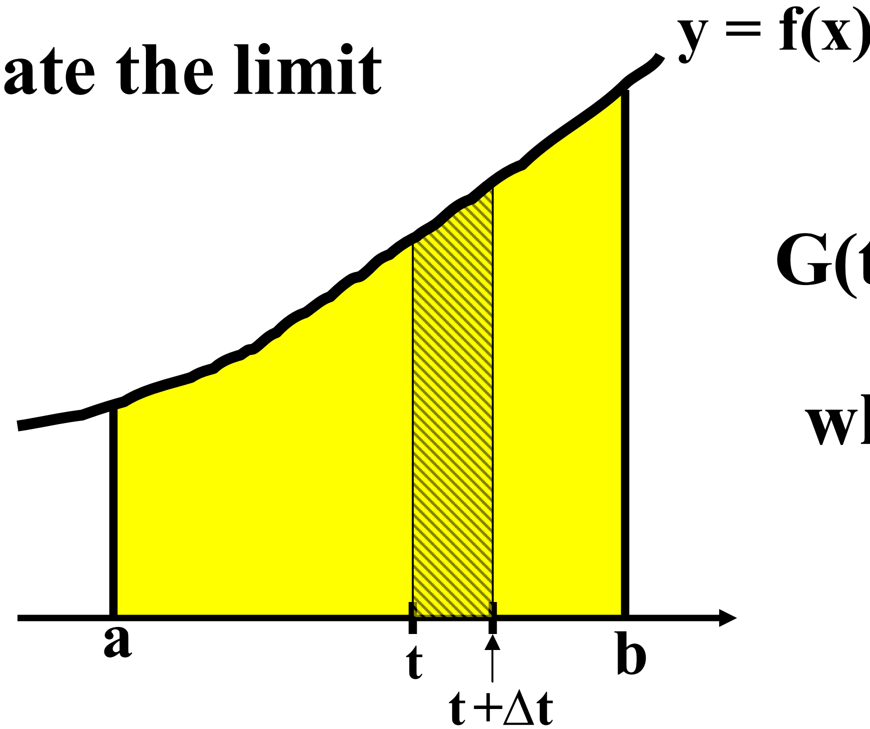

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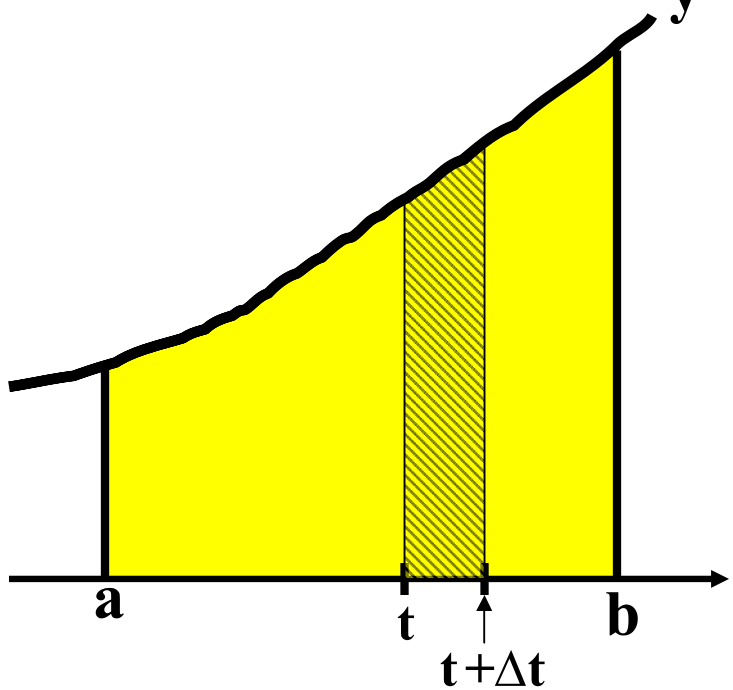

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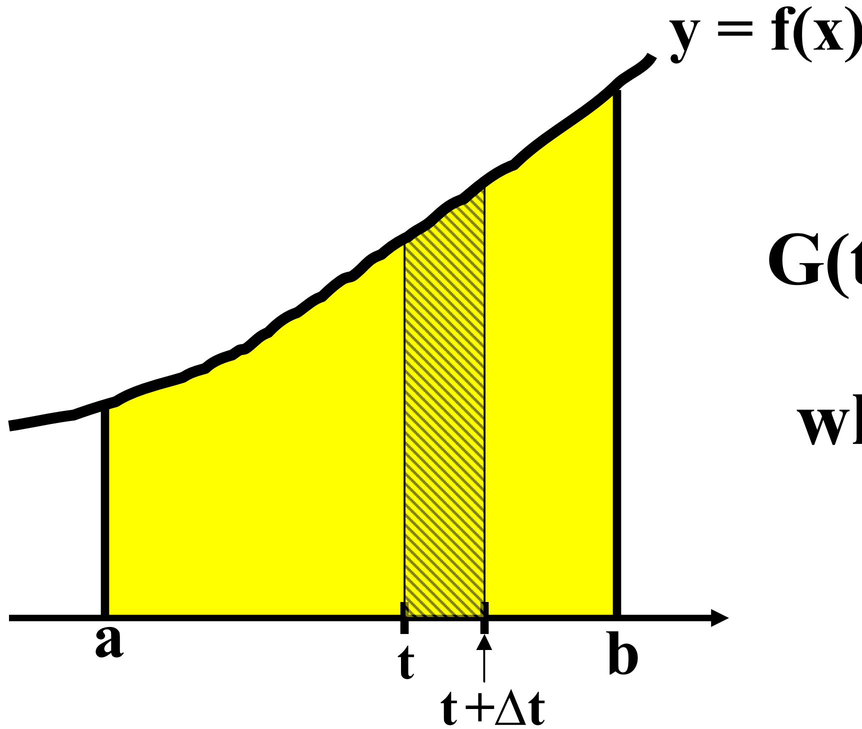

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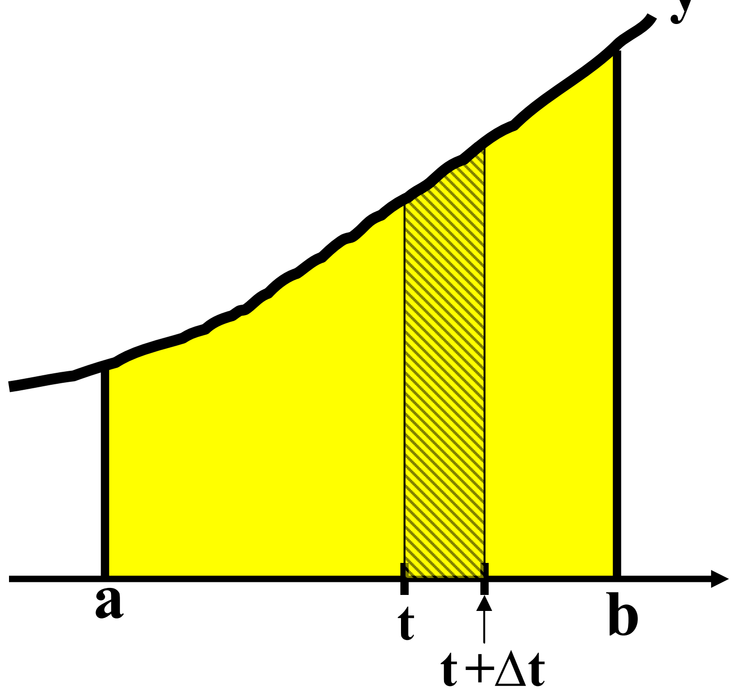

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$f(t) \le G'(t) \le f(t)$ ⇨ $G'(t) = f(t)$!!!

Let’s review.

Calculus Unit 3 The Fundamental Theorems

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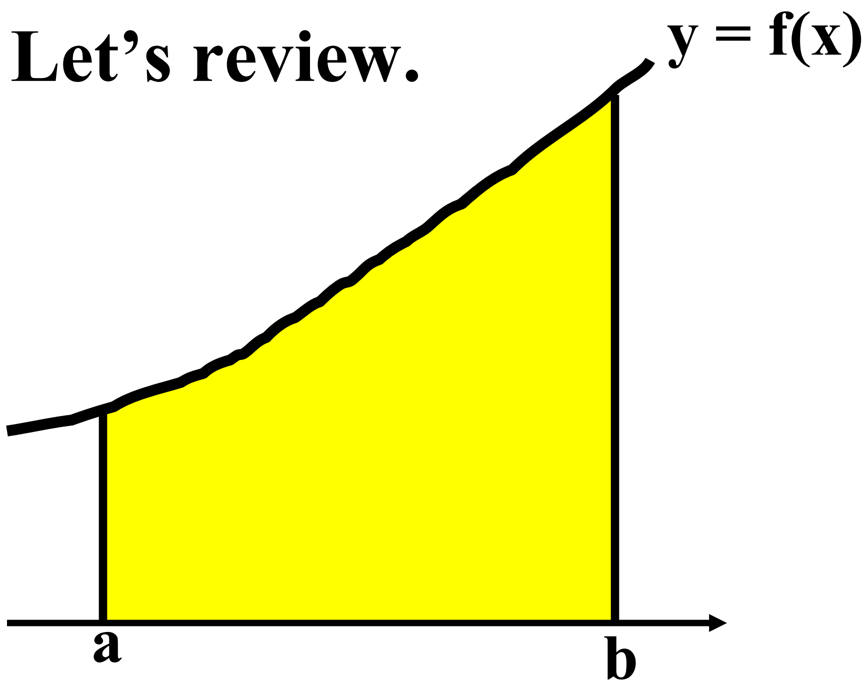

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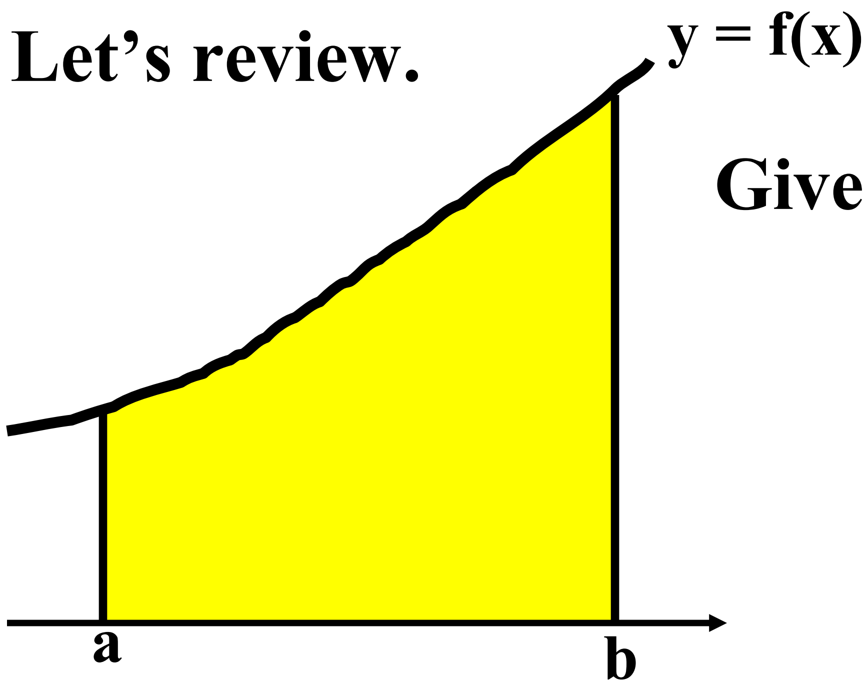

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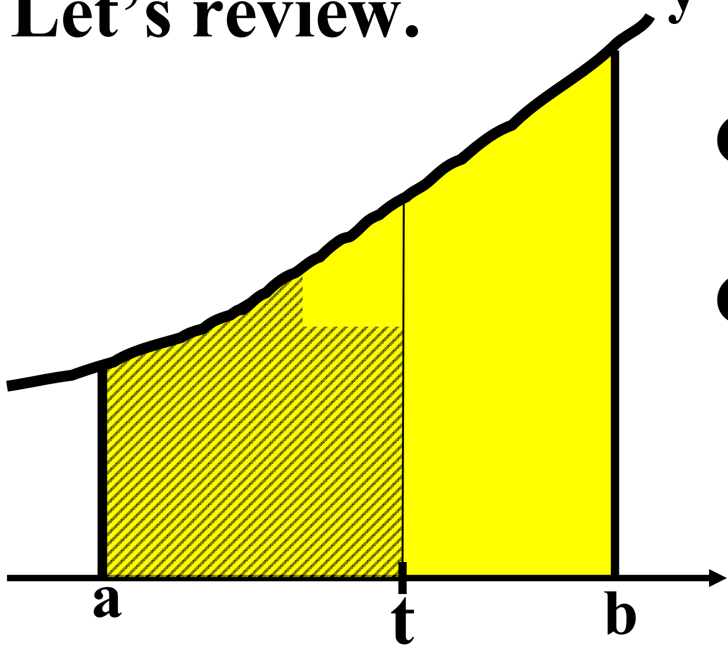

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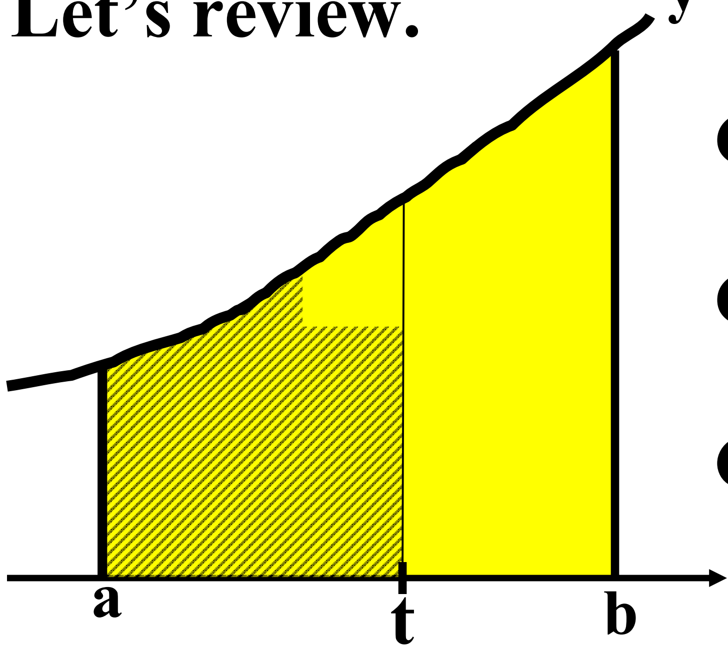

