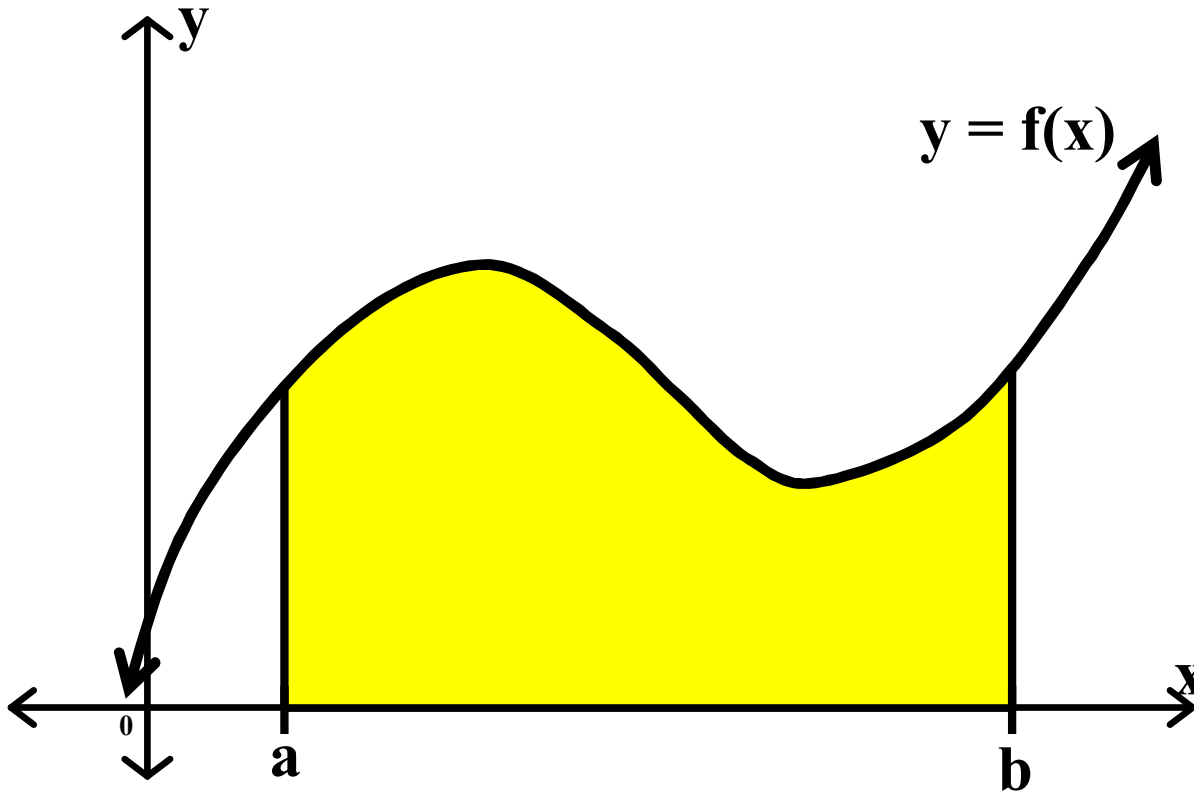


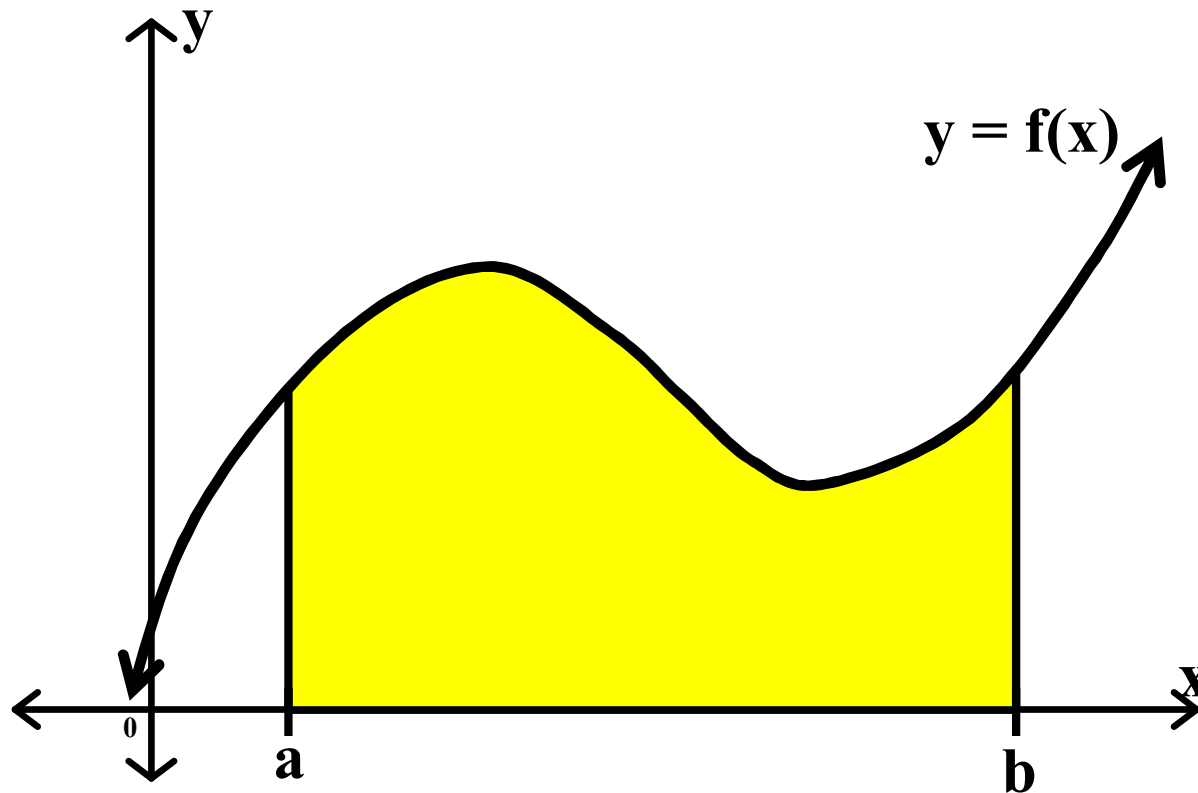
Calculus Lesson #5 Unit 11

Class Worksheet #5

**Numerical Methods for
Approximating Definite Integrals**

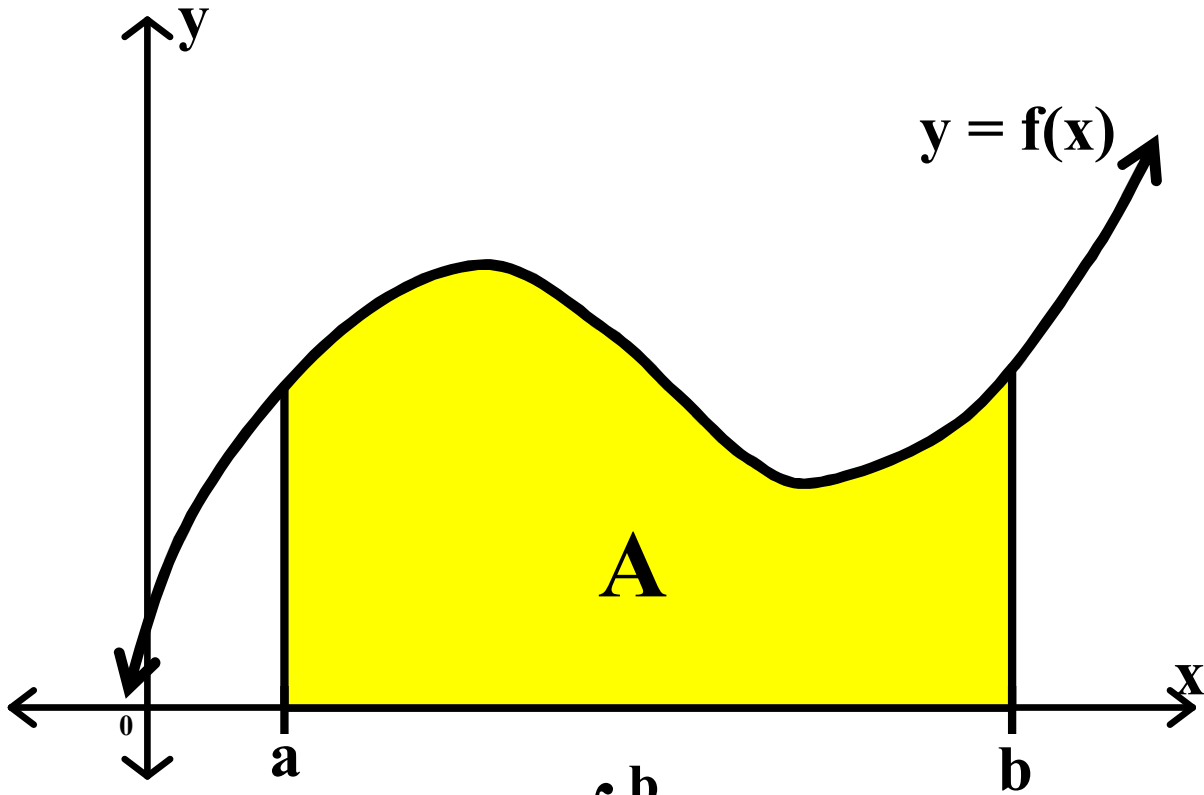


Consider the shaded region between the x-axis, the graph of the function $y = f(x)$, and the vertical lines $x = a$ and $x = b$.

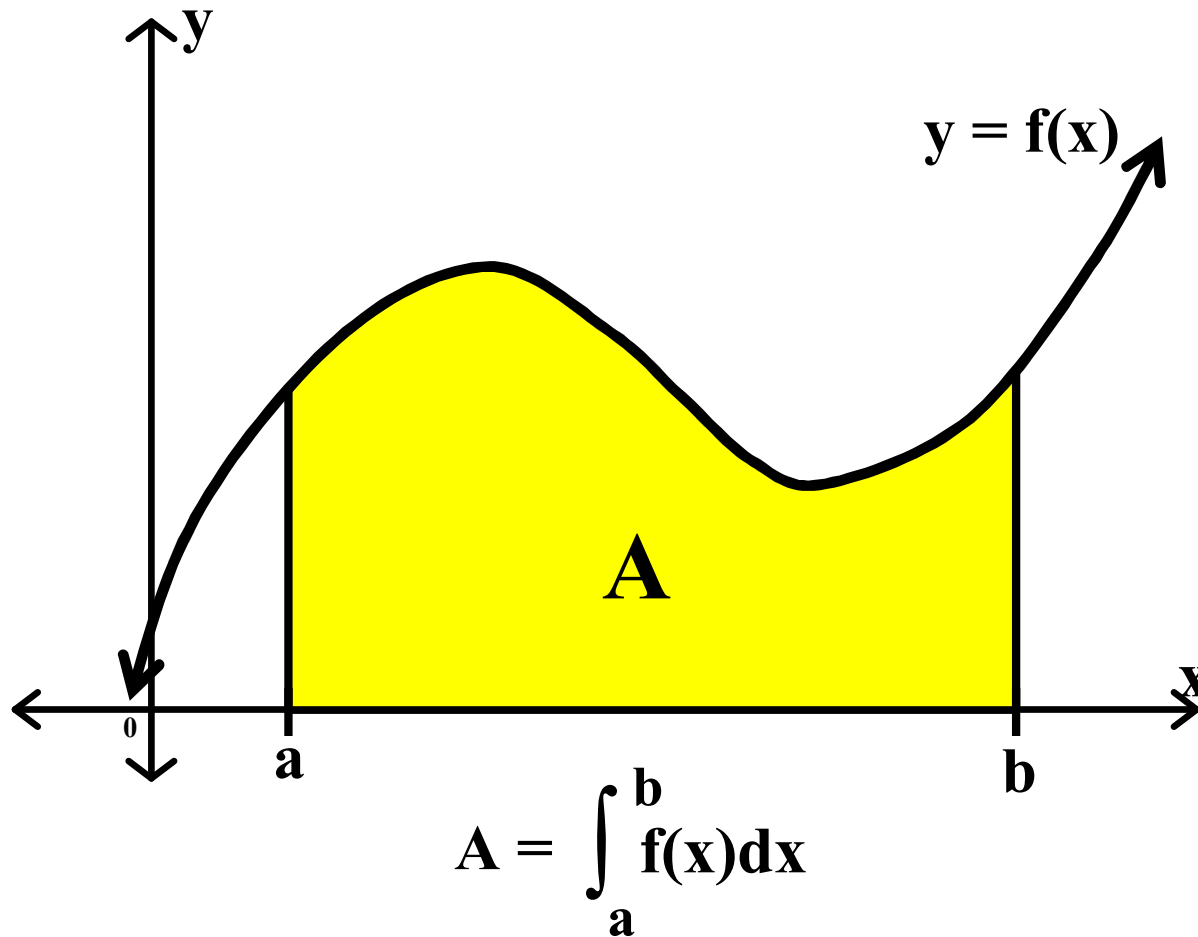


Consider the shaded region between the x-axis, the graph of the function $y = f(x)$, and the vertical lines $x = a$ and $x = b$. The area of this region can be represented by the definite integral

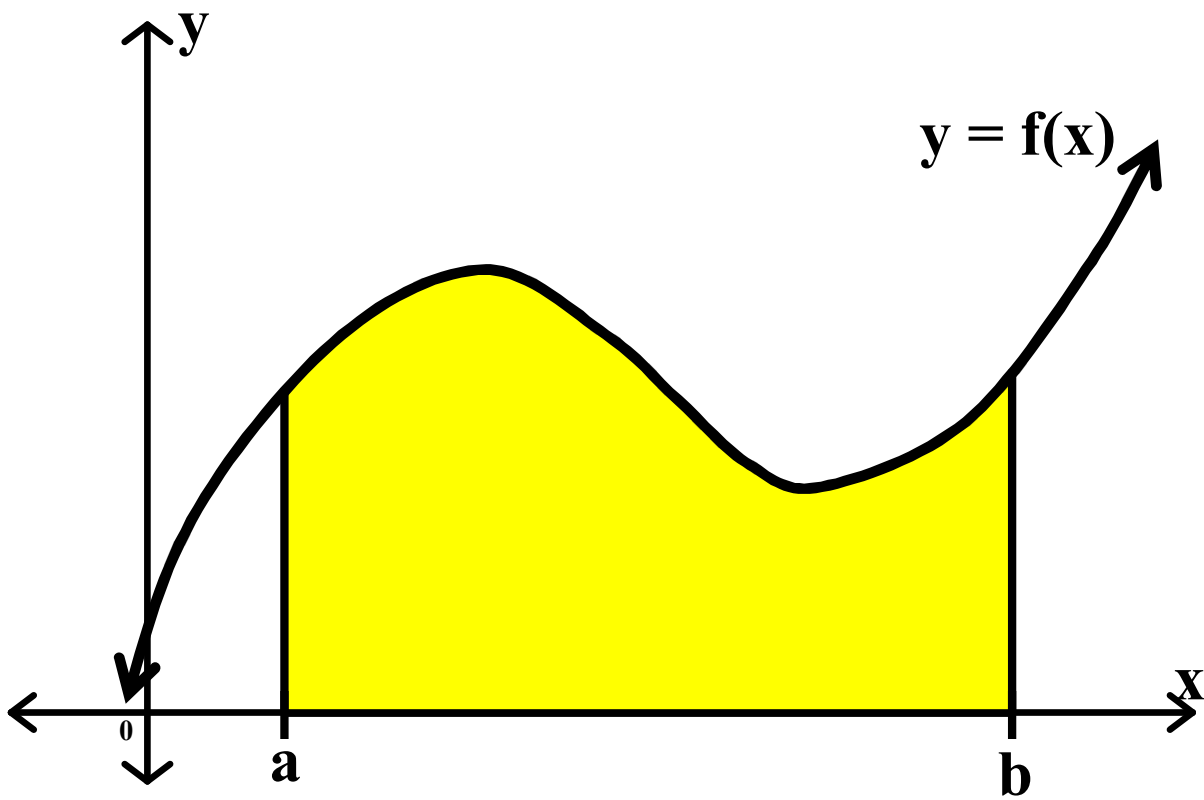
$$\int_a^b f(x)dx.$$

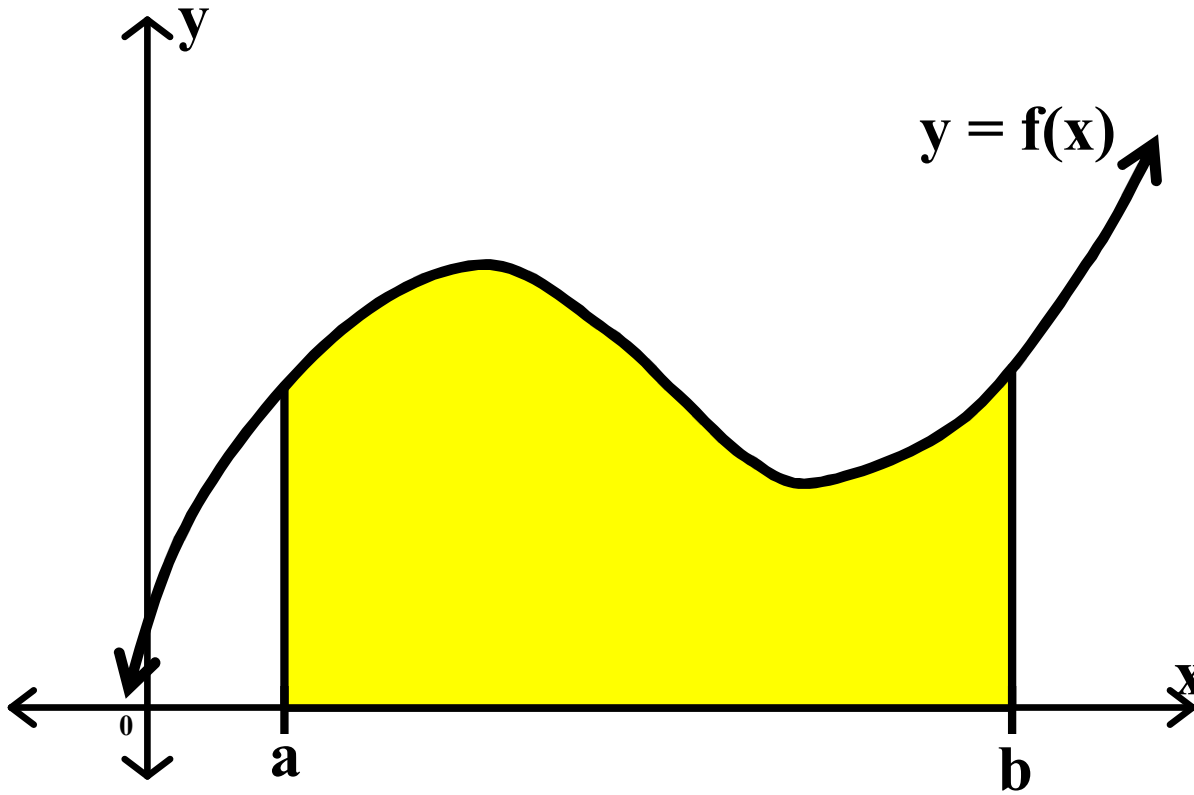


$$A = \int_a^b f(x) dx$$

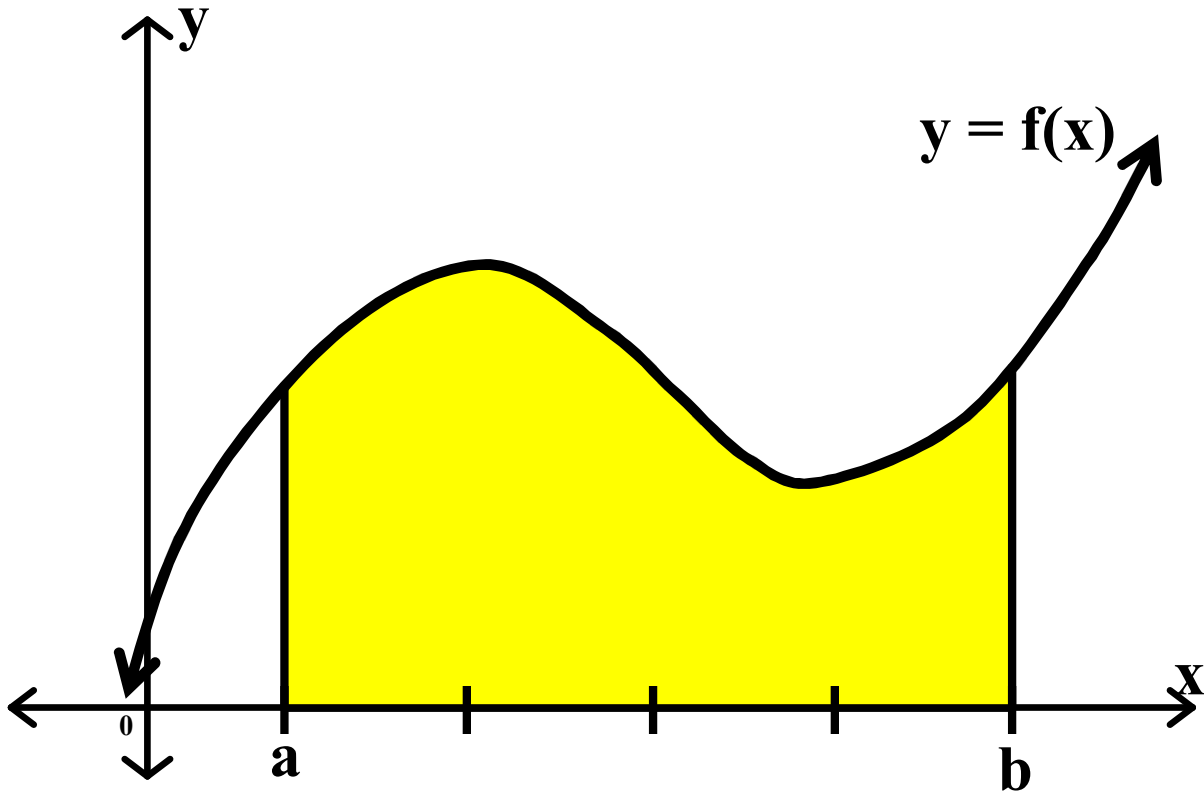


The purpose of this lesson is to introduce several numerical methods that can be used to approximate the value of a definite integral.

