

Calculus Review Unit 12 page 1 _____

Use calculus to find the total amount of work done in each of the following problems. Where appropriate, express your answers rounded to 3 significant figures.

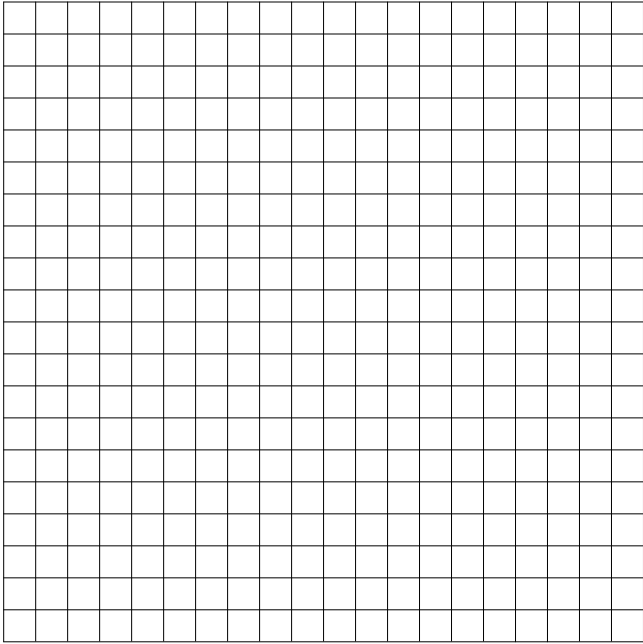
1. A water tank is in the shape of a right, circular cone with axis vertical. The tank is 50 feet tall, and the top has a radius of 15 feet. The tank is empty initially, but water is pumped from a near-by lake through a pipe that goes into the bottom of the tank. If the bottom of the tank is 50 feet above the surface of the lake, then how much work is done in filling the tank. (Assume that a cubic foot of water weighs 62.4 pounds.)

2. Suppose that a large spring has a natural length of 3 feet and that a force of 20 pounds is needed to compress it to a length of 2.8 feet. How much work is being done in compressing the spring from a length of 2.8 feet to a length of 2.5 feet? (According to Hook's Law, the force required to compress the spring is directly proportional to the distance the spring is compressed.)

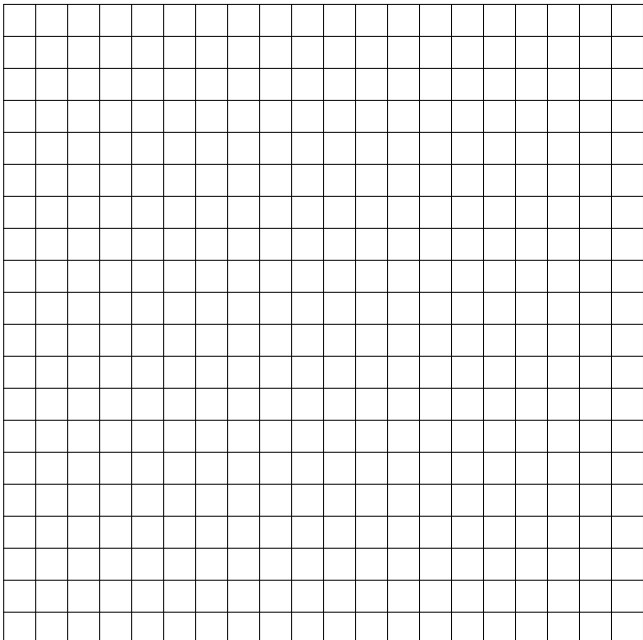
Calculus Review Unit 12 page 2

In each of the following problems, you are given a function $y = f(x)$ and two values of x , $x = a$ and $x = b$. You are to find the length of the graph of $y = f(x)$ from the point $(a, f(a))$ to the point $(b, f(b))$. In each case, draw a graph showing the 'curve' from $x = a$ to $x = b$. Express your answers rounded to 3 significant figures.

3. $y = f(x) = 4 - x^2$; $a = -3$; $b = 1$.



