Calculus Worksheet #3 Unit 2 Selected Solutions

Find the acute angle between the graphs of the given functions at each point where they intersect. Show your work and your solutions neatly organized.

2. $y = x^2 + 2x \rightarrow f_1(x) = x^2 + 2x$

$y = x + 2 \rightarrow f_2(x) = x + 2$

$x^2 + 2x = x + 2$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2$ or $x = 1$

$y = 0$ $y = 3$

$(-2, 0)$ $(1, 3)$

In general, $m_1 = f_1'(x) = 2x + 2$ and $m_2 = f_2'(x) = 1$

At $(-2, 0)$: $m_1 = -2$ and $m_2 = 1$

$$\text{Tan}\,\theta = \left| \frac{m_1 - m_2}{1 + (m_1)(m_2)} \right| \qquad \text{Tan}\,\theta = \left| \frac{-2 - 1}{1 + (-2)(1)} \right| = 3$$

$\theta = \text{Tan}^{-1}(3) \approx 71.6°$

At $(1, 3)$: $m_1 = 4$ and $m_2 = 1$

$$\text{Tan}\,\theta = \left| \frac{m_1 - m_2}{1 + (m_1)(m_2)} \right| \qquad \text{Tan}\,\theta = \left| \frac{4 - 1}{1 + (4)(1)} \right| = 0.6$$

$\theta = \text{Tan}^{-1}(.6) \approx 31.0°$

The angle is about 71.6° at (-2, 0) and about 31° at (1, 3).

4. $y = 2x^2 + x - 10 \rightarrow f_1(x) = 2x^2 + x - 10$

$y = -x^2 - 5x + 14 \rightarrow f_2(x) = -x^2 - 5x + 14$

$2x^2 + x - 10 = -x^2 - 5x + 14$

$3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$

$y = 18$ $y = 0$

$(-4, 18)$ $(2, 0)$

In general, $m_1 = f_1'(x) = 4x + 1$ and $m_2 = f_2'(x) = -2x - 5$

At $(-4, 18)$: $m_1 = -15$ and $m_2 = 3$

$$\text{Tan}\theta = \left| \frac{m_1 - m_2}{1 + (m_1)(m_2)} \right| \qquad \text{Tan}\theta = \left| \frac{-15 - 3}{1 + (-15)(3)} \right| = \frac{9}{22}$$

$\theta = \text{Tan}^{-1}(9/22) \approx 22.2°$

At $(2, 0)$: $m_1 = 9$ and $m_2 = -9$

$$\text{Tan}\theta = \left| \frac{m_1 - m_2}{1 + (m_1)(m_2)} \right| \qquad \text{Tan}\theta = \left| \frac{9 - -9}{1 + (9)(-9)} \right| = \frac{9}{40}$$

$\theta = \text{Tan}^{-1}(9/40) \approx 12.7°$

The angle is about 22.2° at (-4, 18) and about 12.7° at (2, 0).