Calculus Worksheet #6 Unit 11 Selected Solutions

Approximate the following definite integrals using each of the following approximation methods.
(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular),
(d) S_T (Trapezoidal), and (e) S_S (Simpsonôs).
Show your complete solutions neatly organized. In every case, divide the interval [a, b] into 6 sub-intervals.

1. $\int_1^4 (x^2+1)dx$ $\qquad \Delta x = \frac{4-1}{6} = 0.5$

$x_0 = 1$ $\quad f(x_0) = 2$

$x_1 = 1.5$ $\quad f(x_1) = 3.25$

$x_2 = 2$ $\quad f(x_2) = 5$

$x_3 = 2.5$ $\quad f(x_3) = 7.25$

$x_4 = 3$ $\quad f(x_4) = 10$

$x_5 = 3.5$ $\quad f(x_5) = 13.25$

$x_6 = 4$ $\quad f(x_6) = 17$

$x_1^* = 1.25$ $\quad f(x_1^*) = 2.5625$

$x_2^* = 1.75$ $\quad f(x_2^*) = 4.0625$

$x_3^* = 2.25$ $\quad f(x_3^*) = 6.0625$

$x_4^* = 2.75$ $\quad f(x_4^*) = 8.5625$

$x_5^* = 3.25$ $\quad f(x_5^*) = 11.5625$

$x_6^* = 3.75$ $\quad f(x_6^*) = 15.0625$

$S_L = (2 + 3.25 + 5 + 7.25 + 10 + 13.25)(.5) = (40.75)(.5) = 20.375$

$S_R = (3.25 + 5 + 7.25 + 10 + 13.25 + 17)(.5) = (55.75)(.5) = 27.875$

$S_M = (2.5625 + 4.0625 + 6.0625 + 8.5625 + 11.5625 + 15.0625)(.5) = (47.875)(.5) = 23.9375$

$S_T = ((.5)(2) + (3.25 + 5 + 7.25 + 10 + 13.25 + (.5)(17))(.5) = (48.25)(.5) = 24.125$

$S_S = (.5/3)(2 + 2(5 + 10) + 4(3.25 + 7.25 + 13.25) + 17) = (1/6)(144) = 24$