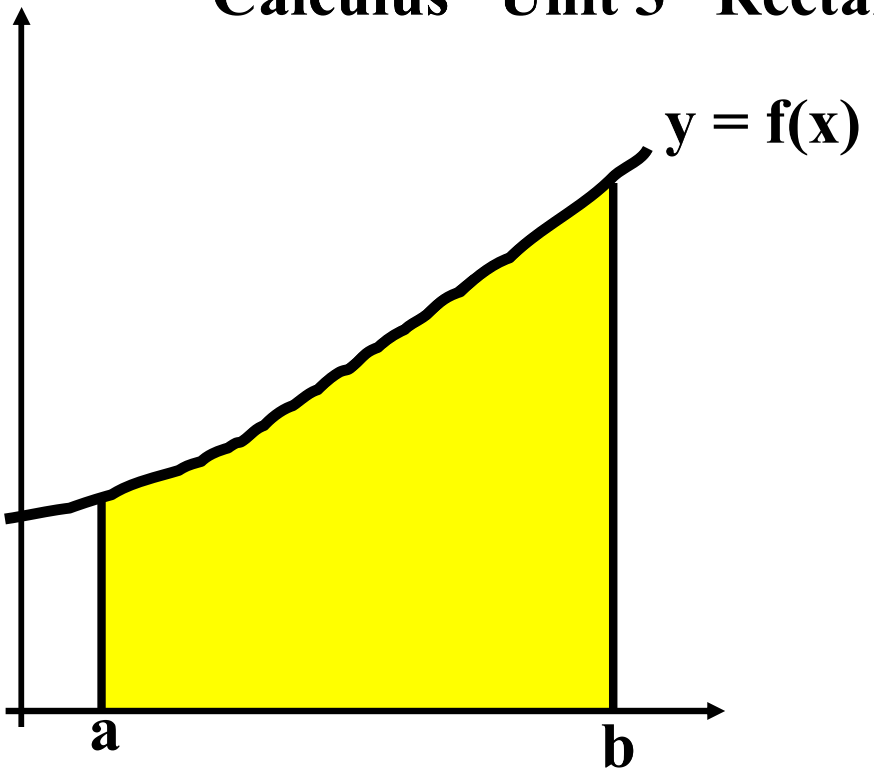
Calculus Lesson #1 Unit 3

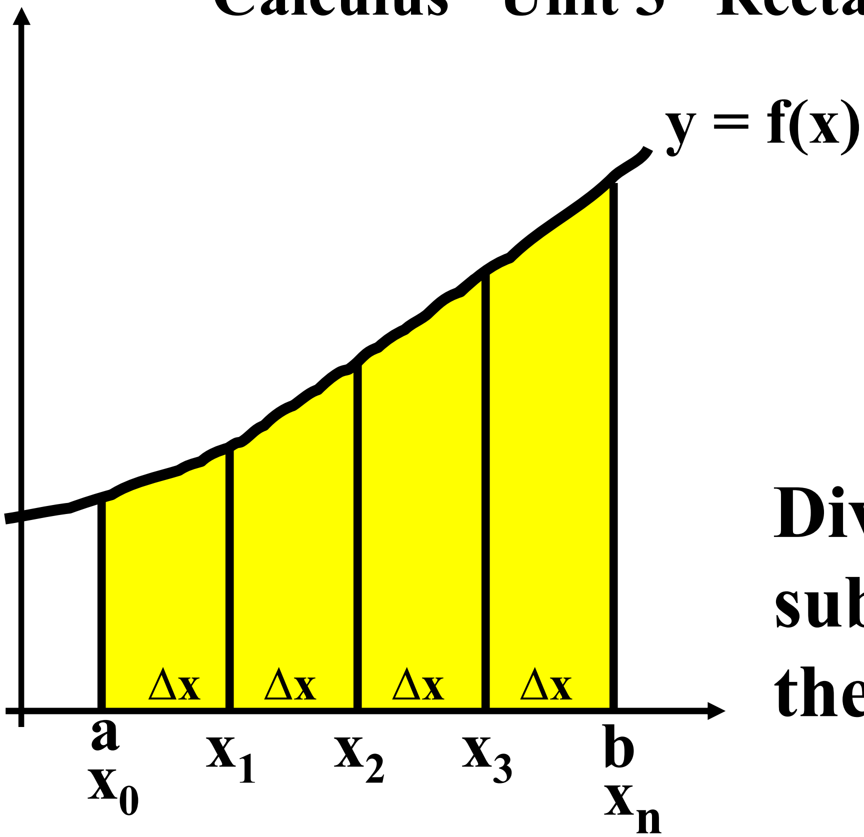
Rectangular Approximations

Calculus Unit 3 Rectangular Approximations



Consider the region bounded by the x-axis, the lines x = a and x = b, and the graph of y = f(x).

Calculus Unit 3 Rectangular Approximations

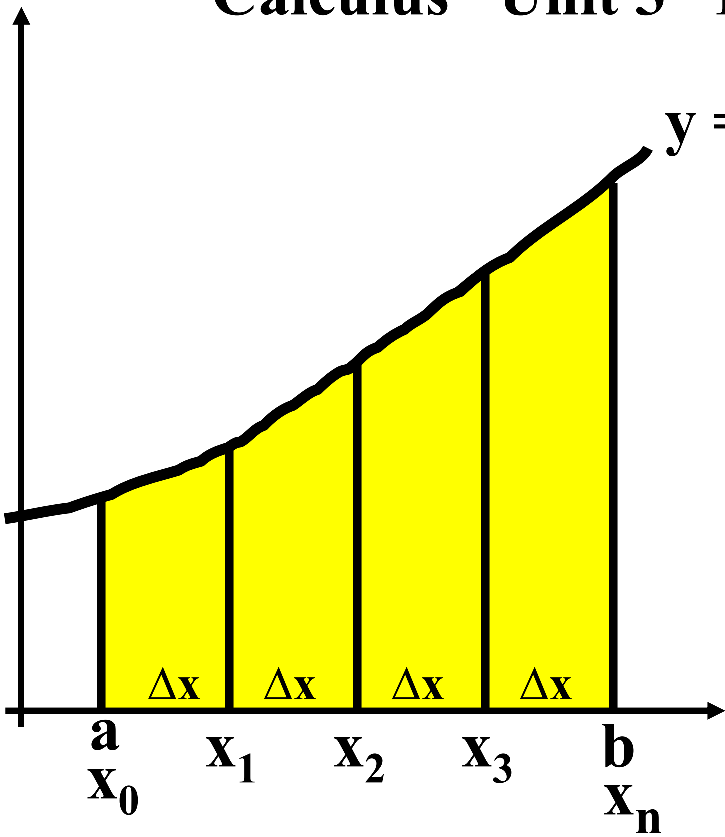


Consider the region bounded by the x-axis, the lines $x = a$ and $x = b$, and the graph of $y = f(x)$.

Divide the interval $[a, b]$ into n subintervals each of width Δx by the numbers

$$a = x_0, x_1, x_2, \ldots, x_n = b.$$

Calculus Unit 3 Rectangular Approximations



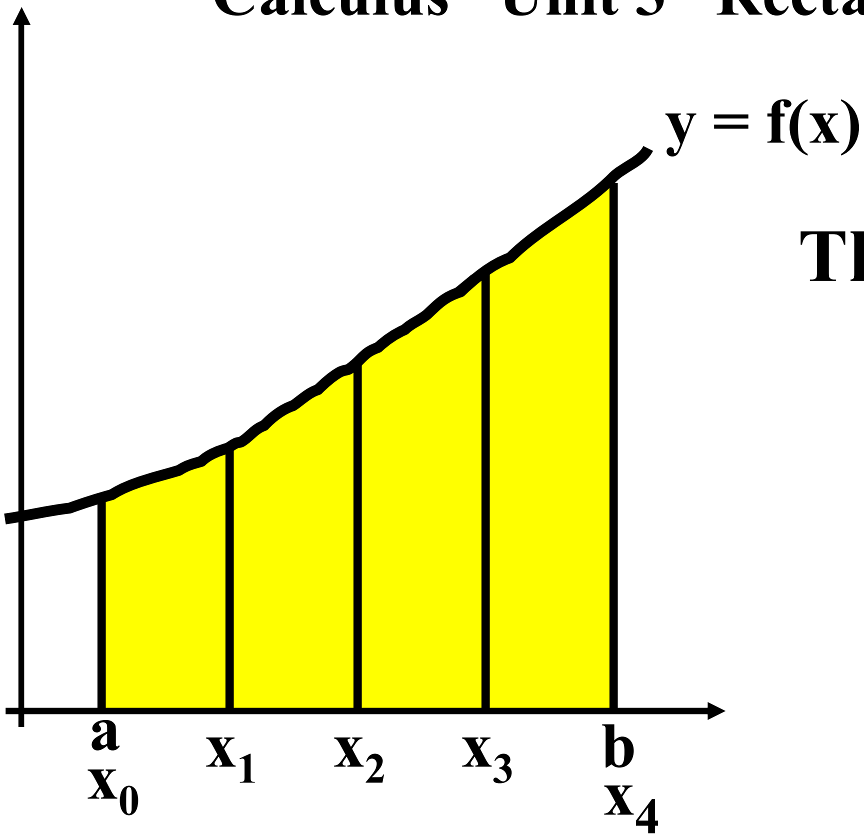
Consider the region bounded by the x-axis, the lines $x = a$ and $x = b$, and the graph of $y = f(x)$.

Divide the interval $[a, b]$ into n subintervals each of width Δx by the numbers

$$a = x_0, x_1, x_2, \ldots, x_n = b.$$

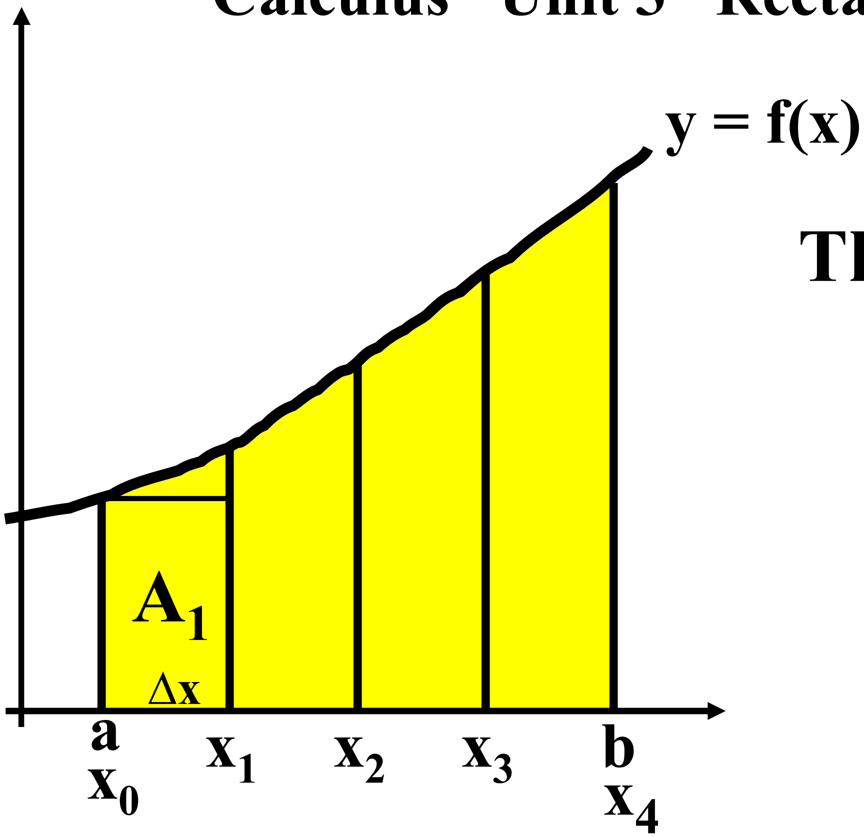
Clearly $\Delta x = \dfrac{b - a}{n}$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

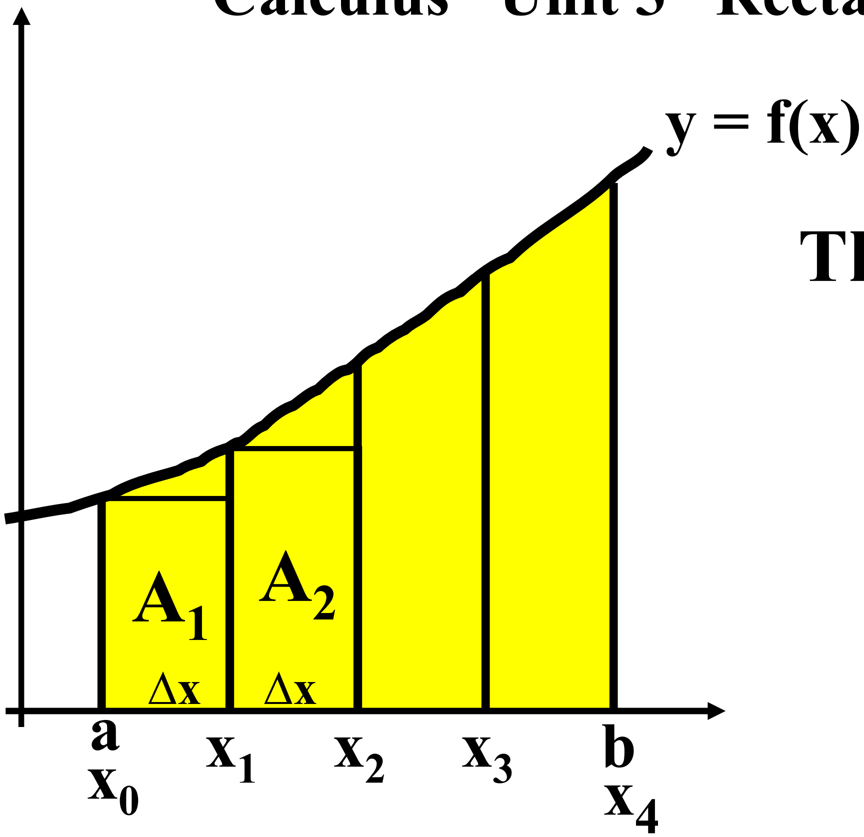
Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$$A_1 = f(x_0)\Delta x$$