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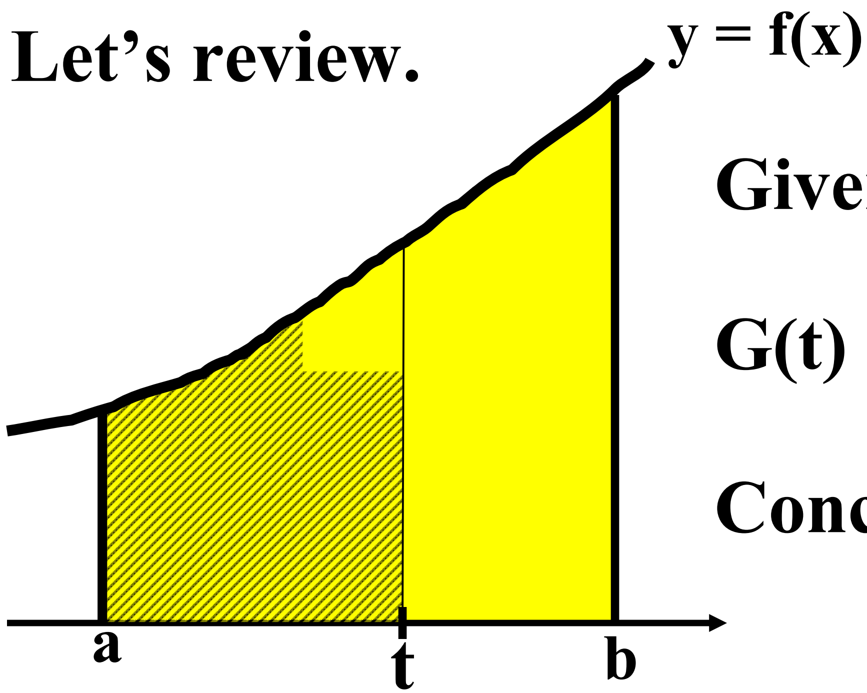

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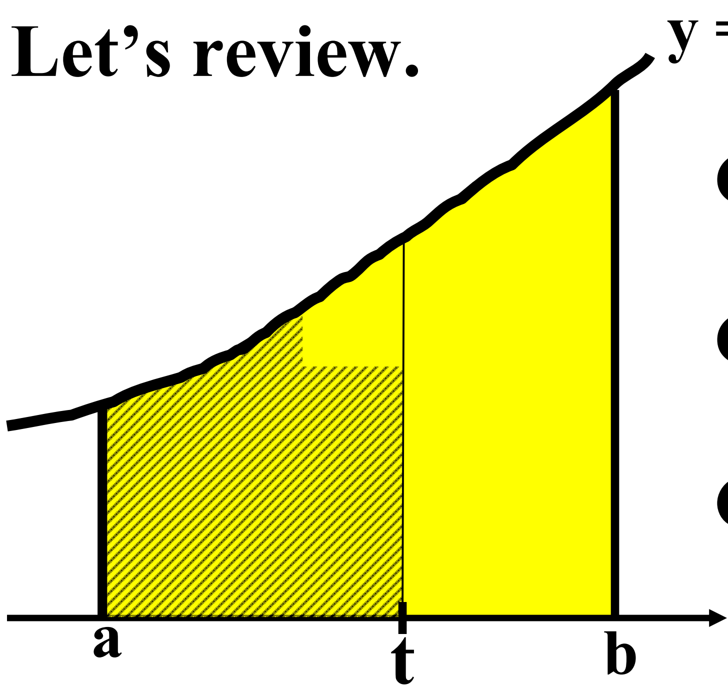

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Conclusion: $G' = f$

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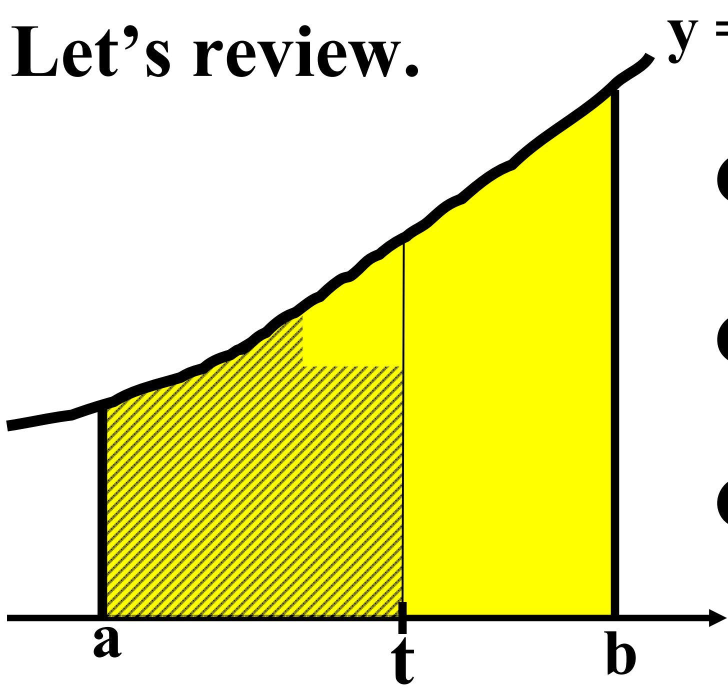

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$$G(t) = \int_a^t f(x)\,dx \quad \text{where } a \leq t \leq b.$$

Conclusion: $G' = f$

This is called the first fundamental theorem of calculus.

Calculus Unit 3 The Fundamental Theorems

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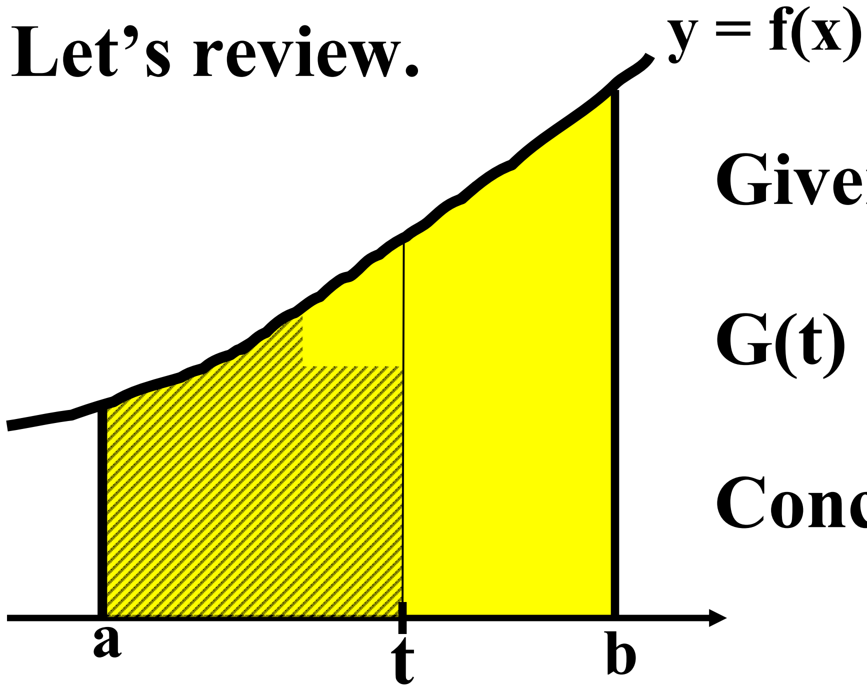

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Since G is a function whose derivative is f,

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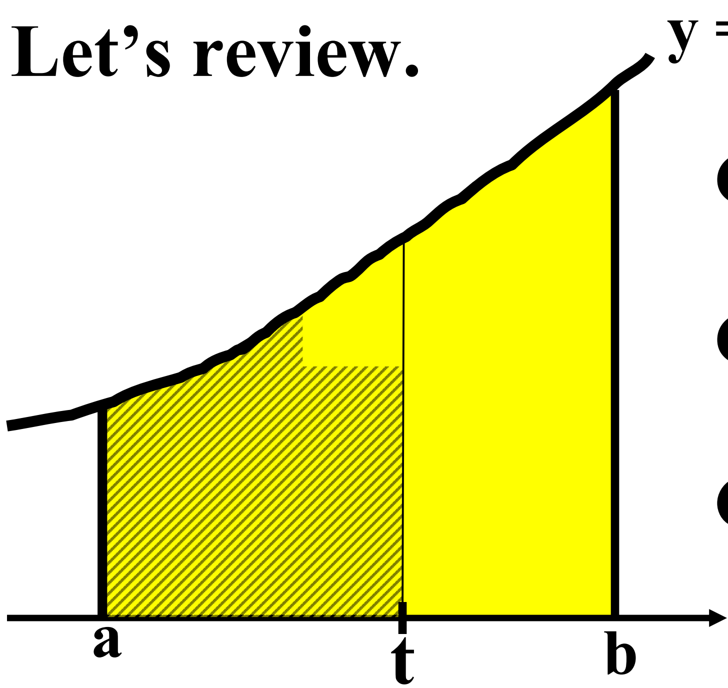

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This is called the first fundamental theorem of calculus. Since G is a function whose derivative is f, it is called an <u>antiderivative</u> of f.

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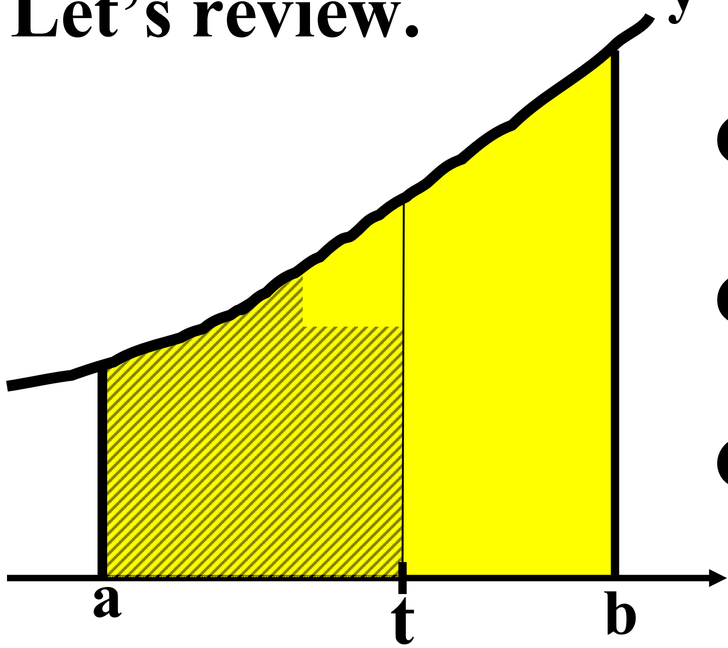

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The process of 'finding' an antiderivative function is called integration.

Calculus Unit 3 The Fundamental Theorems

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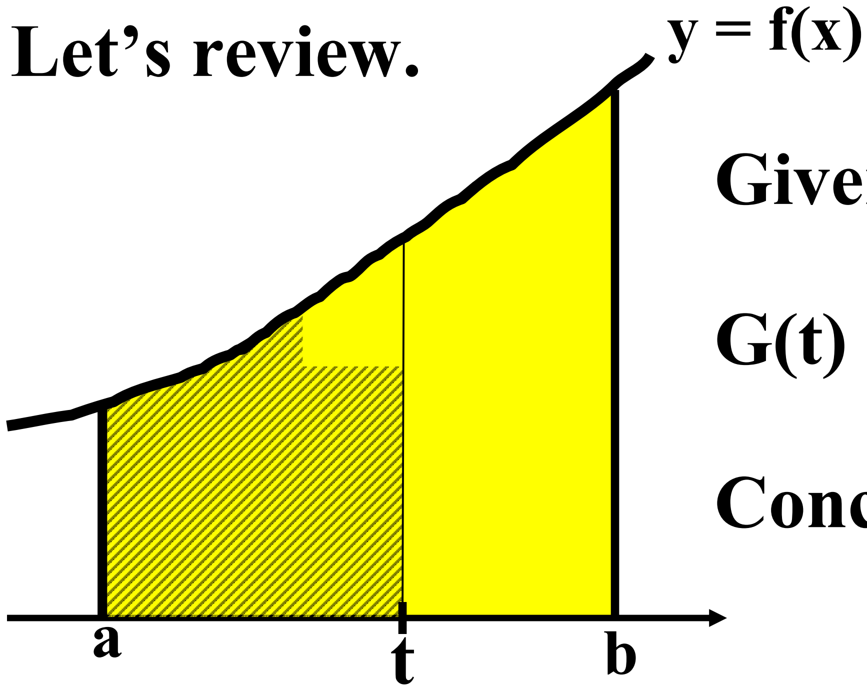

