

Calculus Worksheet #3 Unit 11 Selected Solutions

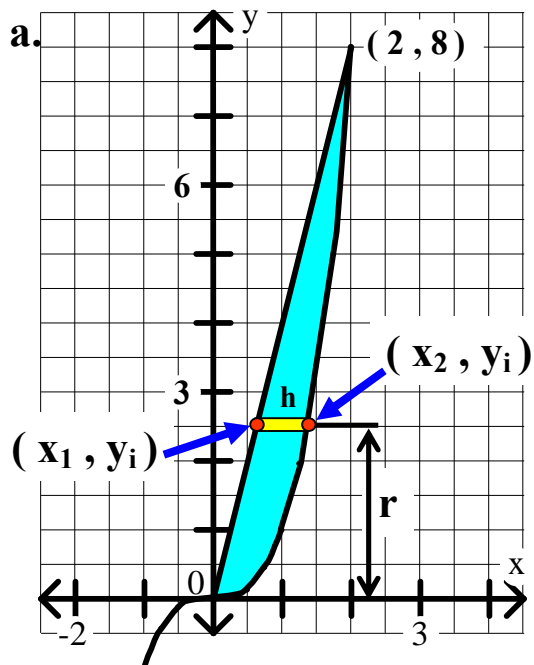
Use shells to find the volume generated by rotating the given region about the given line.

For each problem, you must

- sketch the generating region, showing a typical generating rectangle,
- write an expression for the volume generated by this rectangle,
- express the exact volume of the solid as a definite integral, and
- evaluate the integral.

Show all of your work neatly organized on graph paper.

2. The region in the first quadrant enclosed by $y = x^3$ and $y = 4x$ is rotated about the x-axis.



Shells: $V = 2\pi rh\Delta r$

$$y = x^3 \quad y = 4x$$

$$r = y_i$$

$$x = \sqrt[3]{y} \quad x = \frac{y}{4}$$

$$h = x_2 - x_1 = \sqrt[3]{y_i} - \frac{y_i}{4}$$

$$\Delta r = \Delta y$$

b. $V_i = 2\pi y_i \left(\sqrt[3]{y_i} - \frac{y_i}{4} \right) \Delta y$

c. $V = 2\pi \int_0^8 \left(y \left(\sqrt[3]{y} - \frac{y}{4} \right) \right) dy$

d. $V \approx 76.6 \text{ cu. units}$