Calculus Unit 3 Rectangular Approximations

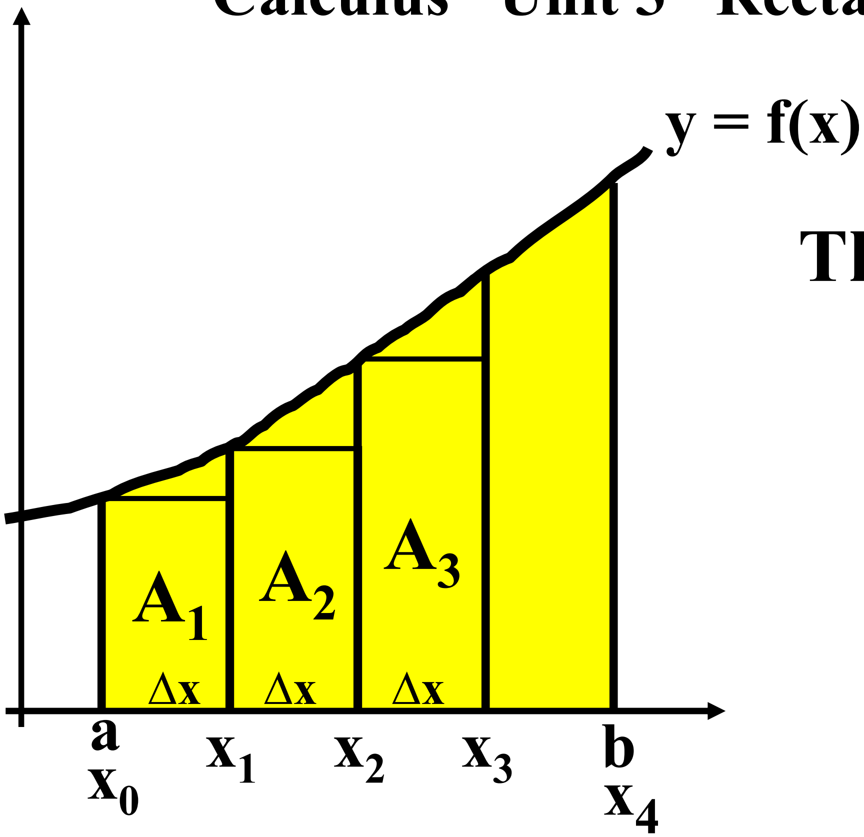


The Lower Rectangular Sum

$$A_1 = f(x_0)\Delta x$$

$$A_2 = f(x_1)\Delta x$$

Calculus Unit 3 Rectangular Approximations



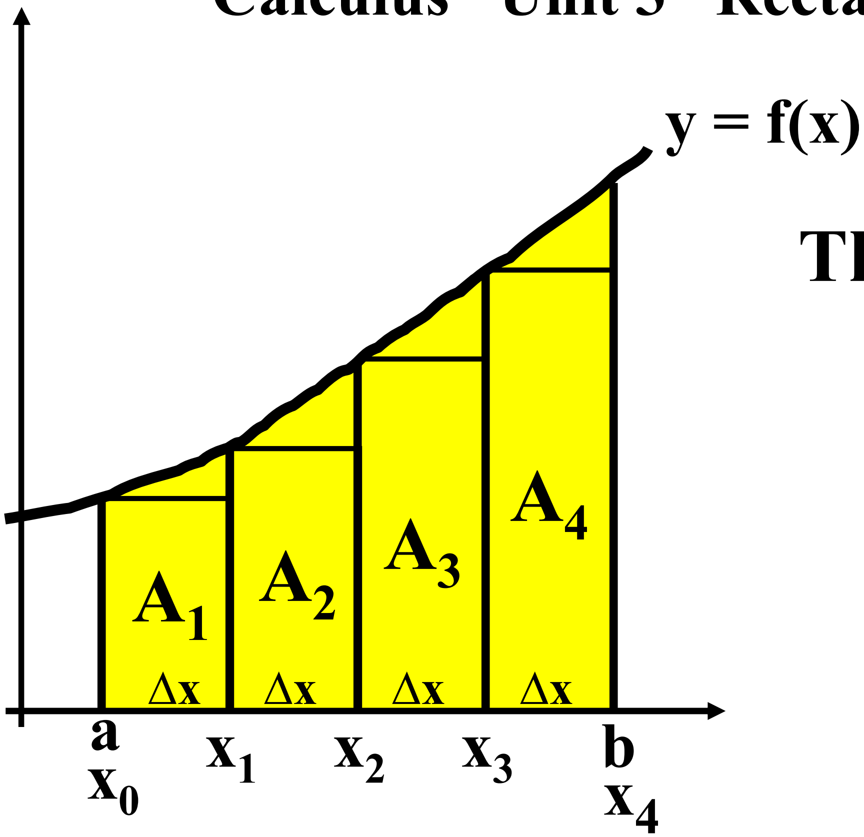
The Lower Rectangular Sum

$$A_1 = f(x_0)\Delta x$$

$$A_2 = f(x_1)\Delta x$$

$$A_3 = f(x_2)\Delta x$$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

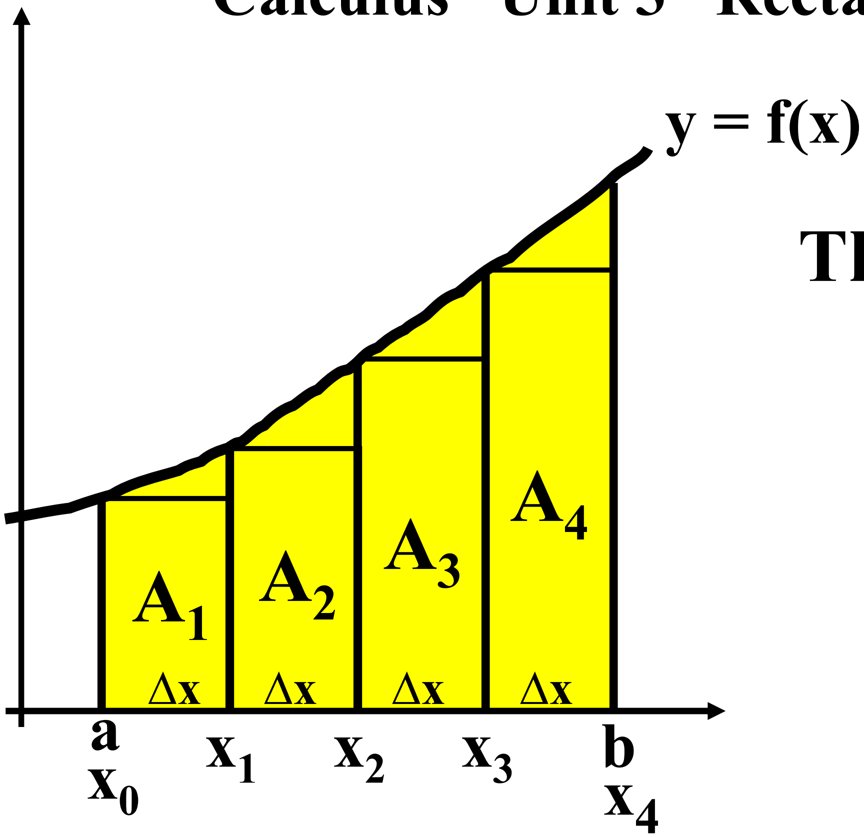
$$A_1 = f(x_0)\Delta x$$

$$A_2 = f(x_1)\Delta x$$

$$A_3 = f(x_2)\Delta x$$

$$A_4 = f(x_3)\Delta x$$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$$A_1 = f(x_0)\Delta x$$

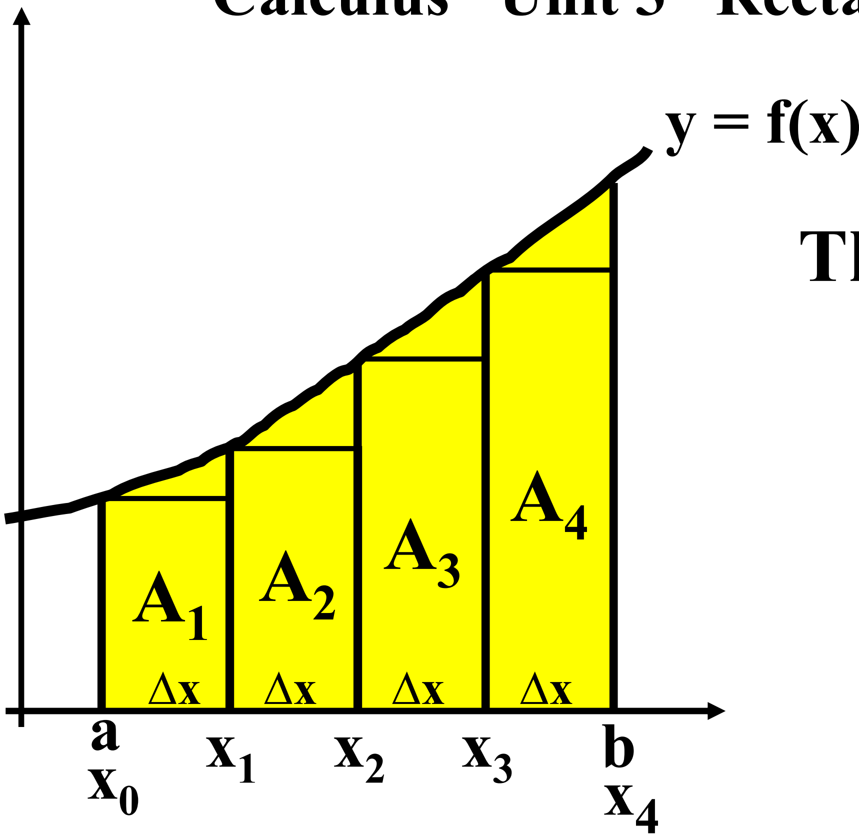
$$A_2 = f(x_1)\Delta x$$

$$A_3 = f(x_2)\Delta x$$

$$A_4 = f(x_3)\Delta x$$

$$S_L =$$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$A_1 = f(x_0)\Delta x$

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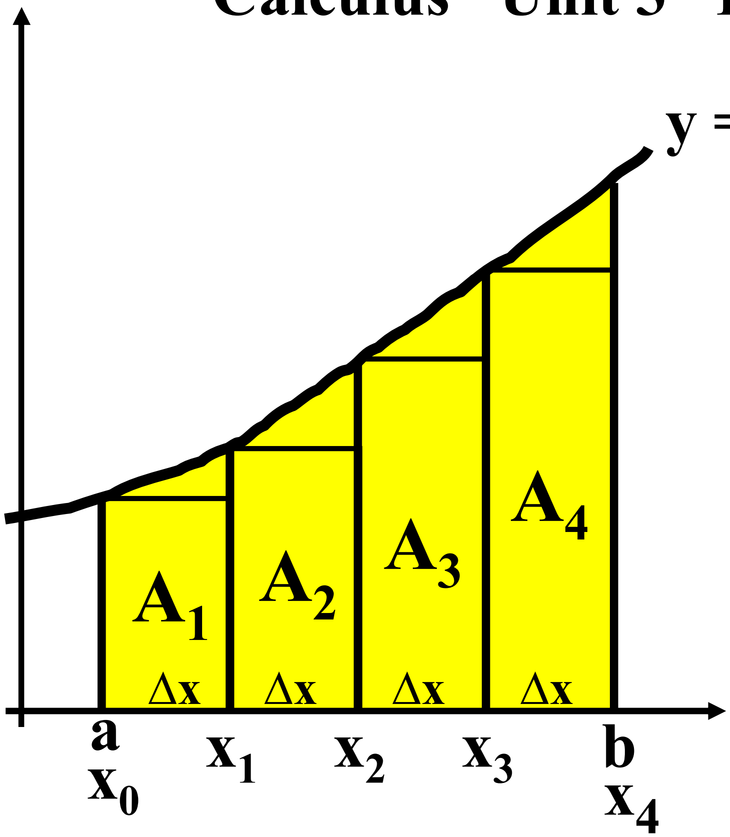
$A_4 = f(x_3)\Delta x$

$S_L = f(x_0)\Delta x$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

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$$A_2 = f(x_1)\Delta x$$

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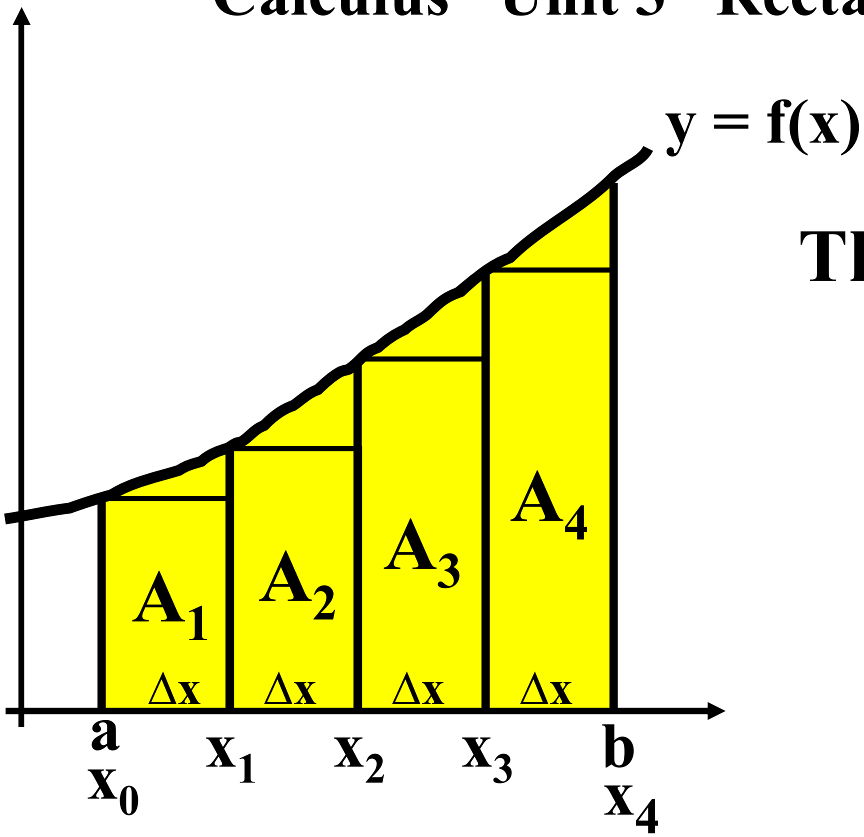
$$A_4 = f(x_3)\Delta x$$

$$S_L = f(x_0)\Delta x +$$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$A_1 = f(x_0)\Delta x$

$A_2 = f(x_1)\Delta x$

$A_3 = f(x_2)\Delta x$

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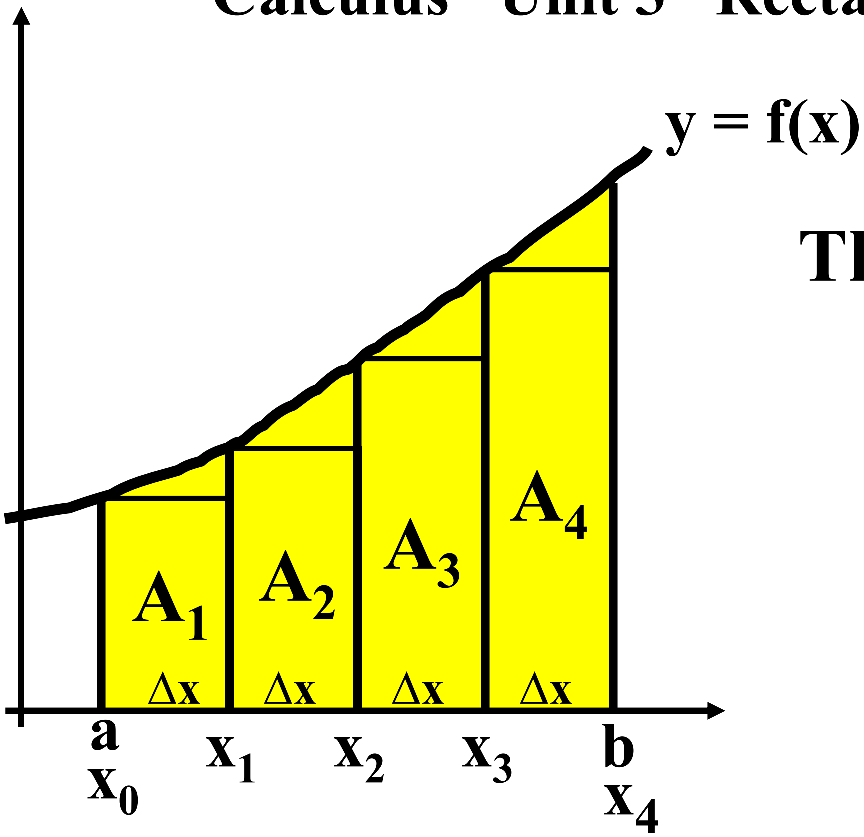
$S_L = f(x_0)\Delta x + f(x_1)\Delta x$