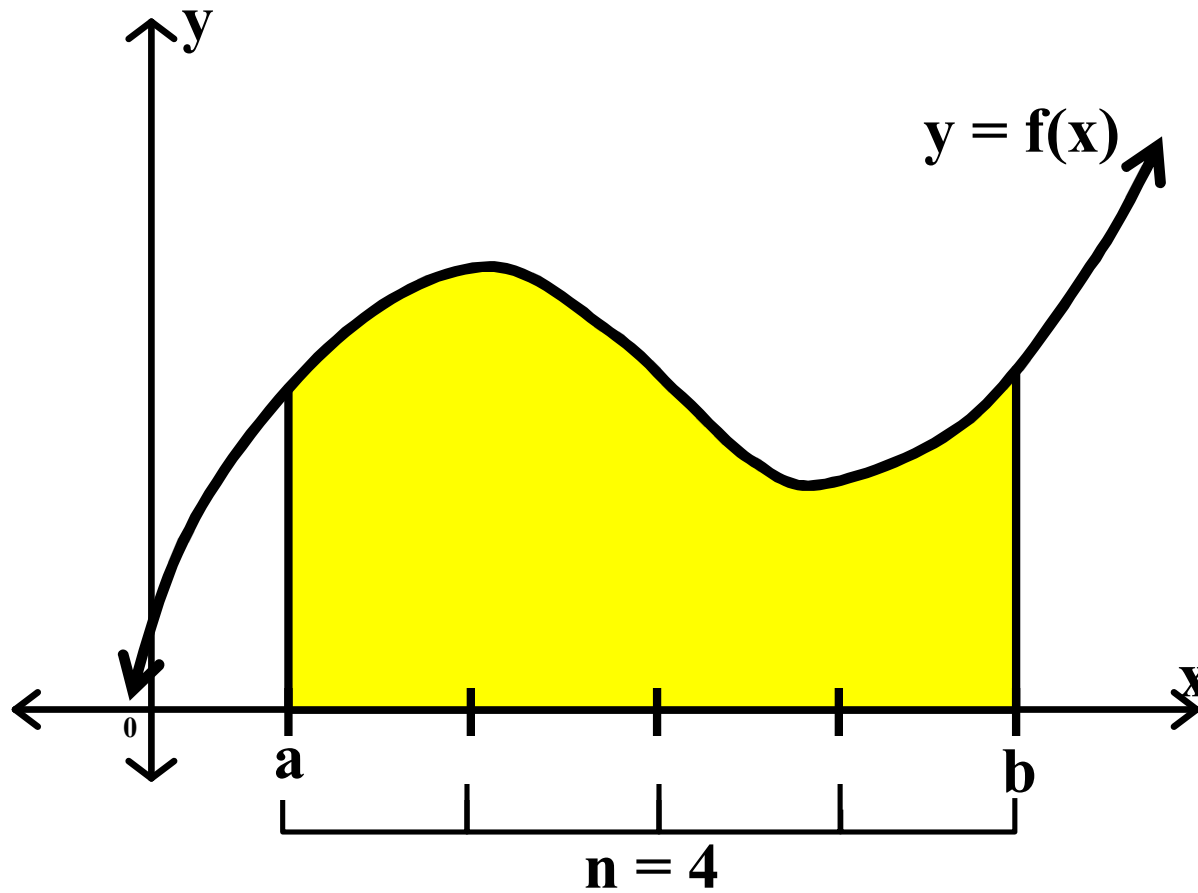




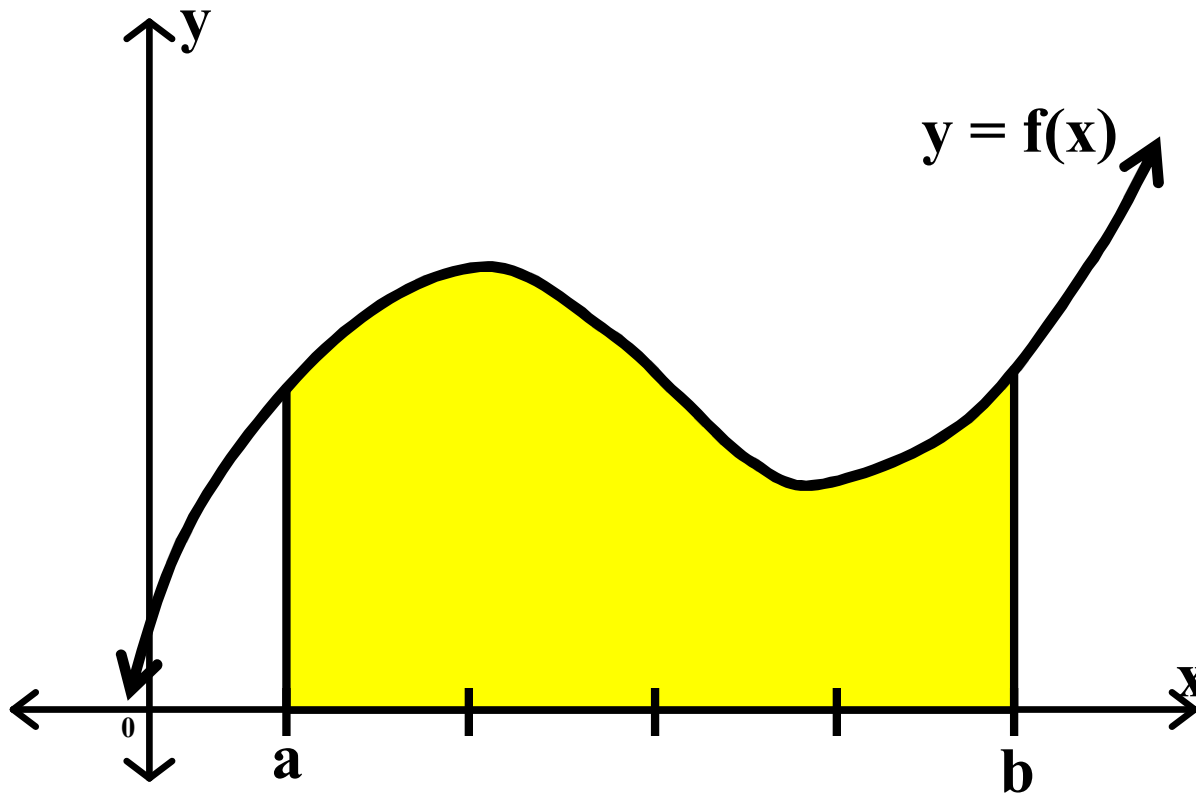
Divide the interval $[a, b]$ into n sub-intervals



Divide the interval $[a, b]$ into n sub-intervals

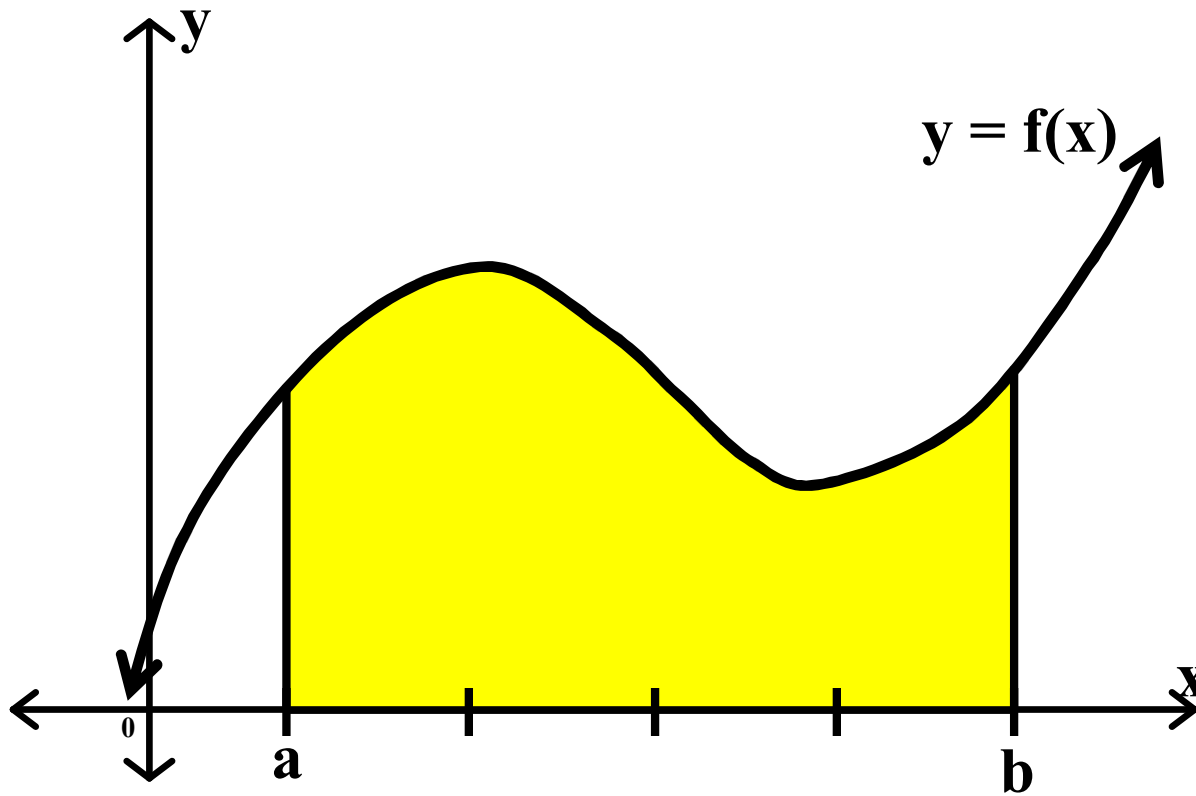


Divide the interval $[a, b]$ into n sub-intervals



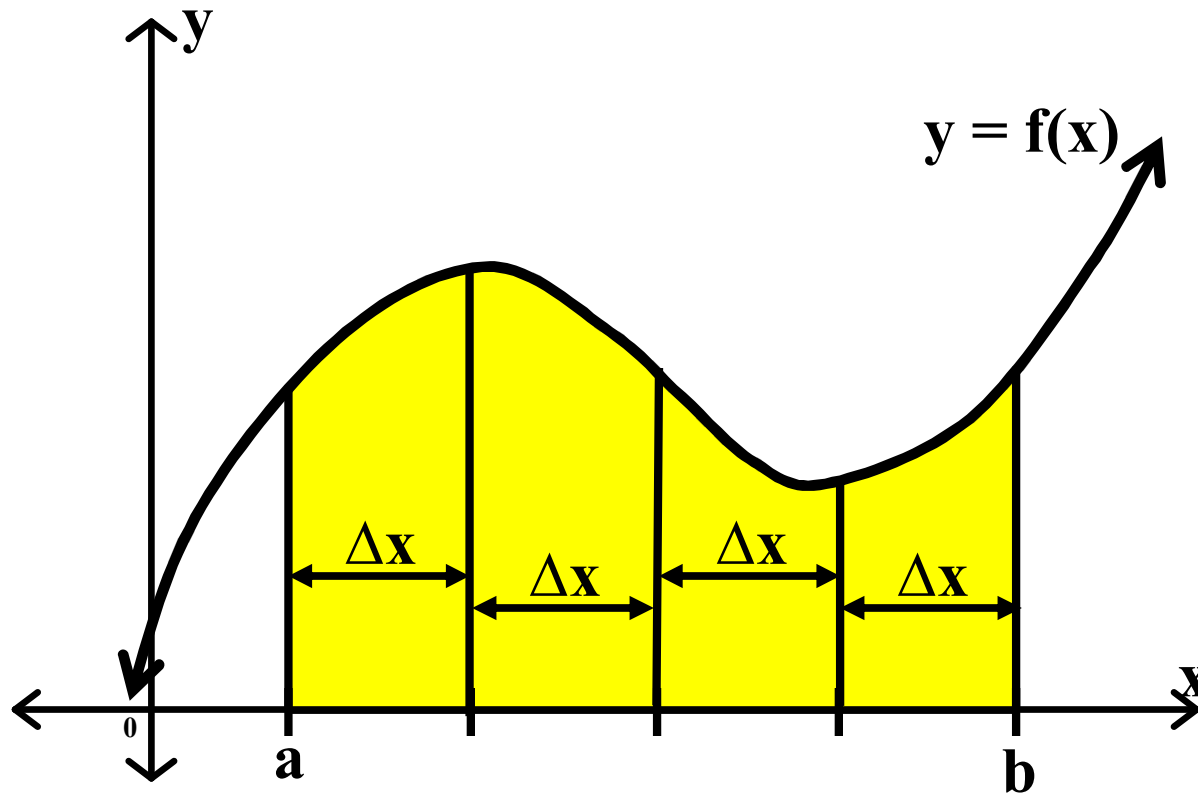
$$n = 4$$

Divide the interval $[a, b]$ into n sub-intervals



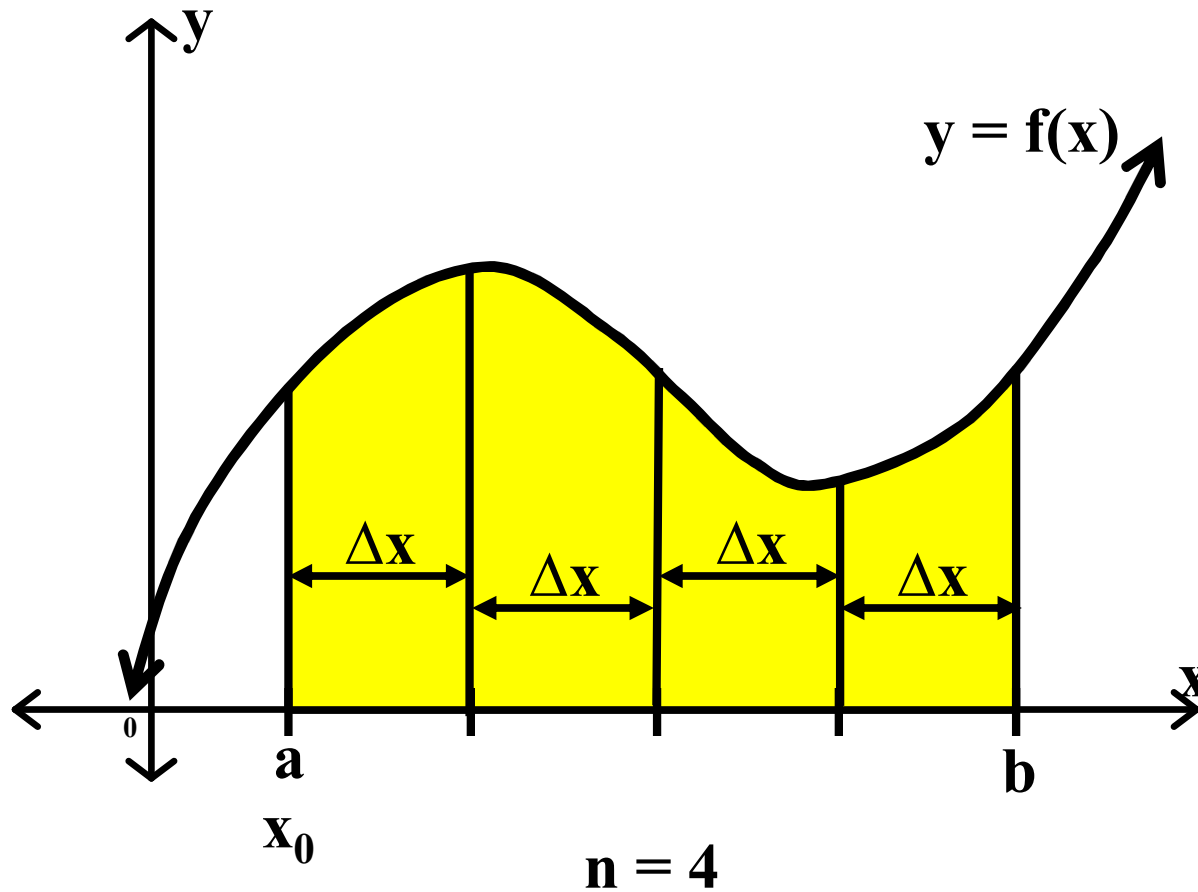
$$n = 4$$

Divide the interval $[a, b]$ into n sub-intervals each of width Δx

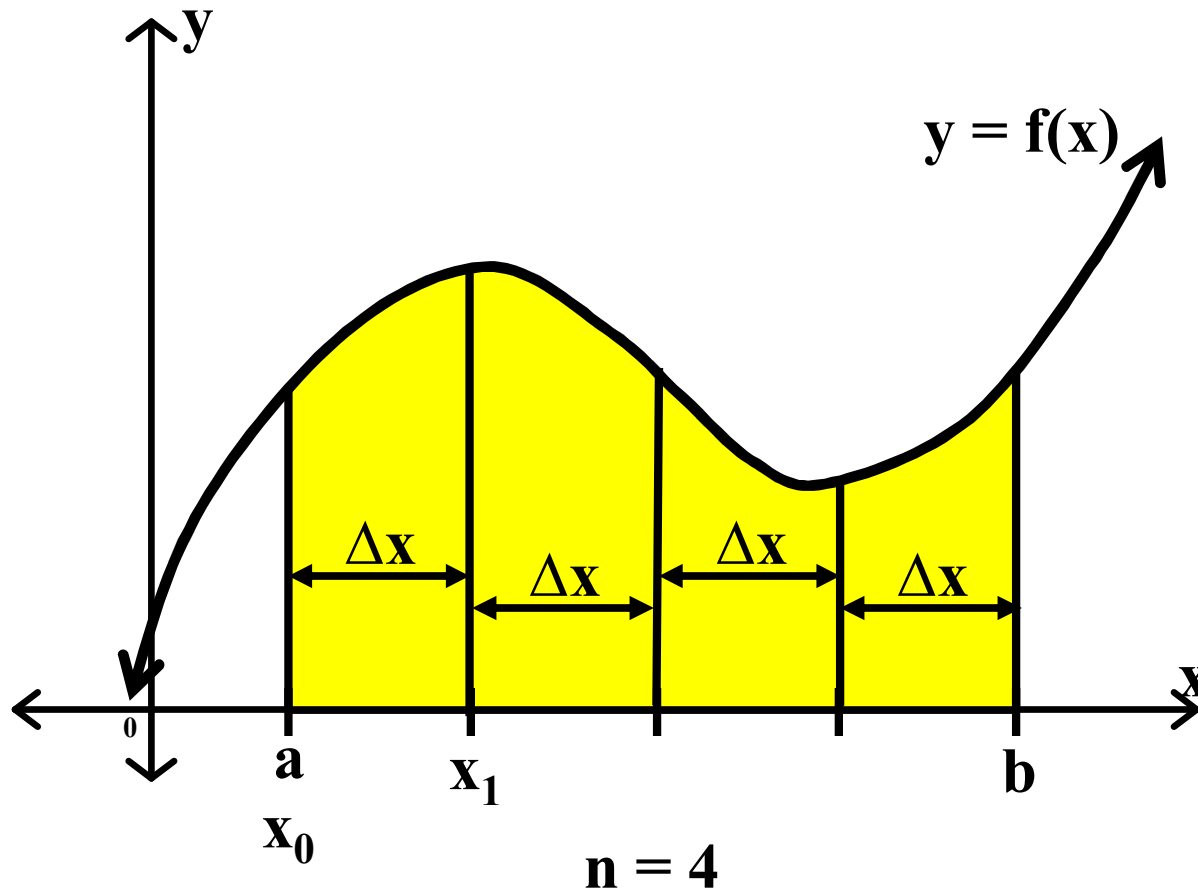


$$n = 4$$

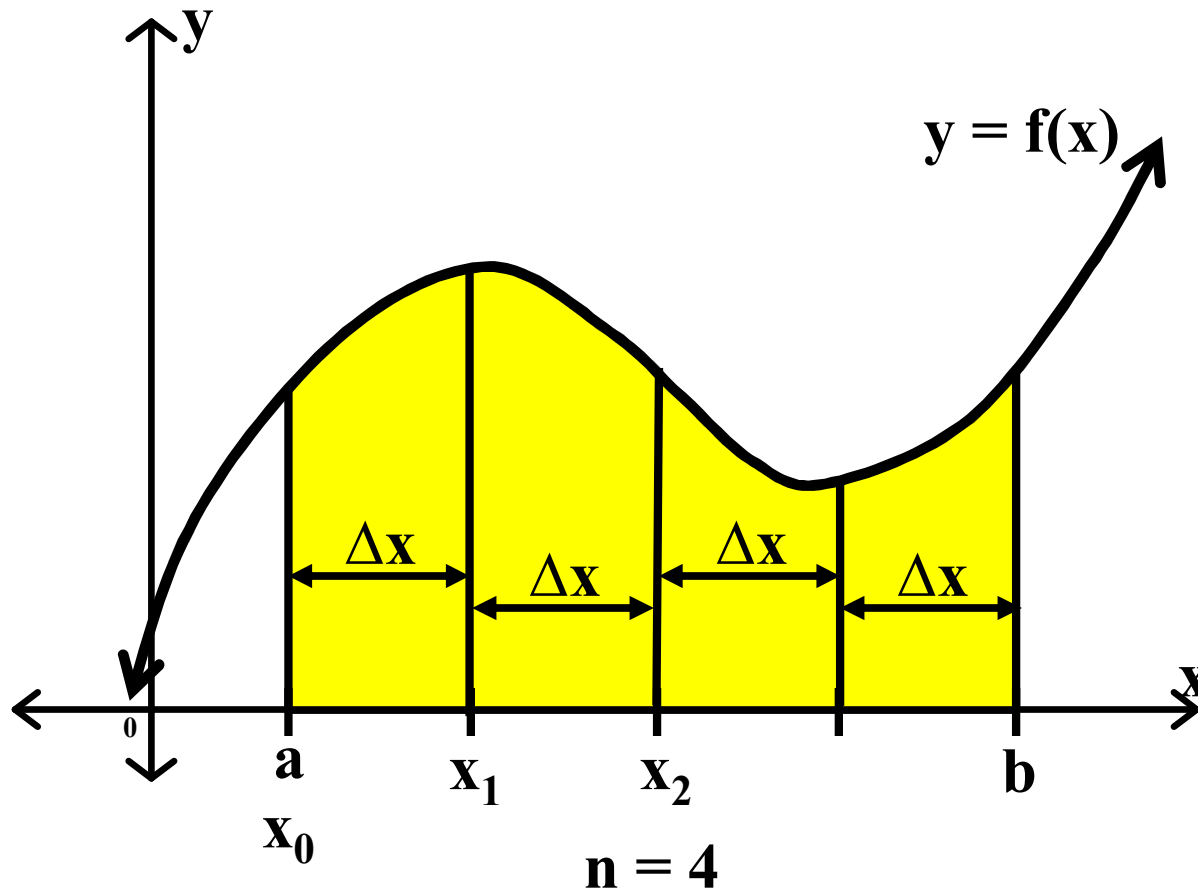
Divide the interval $[a, b]$ into n sub-intervals each of width Δx