4. $y = f(x) = x^3 - 3x^2 + x + 2$; $a = -1$; $b = 3$.

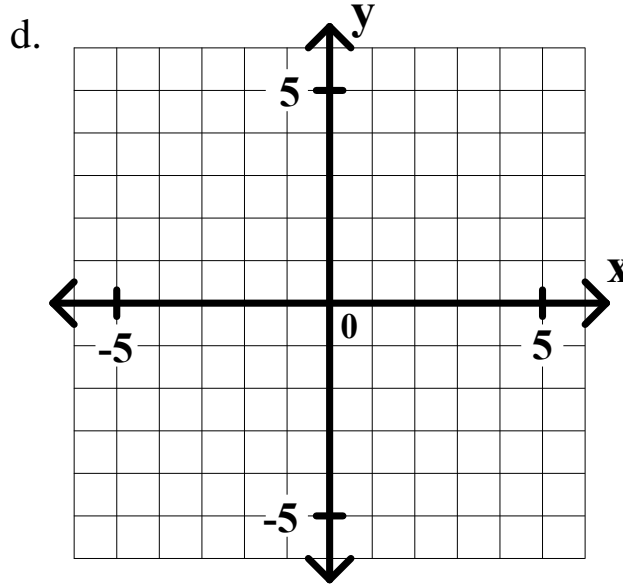
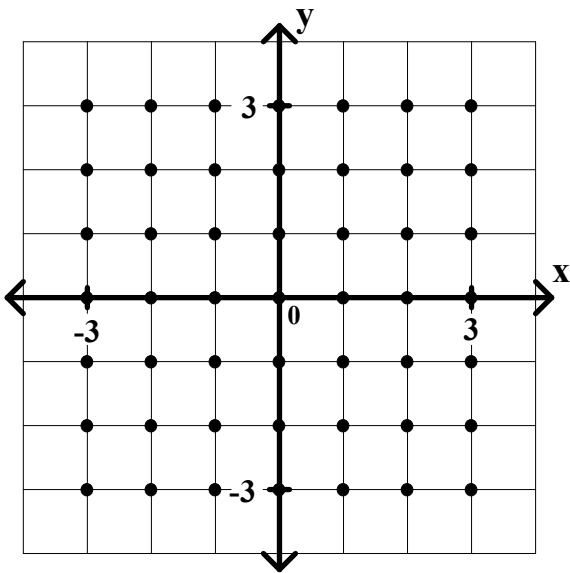


Calculus Review Unit 12 page 3

In the following problem, you are given a differential equation and a point. Do each of the following. Show your work in the space provided.

- a. Sketch a slope field on the axes provided at the indicated points.
- b. Find the general solution of the differential equation.
- c. Find the specific solution that would contain the given point.
- d. Graph the specific solution.

5. $\frac{dy}{dx} = \frac{-x}{4y}$; (0, -3)



b. _____

c. _____

Calculus Review Unit 12 page 4

Find the general solution of each of the following differential equations. Then find the specific solution that would contain the given point.

6. $3y^2 \sin^2 x \, dy = dx$; $(\frac{\pi}{6}, \sqrt{3})$

7. $\csc x \, dy - y \, dx = 0$; $(\frac{\pi}{3}, e^3)$

8. $dy/dx = 0.025y$; $(0, 4000)$

9. $\sqrt{xy} \, dy = dx$; $(4, 9)$

Calculus Review Unit 12 page 5

Solve the following problem. Show your complete solution neatly organized.

10. Water is leaking from the bottom of a tall rectangular water tank at a rate that is proportional to the square of the depth of the water remaining in the tank. If the depth drops from 10 feet to 8 feet in one hour, then how much longer will it take for the water level to drop to 5 feet?

11. The rotation of a flywheel is opposed by a frictional force which produces a negative acceleration that is proportional to square root of the velocity. If a wheel slows from 2500 r.p.m. to 1600 r.p.m. in 10 minutes, then how fast will it be turning after another 10 minutes?