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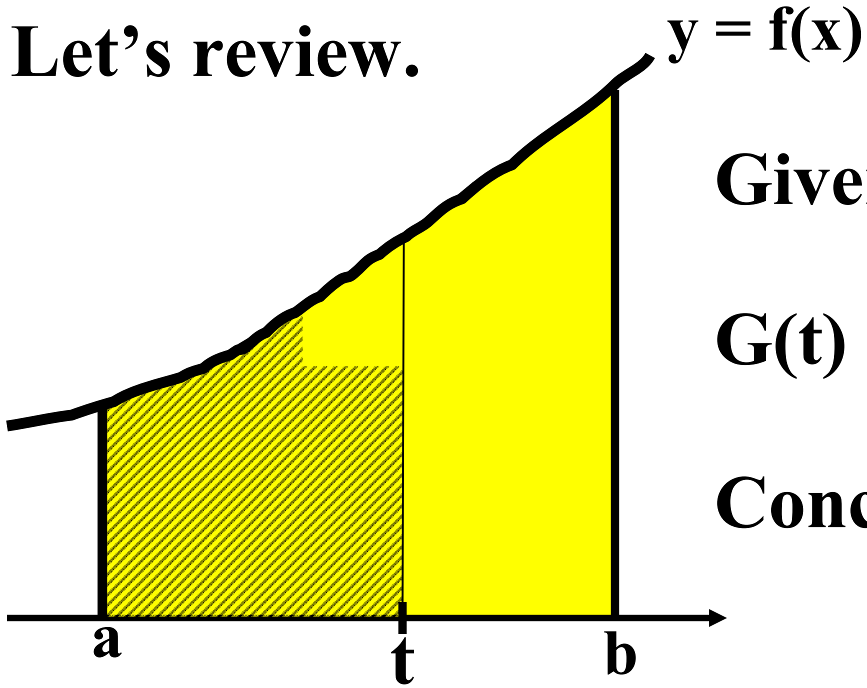

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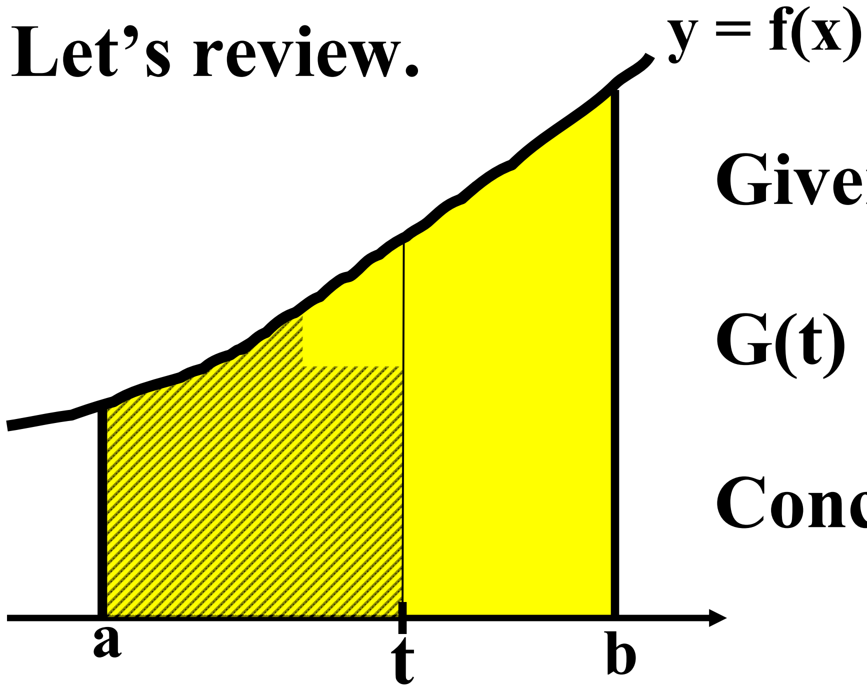

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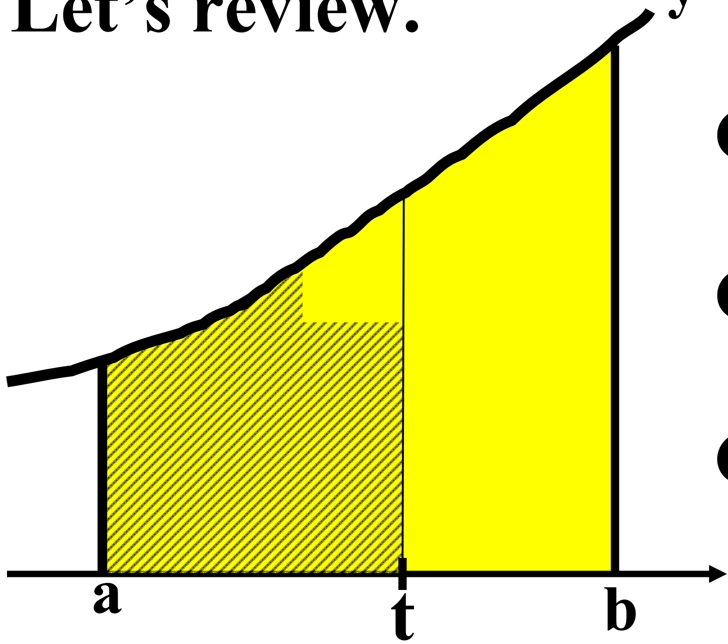

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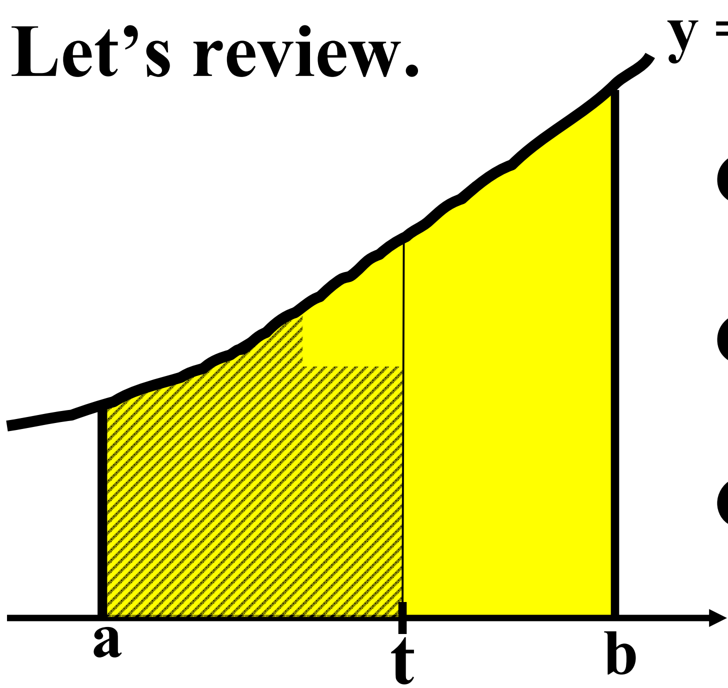

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Make sure you get this !!

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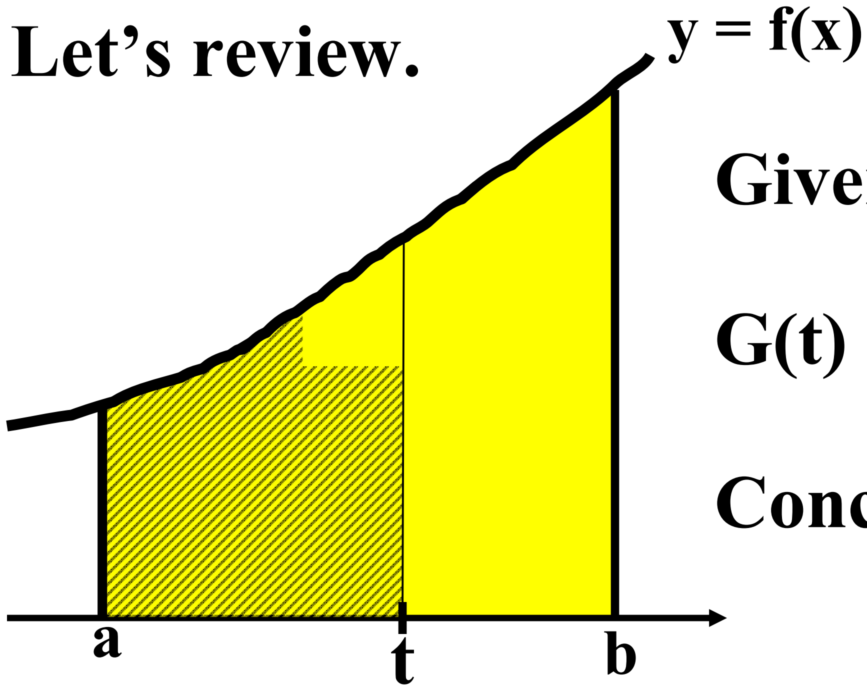

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Make sure you get this !! Remember, C is a constant.

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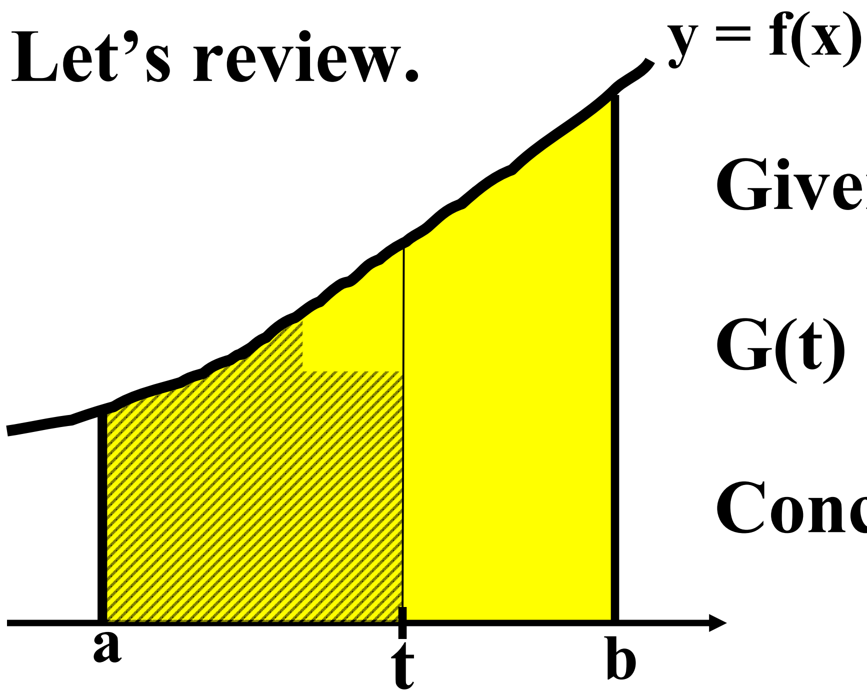

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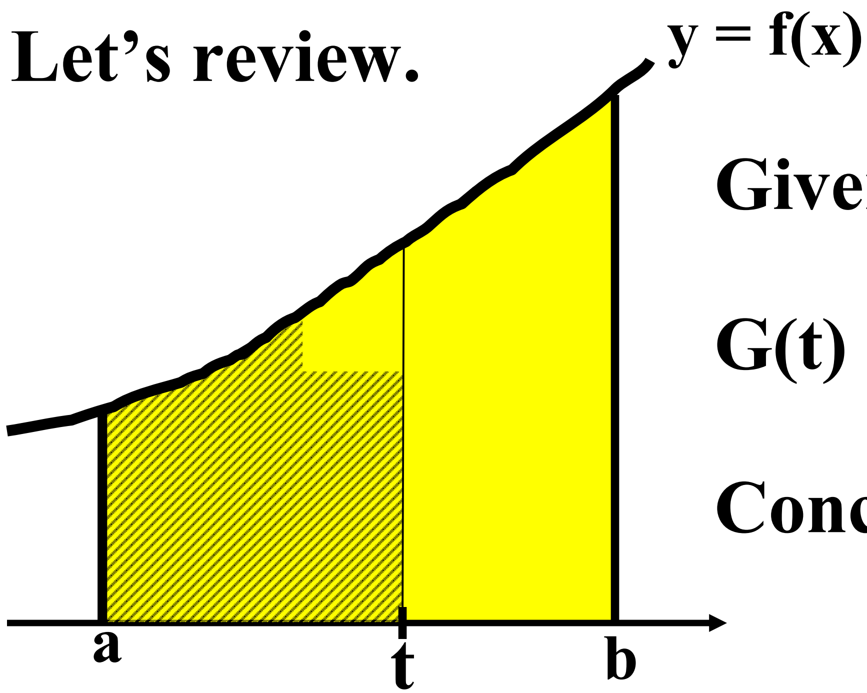

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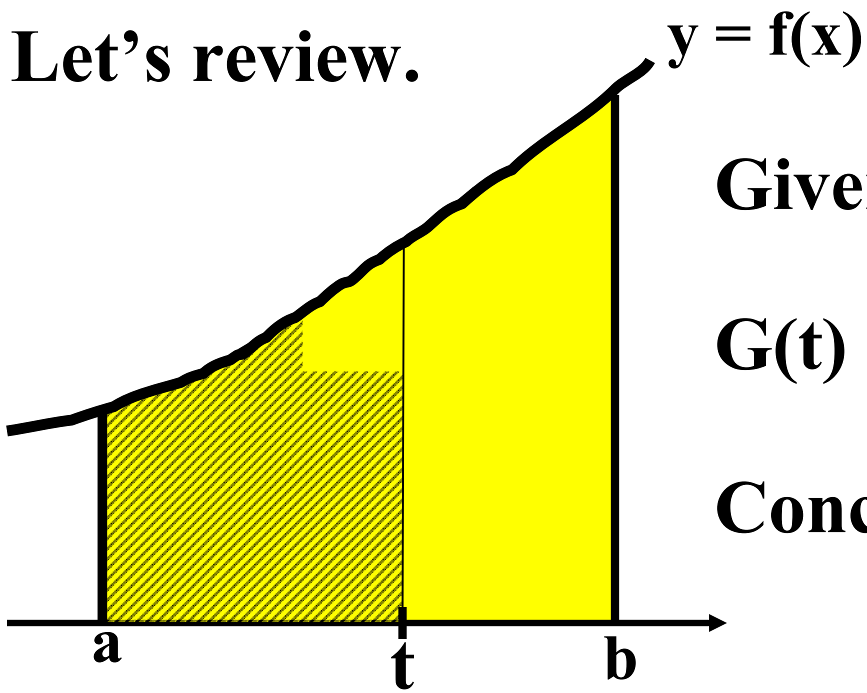