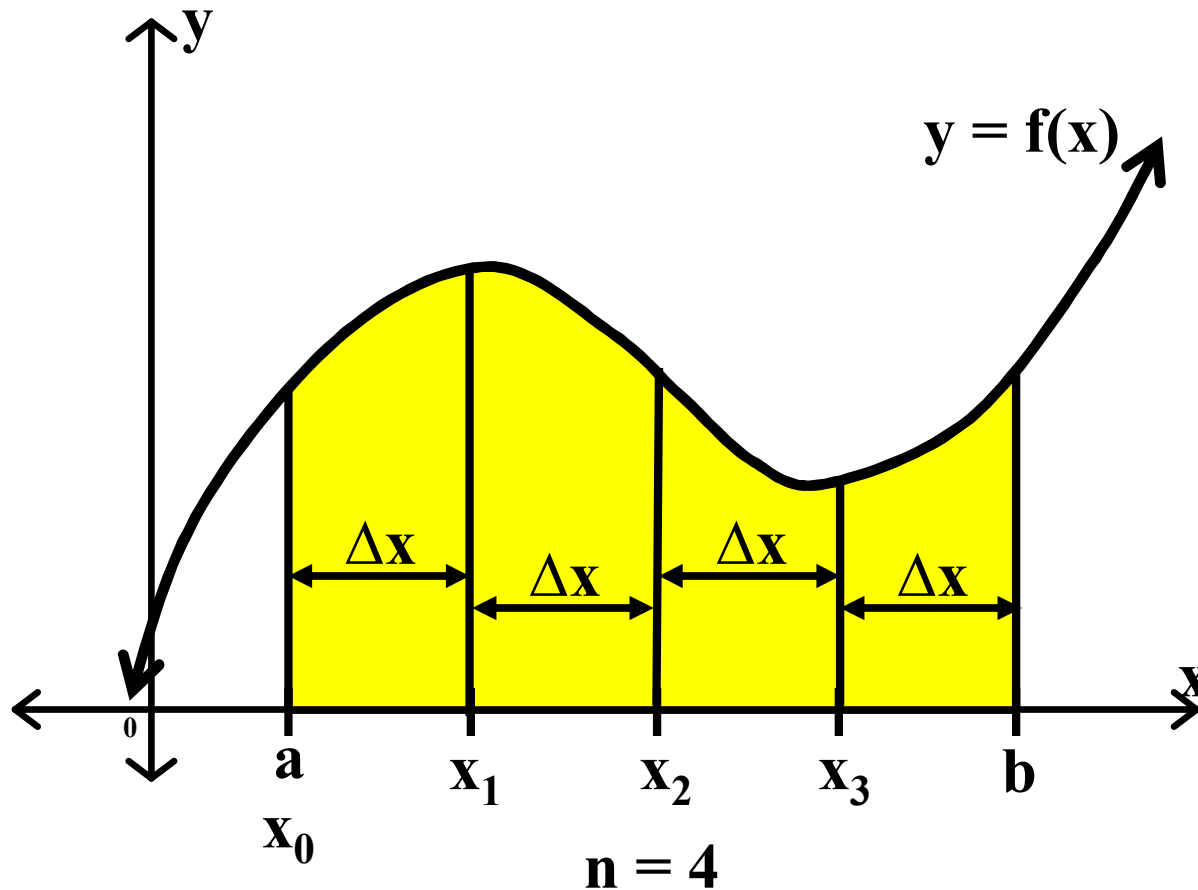
Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a$,



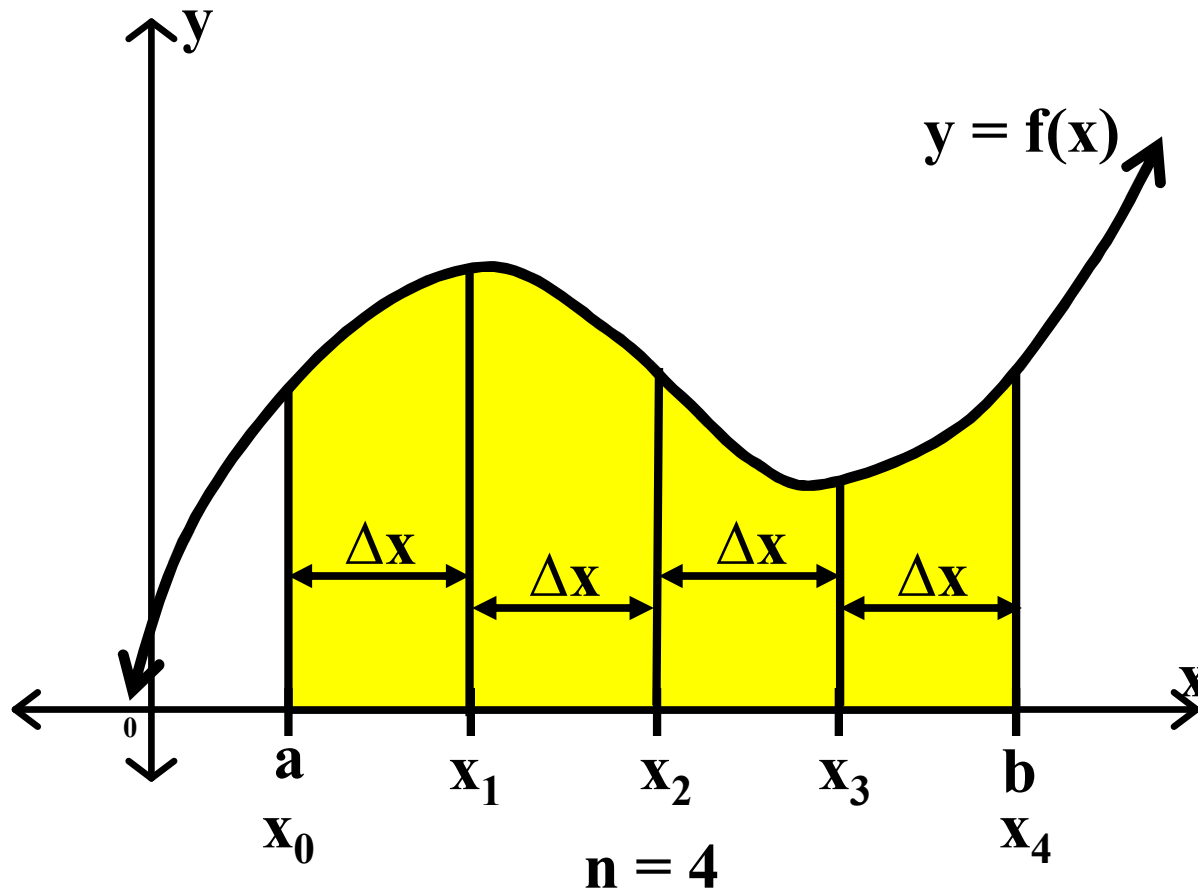
Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a$, x_1 ,



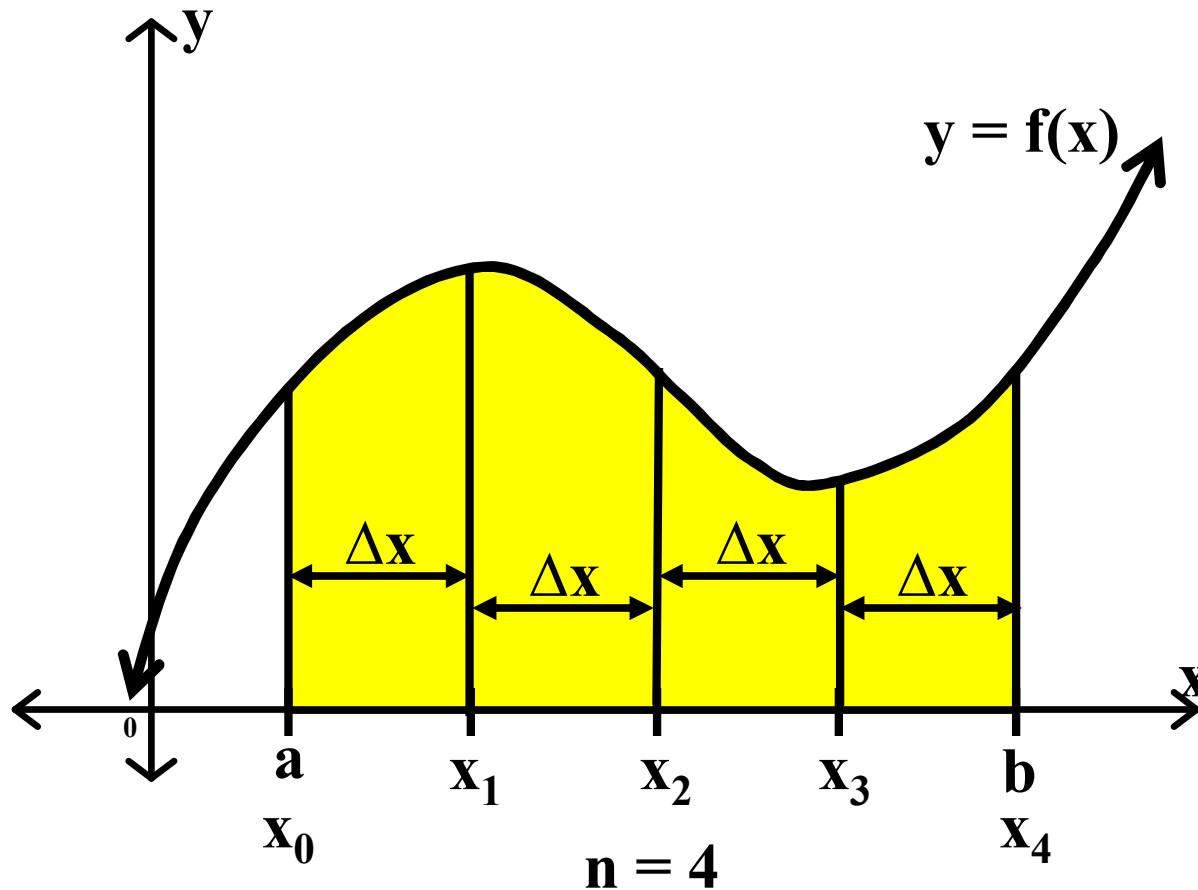
Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a, x_1, x_2,$



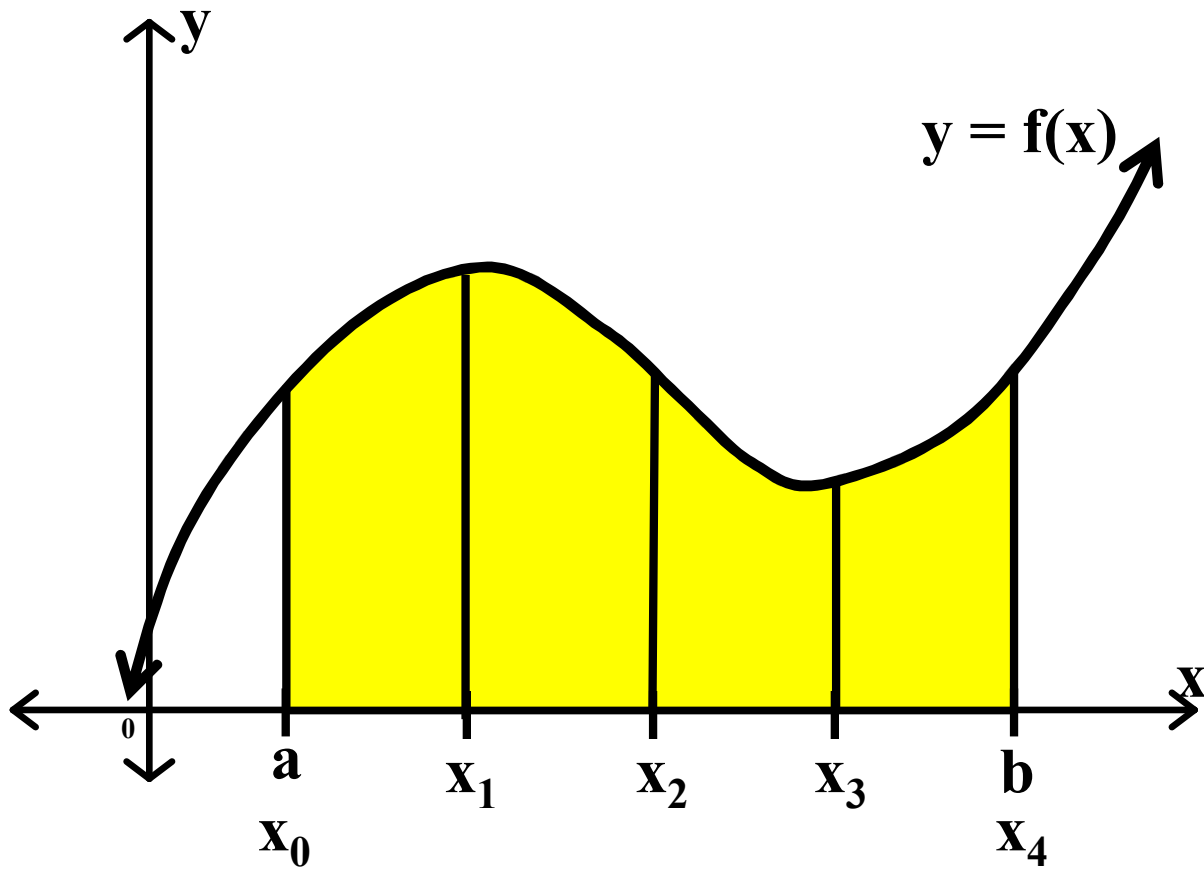
Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a, x_1, x_2, \dots,$

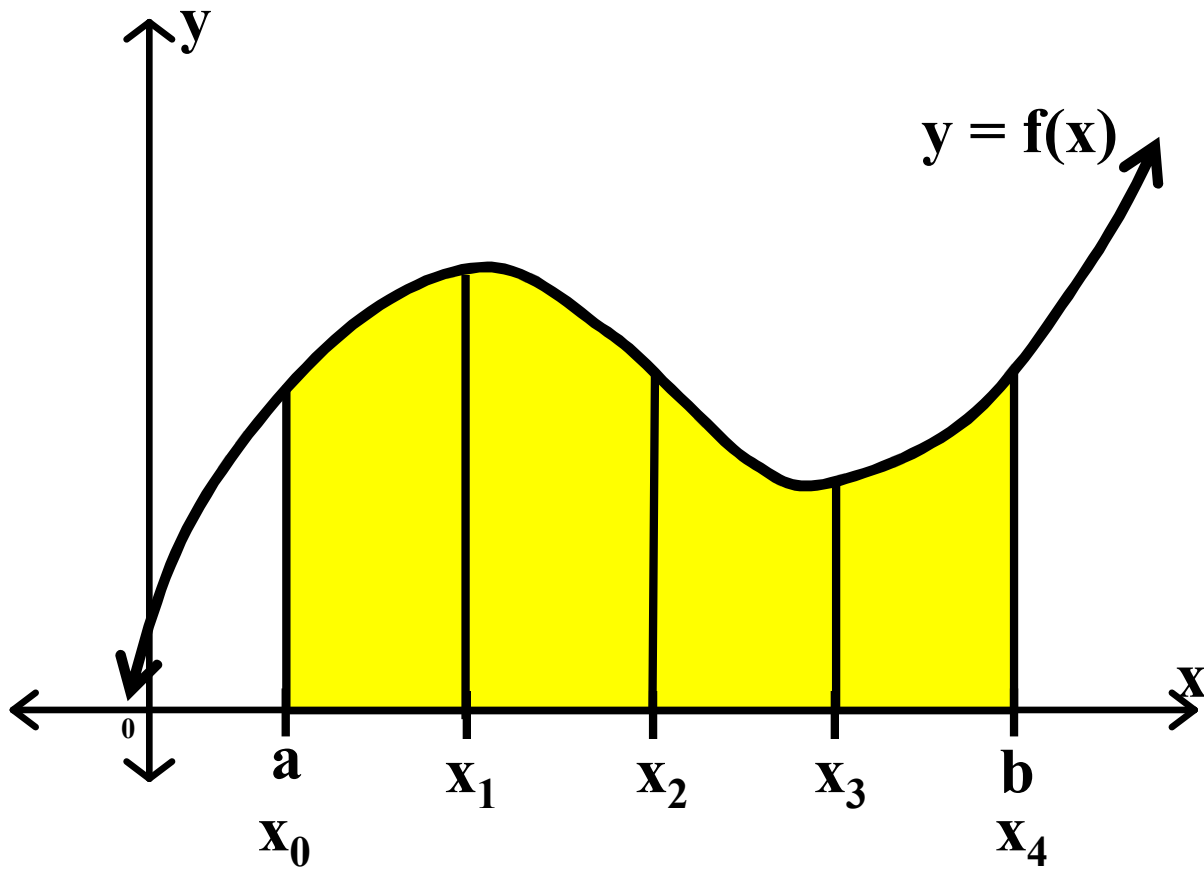


Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a, x_1, x_2, \dots, x_n = b$.