Calculus Unit 3 Rectangular Approximations



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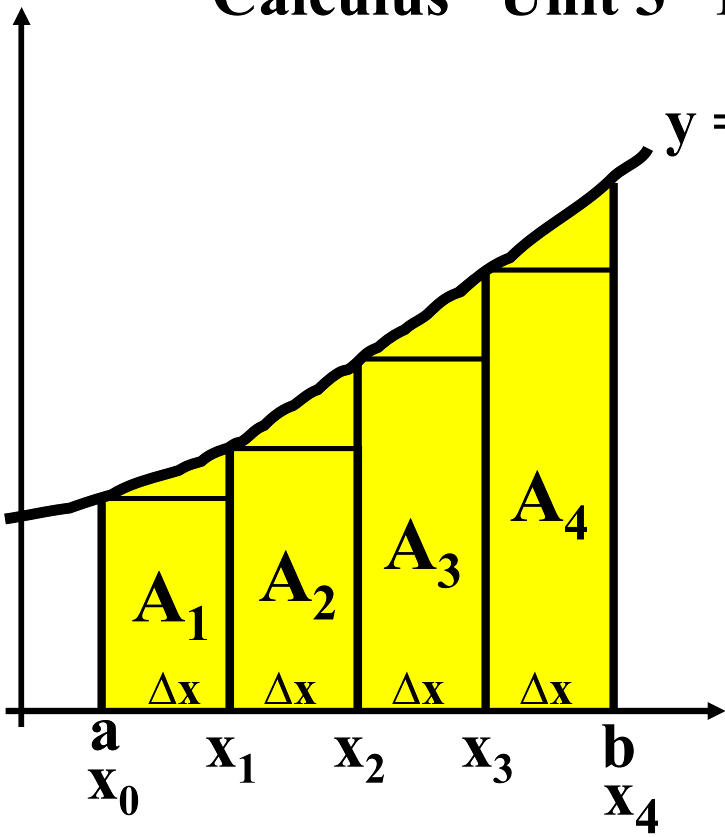
Calculus Unit 3 Rectangular Approximations



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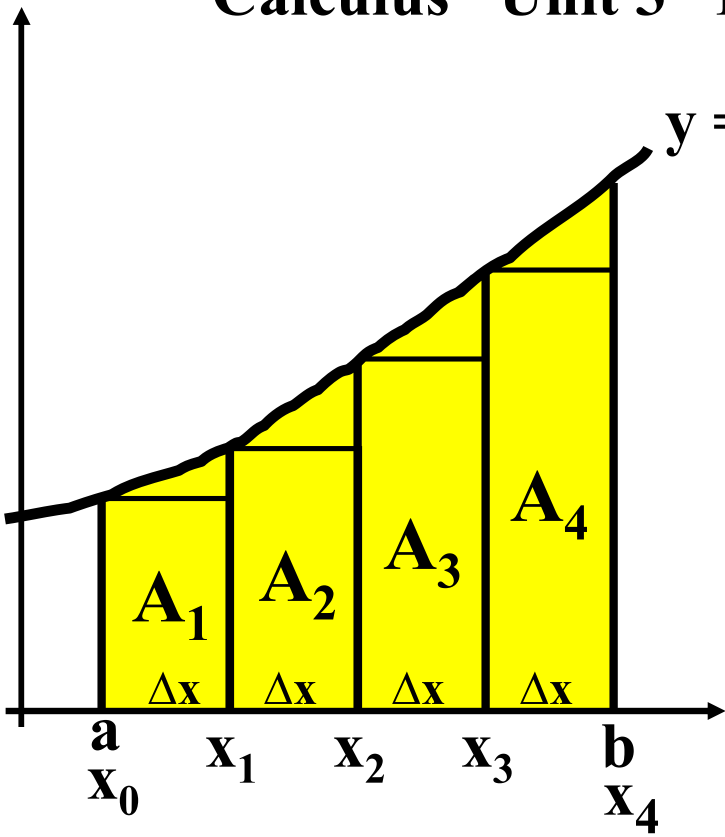
$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x$$

Calculus Unit 3 Rectangular Approximations



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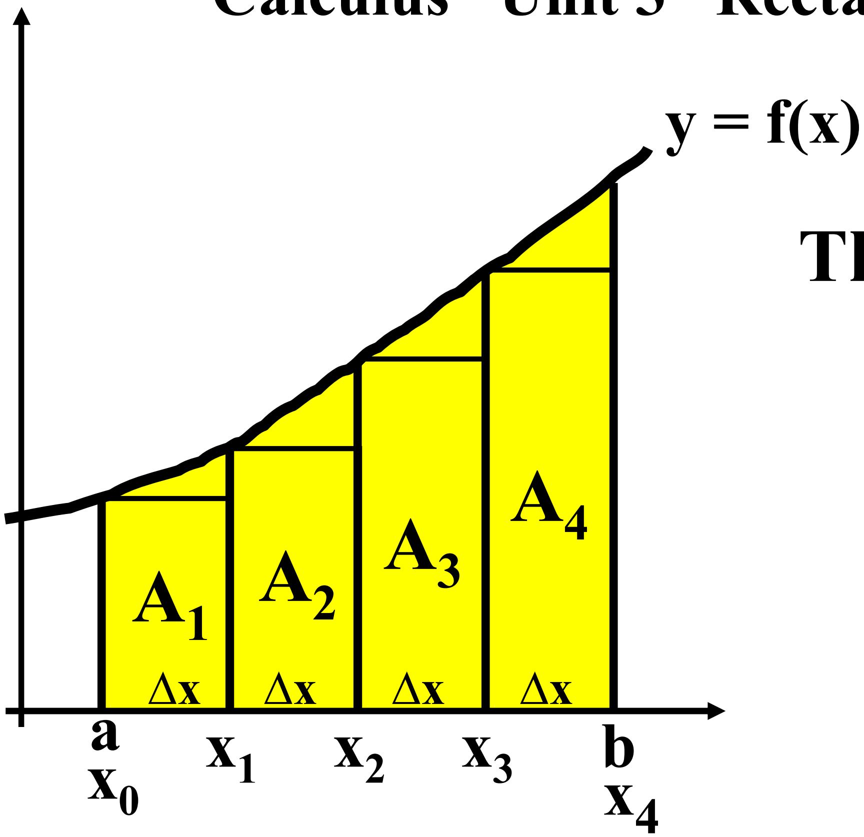
Calculus Unit 3 Rectangular Approximations



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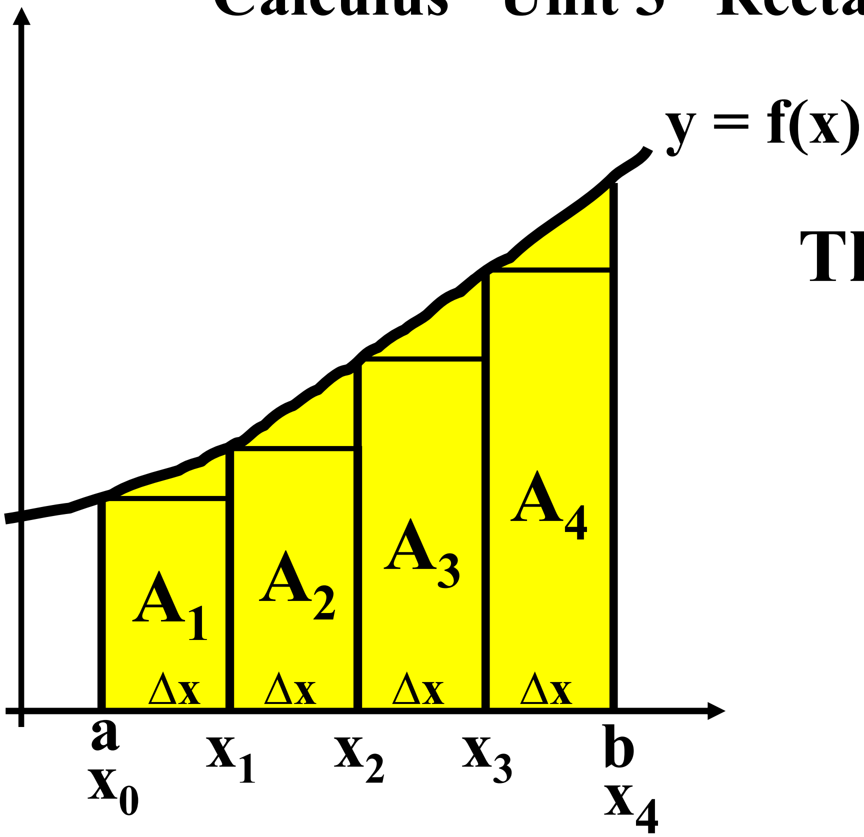
Calculus Unit 3 Rectangular Approximations



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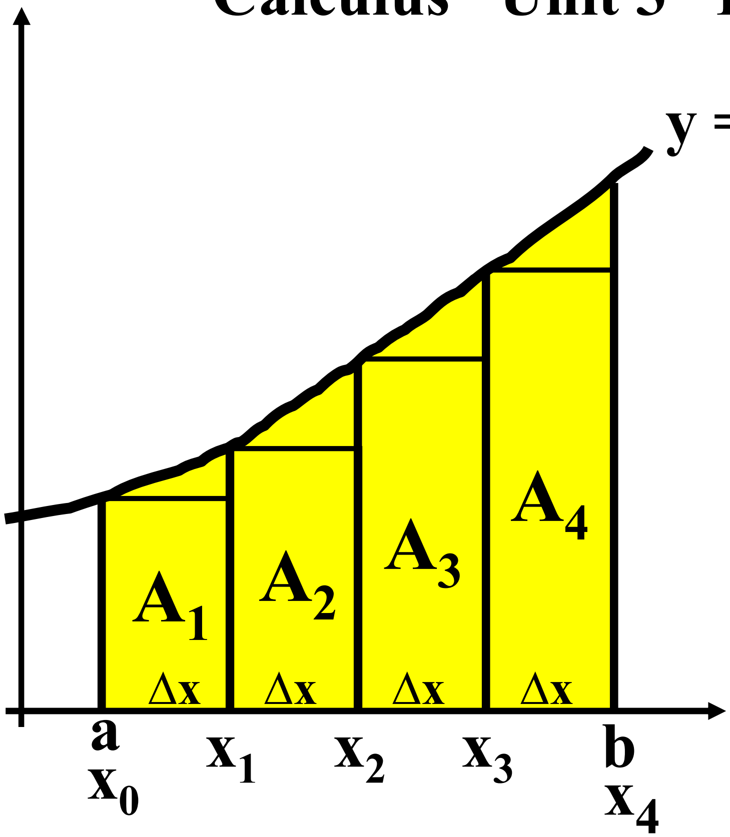
$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

In general,

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$A_1 = f(x_0)\Delta x$

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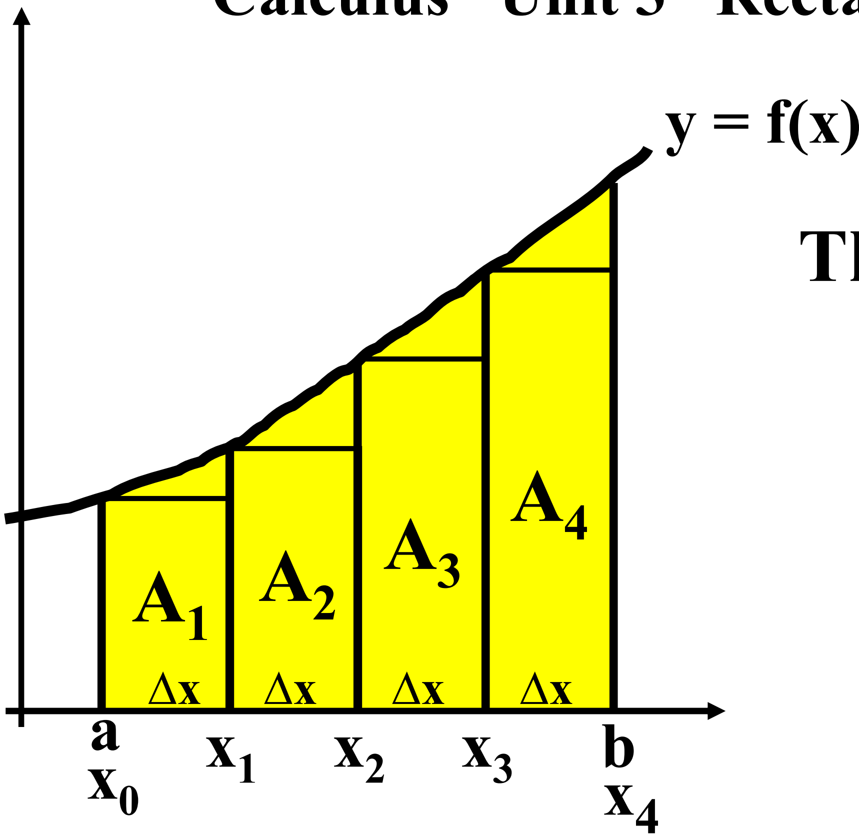
$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

In general, $S_L =$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$$A_1 = f(x_0)\Delta x$$

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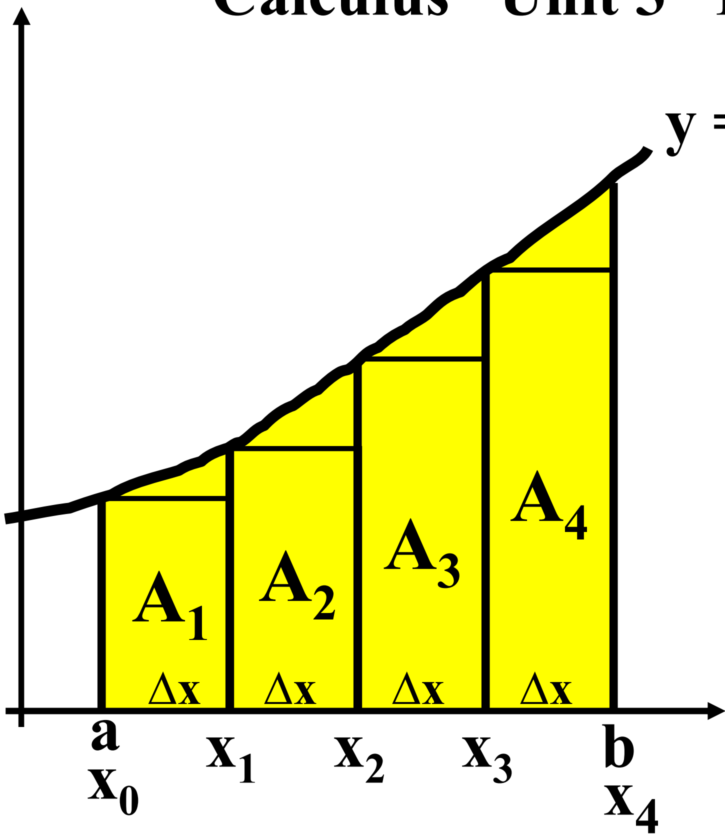
$$A_3 = f(x_2)\Delta x$$

