

Calculus Worksheet #2 Unit 2 Selected Solutions

Write the equation of any line that contains the given point and is tangent to the given function. Give the point of tangency with each equation.

3. $(1, -2)$; $f(x) = x^2 - 2x + 3$

Let $T(x, y)$ represent any point of tangency.

At point T , $y = f(x) = x^2 - 2x + 3$

Also, $\frac{y - y_1}{x - x_1} = f'(x)$

$f'(x) = 2x - 2$

$\frac{y + 2}{x - 1} = 2x - 2$

$y + 2 = (x - 1)(2x - 2)$

$y + 2 = 2x^2 - 4x + 2$

$y = 2x^2 - 4x$

Therefore, $2x^2 - 4x = x^2 - 2x + 3$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3$ or $x = -1$

$y = f(3) = 6$

point $(3, 6)$ $m_t = f'(3) = 4$

$y - 6 = 4(x - 3)$

$y - 6 = 4x - 12$

$y = 4x - 6$ at $(3, 6)$

$y = f(-1) = 6$

point $(-1, 6)$ $m_t = f'(-1) = -4$

$y - 6 = -4(x + 1)$

$y - 6 = -4x - 4$

$y = -4x + 2$ at $(-1, 6)$

5. $(2, -5)$; $f(x) = x^3 - 6x^2 + 4x + 1$

Let $T(x, y)$ represent any point of tangency.

At point T , $y = f(x) = x^3 - 6x^2 + 4x + 1$

Also, $\frac{y - y_1}{x - x_1} = f'(x)$

$f'(x) = 3x^2 - 12x + 4$

$\frac{y + 5}{x - 2} = 3x^2 - 12x + 4$

$y + 5 = (x - 2)(3x^2 - 12x + 4)$

$y + 5 = 3x^3 - 18x^2 + 28x - 8$

$y = 3x^3 - 18x^2 + 28x - 13$

Therefore,

$3x^3 - 18x^2 + 28x - 13 = x^3 - 6x^2 + 4x + 1$

$2x^3 - 12x^2 + 24x - 14 = 0$

$x^3 - 6x^2 + 12x - 7 = 0$

$(x - 1)(x^2 - 5x + 7) = 0$

$x = 1$ or $x = \frac{5 \pm \sqrt{25 - 28}}{2}$

$y = f(1) = 0$

point $(1, 0)$ $m_t = f'(1) = -5$

$y - 0 = -5(x - 1)$

$y = -5x + 5$ at $(1, 0)$