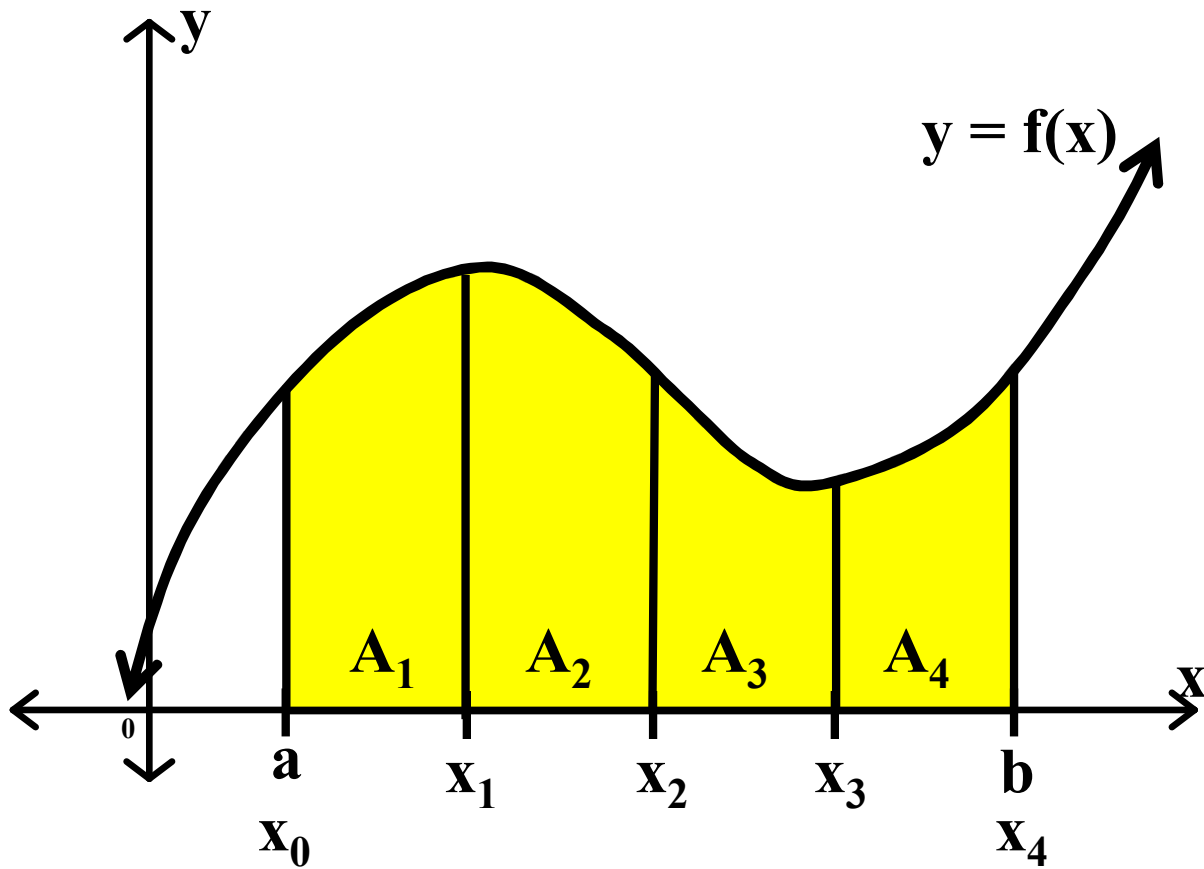


Divide the interval $[a, b]$ into n sub-intervals each of width Δx by the numbers $x_0 = a, x_1, x_2, \dots, x_n = b$. Clearly, $\Delta x = (b - a)/n$.

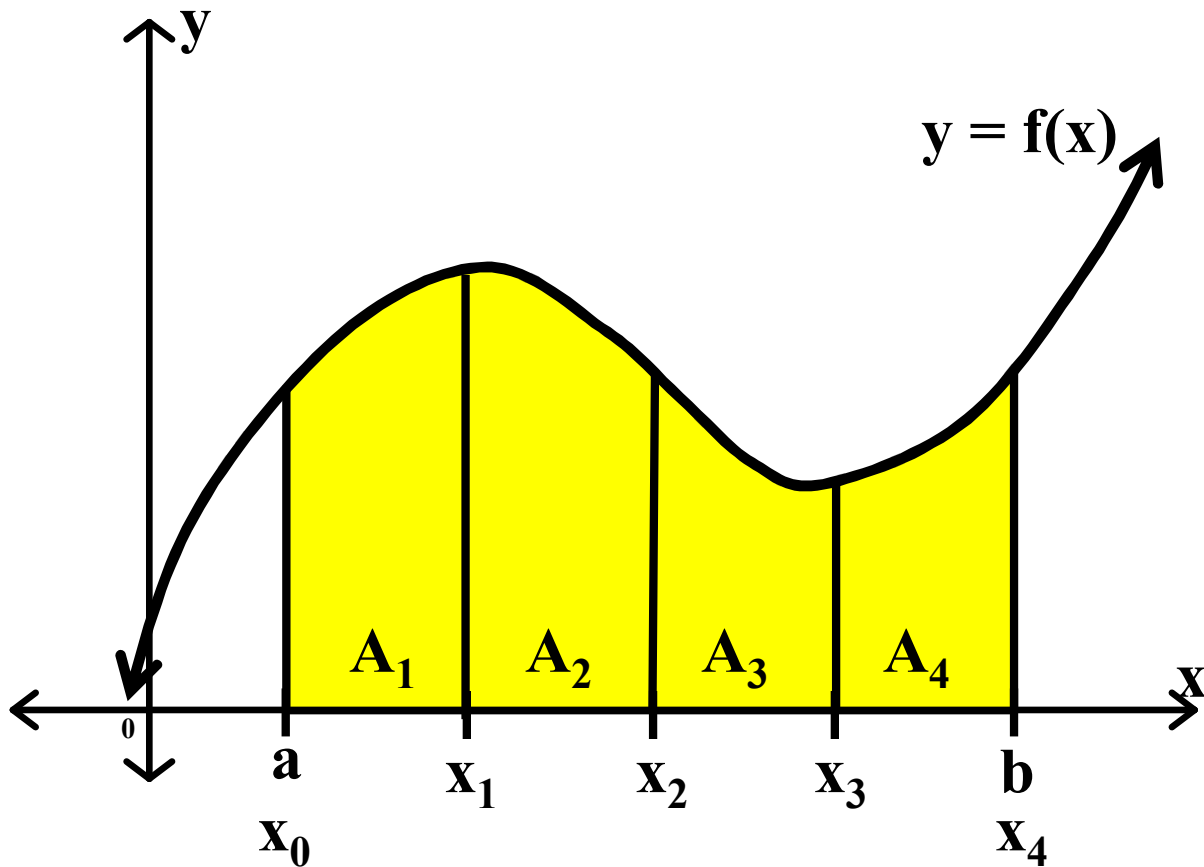




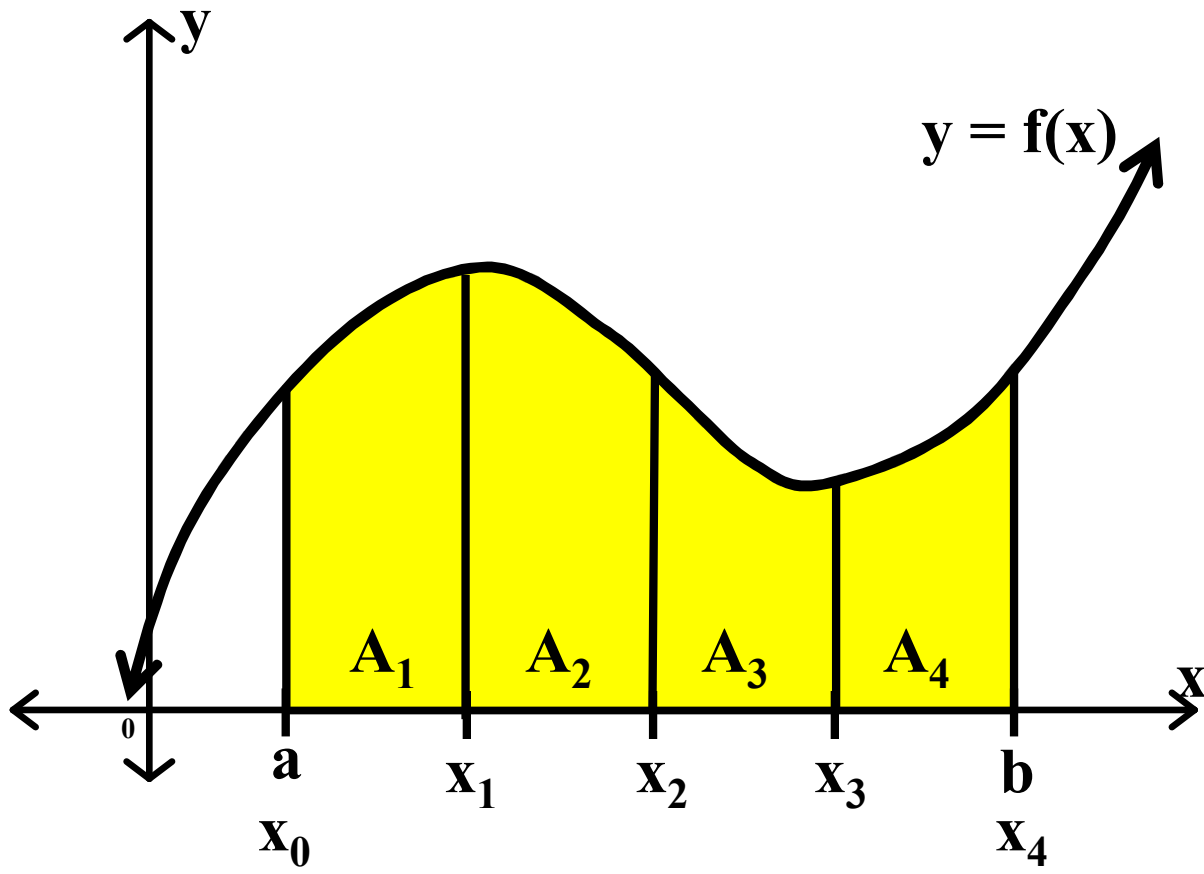
Notice that the region is divided into n 'strips',

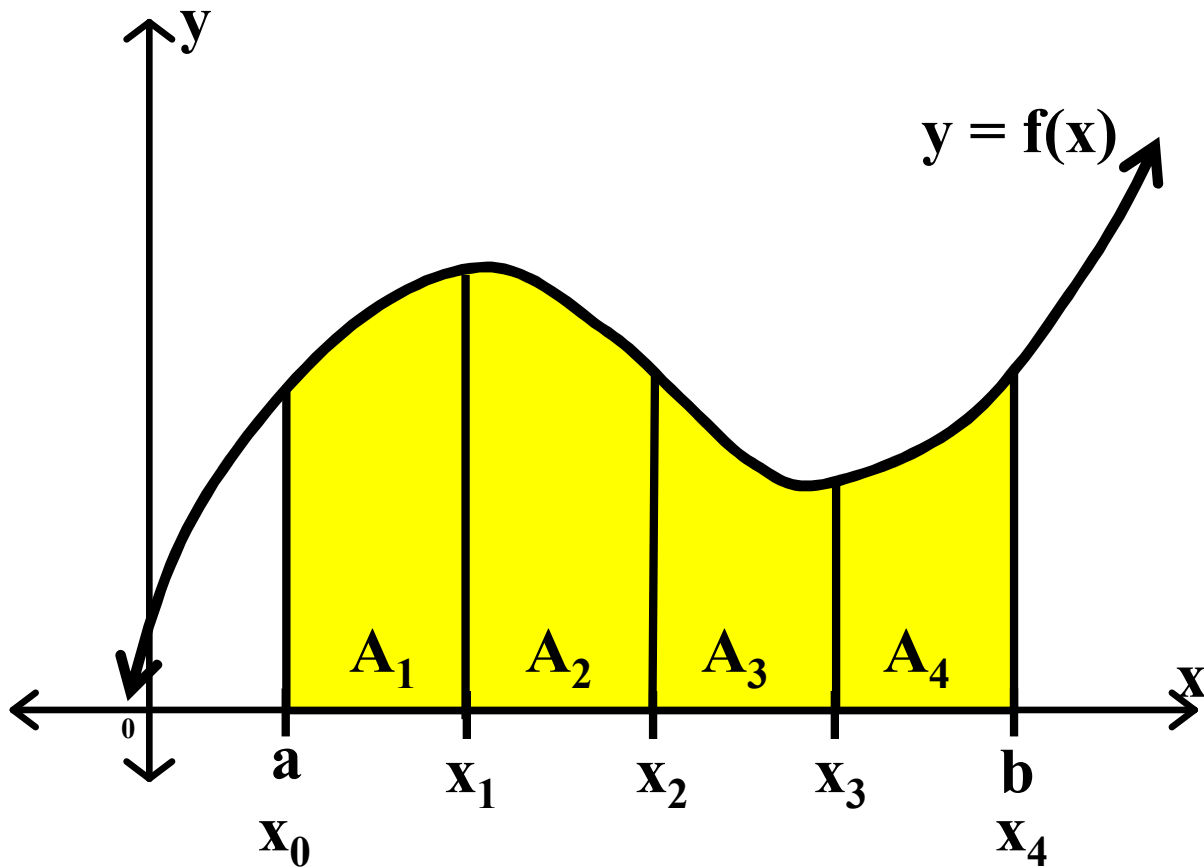


Notice that the region is divided into n ‘strips’, with areas $A_1, A_2, A_3, \dots, A_n$.

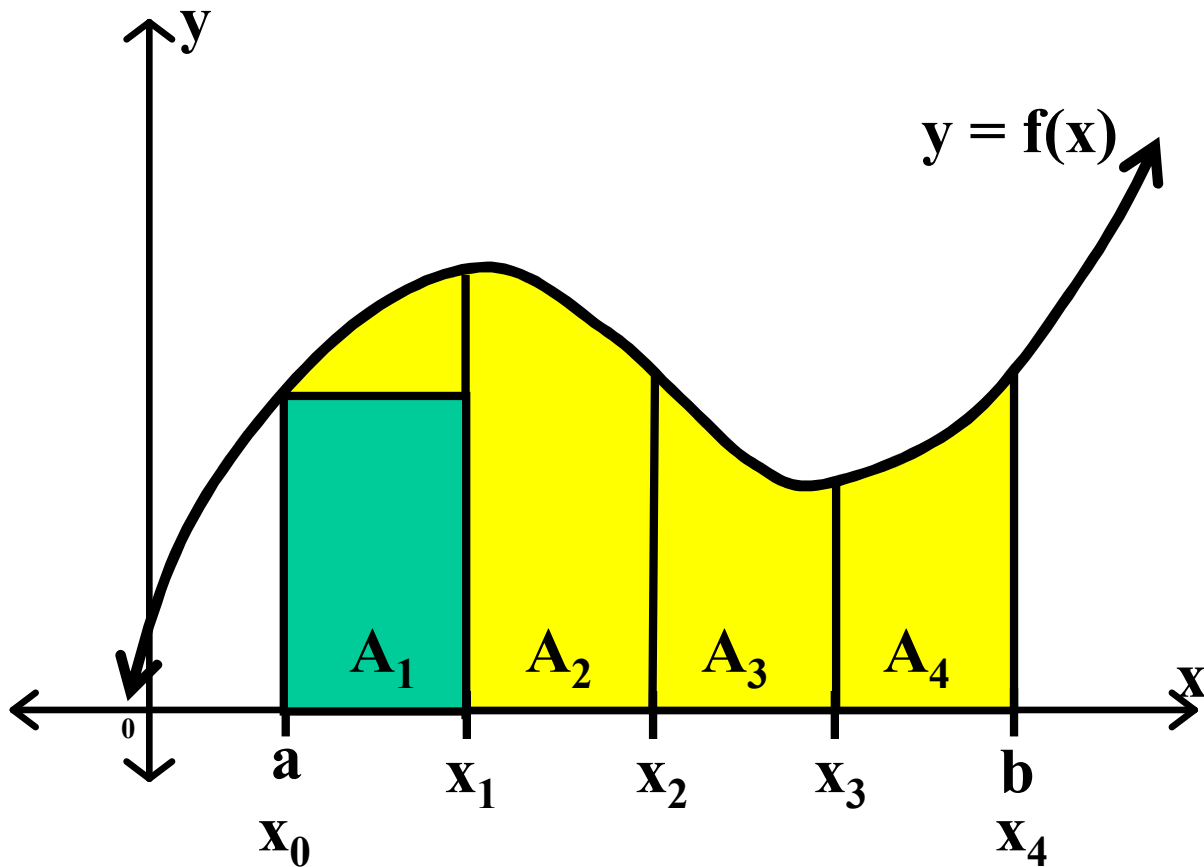


Notice that the region is divided into n ‘strips’, with areas $A_1, A_2, A_3, \dots, A_n$. Rectangles can be used to approximate the area of these strips.

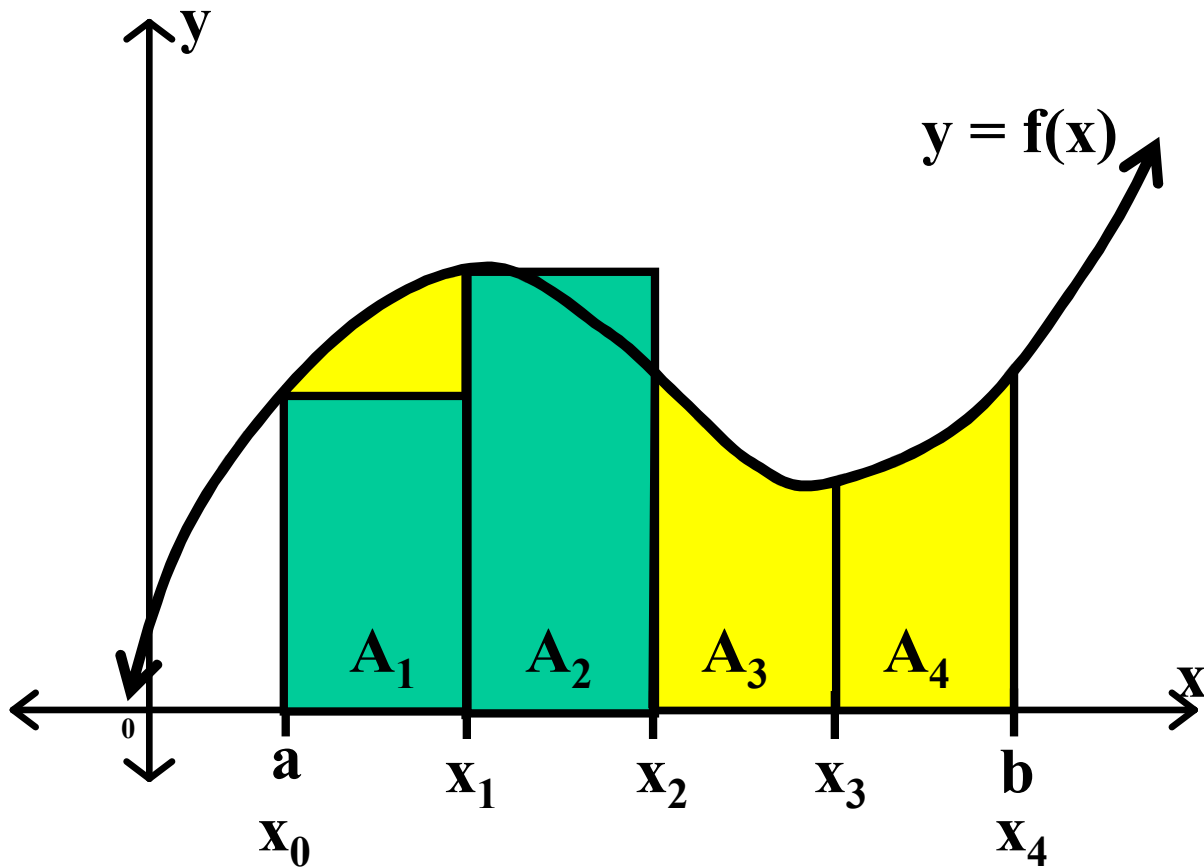




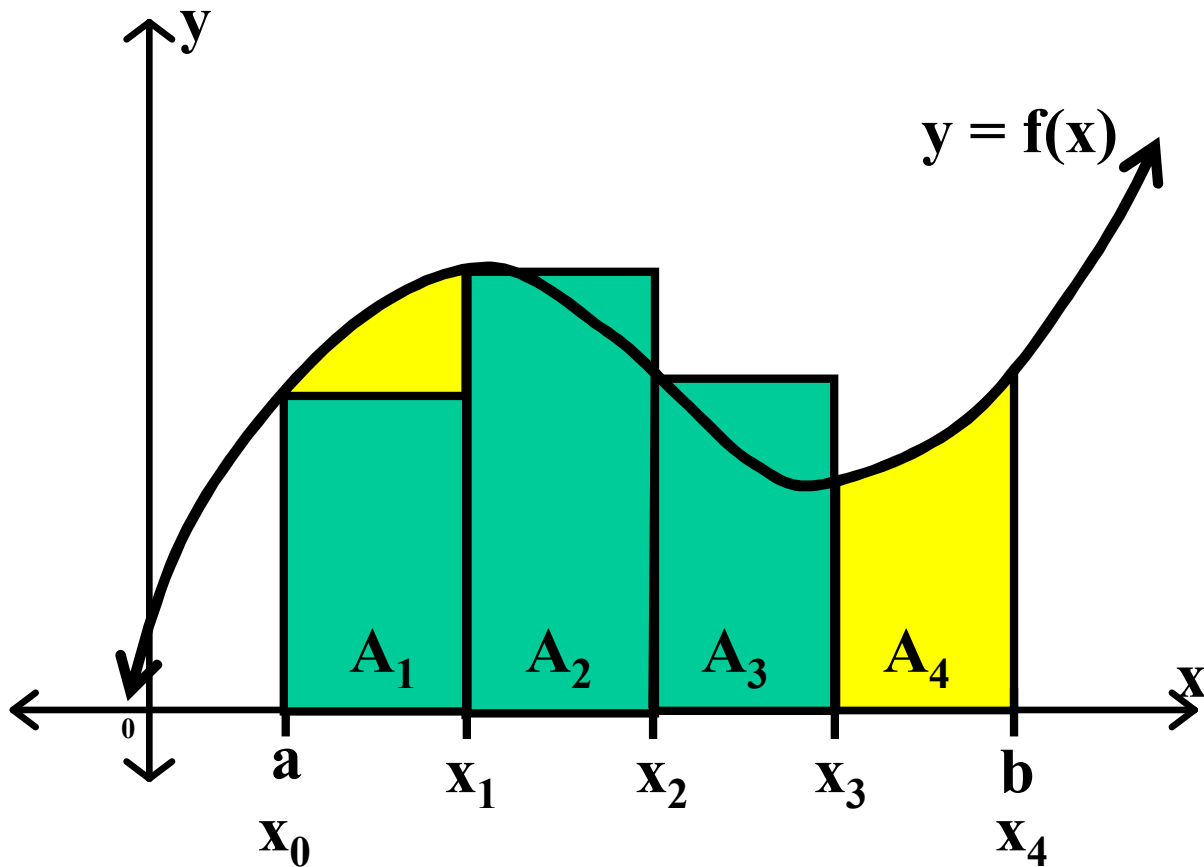
The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle.