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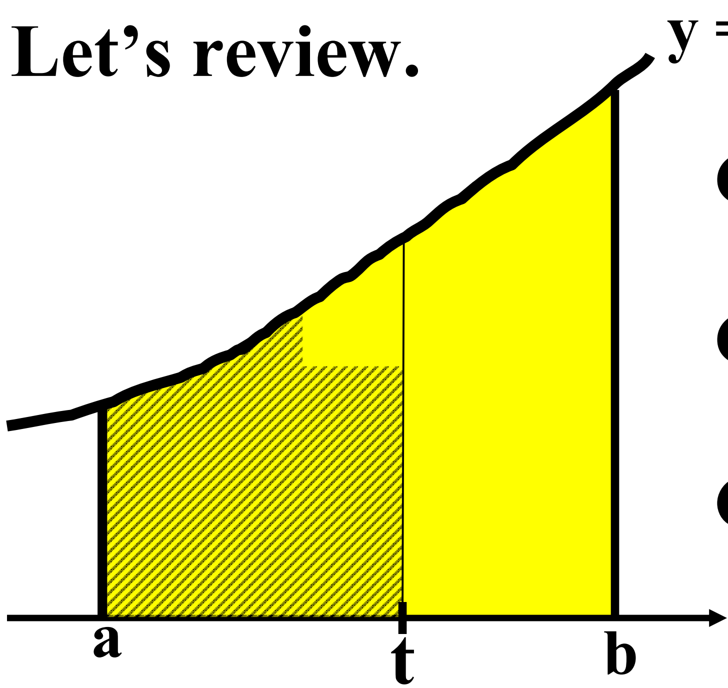

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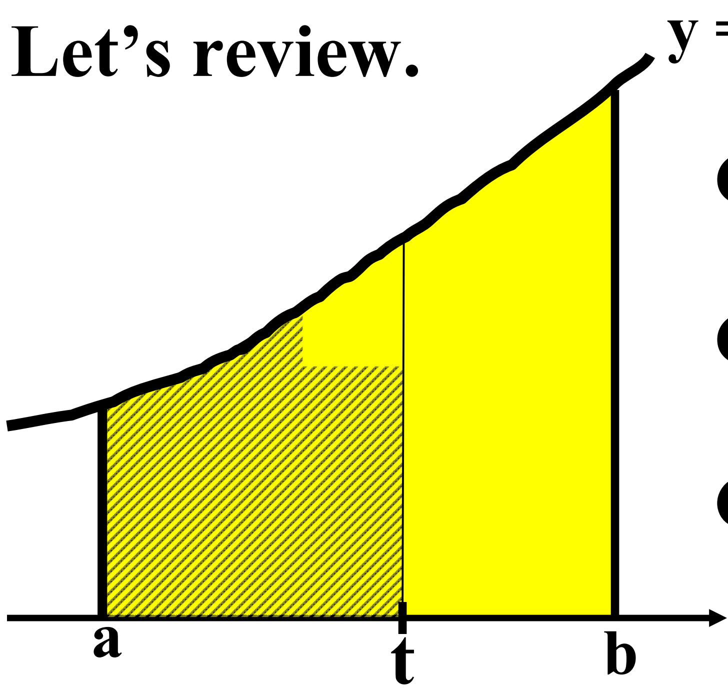

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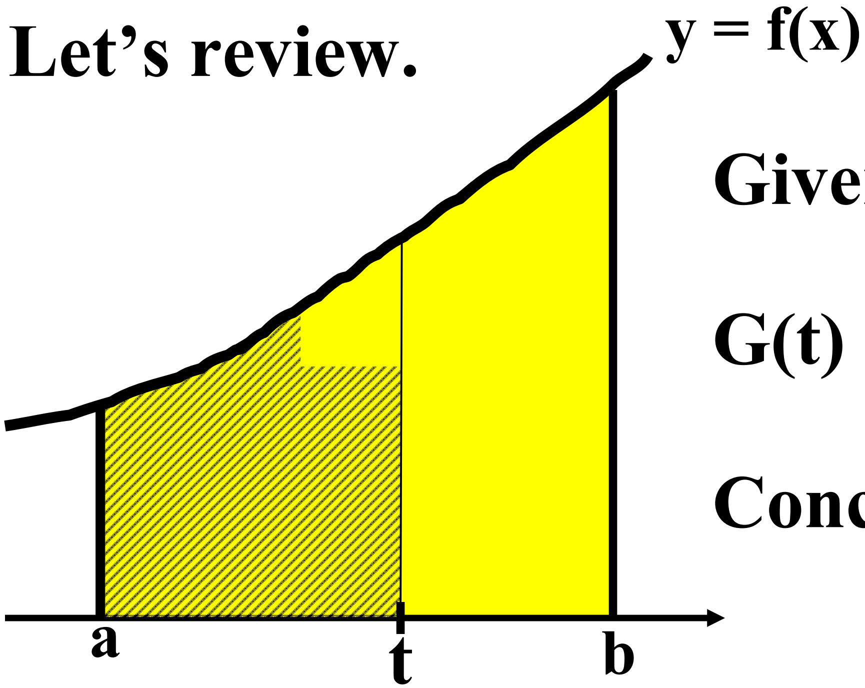

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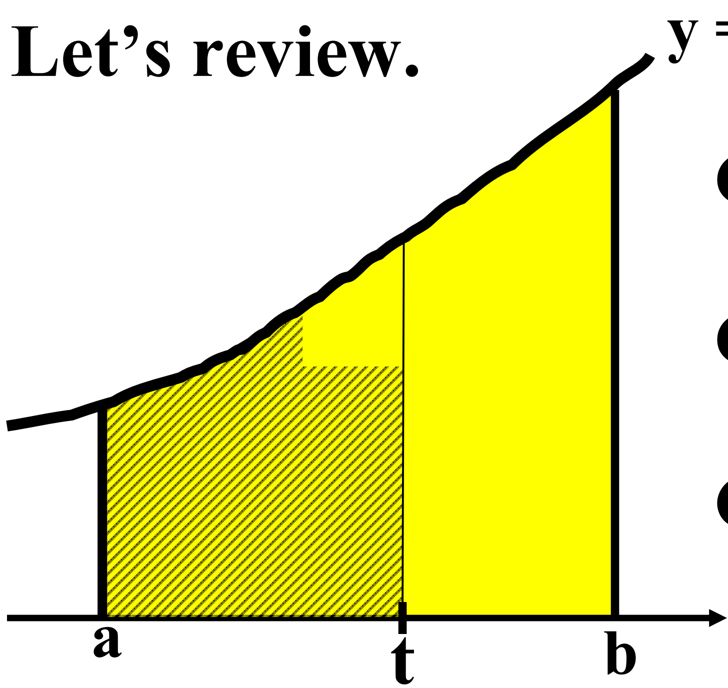

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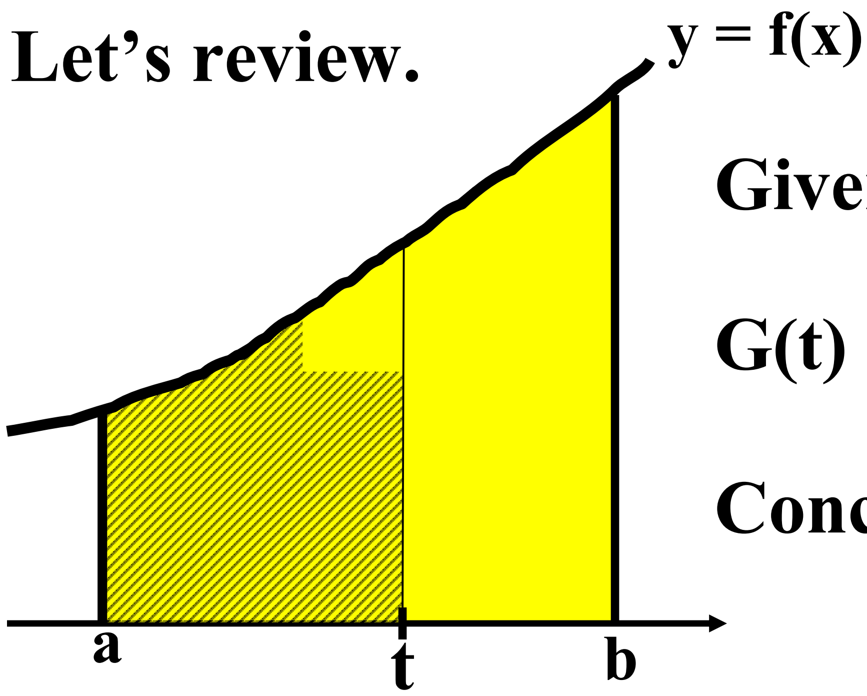

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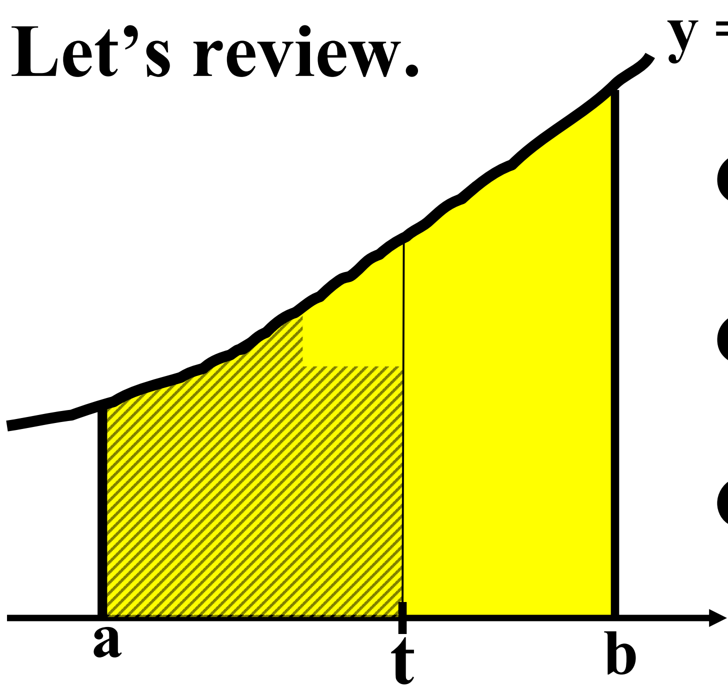

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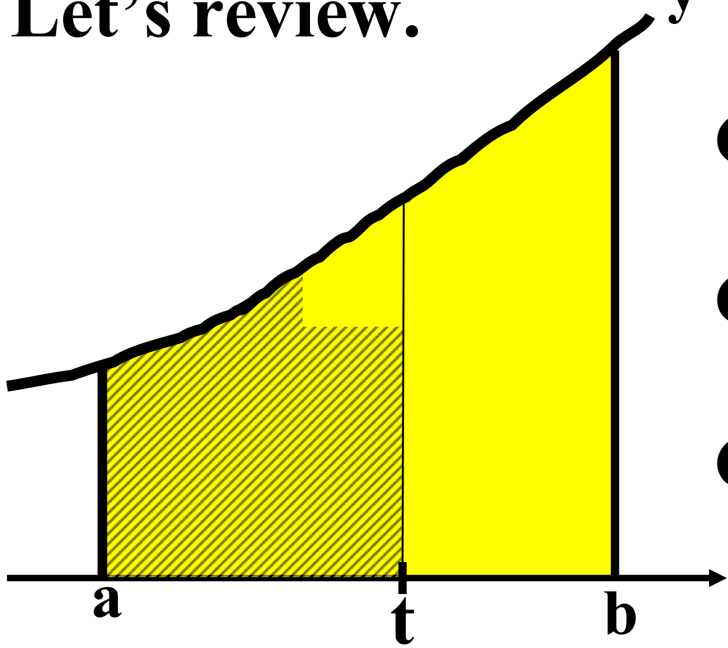

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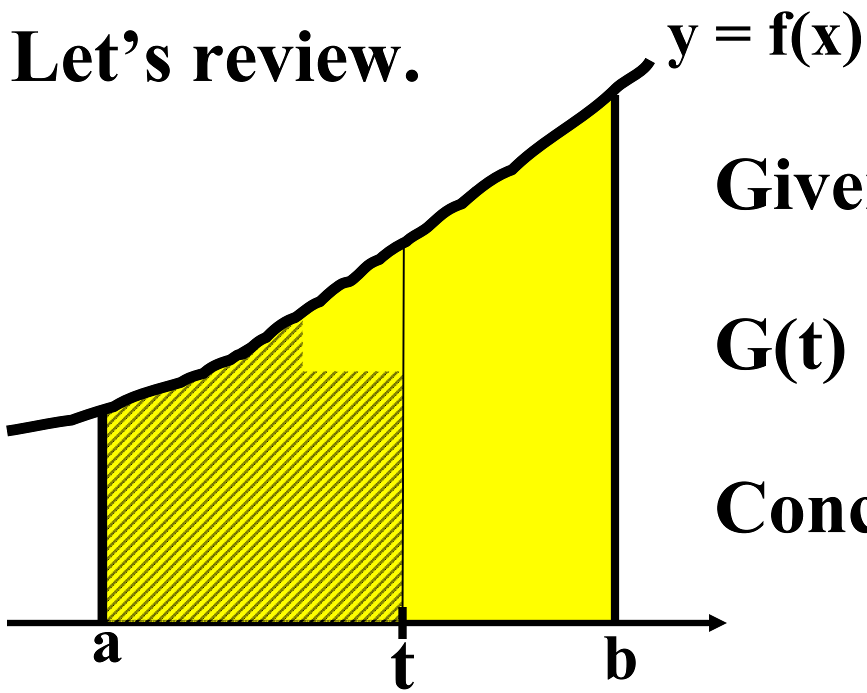