$$A_4 = f(x_3)\Delta x$$

$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

In general, $S_L = \sum_{i=1}^{n}$

Calculus Unit 3 Rectangular Approximations



The Lower Rectangular Sum

$A_1 = f(x_0)\Delta x$

$A_2 = f(x_1)\Delta x$

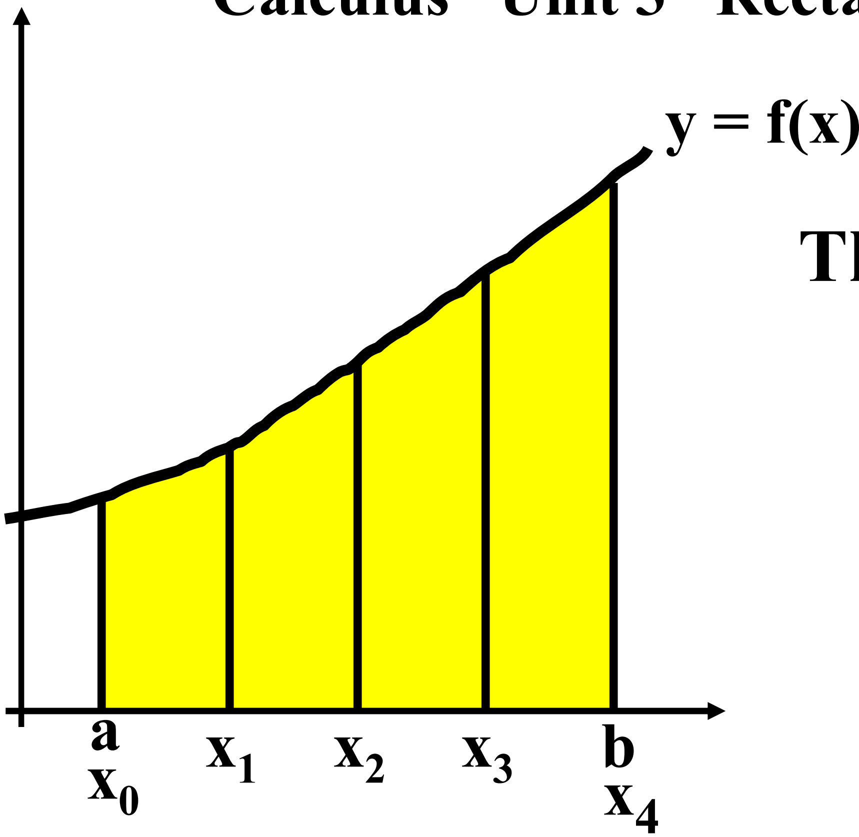
$A_3 = f(x_2)\Delta x$

$A_4 = f(x_3)\Delta x$

$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

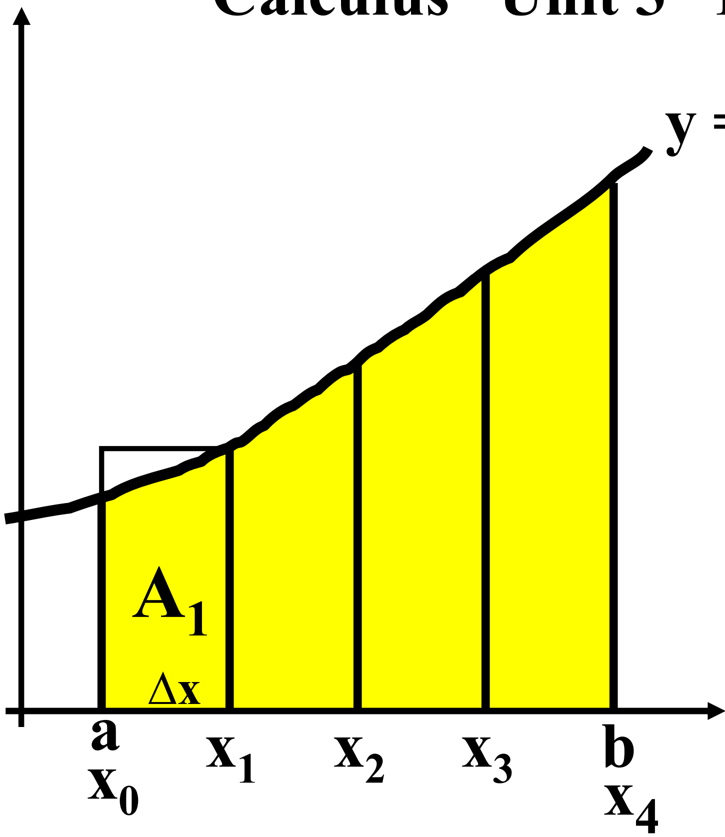
In general, $S_L = \sum_{i=1}^{n} f(x_{i-1})\Delta x$

Calculus Unit 3 Rectangular Approximations



The Upper Rectangular Sum

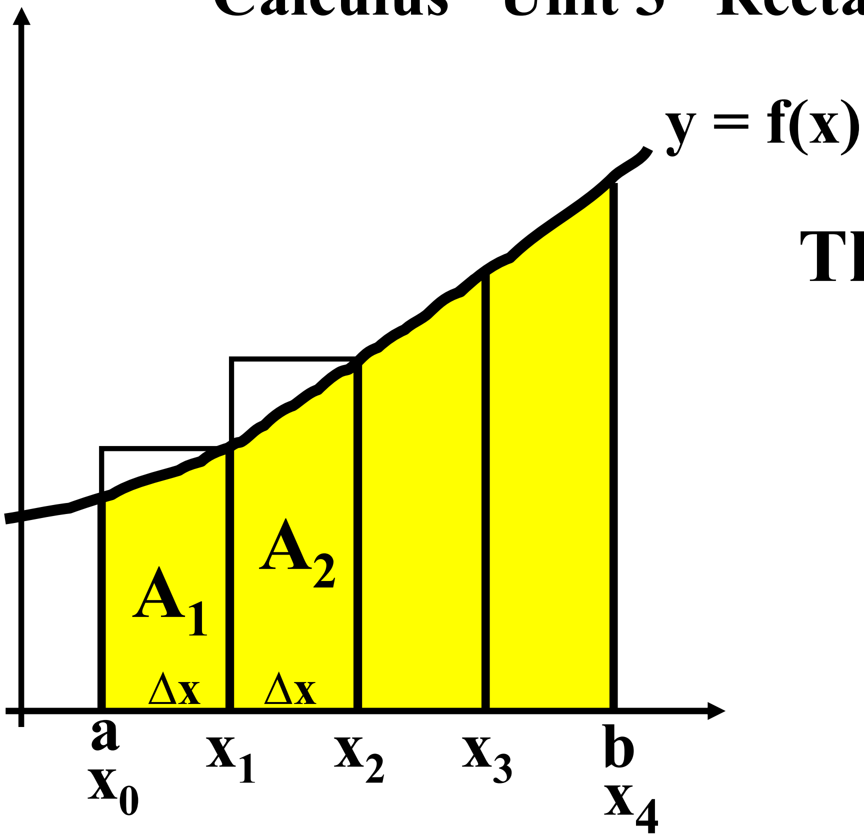
Calculus Unit 3 Rectangular Approximations



The Upper Rectangular Sum

$$A_1 = f(x_1)\Delta x$$

Calculus Unit 3 Rectangular Approximations



The Upper Rectangular Sum

$$A_1 = f(x_1)\Delta x$$

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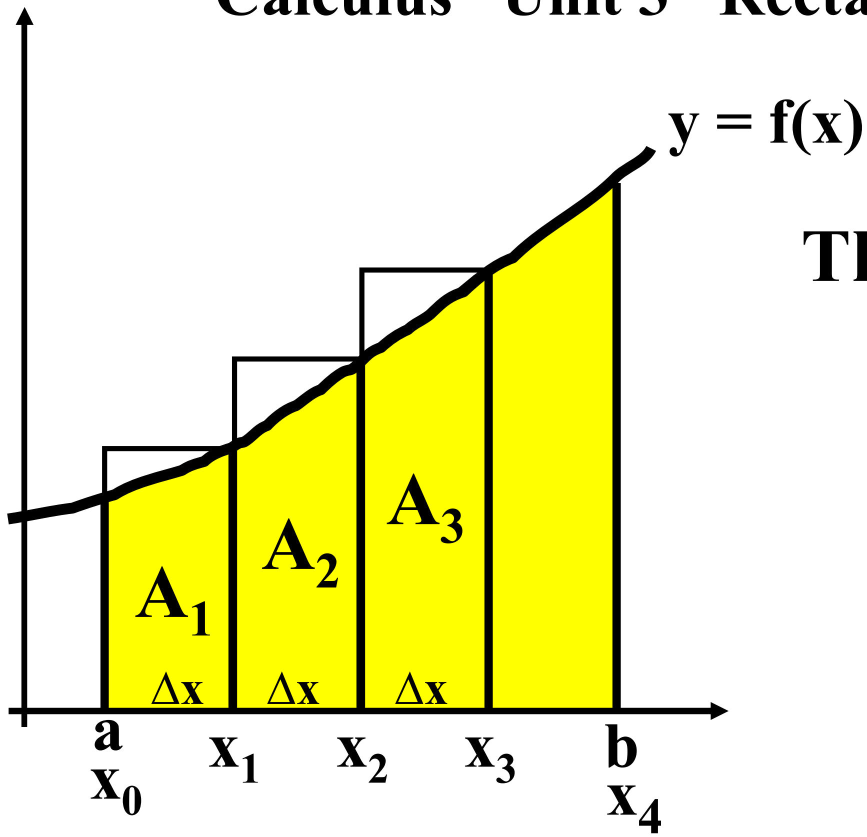
Calculus Unit 3 Rectangular Approximations



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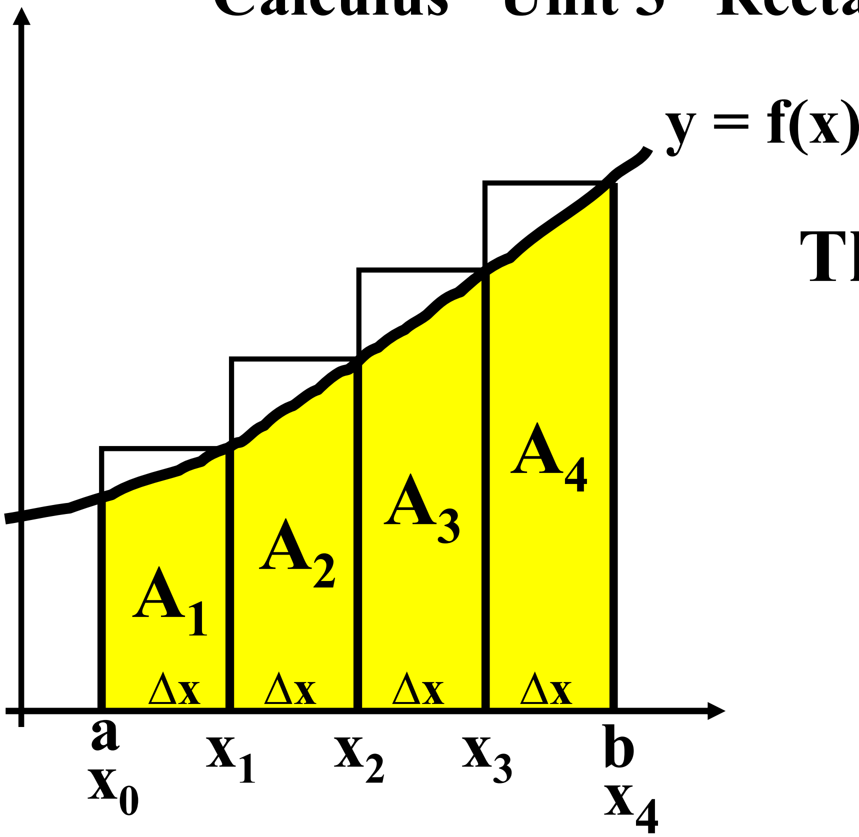
Calculus Unit 3 Rectangular Approximations



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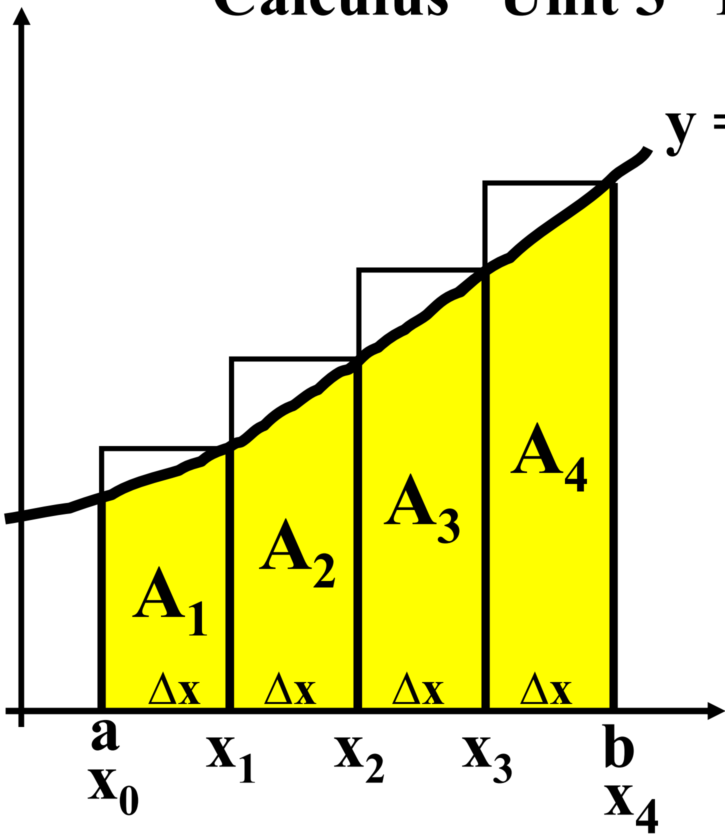
Calculus Unit 3 Rectangular Approximations



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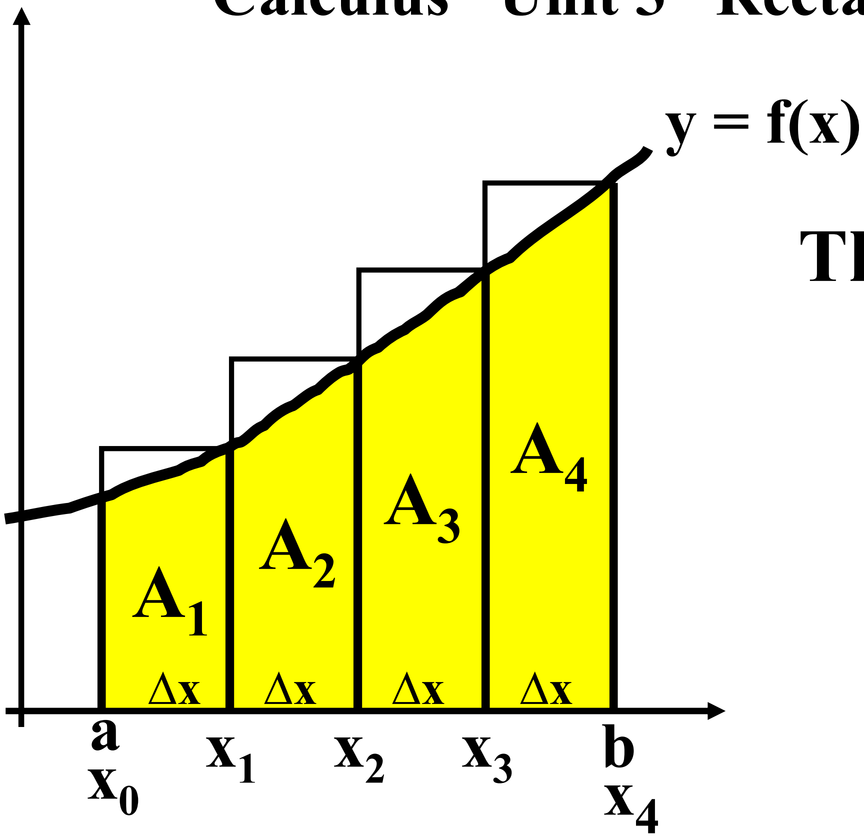
$$A_3 = f(x_3)\Delta x$$

$$A_4 = f(x_4)\Delta x$$

$$S_U =$$

Calculus Unit 3 Rectangular Approximations



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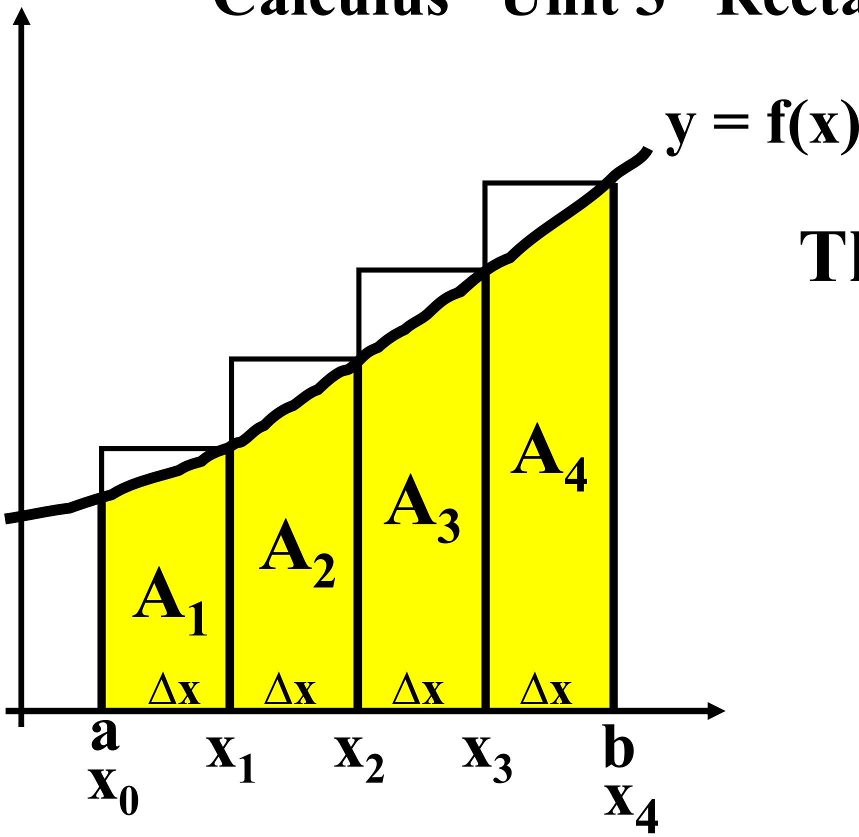
Calculus Unit 3 Rectangular Approximations