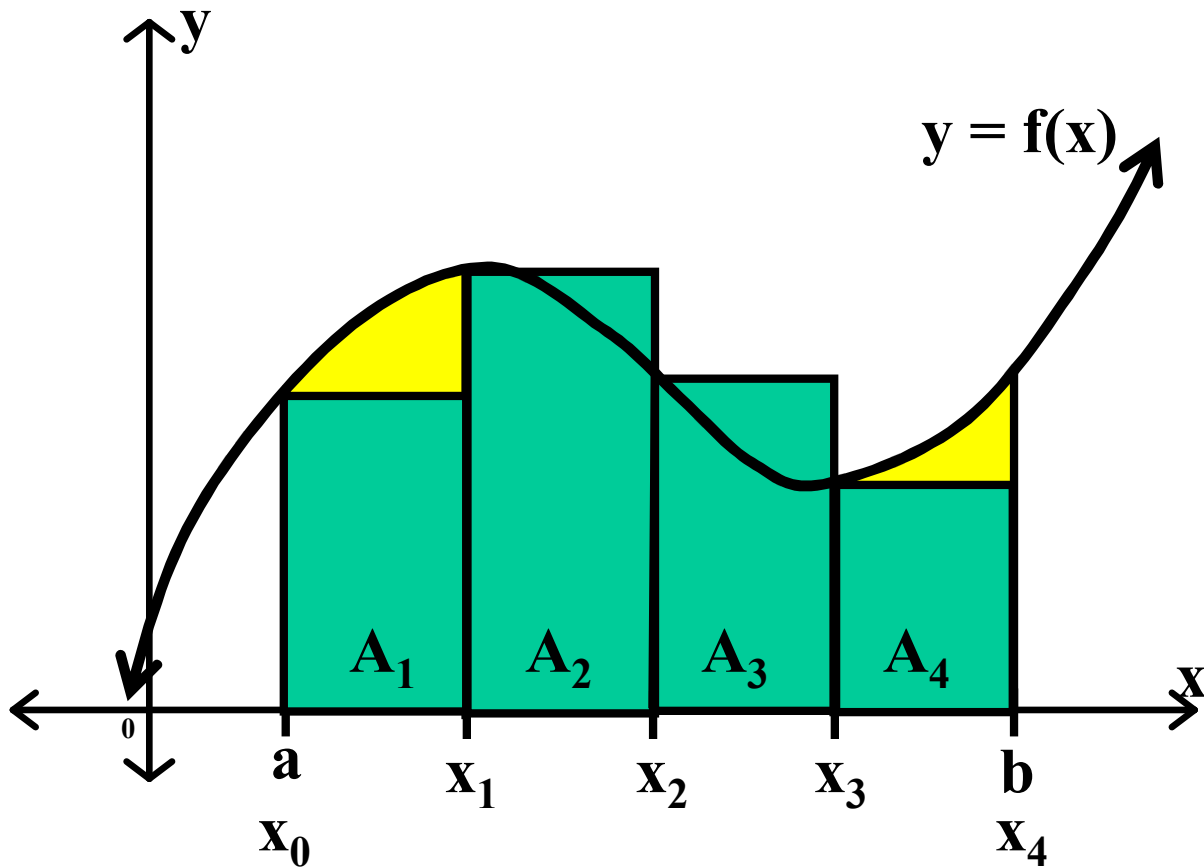
The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle.



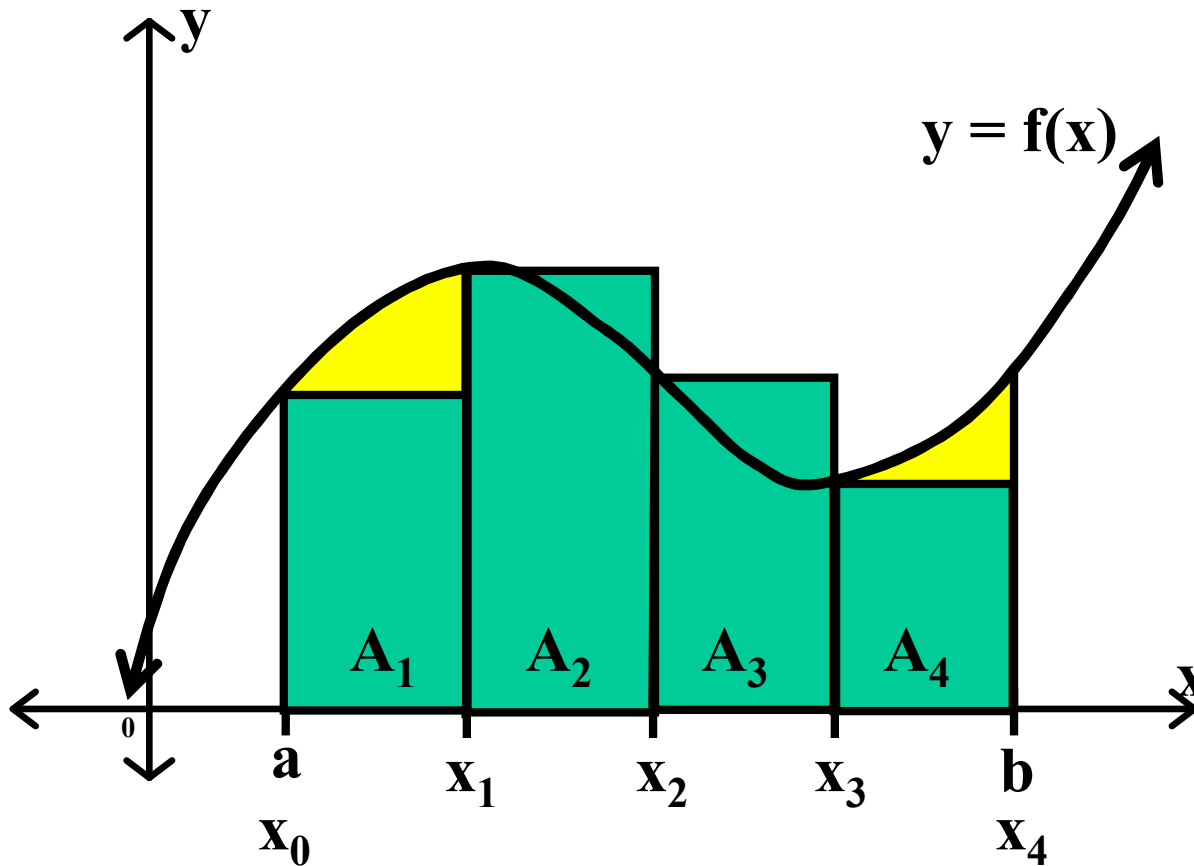
The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle.



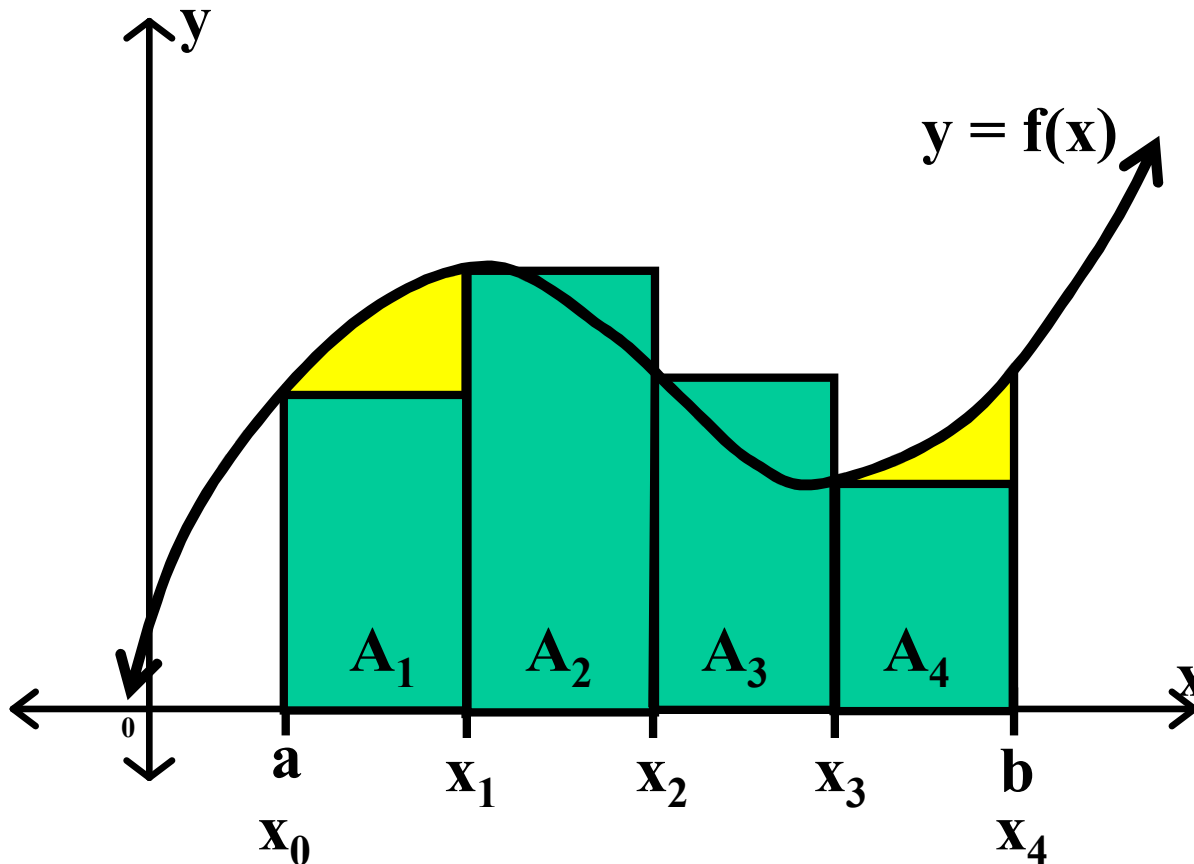
The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle.



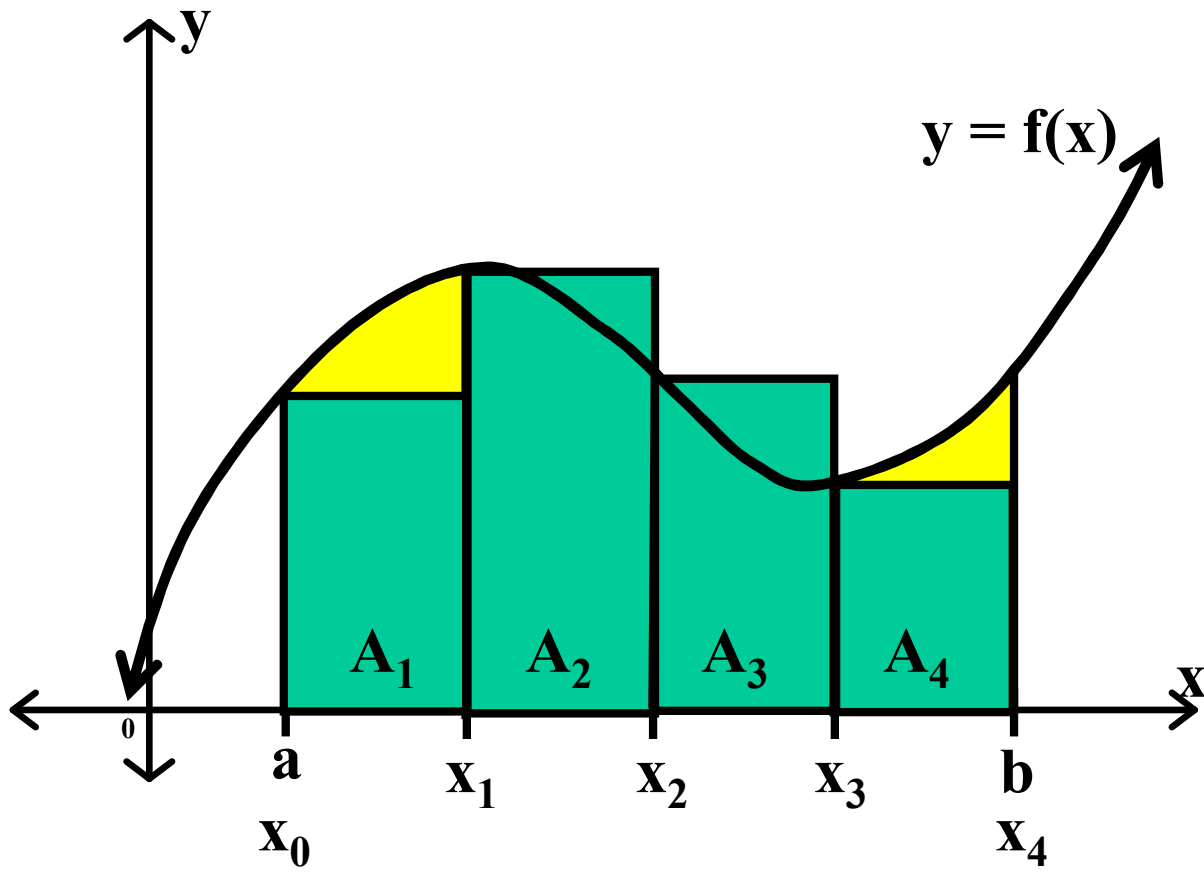
The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle.

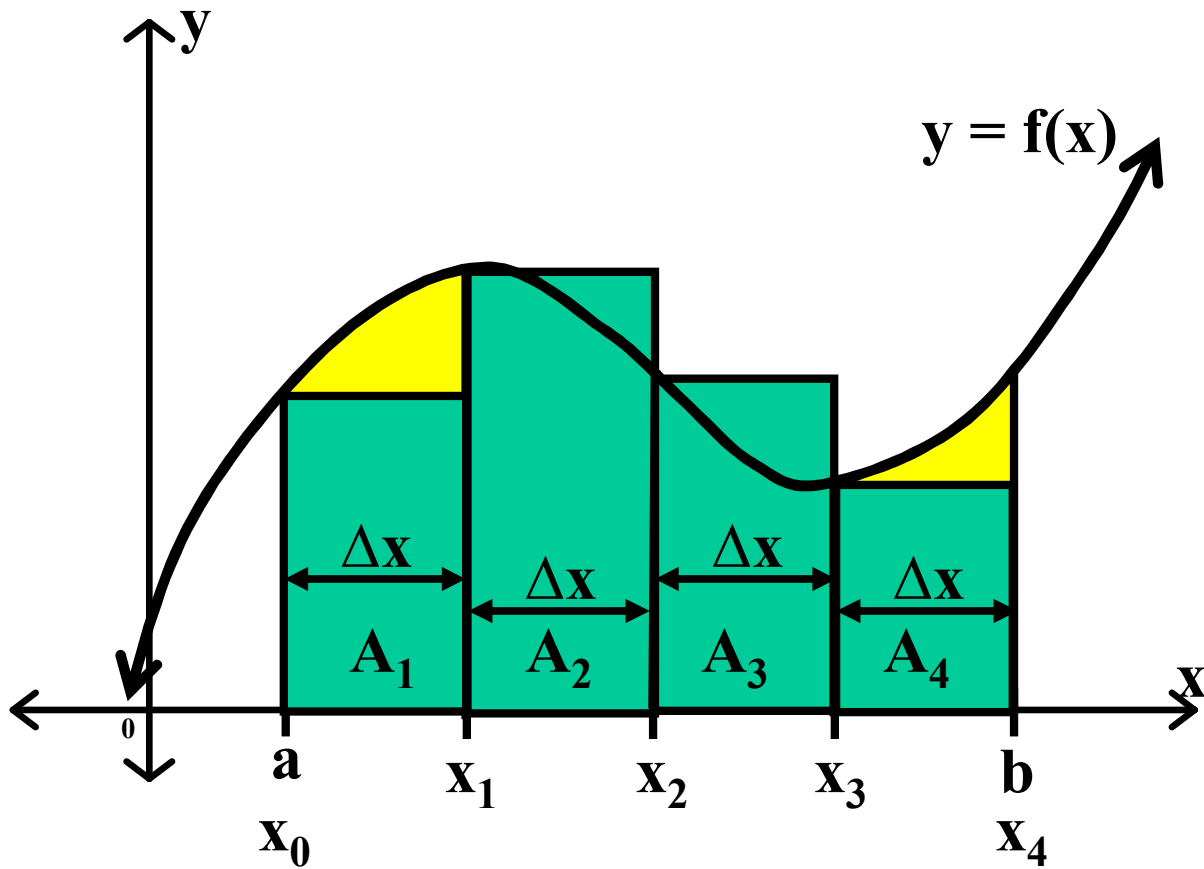


The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle. This is called the ‘left rectangular’ approximation.

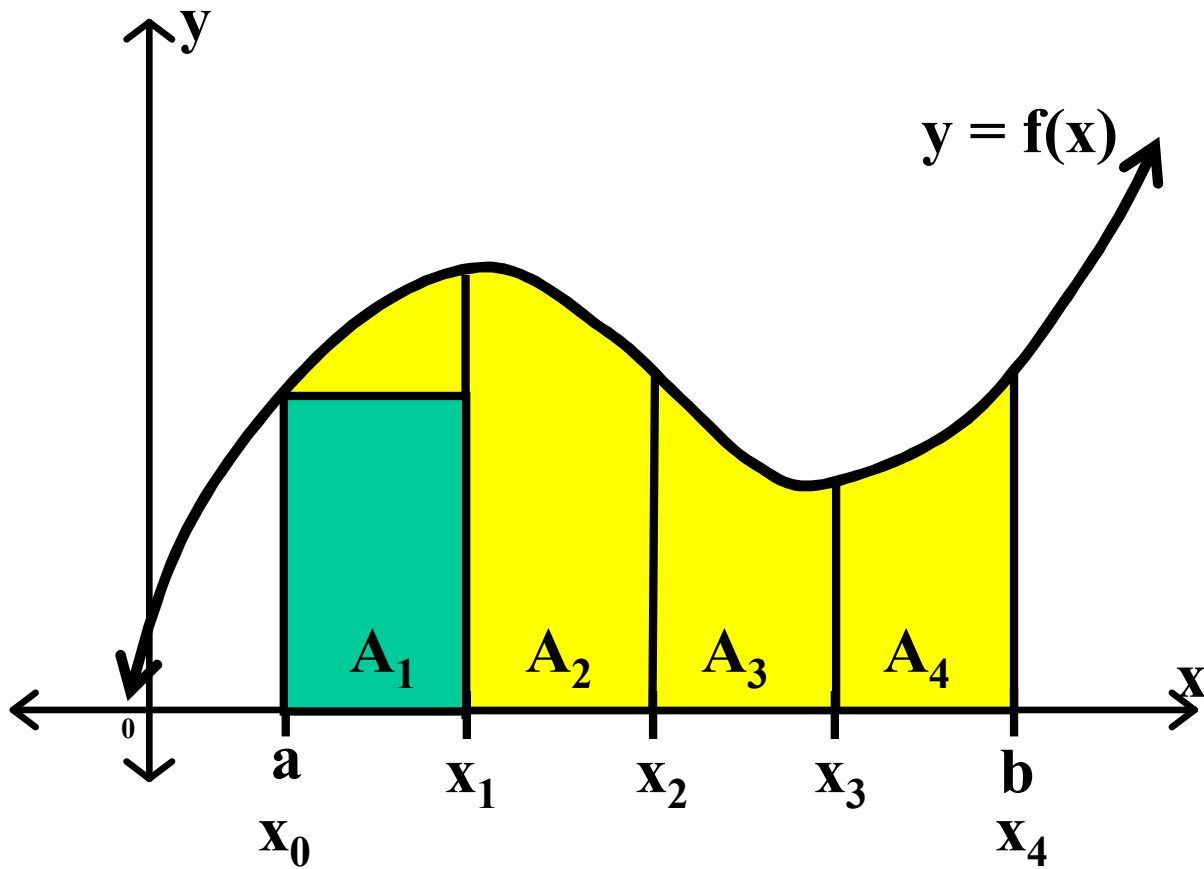


The first of the ‘rectangular’ approximations uses the length of the left hand side of each strip as the length of the rectangle. This is called the ‘left rectangular’ approximation, S_L .