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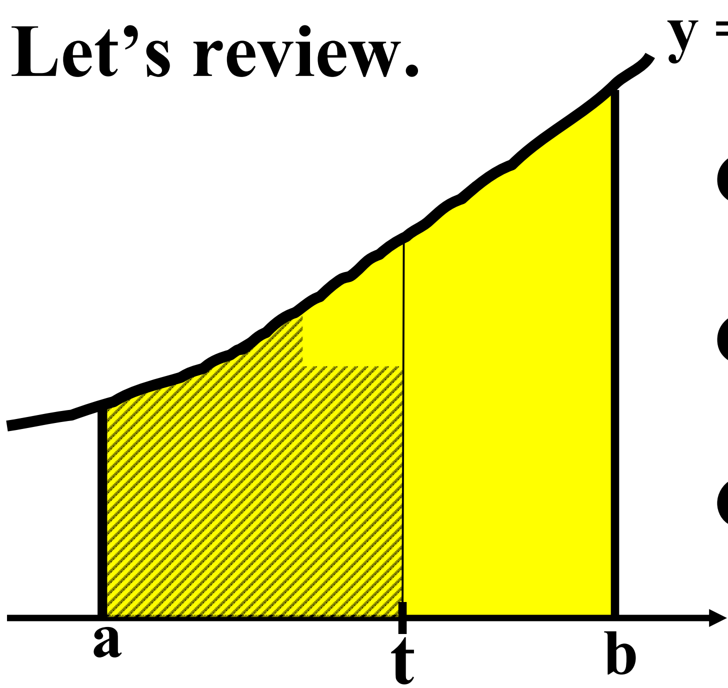

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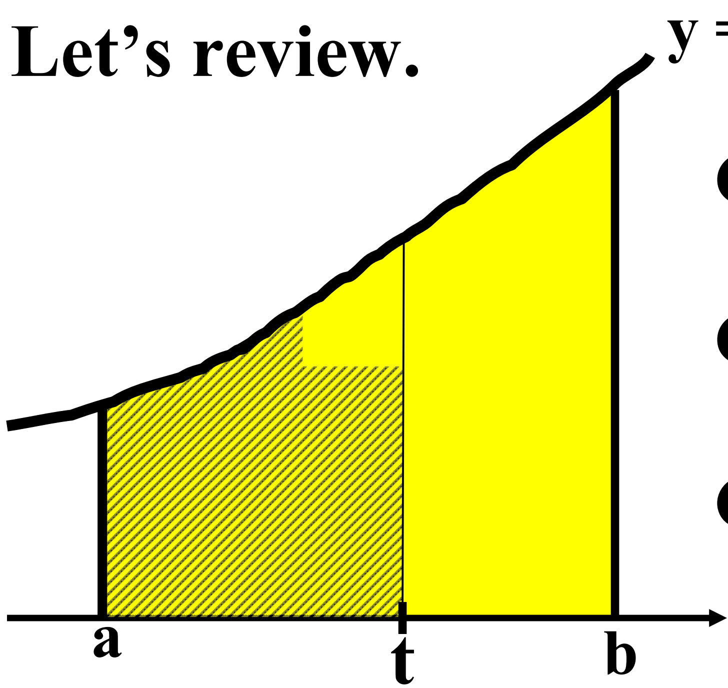

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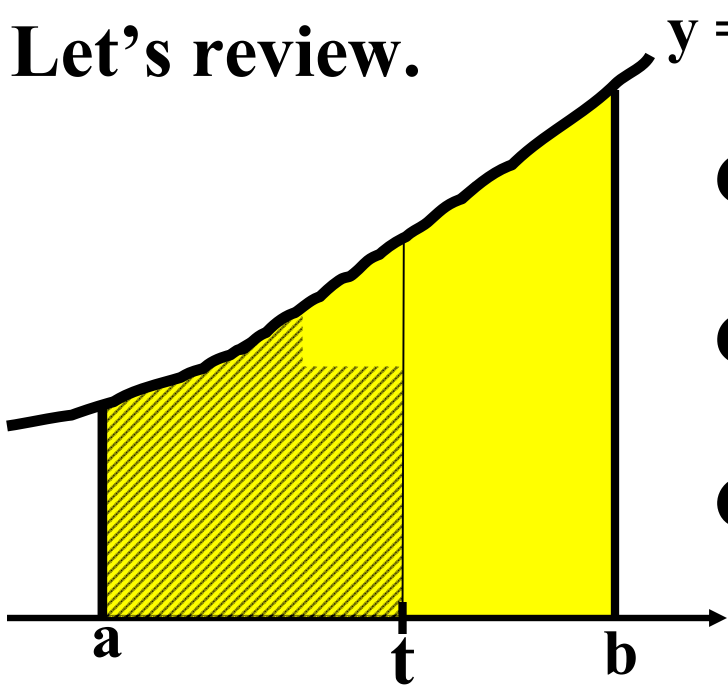

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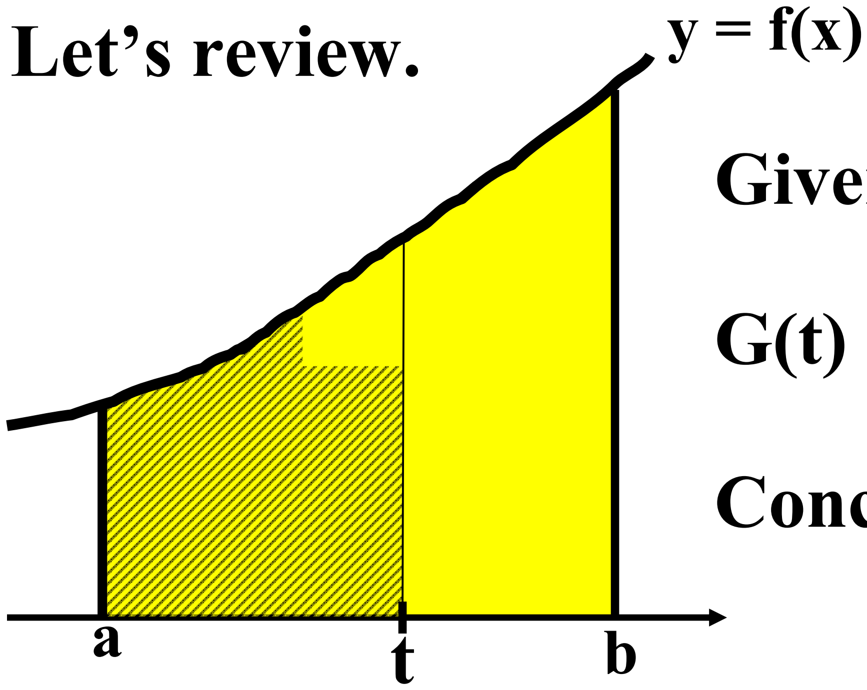

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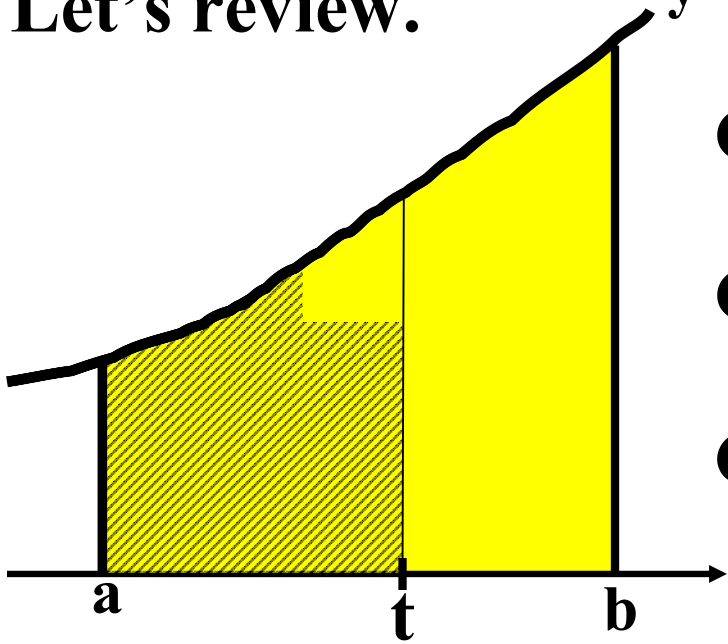

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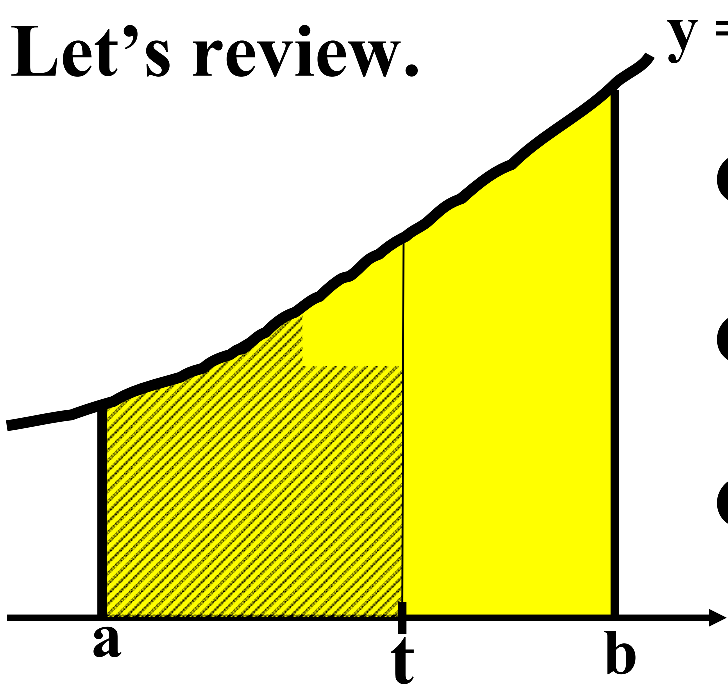

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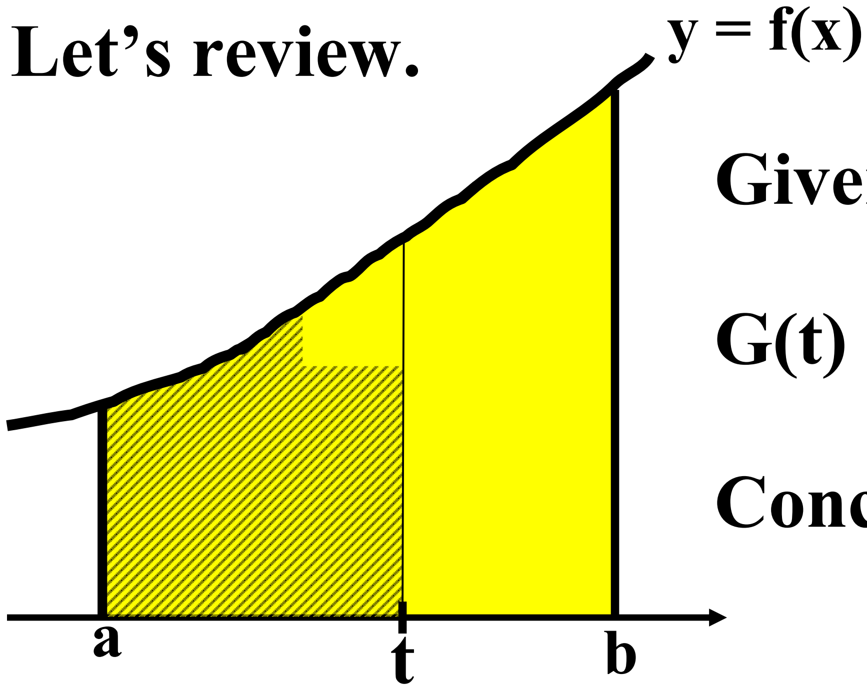

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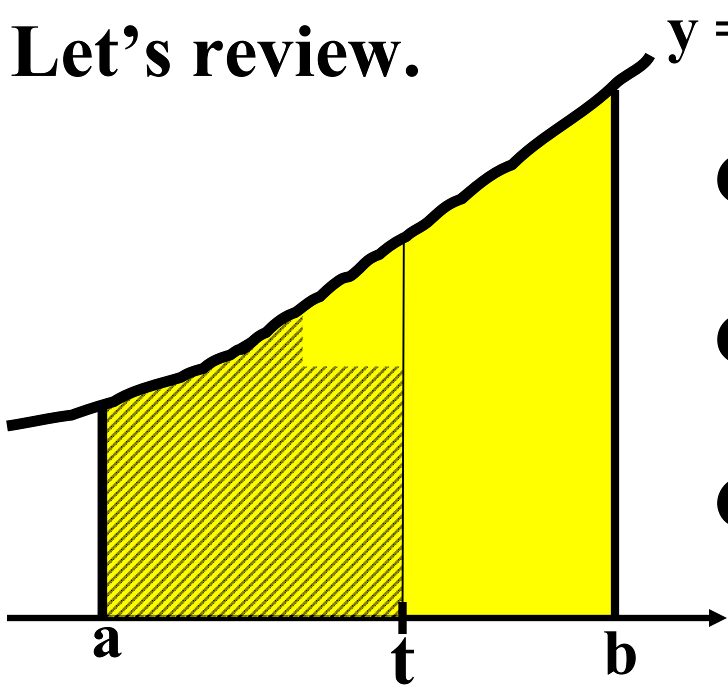