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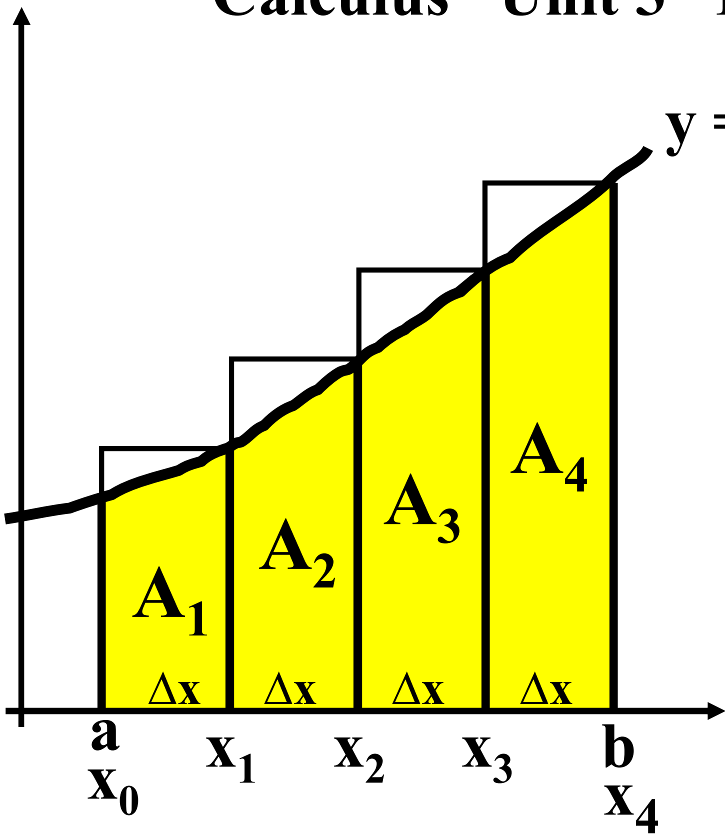
Calculus Unit 3 Rectangular Approximations



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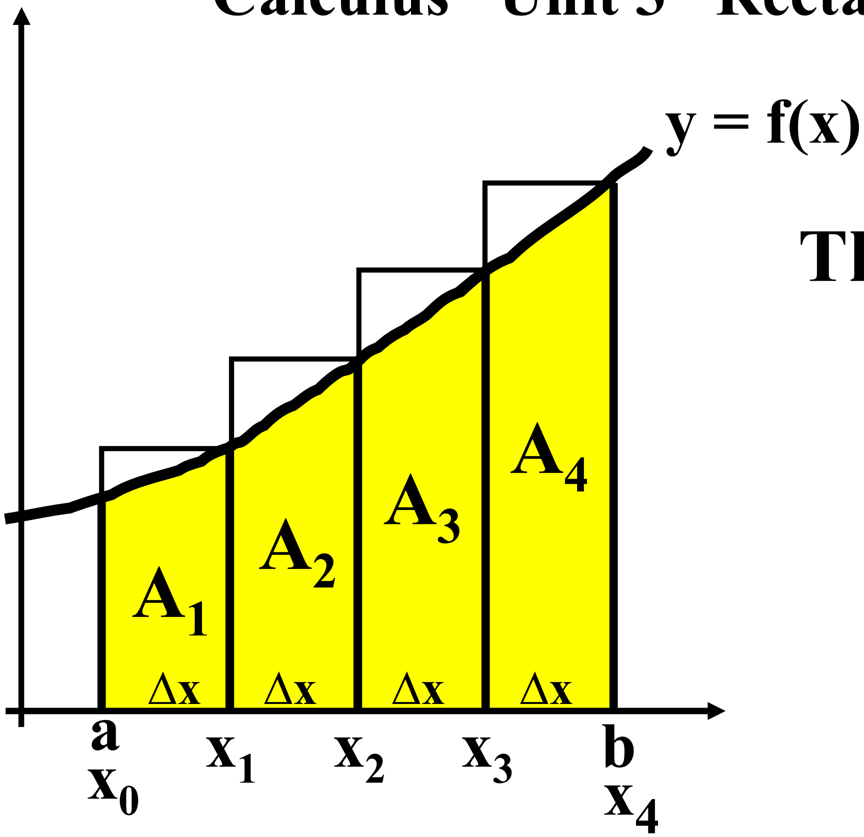
Calculus Unit 3 Rectangular Approximations



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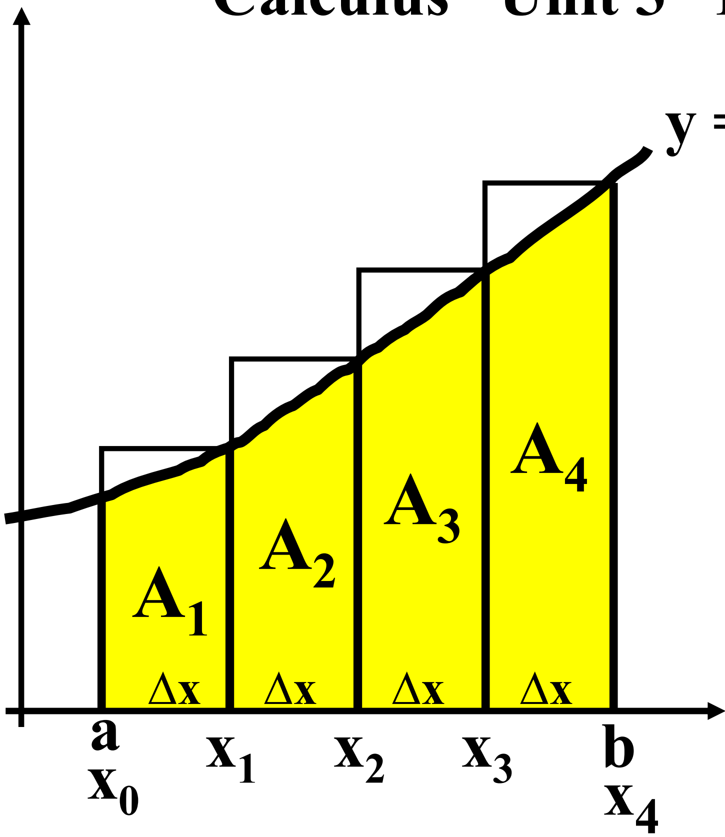
Calculus Unit 3 Rectangular Approximations



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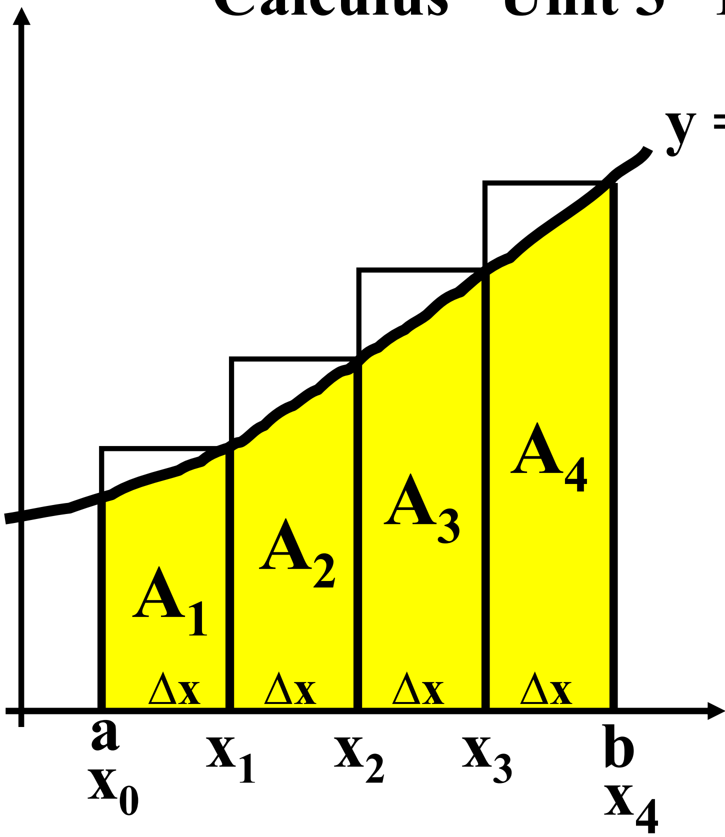
Calculus Unit 3 Rectangular Approximations



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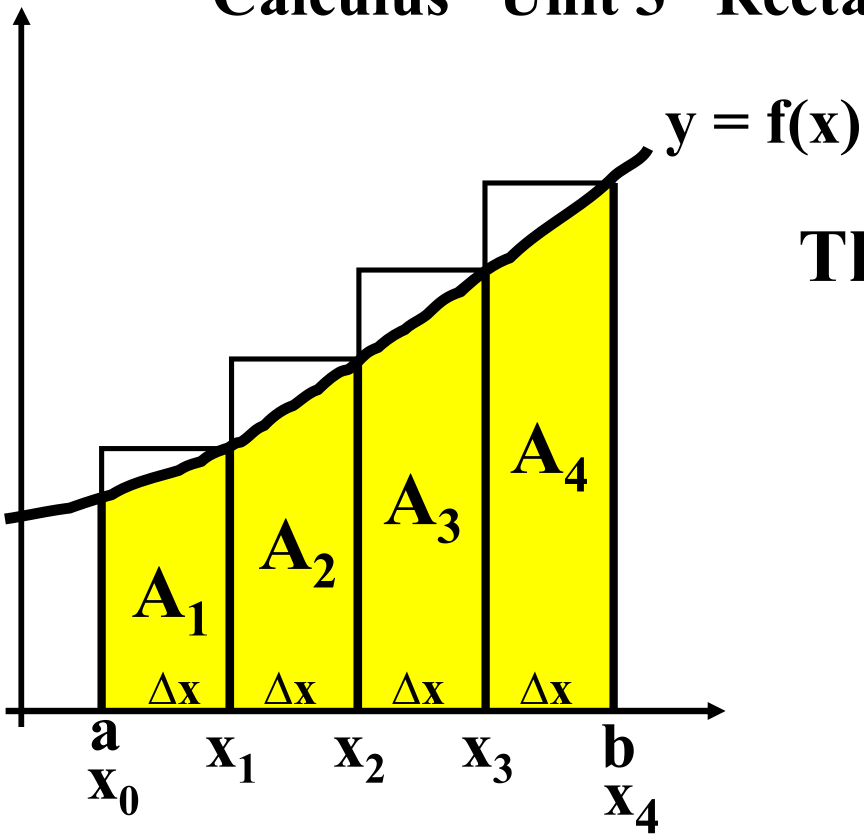
Calculus Unit 3 Rectangular Approximations



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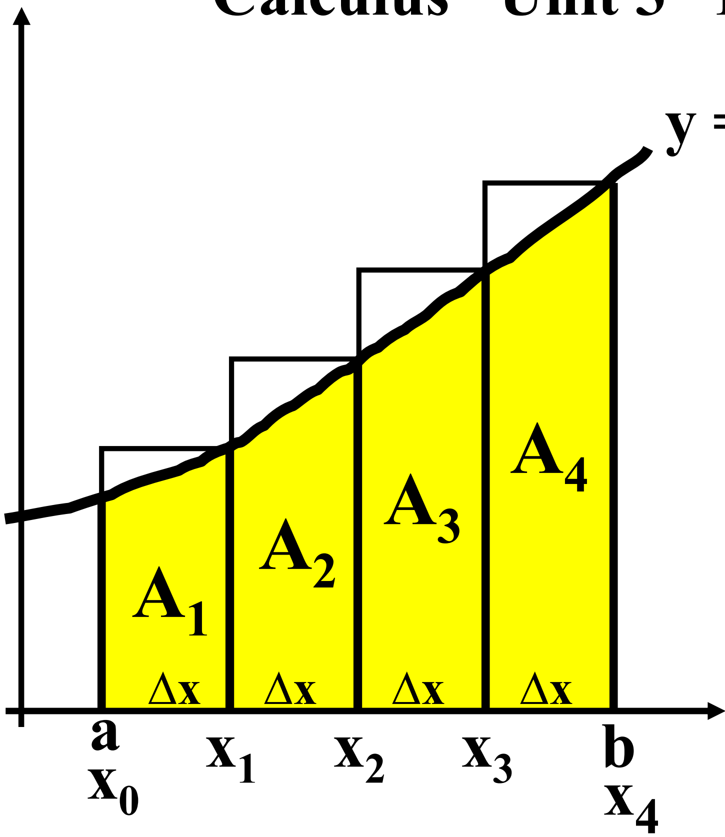
Calculus Unit 3 Rectangular Approximations



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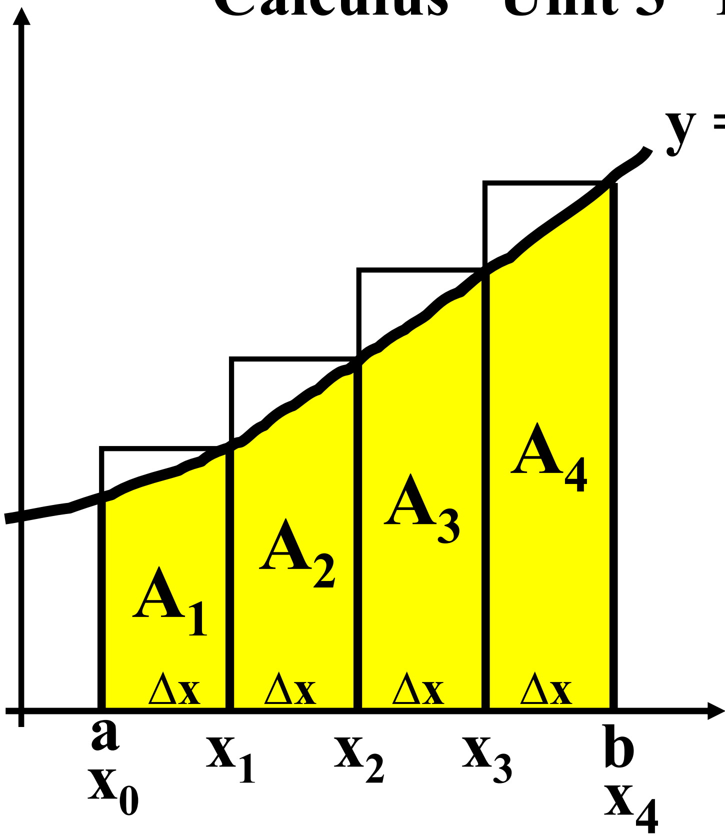
In general,