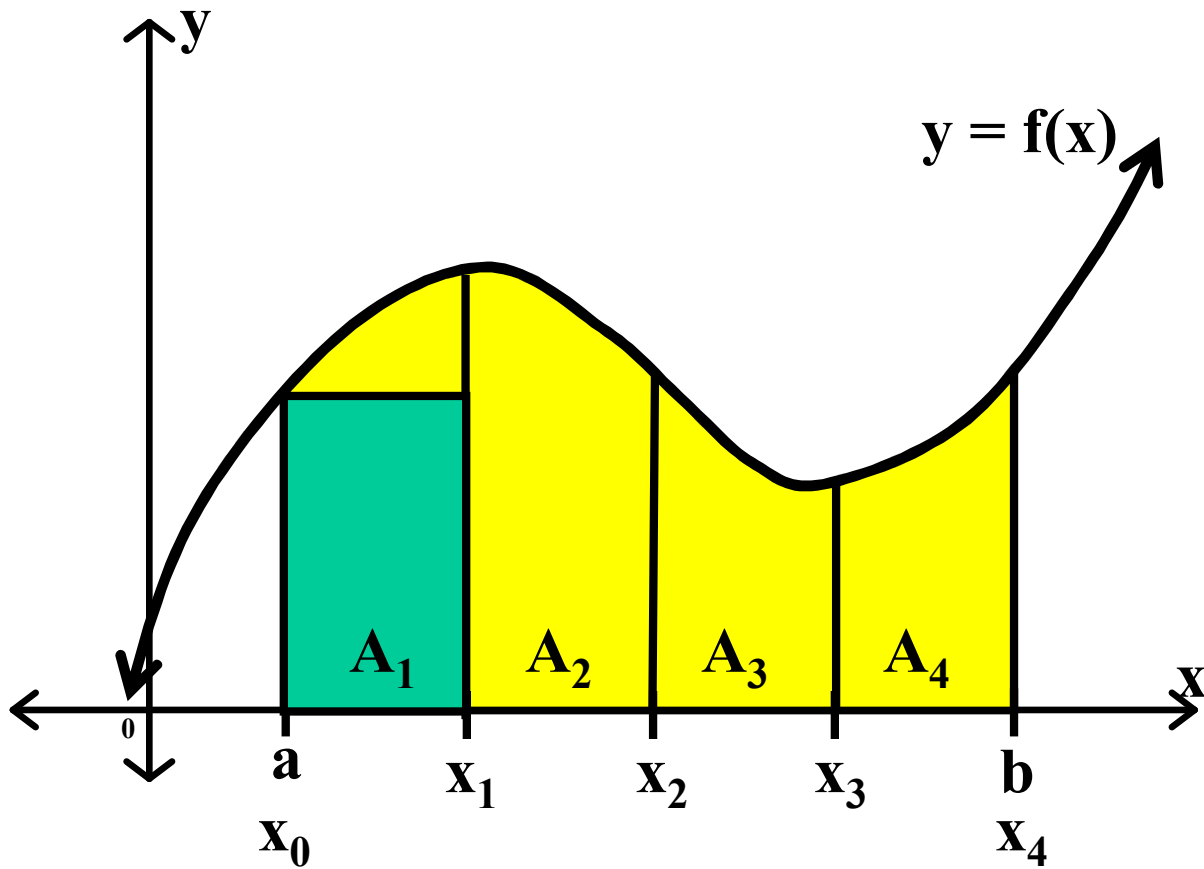




The width of each rectangle is Δx .

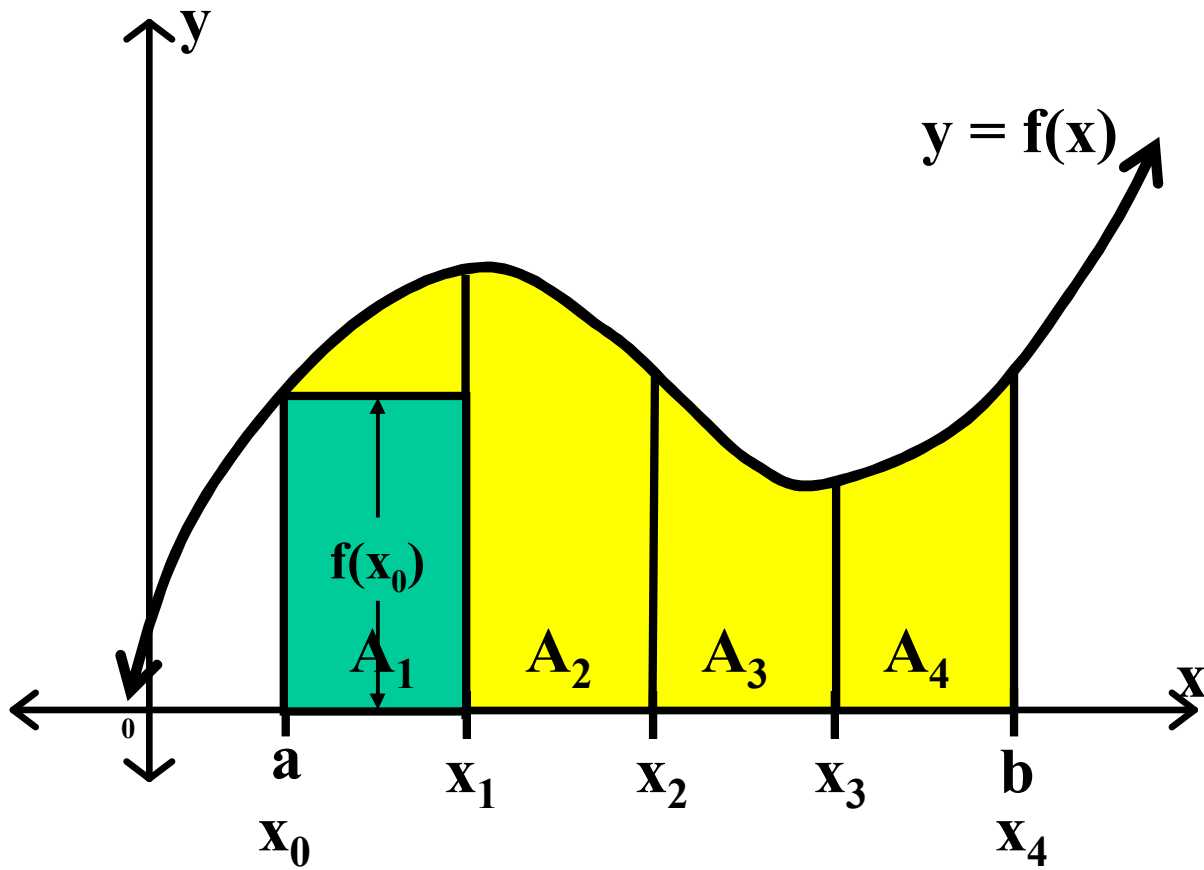


The width of each rectangle is Δx .



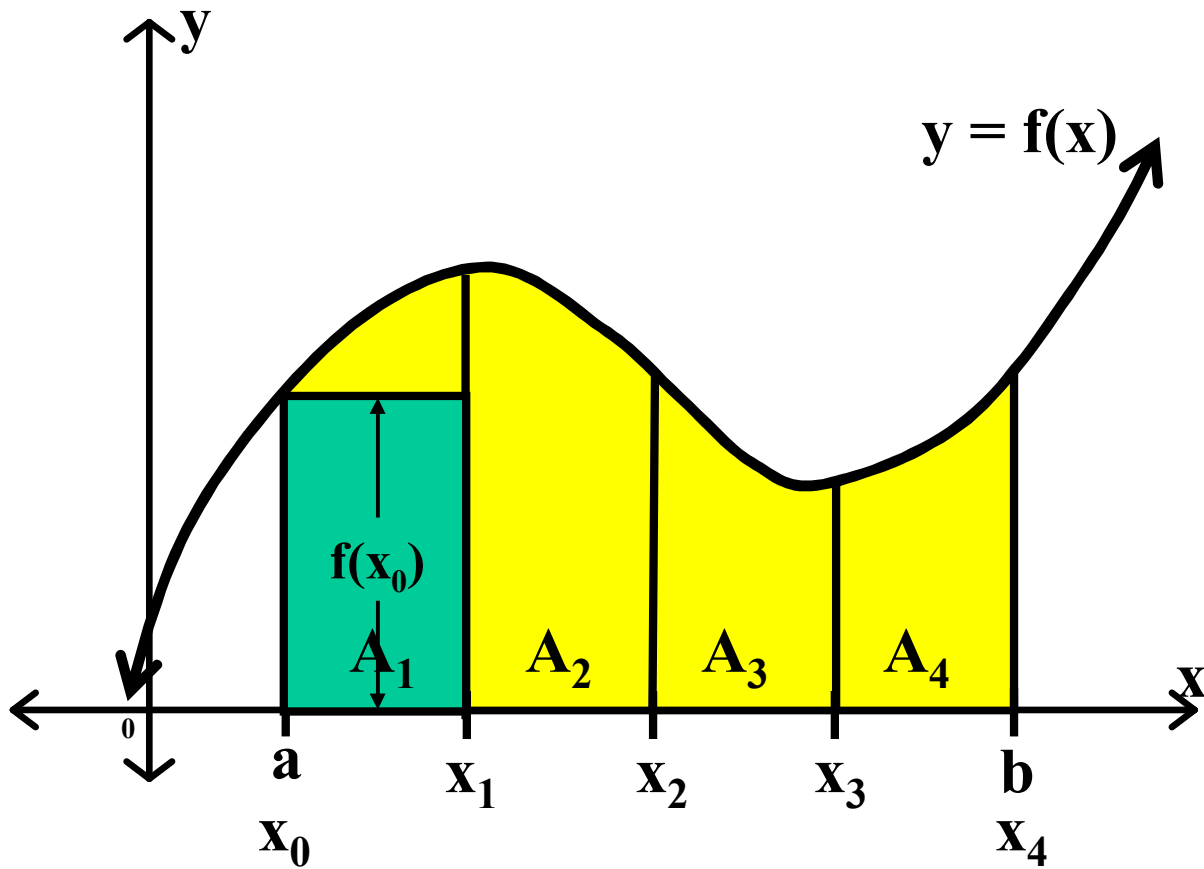
The width of each rectangle is Δx .

$$A_1 \approx$$



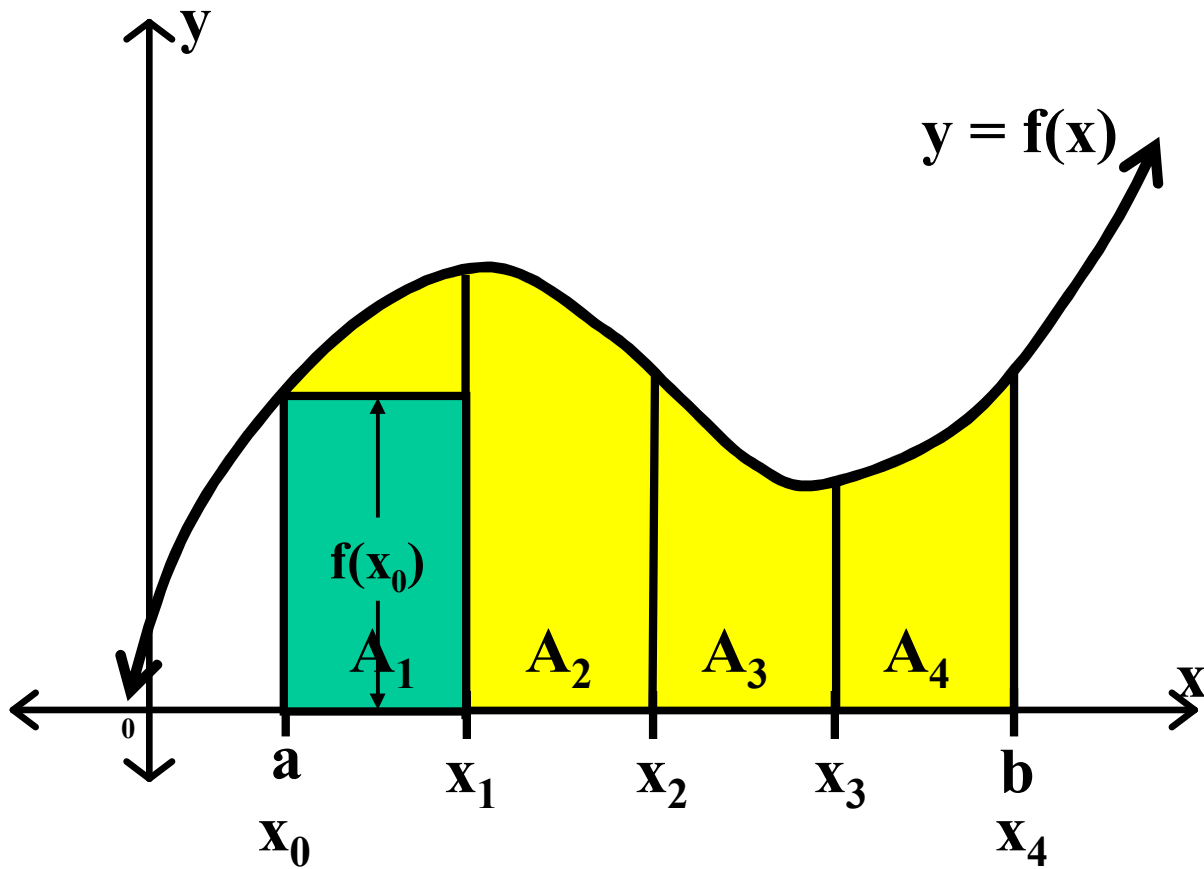
The width of each rectangle is Δx .

$$A_1 \approx$$



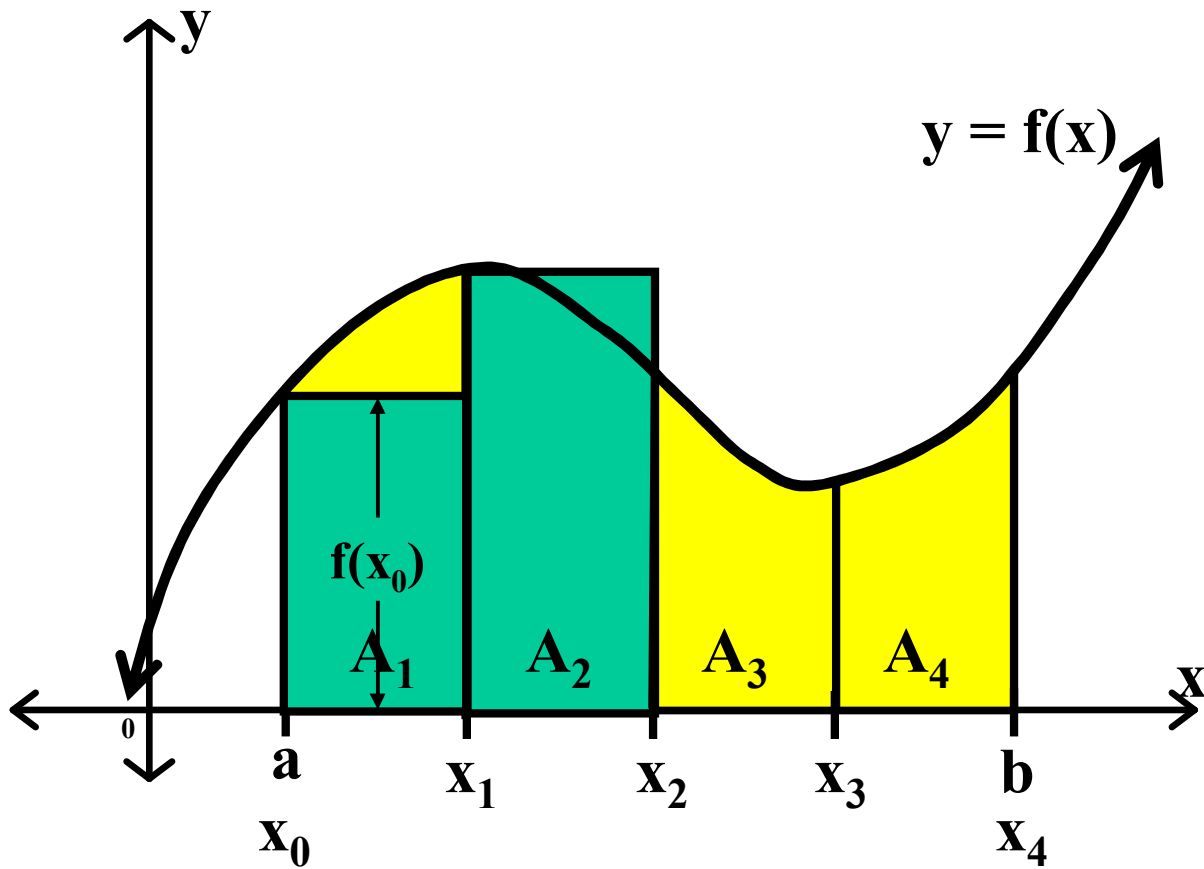
The width of each rectangle is Δx .

$$A_1 \approx f(x_0) \Delta x$$



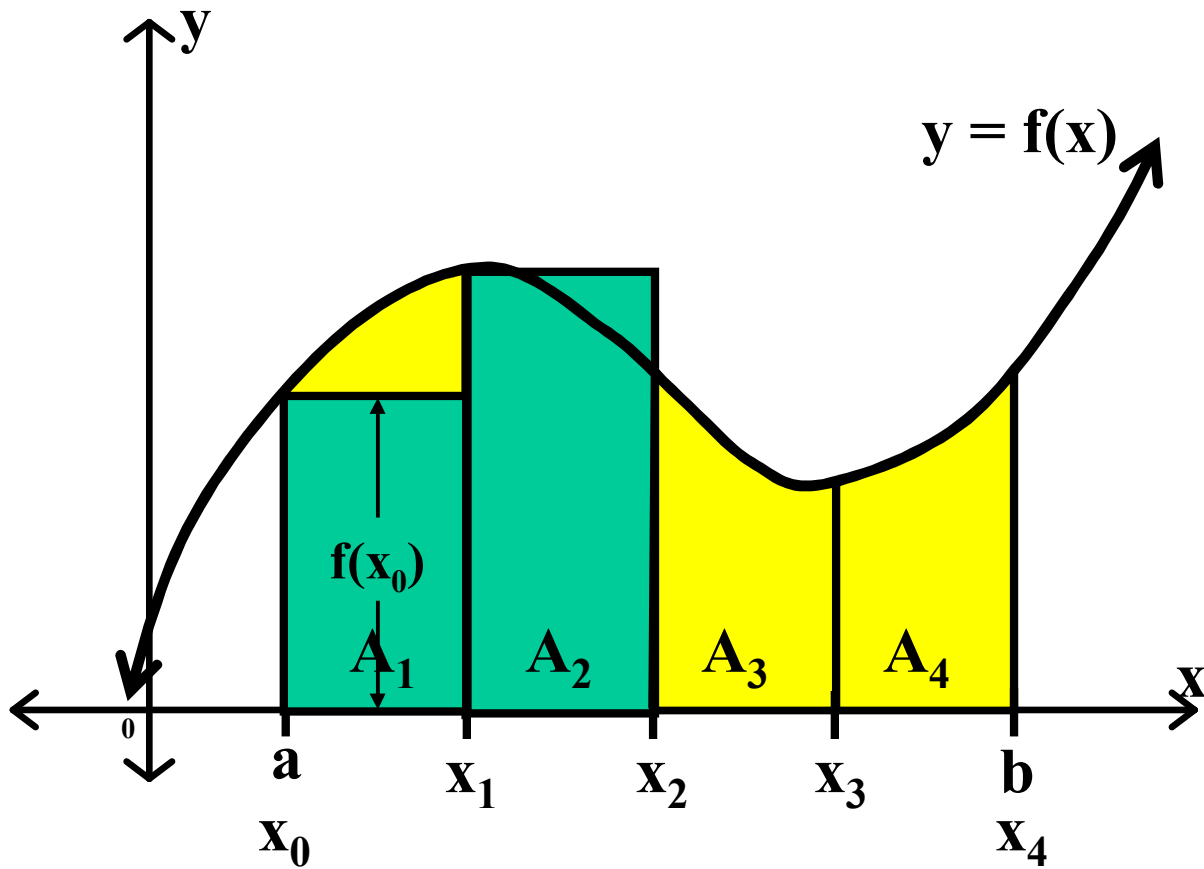
The width of each rectangle is Δx .

