Calculus Worksheet #4 Unit 2 Selected Solutions

Find the equation of (a) the line that is tangent to and (b) the line that is normal to each of the following functions at the point on the graph with the given x-coordinate.

2. $f(x) = x^3 - 3x^2 - 9x - 3 \quad ; \quad x = -1$

$y = f(-1) = 2$ Point $(-1, 2)$ $f'(x) = 3x^2 - 6x - 9$

Tangent: $m_t = f'(-1) = 0$ (horizontal line) Normal (vertical line)

(a) <u>tangent line: $y = 2$</u> (b) <u>normal line: $x = -1$</u>

Write the equation of any line which contains the given point and is tangent to the graph of the given function. Give the point of tangency with each equation.

7. $(4, -7) \quad ; \quad f(x) = x^2 - 6x + 5$

Let $T(x, y)$ represent any point of tangency.

At point T, $y = f(x) = x^2 - 6x + 5$

Also, $\dfrac{y - y_1}{x - x_1} = f'(x)$

$f'(x) = 2x - 6$

$\dfrac{y + 7}{x - 4} = 2x - 6$

$y + 7 = (x - 4)(2x - 6)$

$y + 7 = 2x^2 - 14x + 24$

$y = 2x^2 - 14x + 17$

Therefore, $2x^2 - 14x + 17 = x^2 - 6x + 5$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2$ or $x = 6$

$x = 2$

$y = f(2) = -3$

point $(2, -3)$ $m_t = f'(2) = -2$

$y + 3 = -2(x - 2)$

$y + 3 = -2x + 4$

$\boxed{y = -2x + 1 \text{ at } (2, -3)}$

$x = 6$

$y = f(6) = 5$

point $(6, 5)$ $m_t = f'(6) = 6$

$y - 5 = 6(x - 6)$

$y - 5 = 6x - 36$

$\boxed{y = 6x - 31 \text{ at } (6, 5)}$

Find the acute angle between the graphs of the given functions at each point where they intersect.

11. $y = x^2 - 2x \;\rightarrow\; f_1(x) = x^2 + 2x$

$y = -x + 6 \;\rightarrow\; f_2(x) = x + 2$

$x^2 - 2x = -x + 6$

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$x = -2$ or $x = 3$

$y = 8$ $\quad$ $y = 3$

$(-2, 8)$ $\quad$ $(3, 3)$

In general, $m_1 = f_1'(x) = 2x - 2$ and $m_2 = f_2'(x) = -1$

At $(-2, 8)$

$m_1 = -6$ and $m_2 = -1$

$$\text{Tan}\,\theta = \left| \frac{-6 + 1}{1 + (-6)(-1)} \right| = \frac{5}{13}$$

$\theta = \text{Tan}^{-1}(5/13) \approx 21.0°$

At $(3, 3)$

$m_1 = 4$ and $m_2 = -1$

$$\text{Tan}\,\theta = \left| \frac{4 + 1}{1 + (4)(-1)} \right| = \frac{5}{3}$$

$\theta = \text{Tan}^{-1}(5/3) \approx 59.0°$

The angle is about 21° at $(-2, 8)$ and about 59° at $(3, 3)$.