

Calculus Worksheet #7 Unit 11 Selected Solutions

Approximate the following definite integrals using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$1. \int_0^3 x^3 dx \quad \Delta x = \frac{3-0}{6} = 0.5$$

$x_0 = 0$	$f(x_0) = 0$	$x_1^* = 1/4$	$f(x_1^*) = 1/64$
$x_1 = .5$	$f(x_1) = .125$	$x_2^* = 3/4$	$f(x_2^*) = 27/64$
$x_2 = 1$	$f(x_2) = 1$	$x_3^* = 5/4$	$f(x_3^*) = 125/64$
$x_3 = 1.5$	$f(x_3) = 3.375$	$x_4^* = 7/4$	$f(x_4^*) = 343/64$
$x_4 = 2$	$f(x_4) = 8$	$x_5^* = 9/4$	$f(x_5^*) = 729/64$
$x_5 = 2.5$	$f(x_5) = 15.625$	$x_6^* = 11/4$	$f(x_6^*) = 1331/64$
$x_6 = 3$	$f(x_6) = 27$		

$$S_L = (0 + .125 + 1 + 3.375 + 8 + 15.625)(.5) = (28.125)(.5) = 14.0625$$

$$S_R = (.125 + 1 + 3.375 + 8 + 15.625 + 27)(.5) = (55.125)(.5) = 27.5625$$

$$S_M = (1/64 + 27/64 + 125/64 + 343/64 + 729/64 + 1331/64)(1/2) = \\ S_M = (2556/64)(1/2) = 2556/128 = 19.96875$$

$$S_T = ((.5)(0) + .125 + 1 + 3.375 + 8 + 15.625 + (.5)(27))(.5) = (41.625)(.5) = 20.8125$$

$$S_S = (.5/3)(0 + 2(1 + 8) + 4(.125 + 3.375 + 15.625) + 27) = (1/6)(121.5) = 20.25$$