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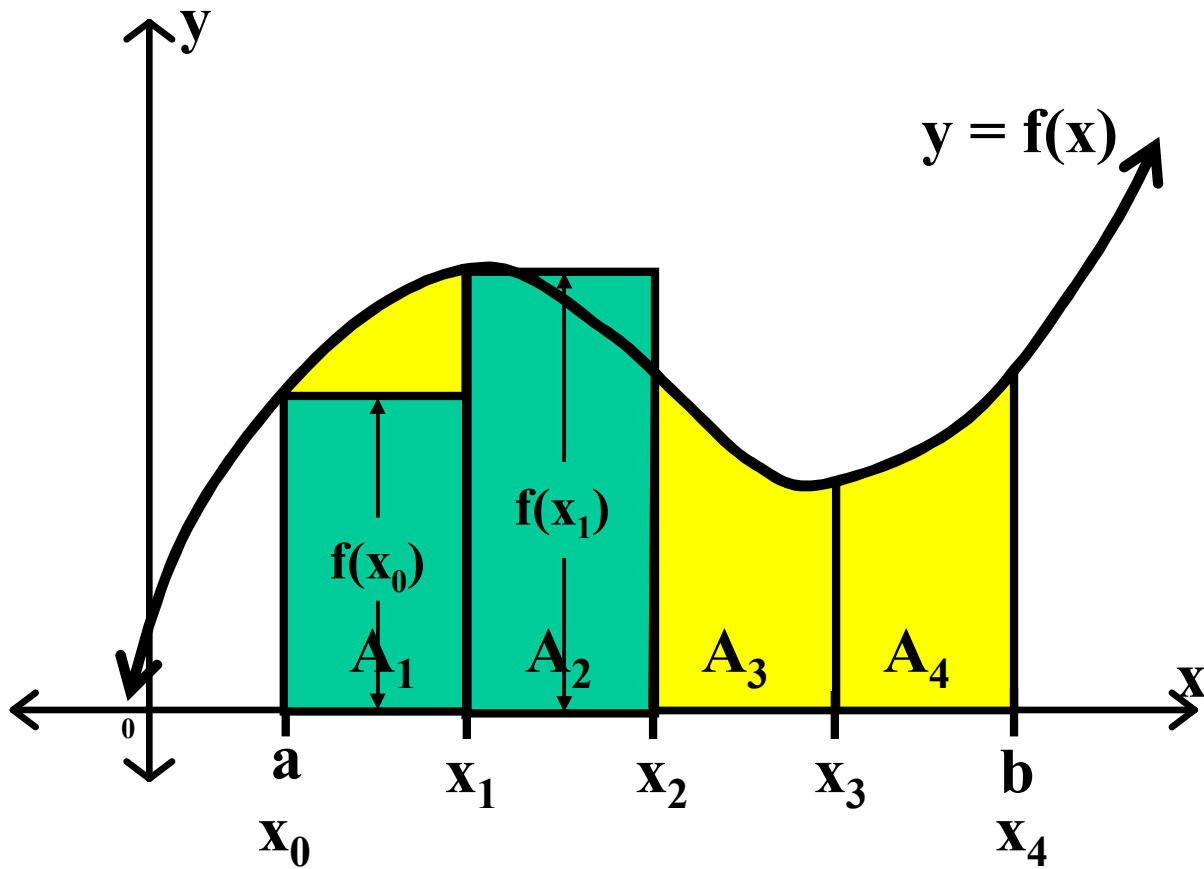
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x$$



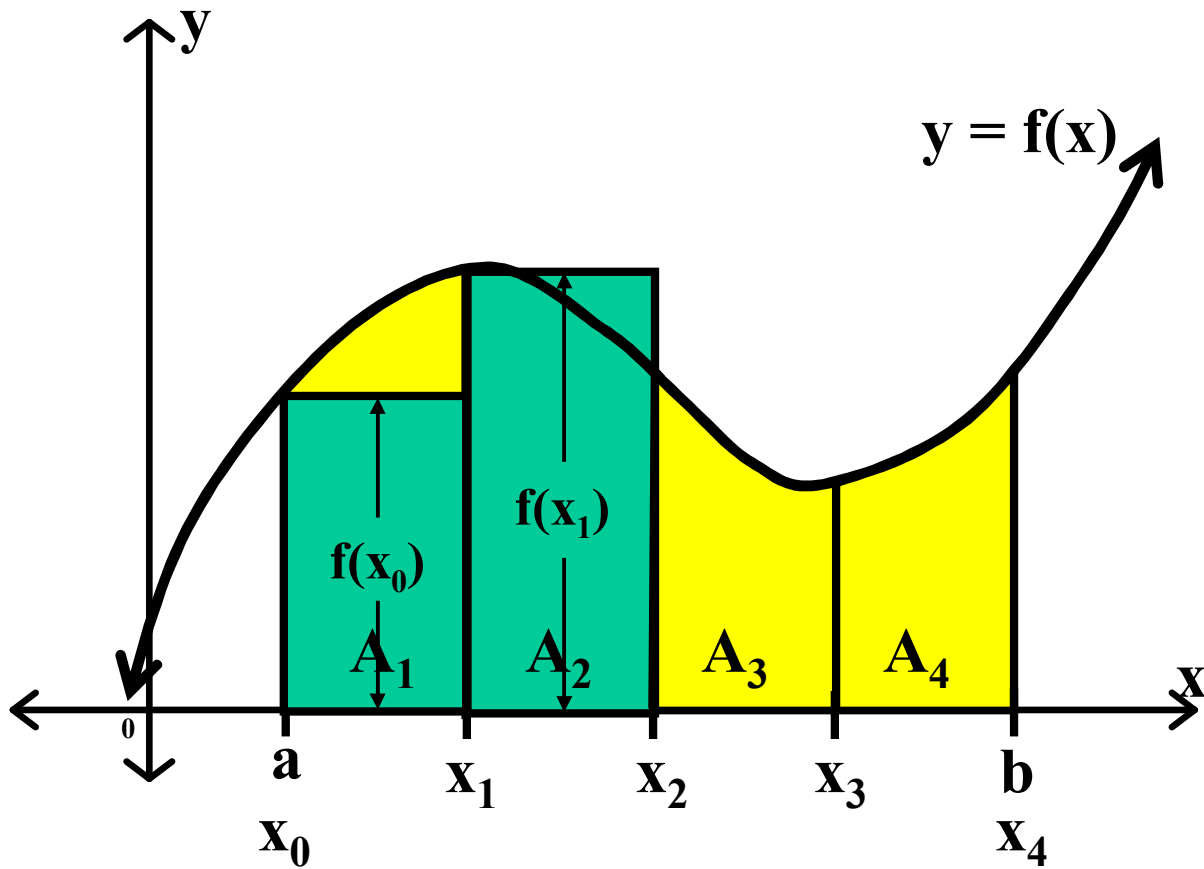
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx$$



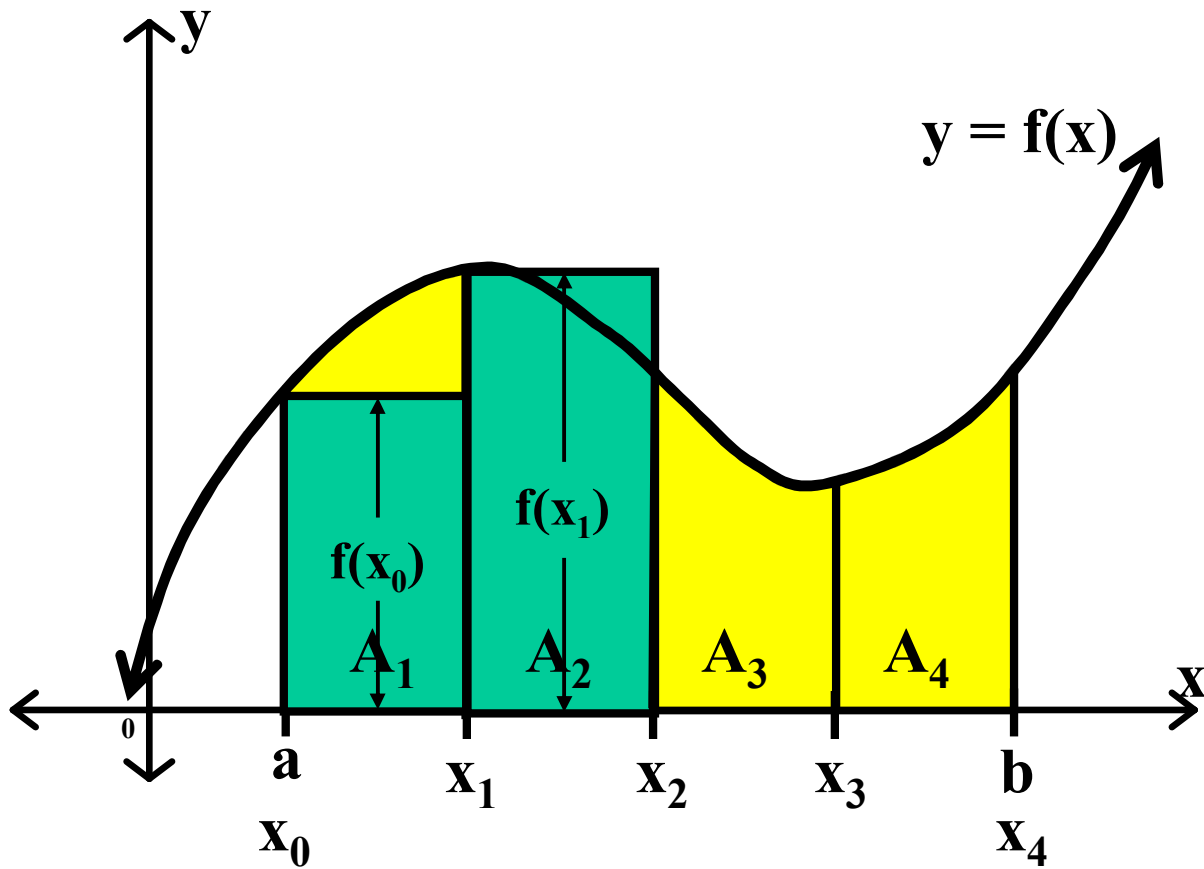
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx$$



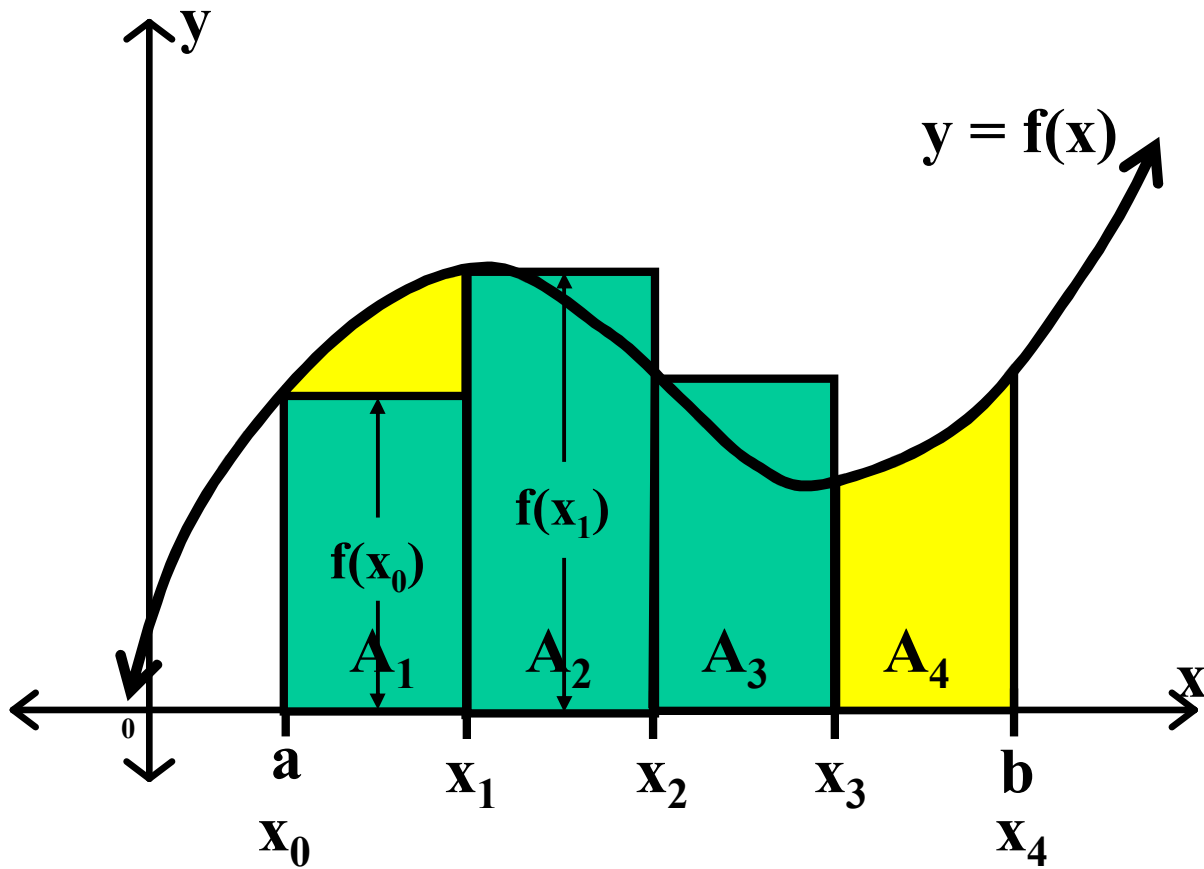
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x$$



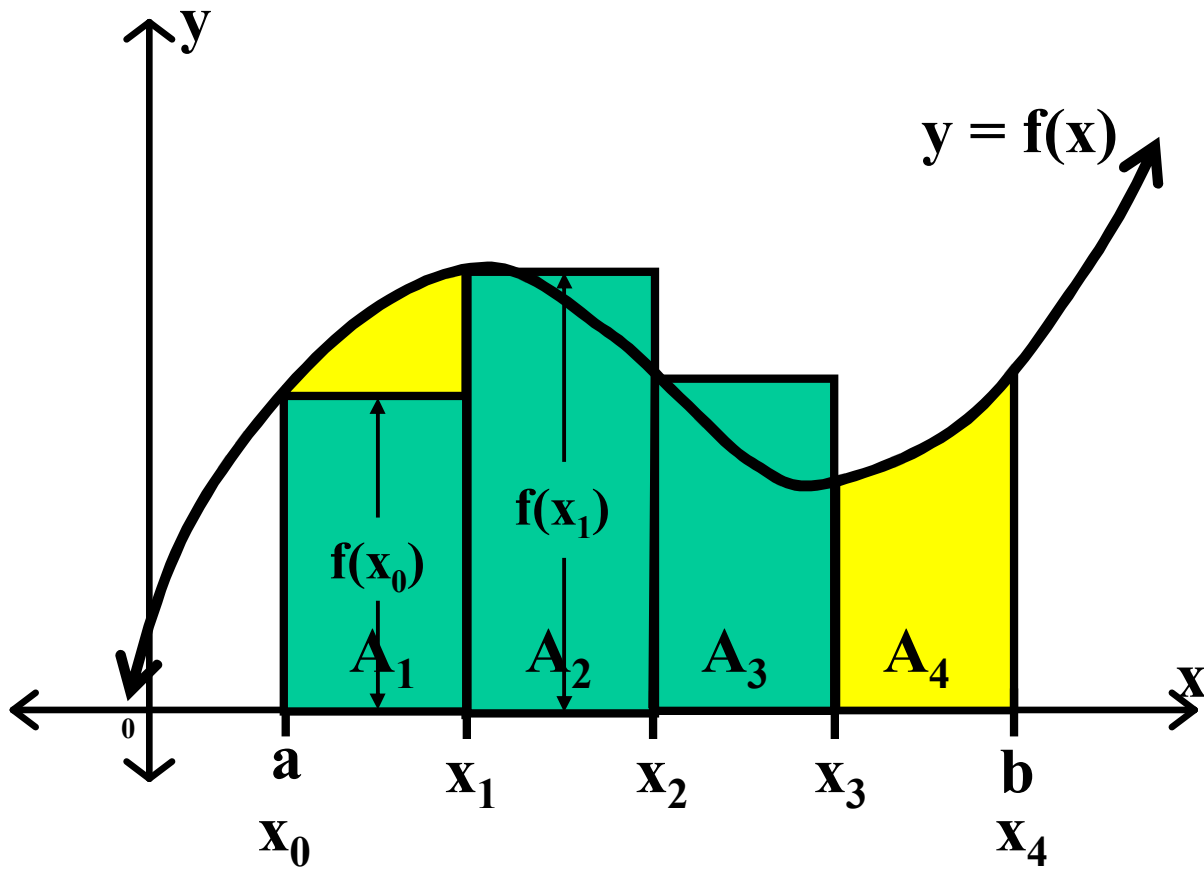
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x$$



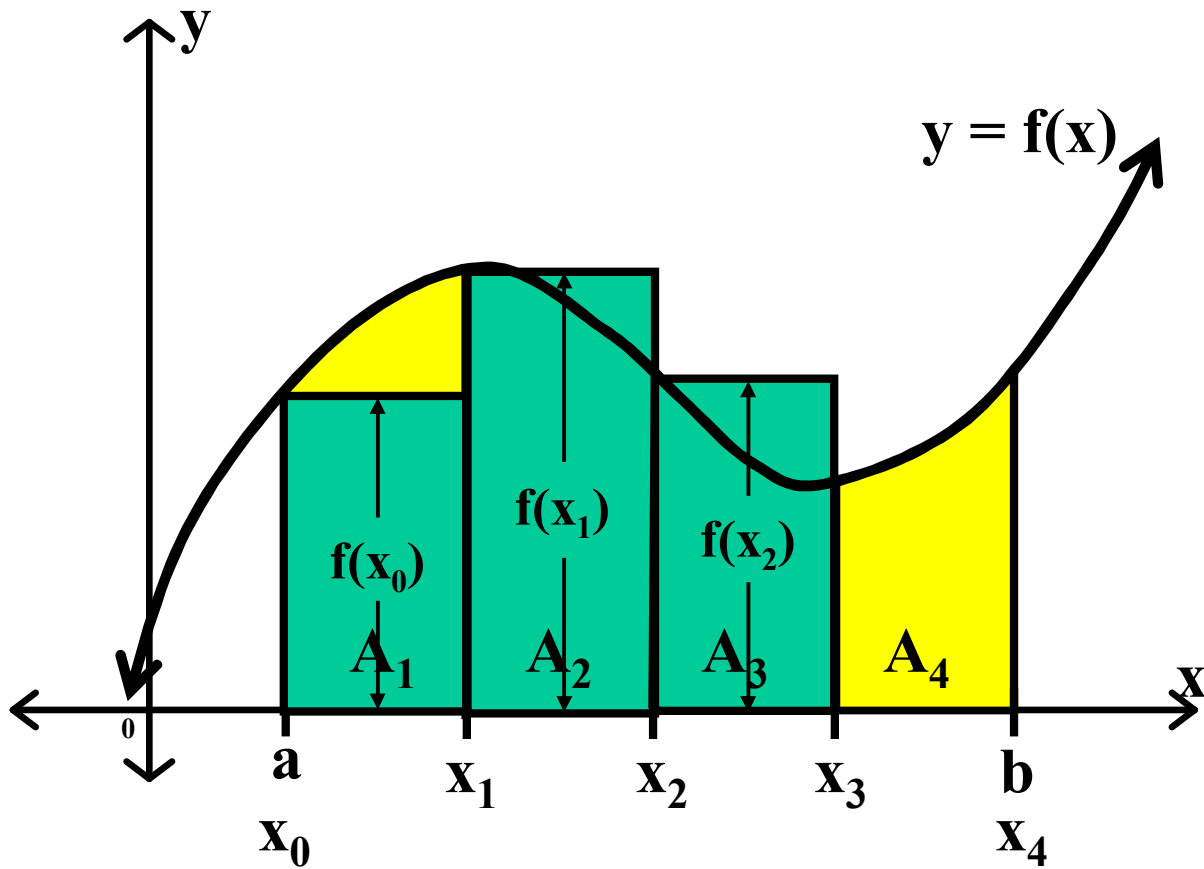
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x$$



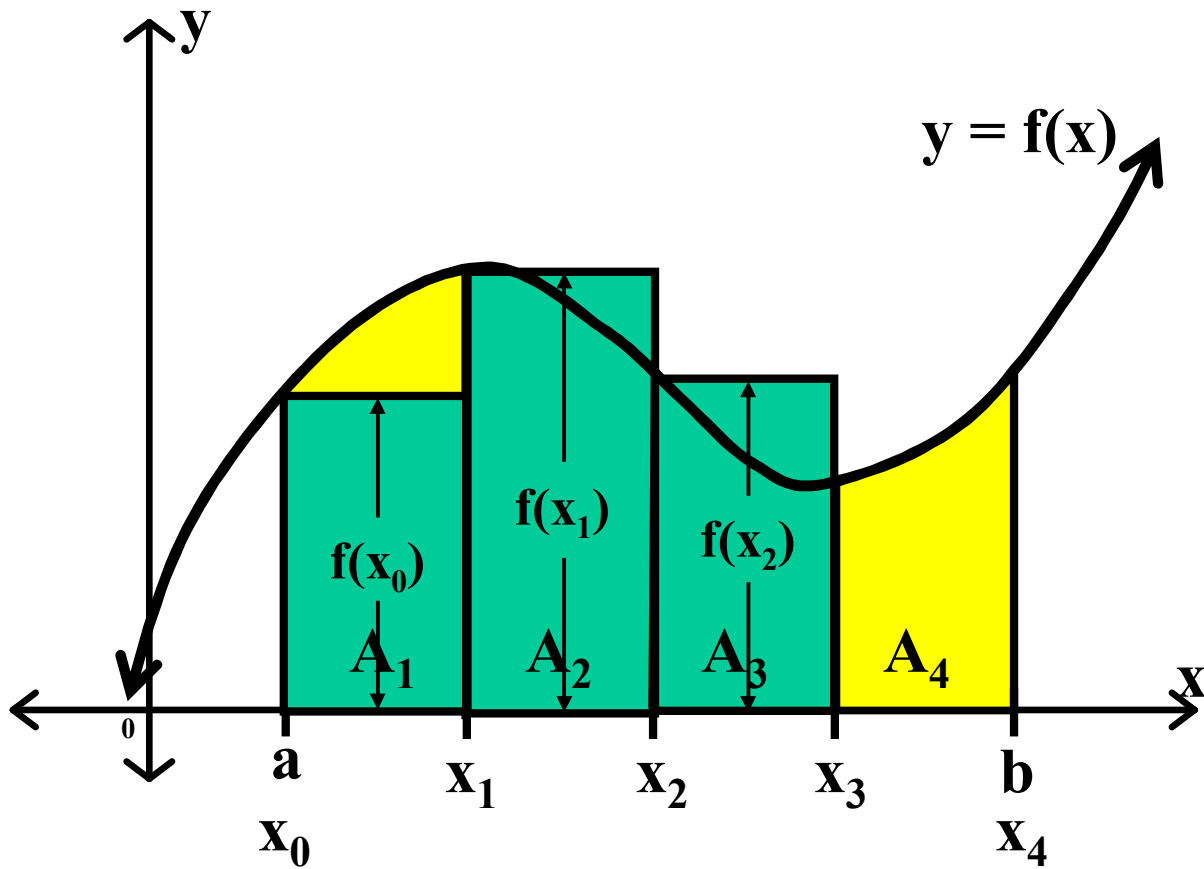
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx$$