Calculus Unit 3 Rectangular Approximations



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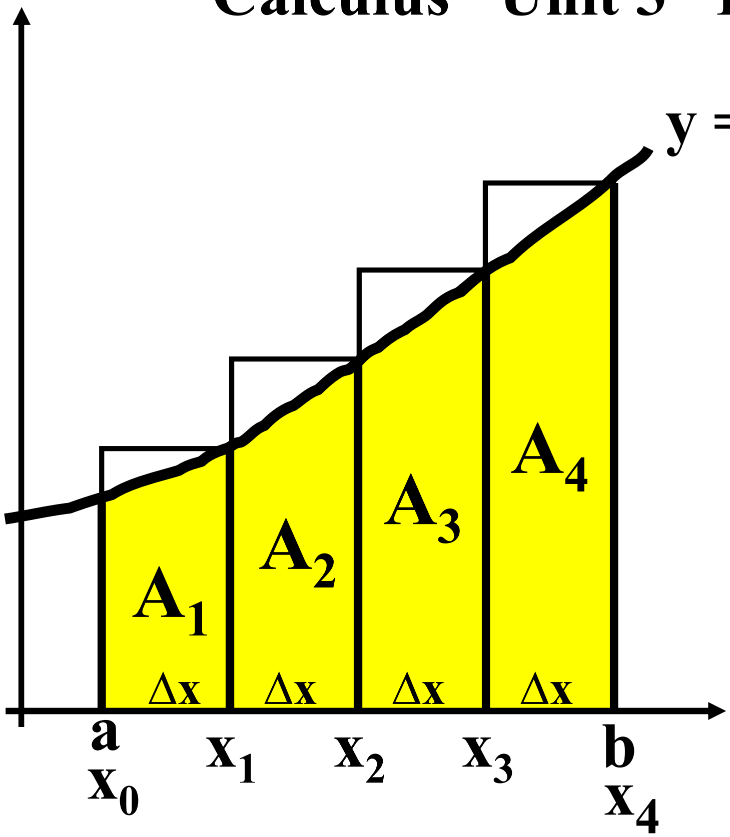
In general, $S_U =$

Calculus Unit 3 Rectangular Approximations



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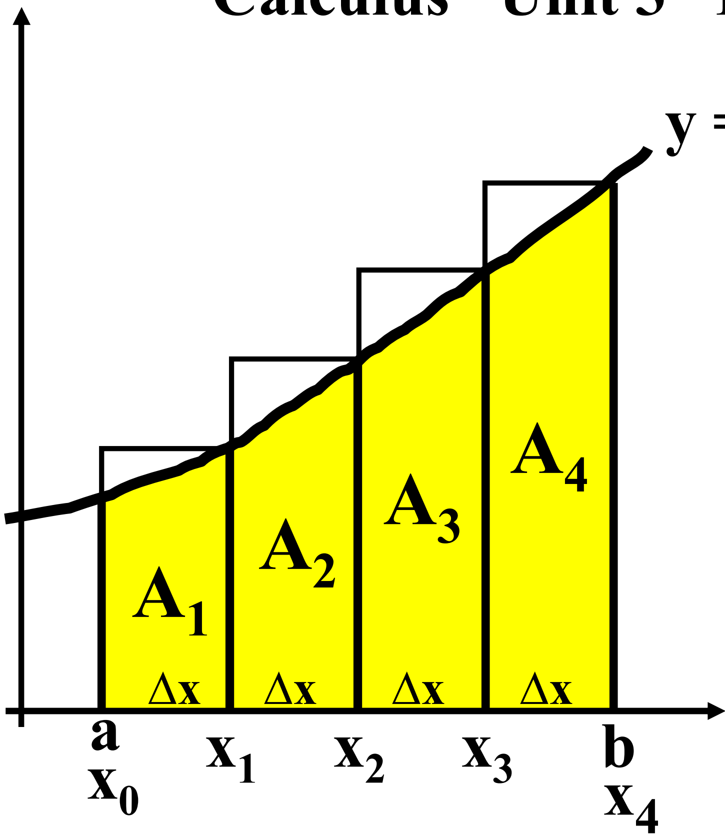
In general, $S_U = \sum_{i=1}^{n}$

Calculus Unit 3 Rectangular Approximations



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$A_1 = f(x_1)\Delta x$

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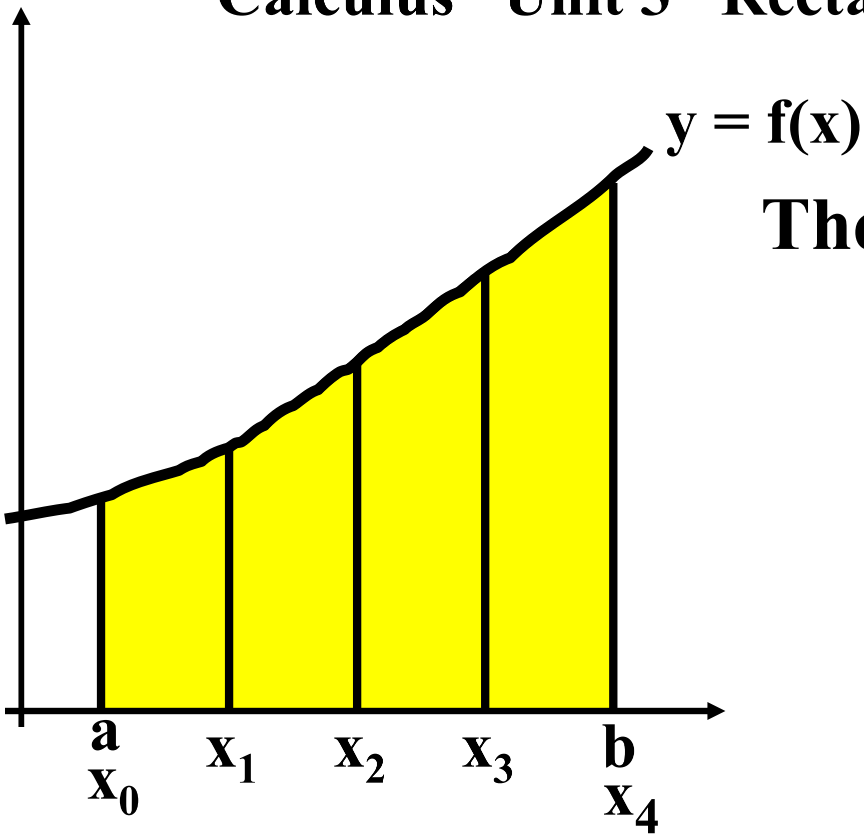
$$S_U = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

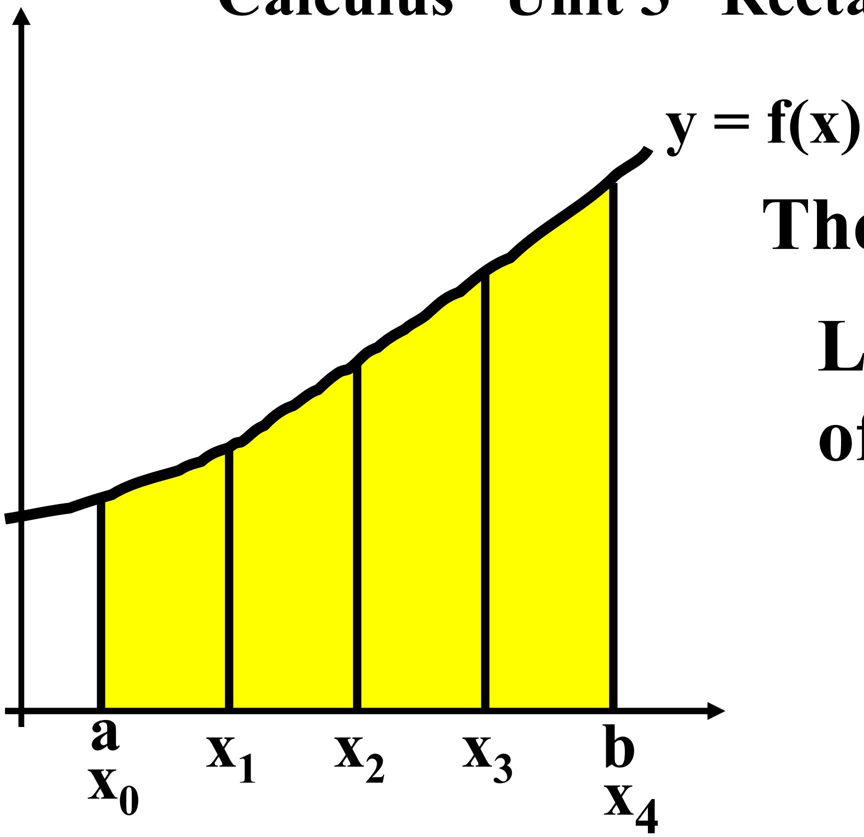
In general, $S_U = \sum_{i=1}^{n} f(x_i)\Delta x$

Calculus Unit 3 Rectangular Approximations



The Midpoint Rectangular Sum

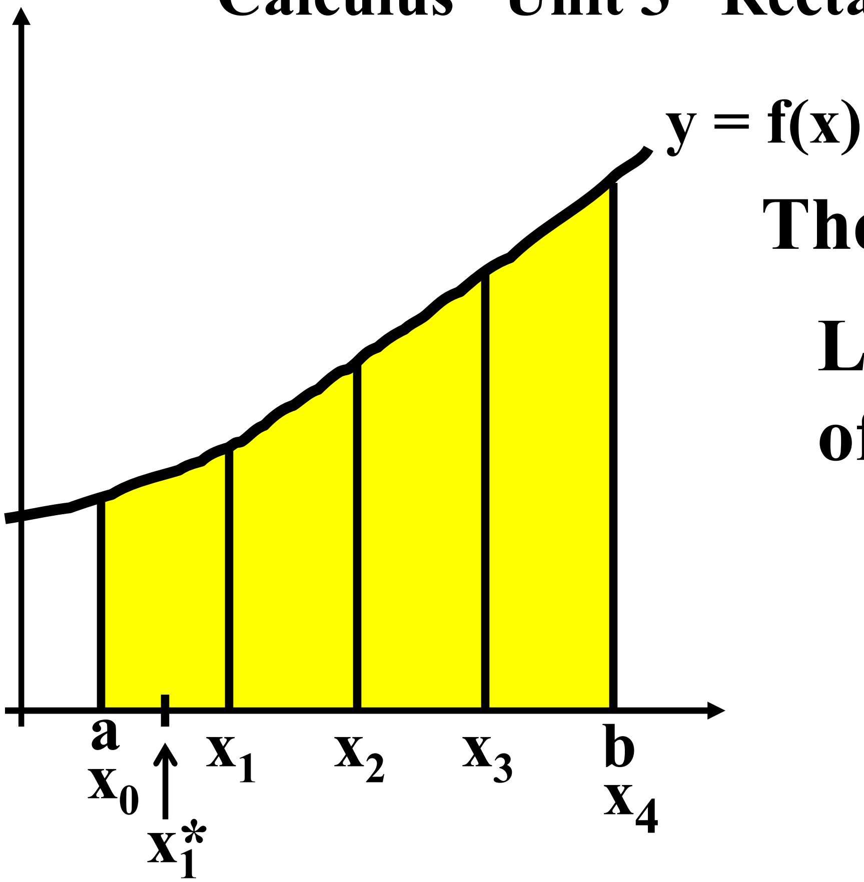
Calculus Unit 3 Rectangular Approximations



The Midpoint Rectangular Sum

Let x_i^* represent the midpoint of the i^{th} sub-interval.

Calculus Unit 3 Rectangular Approximations



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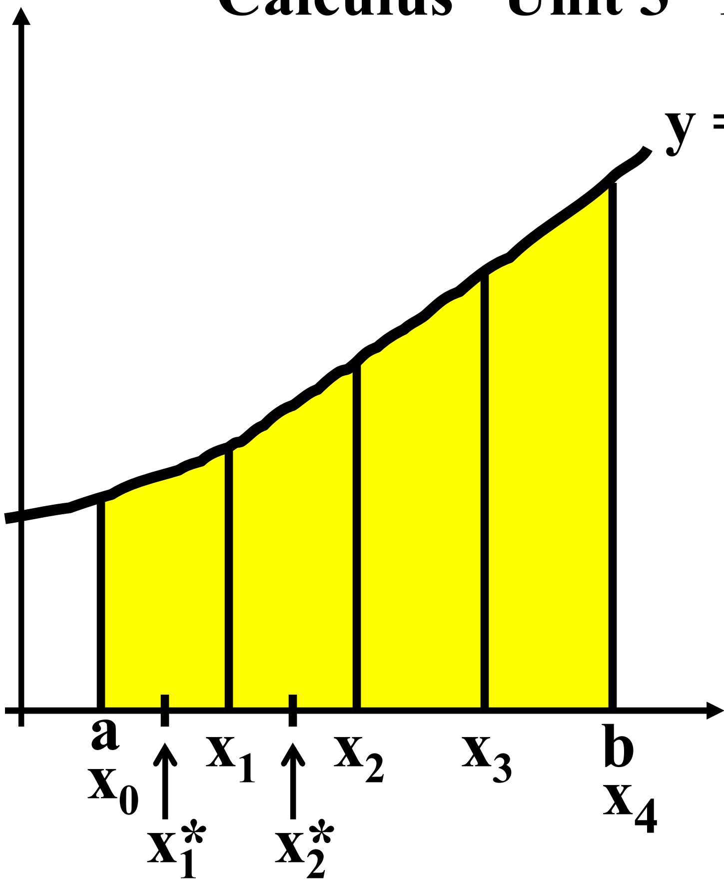

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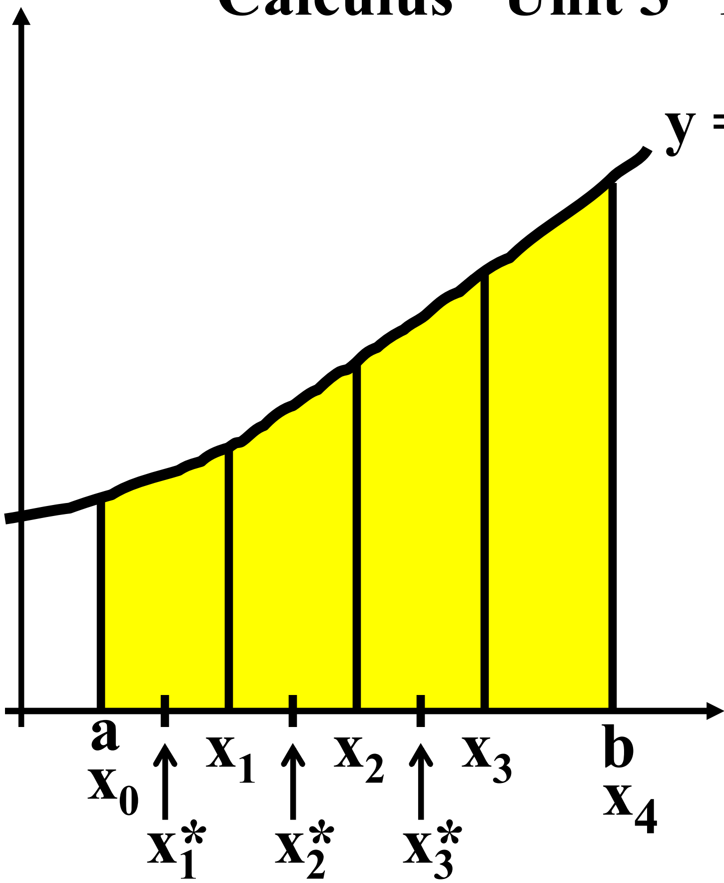
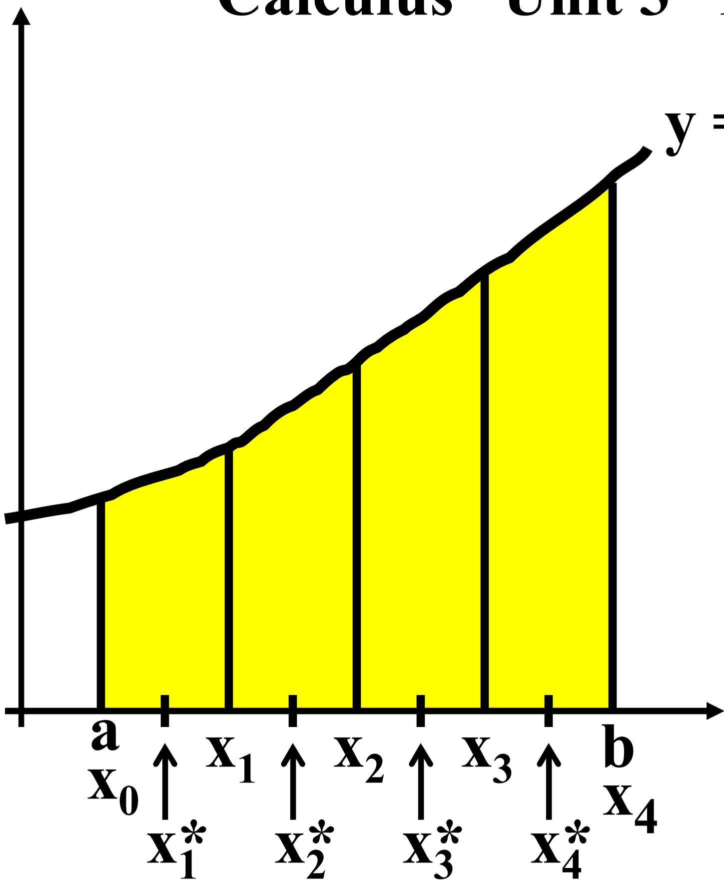


The Midpoint Rectangular Sum

Let x_i^* represent the midpoint of the i^{th} sub-interval.