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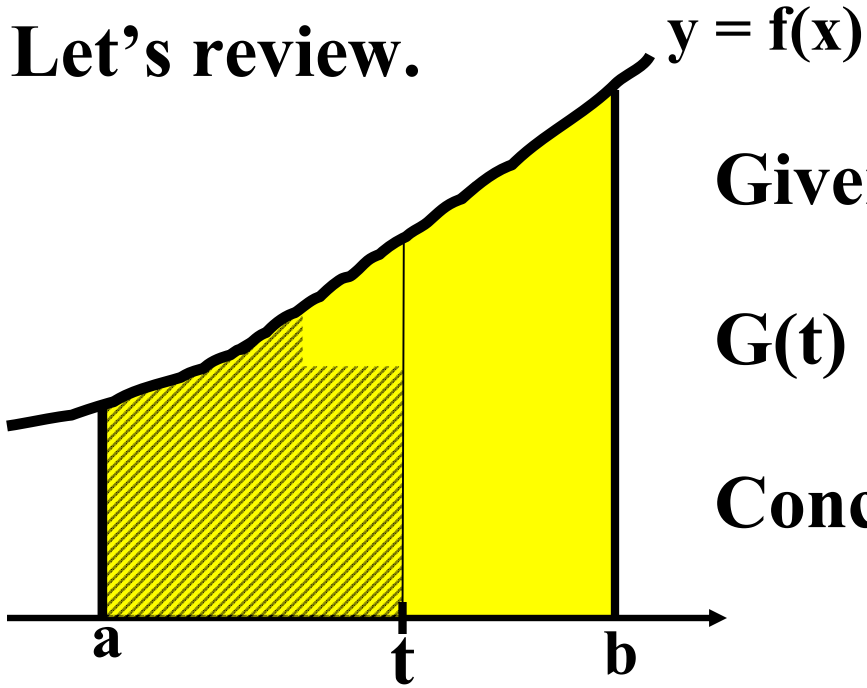

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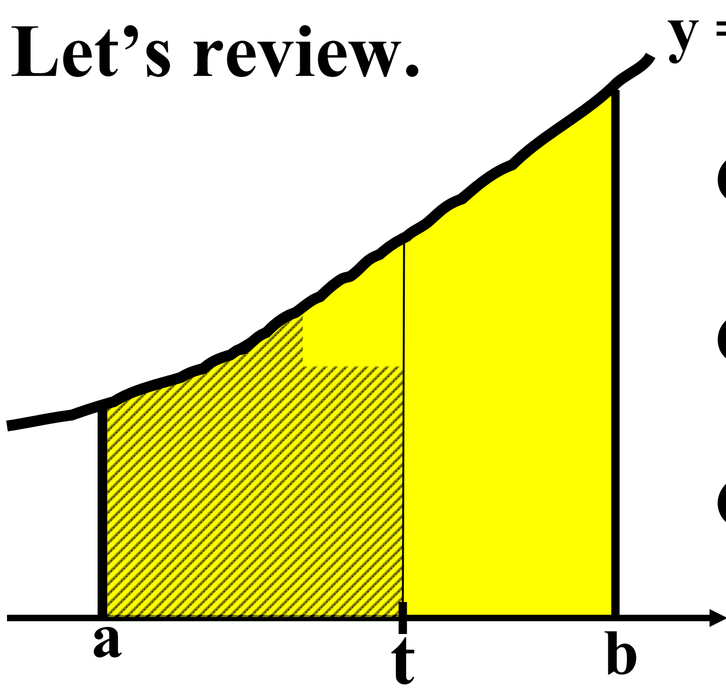

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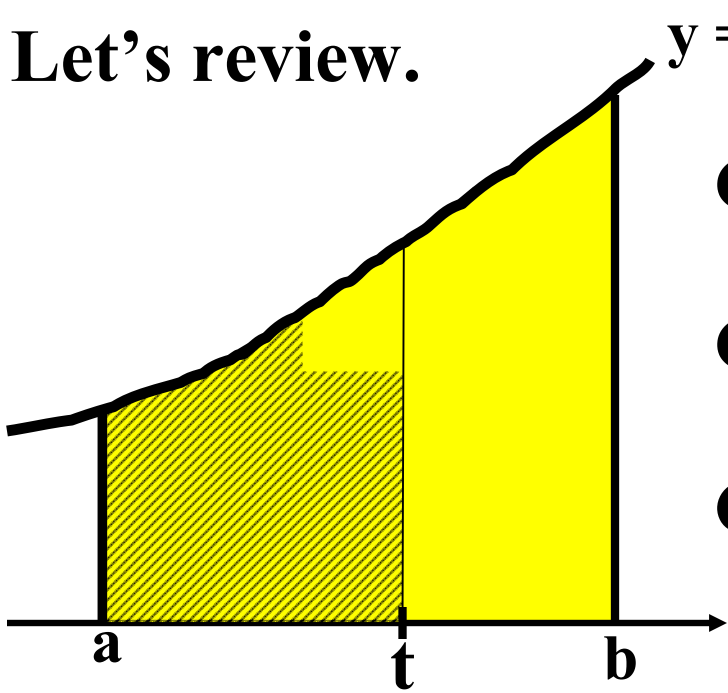

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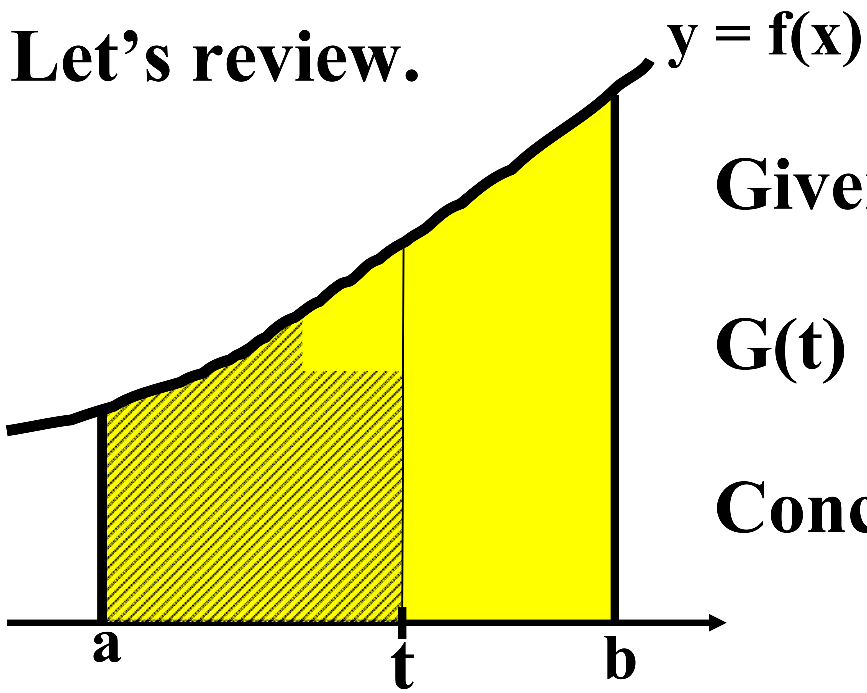

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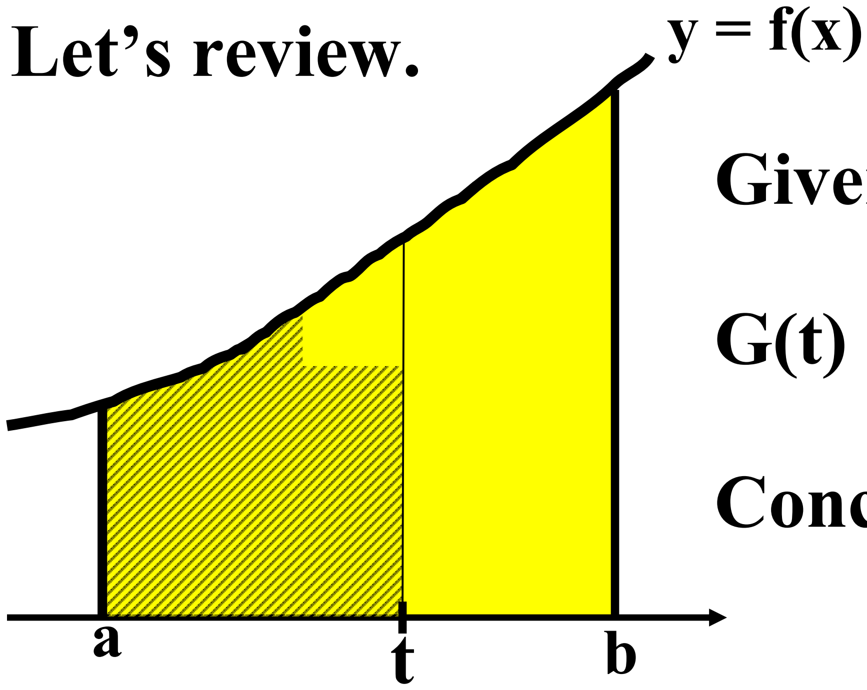

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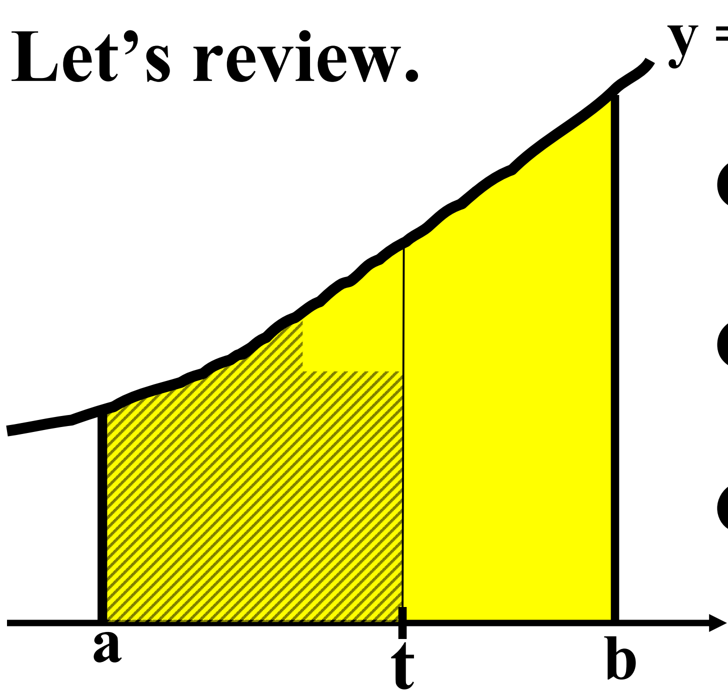

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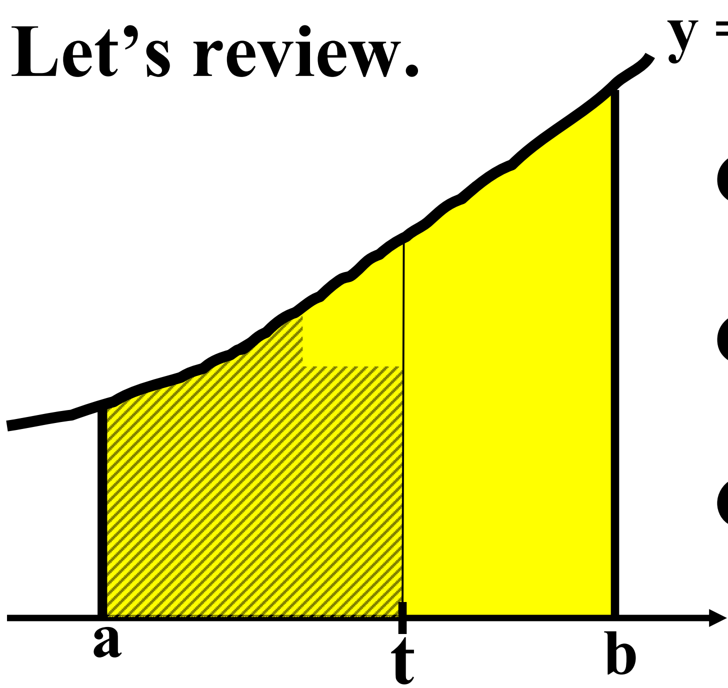

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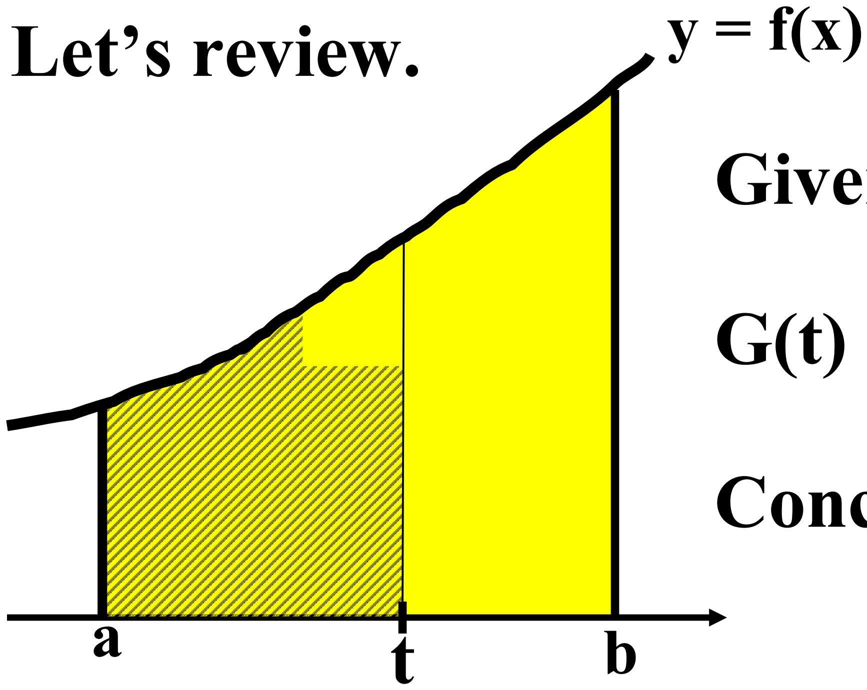

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Calculus Unit 3 The Fundamental Theorems