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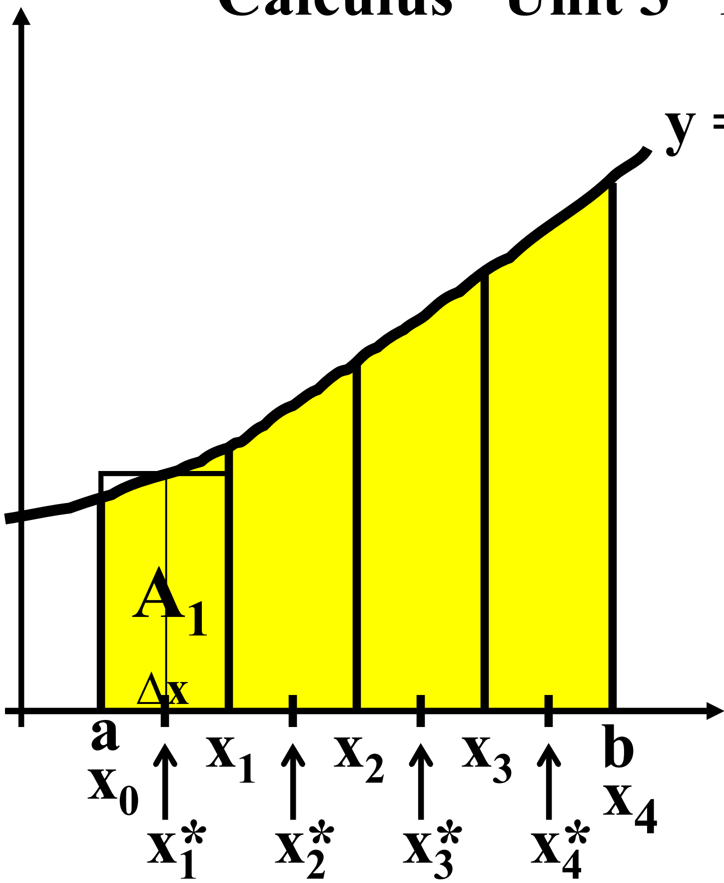
Calculus Unit 3 Rectangular Approximations

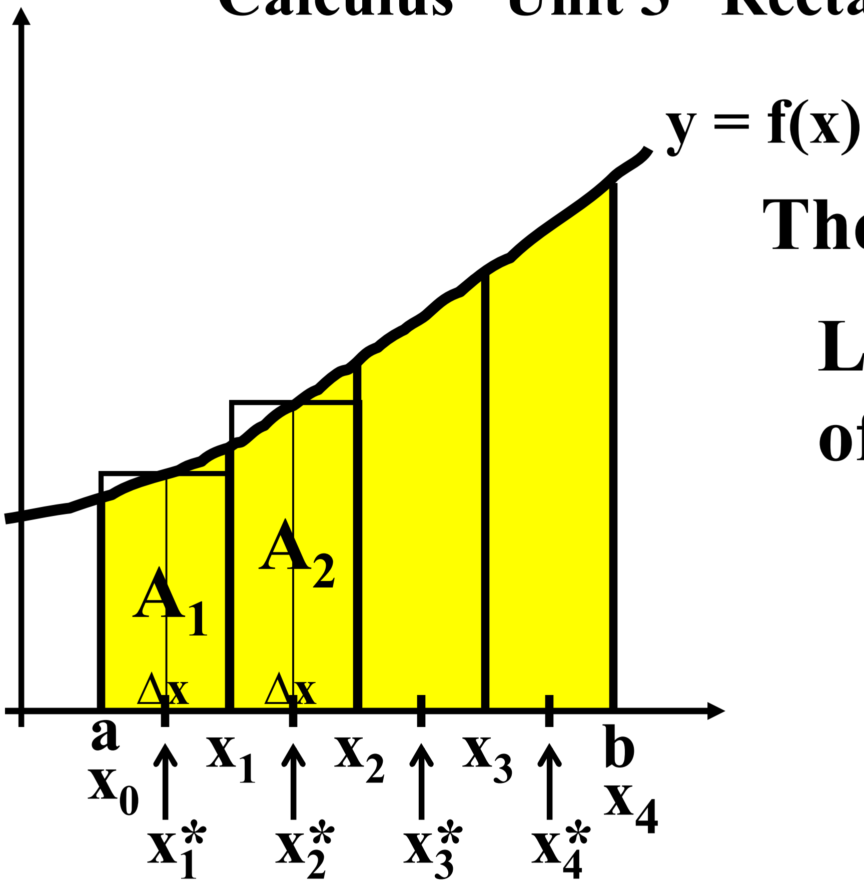


The Midpoint Rectangular Sum

Let x_i^* represent the midpoint of the i^{th} sub-interval.

$$A_1 = f(x_1^*)\Delta x$$

Calculus Unit 3 Rectangular Approximations



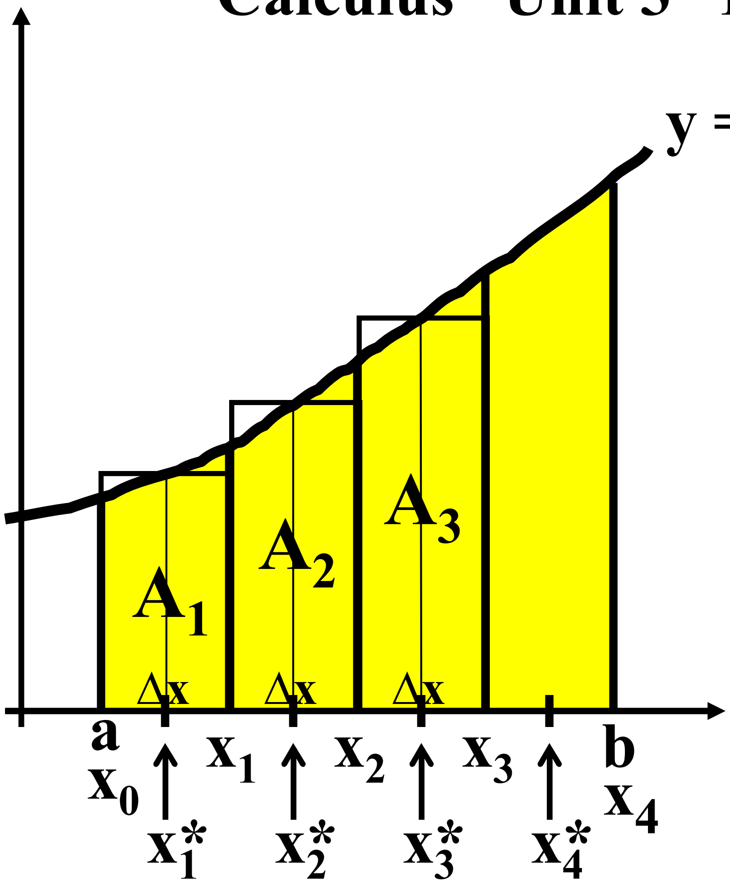
The Midpoint Rectangular Sum

Let x_i^* represent the midpoint of the i^{th} sub-interval.

$$A_1 = f(x_1^*)\Delta x$$

$$A_2 = f(x_2^*)\Delta x$$

Calculus Unit 3 Rectangular Approximations



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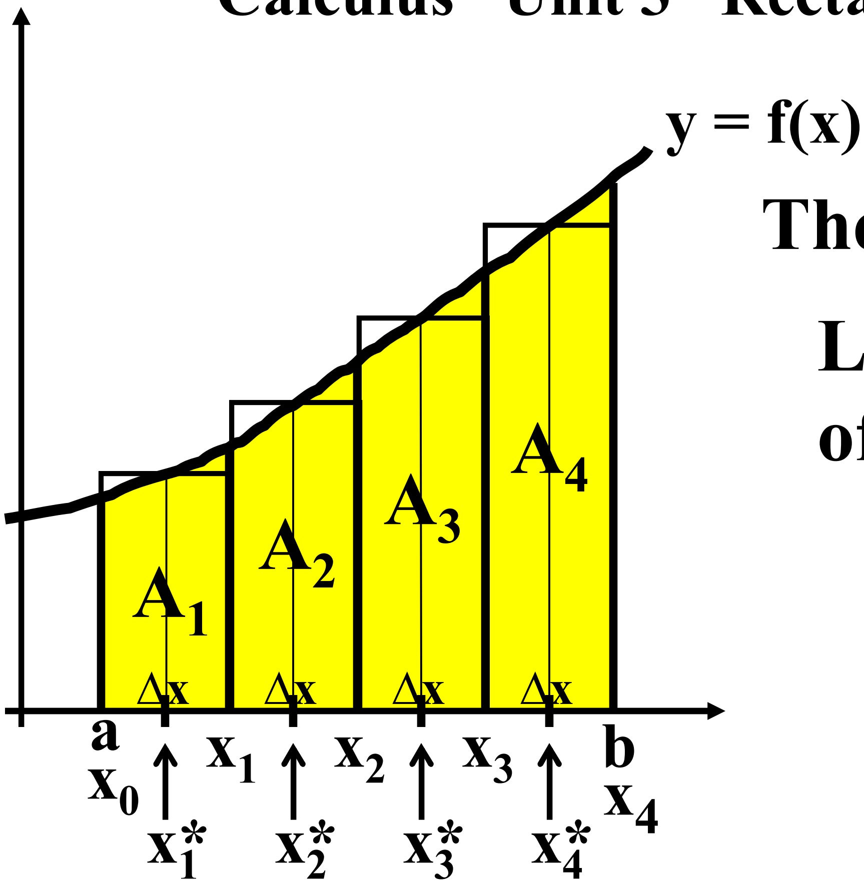
Calculus Unit 3 Rectangular Approximations



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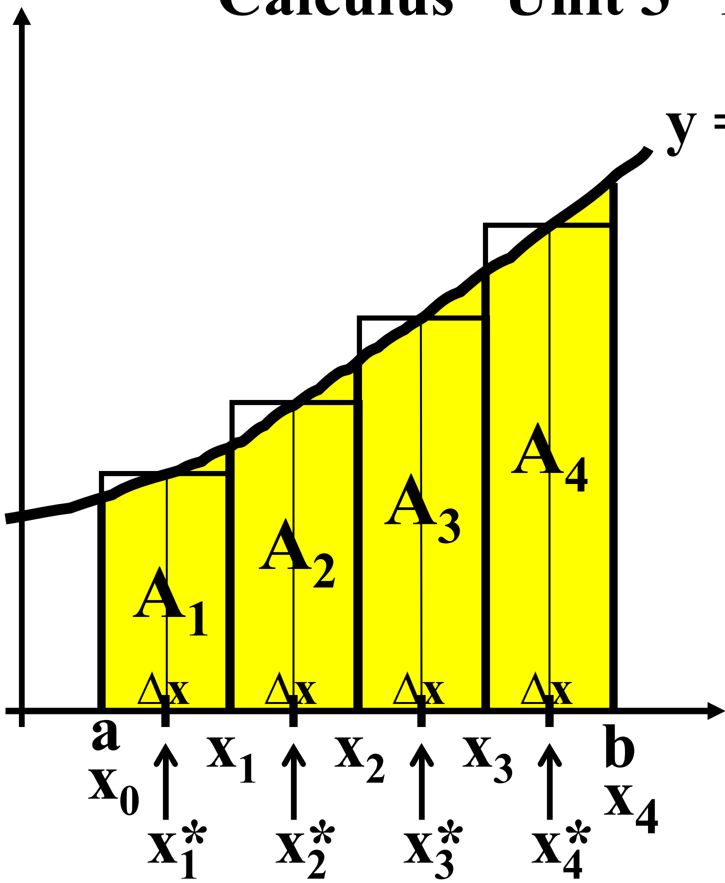
$$A_4 = f(x_4^*)\Delta x$$