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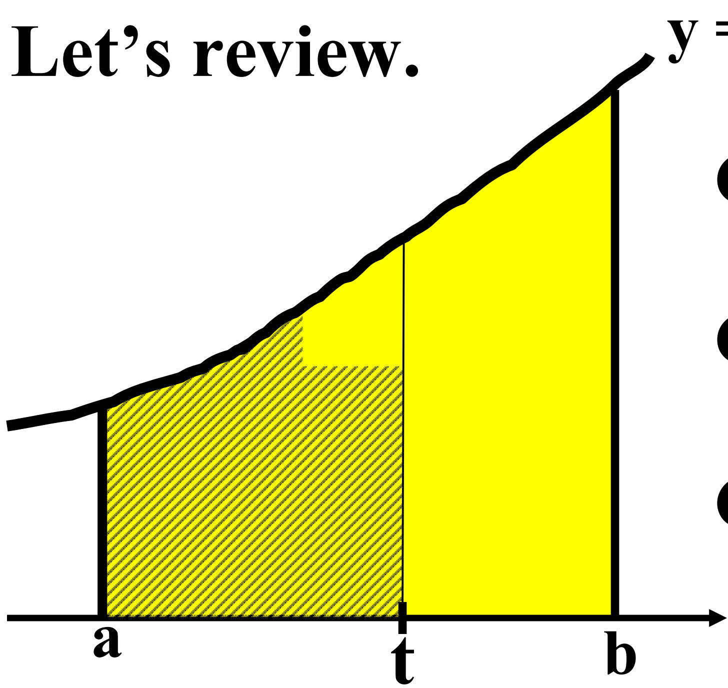

Given:

$$G(t) = \int_a^t f(x)\,dx \quad \text{where } a \le t \le b.$$

Conclusion: $G' = f$

$$F(t) = \int_a^t f(x)\,dx + F(a) \quad \text{where } a \le t \le b$$

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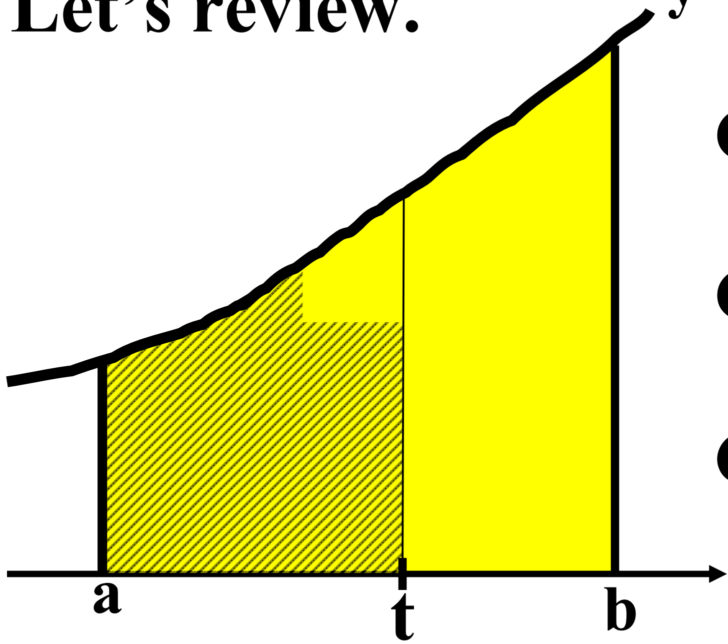

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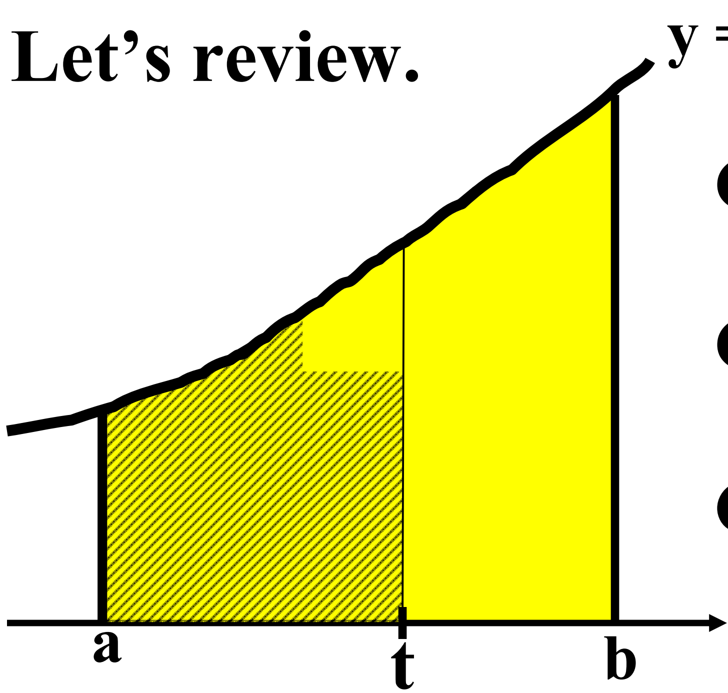

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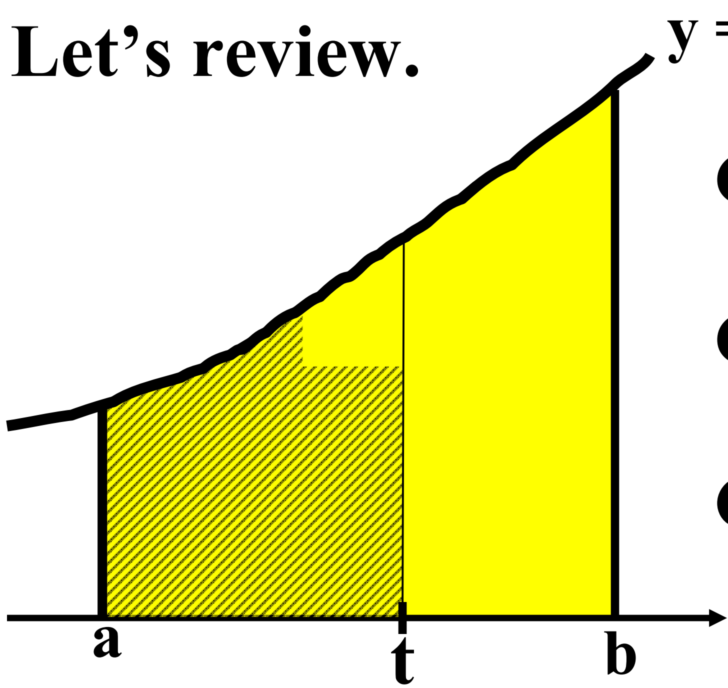

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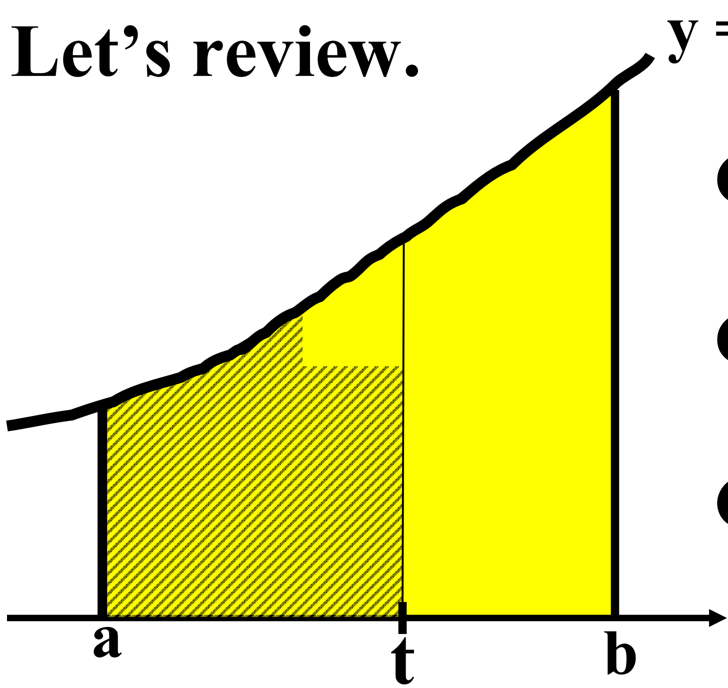

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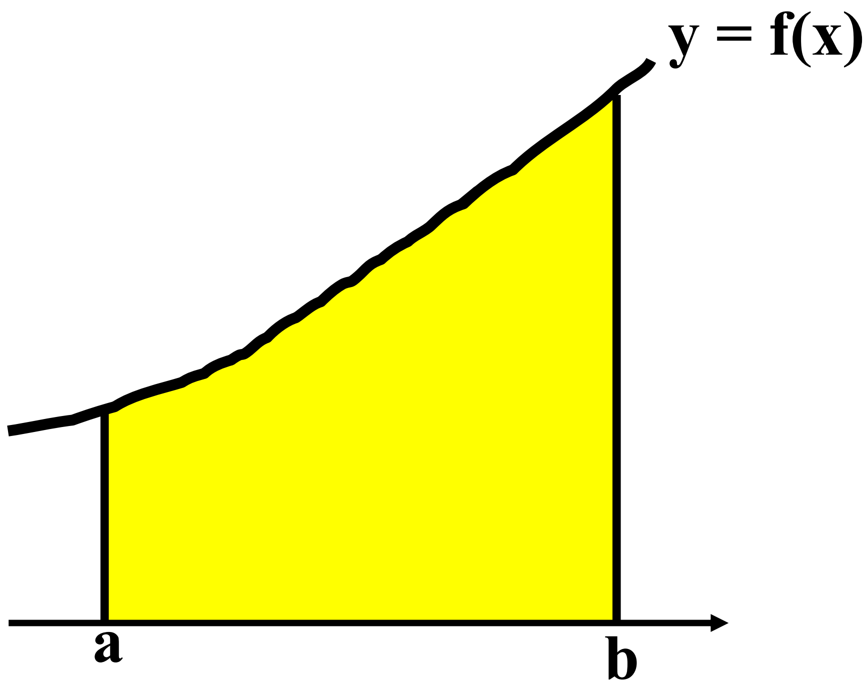

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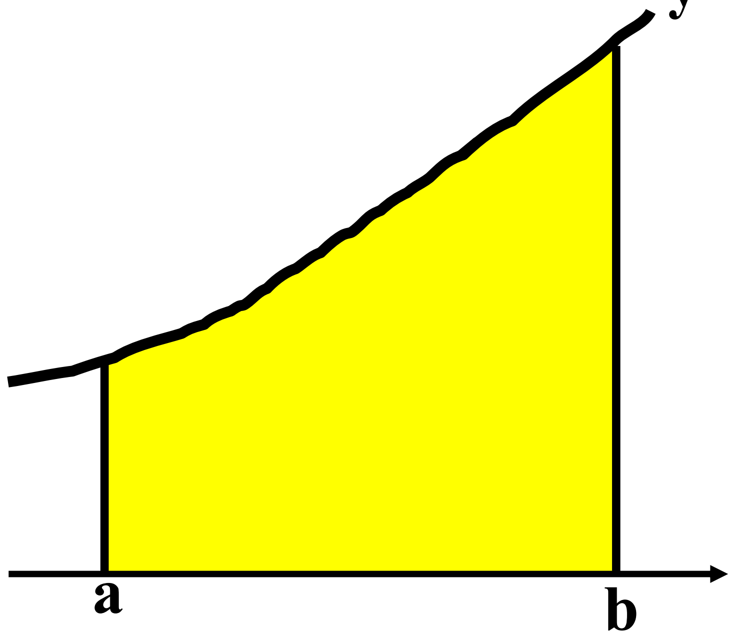

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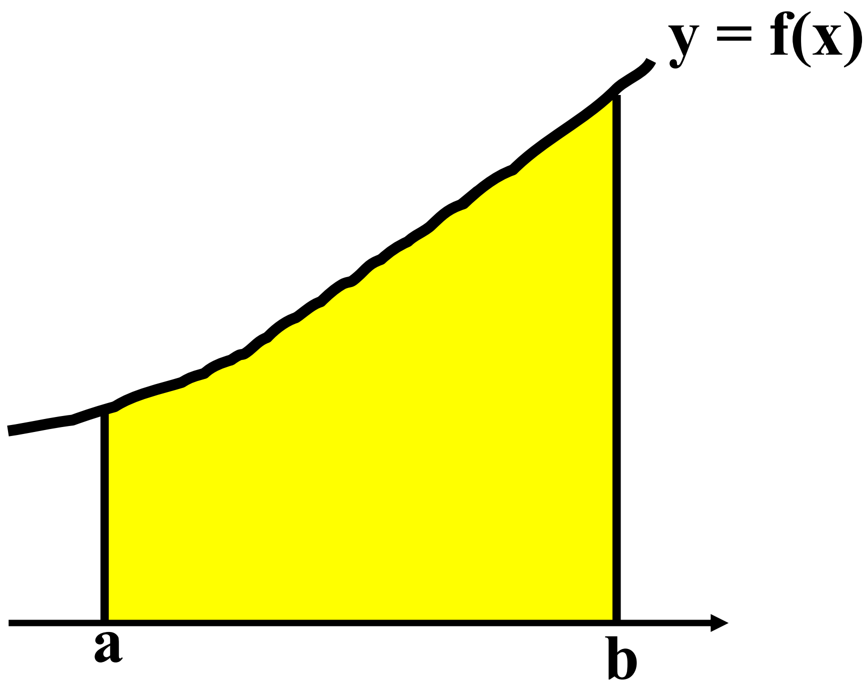

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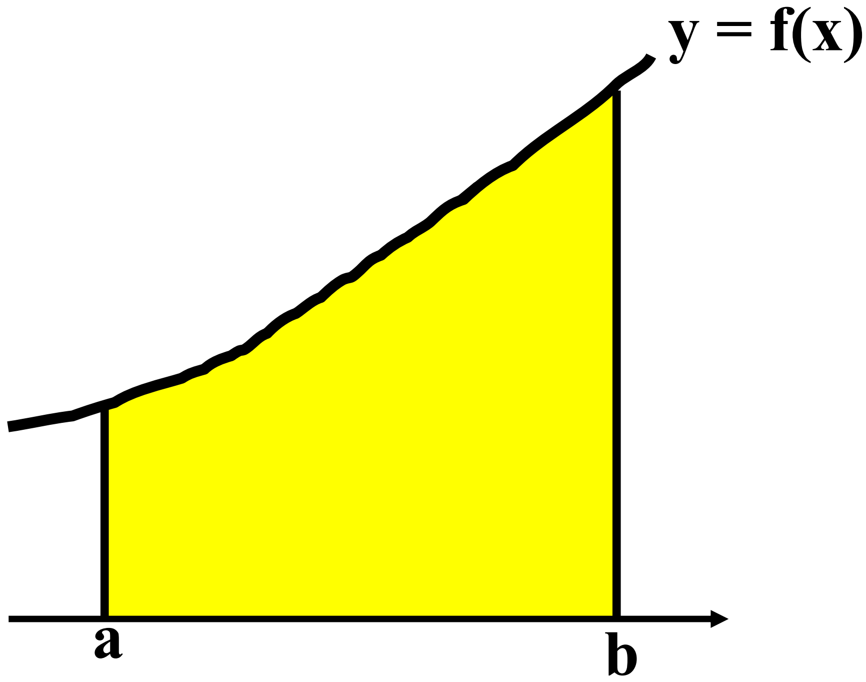