Calculus Unit 3 Rectangular Approximations



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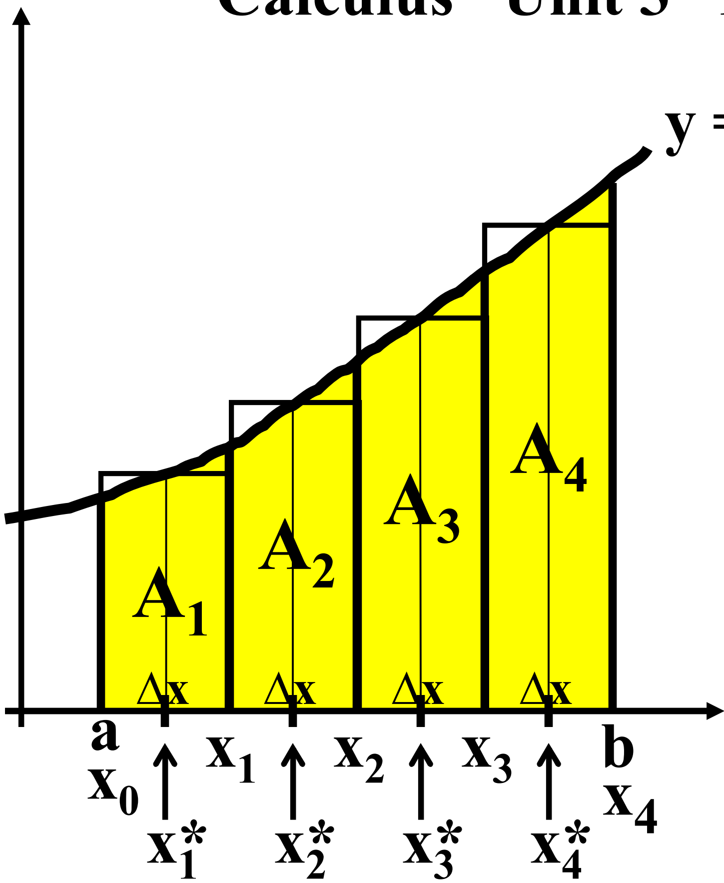
$$A_3 = f(x_3^*)\Delta x$$

$$A_4 = f(x_4^*)\Delta x$$

$$S_M =$$

Calculus Unit 3 Rectangular Approximations



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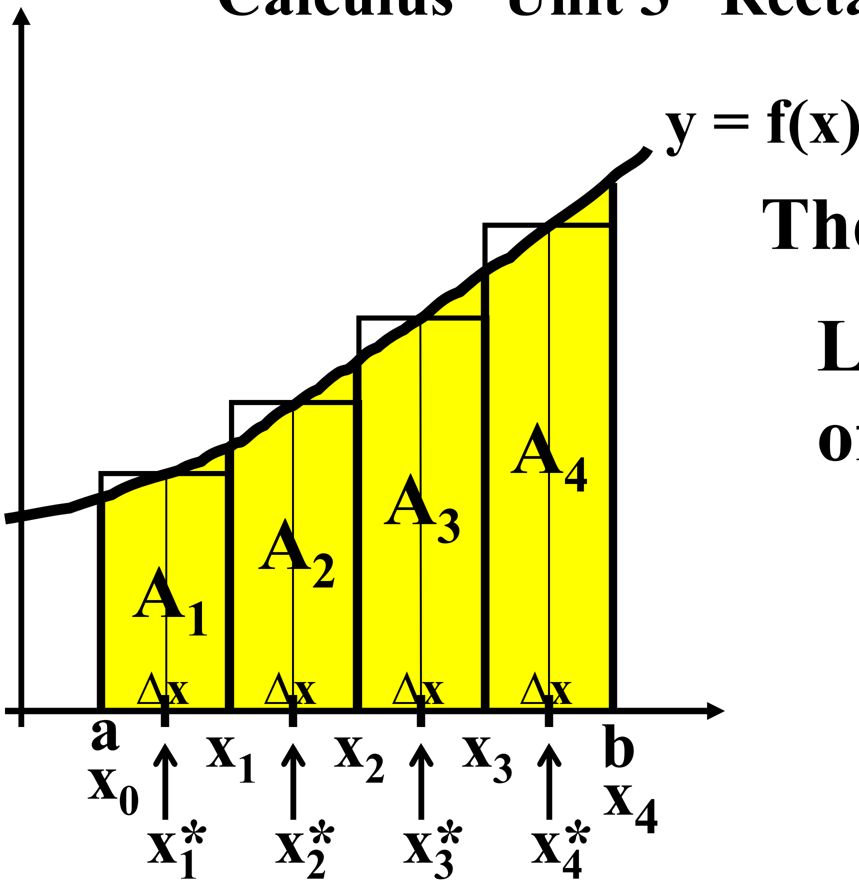
$$S_M = f(x_1^*)\Delta x$$

Calculus Unit 3 Rectangular Approximations



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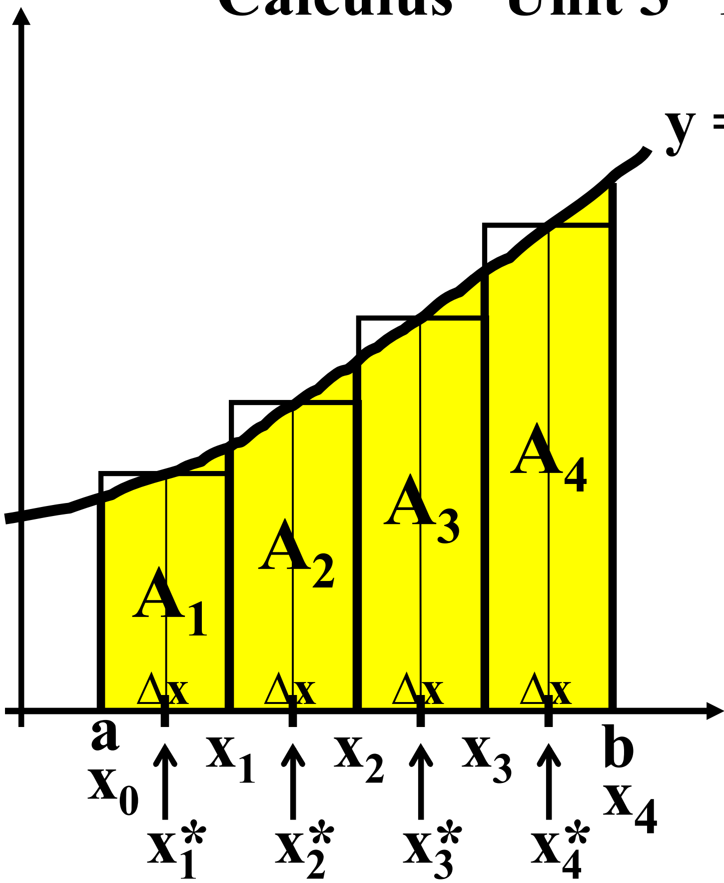
$$S_M = f(x_1^*)\Delta x + f(x_2^*)\Delta x$$

Calculus Unit 3 Rectangular Approximations



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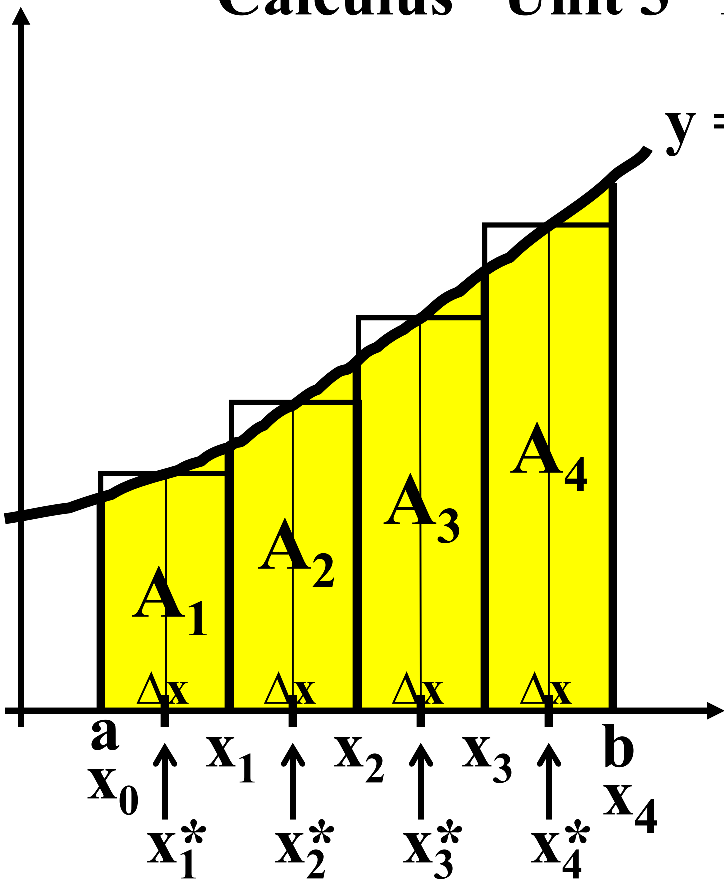
Calculus Unit 3 Rectangular Approximations



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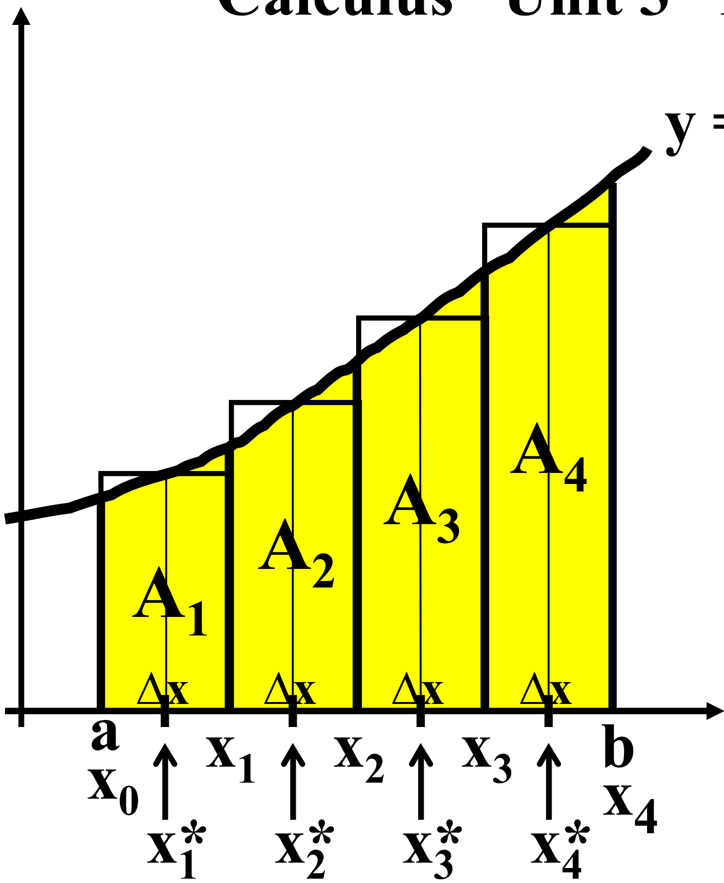
Calculus Unit 3 Rectangular Approximations



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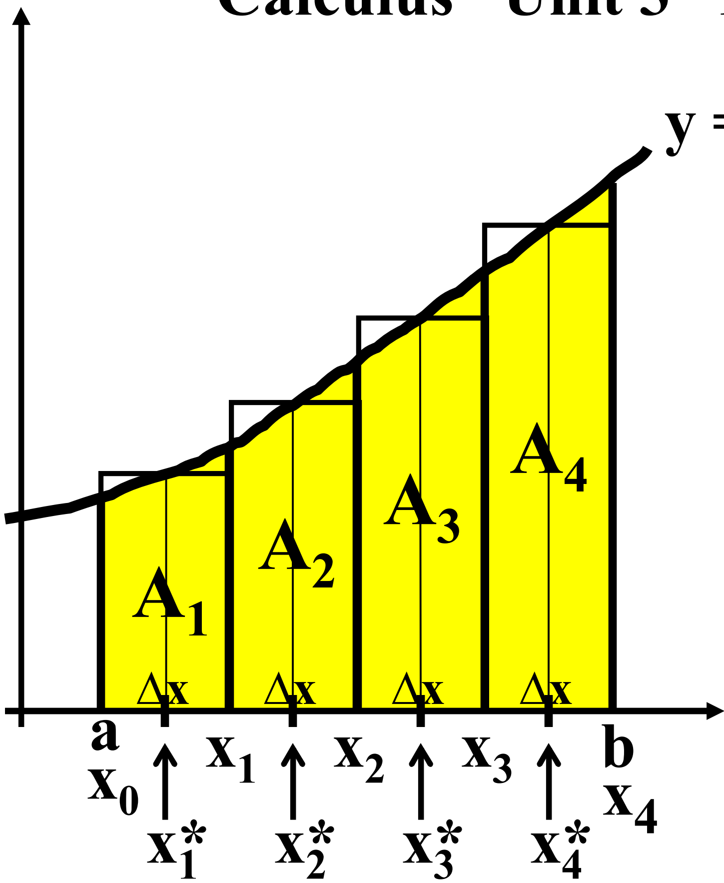
$$S_M = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$$

In general,



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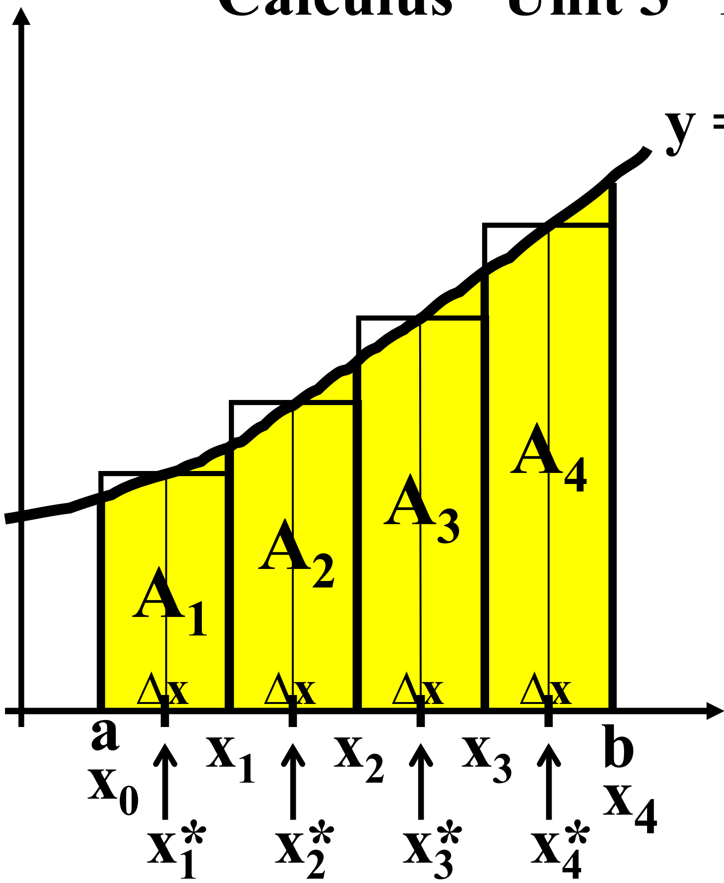
In general, $S_M =$



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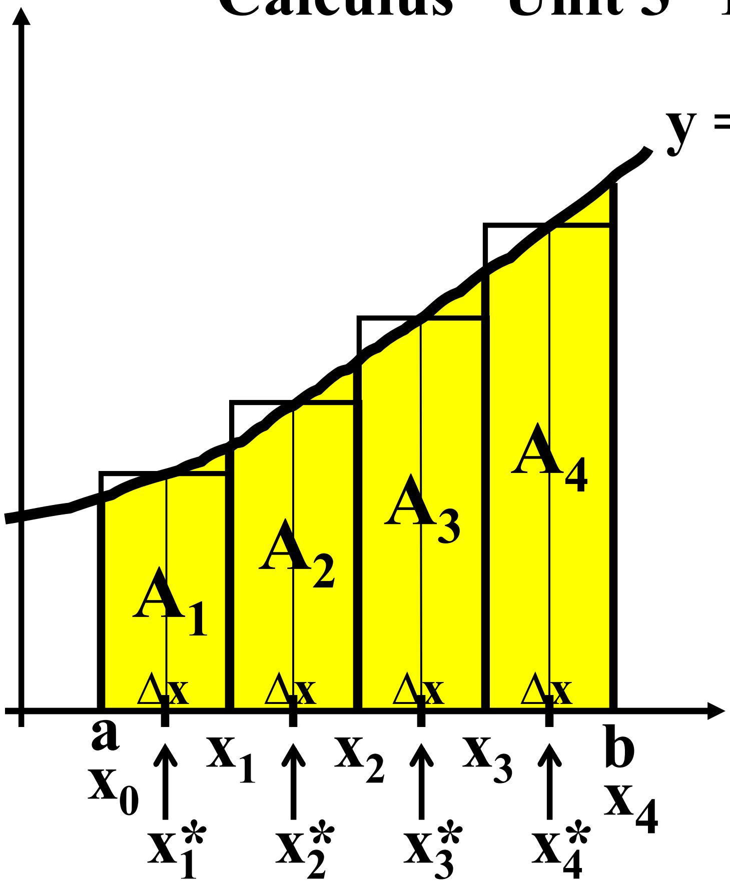
In general, $S_M = \sum_{i=1}^{n}$



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$$A_4 = f(x_4^*)\Delta x$$

$$S_M = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$$

In general, $S_M = \sum_{i=1}^{n} f(x_i^*)\Delta x$

Calculus Lesson #2 Unit 3

The Fundamental Theorems