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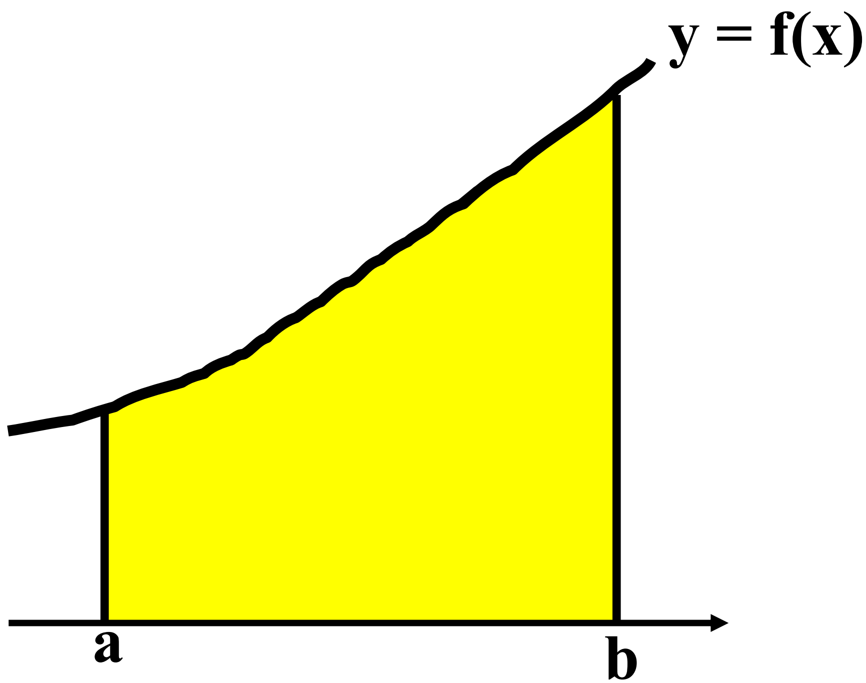

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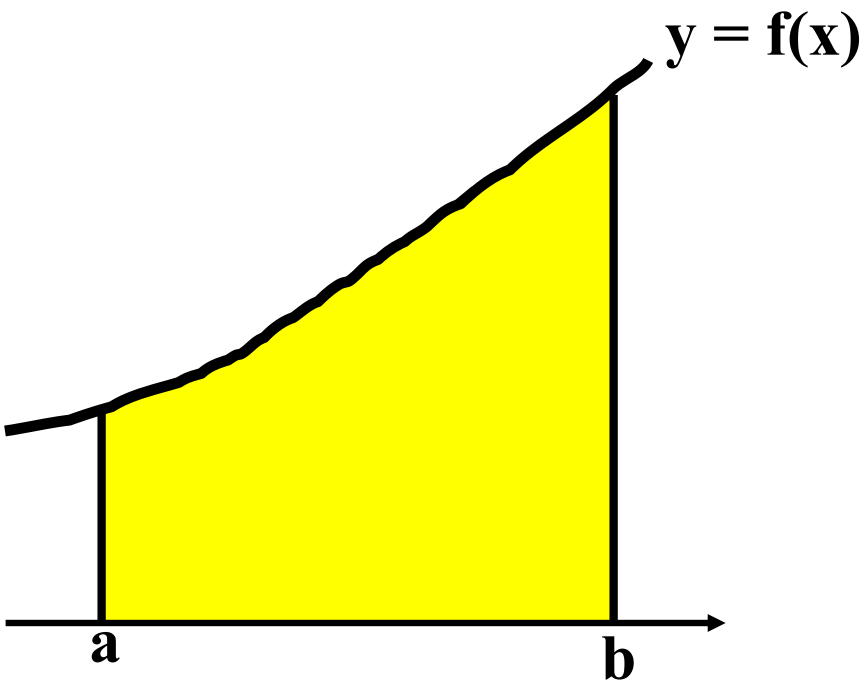

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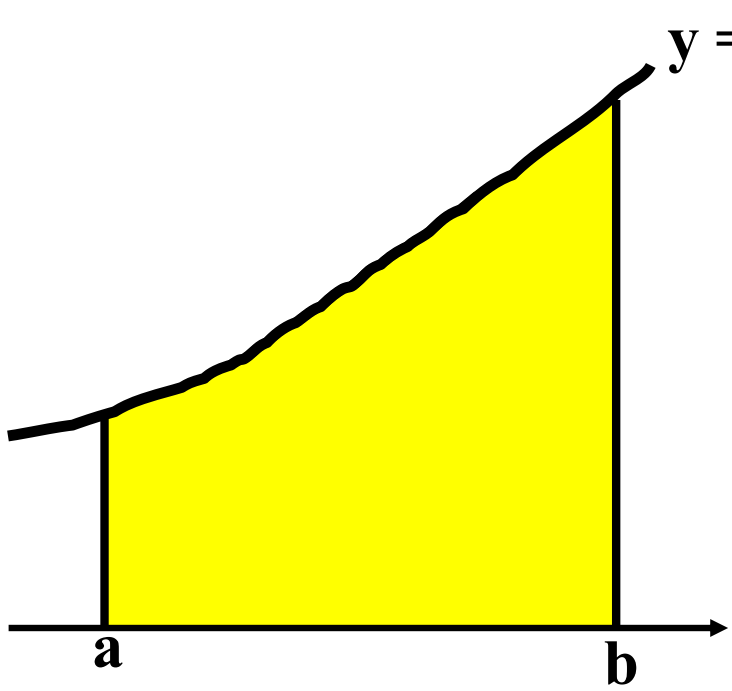

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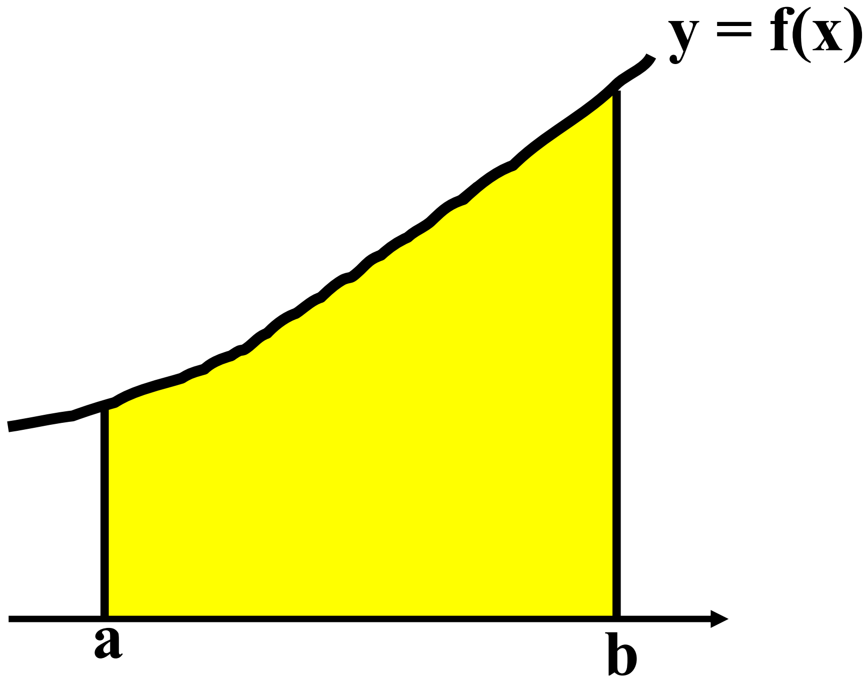

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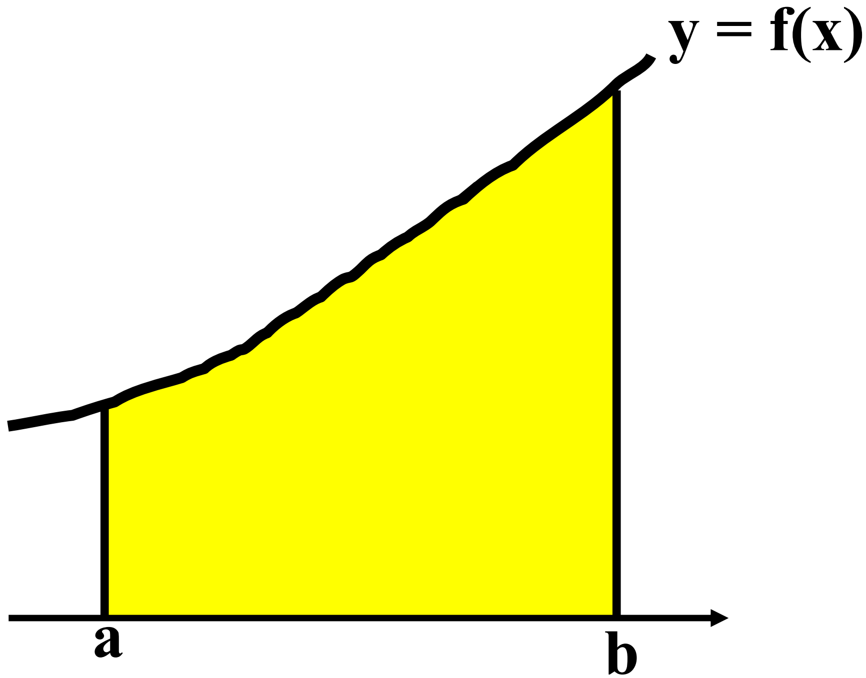

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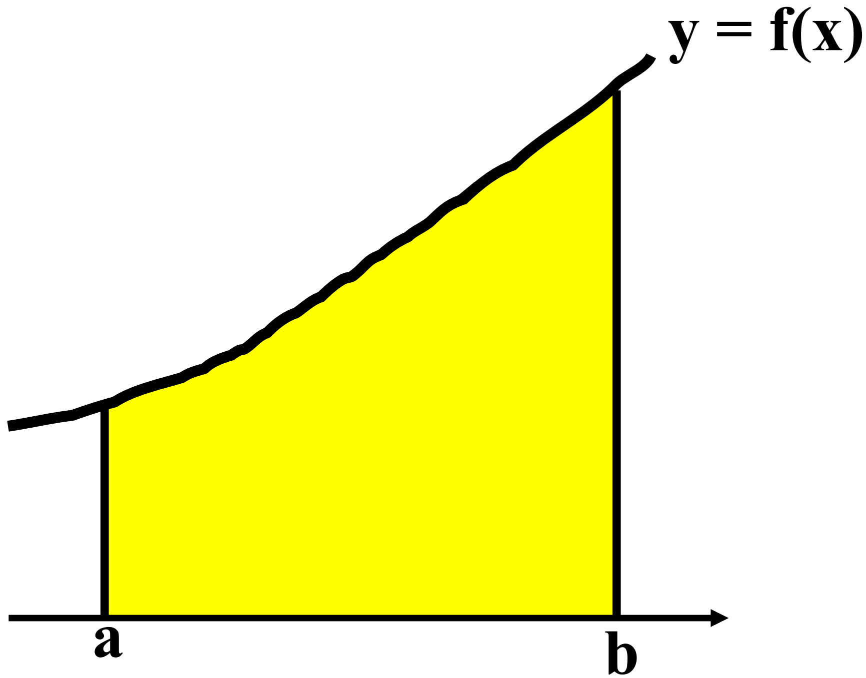

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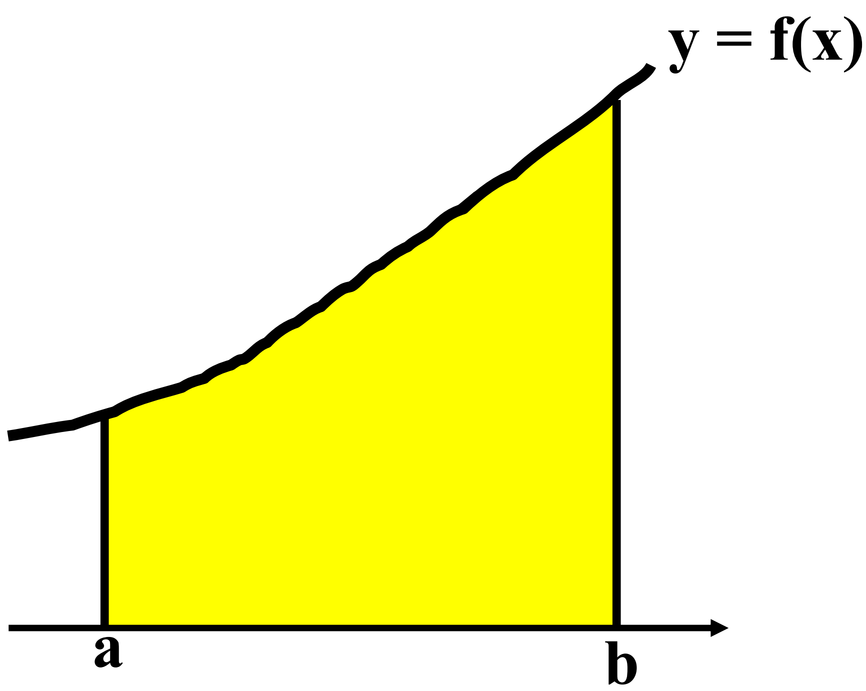

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Let's practice !!!