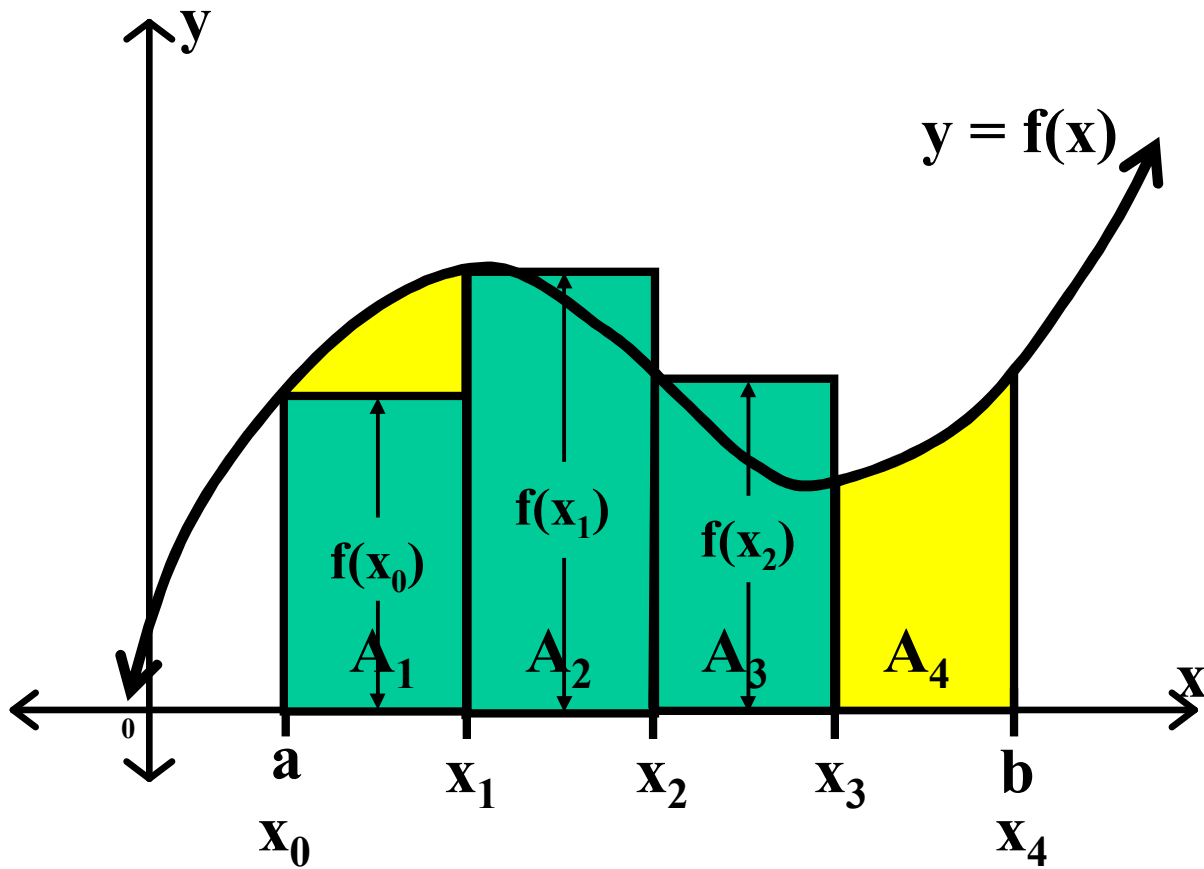
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx$$



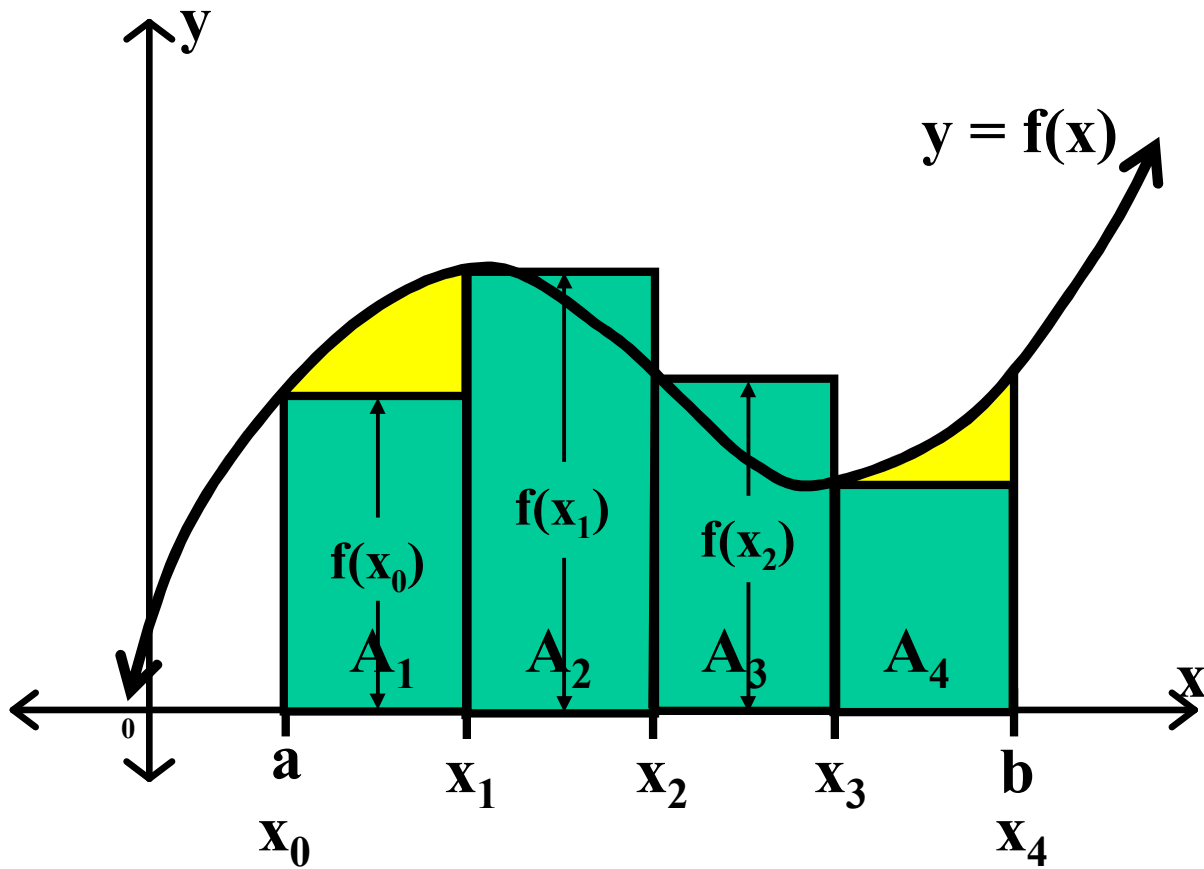
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x$$



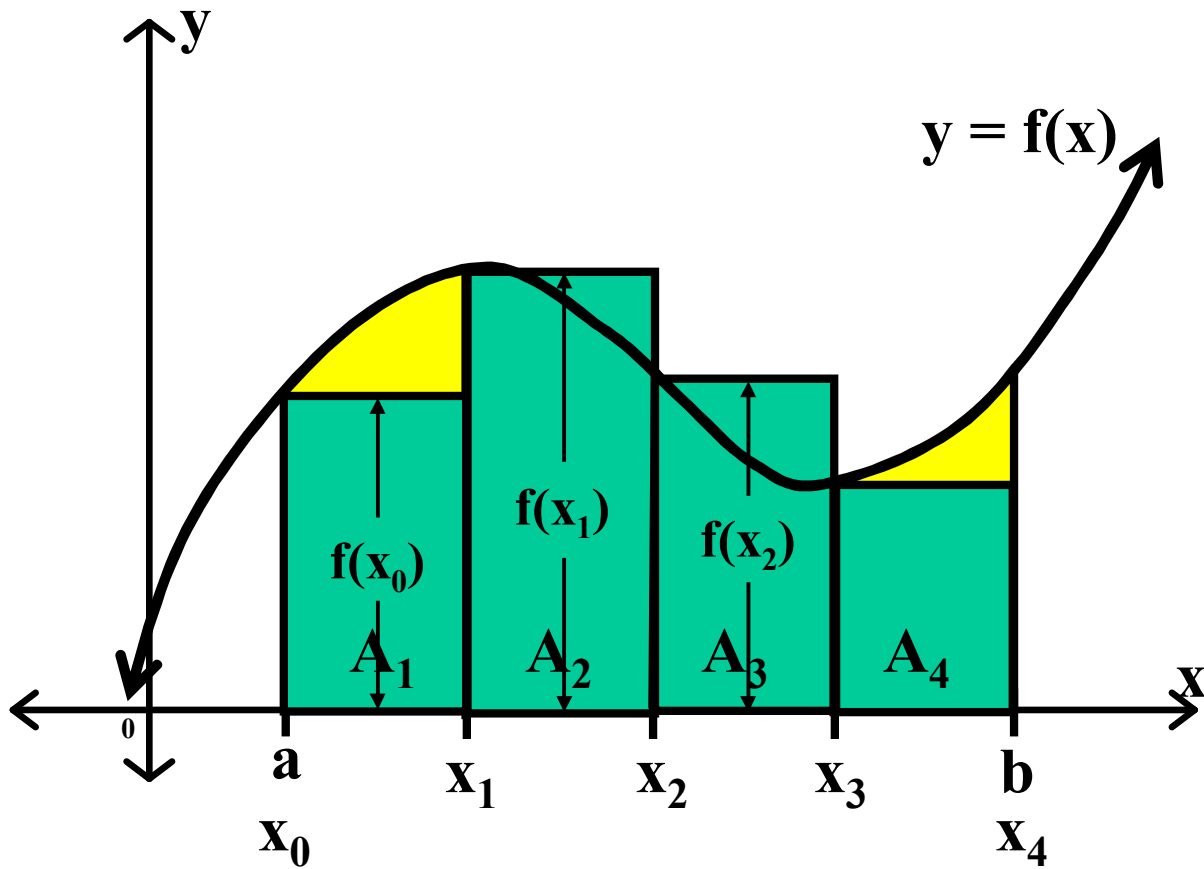
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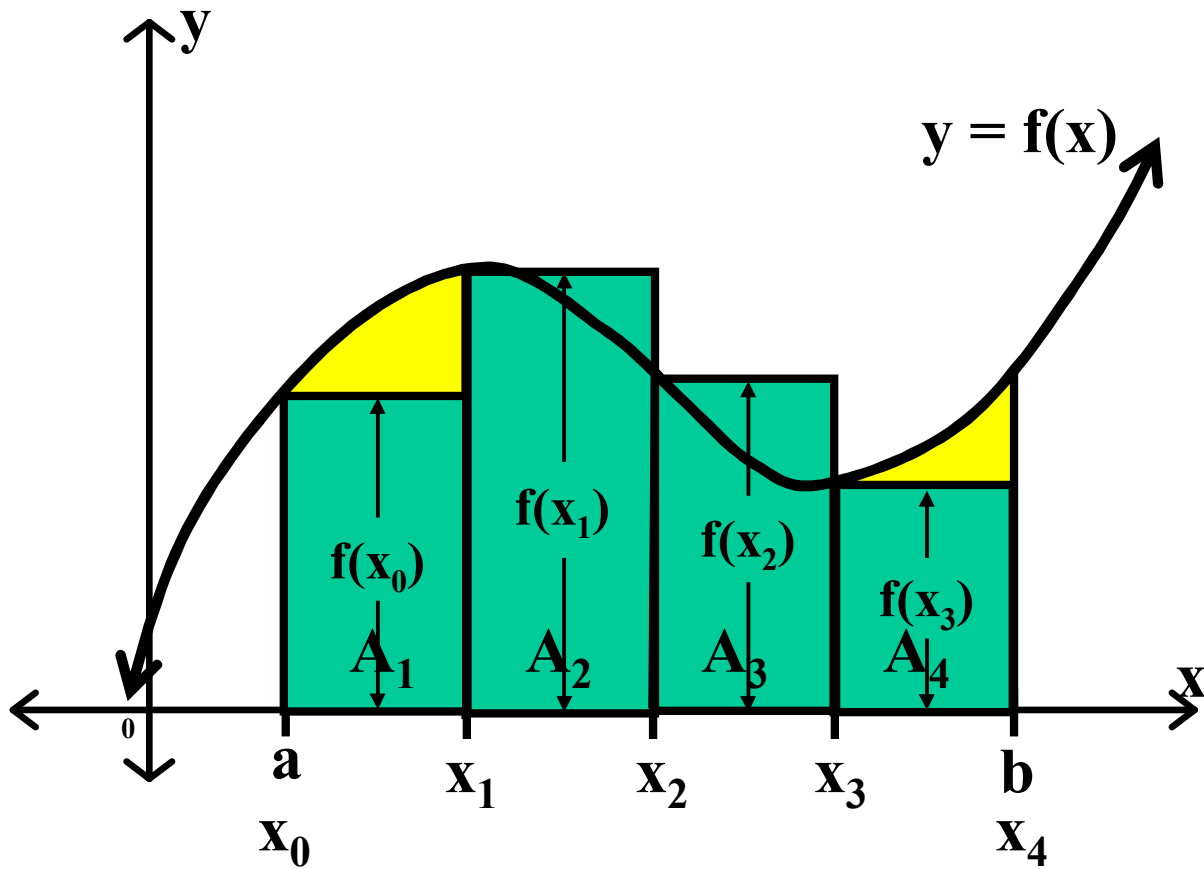
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x$$



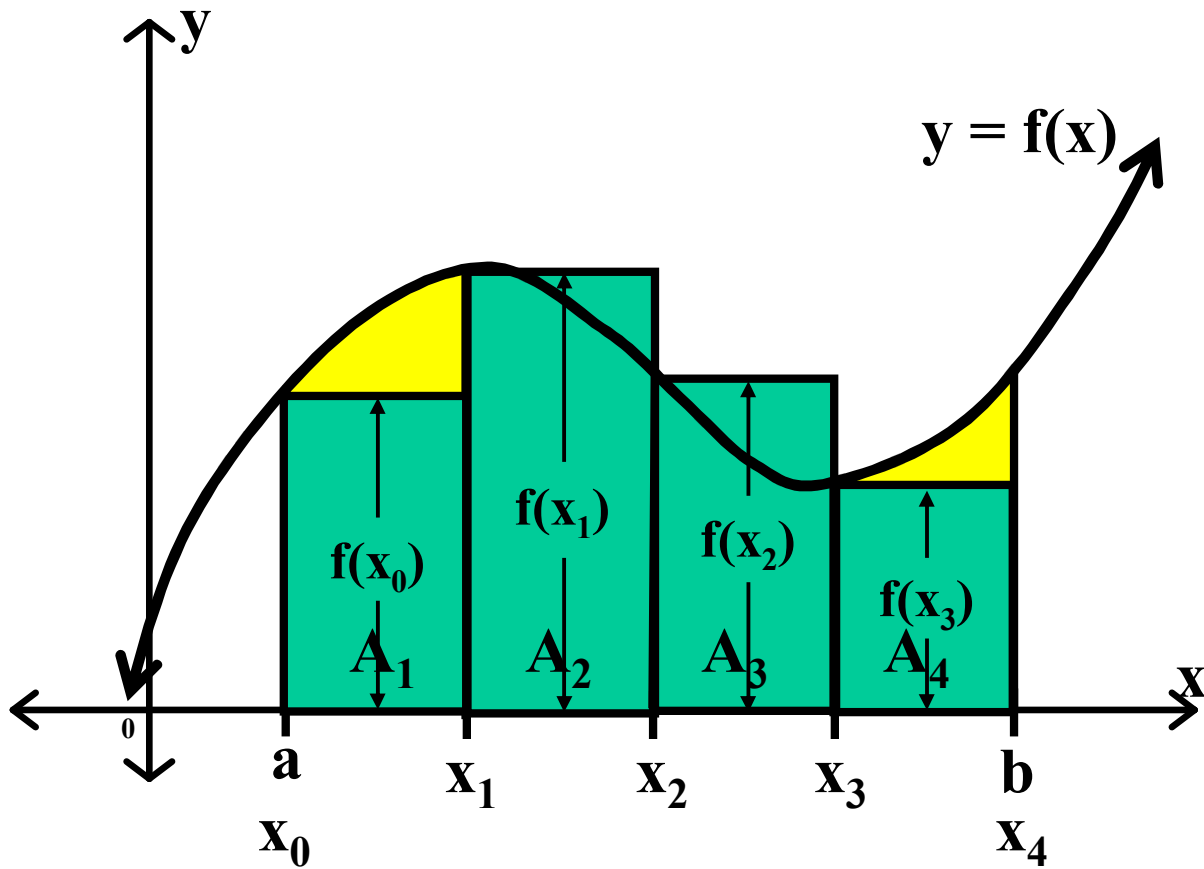
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx$$



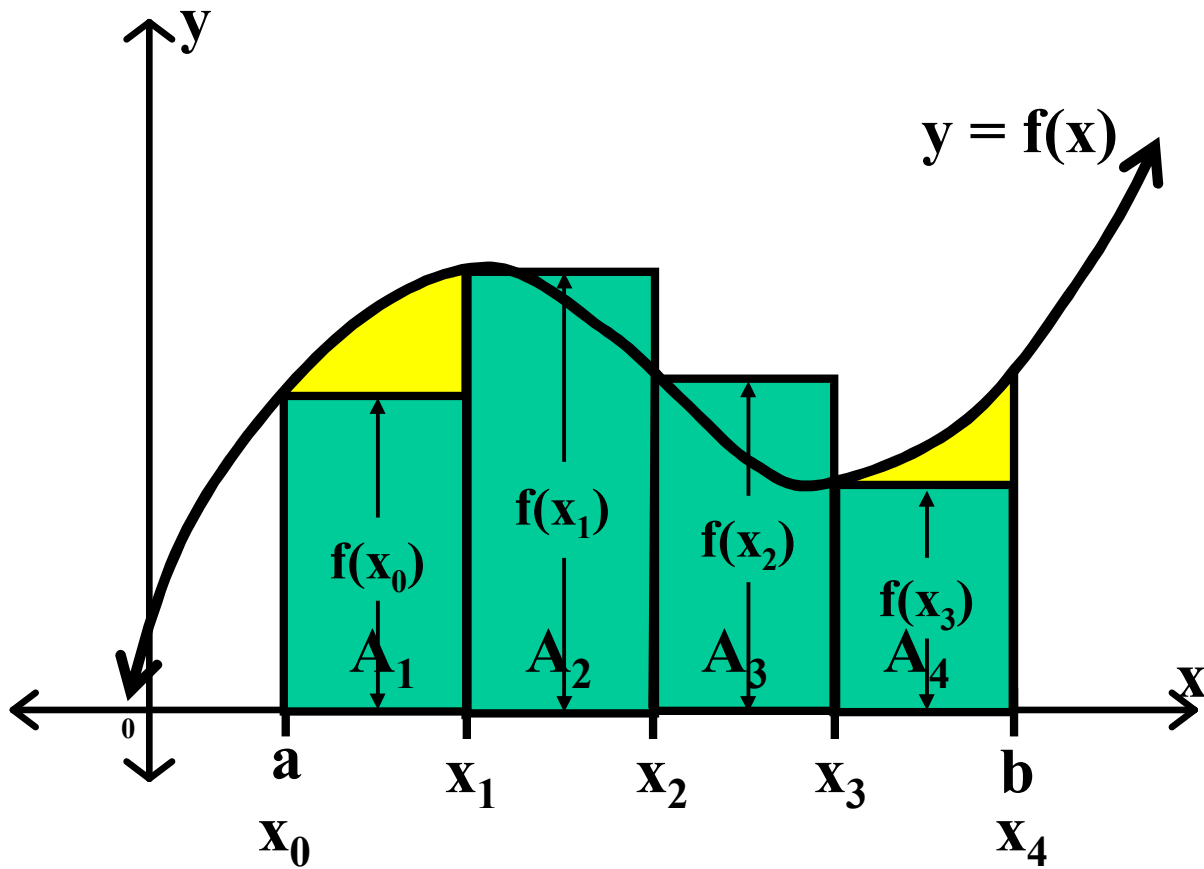
The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx$$



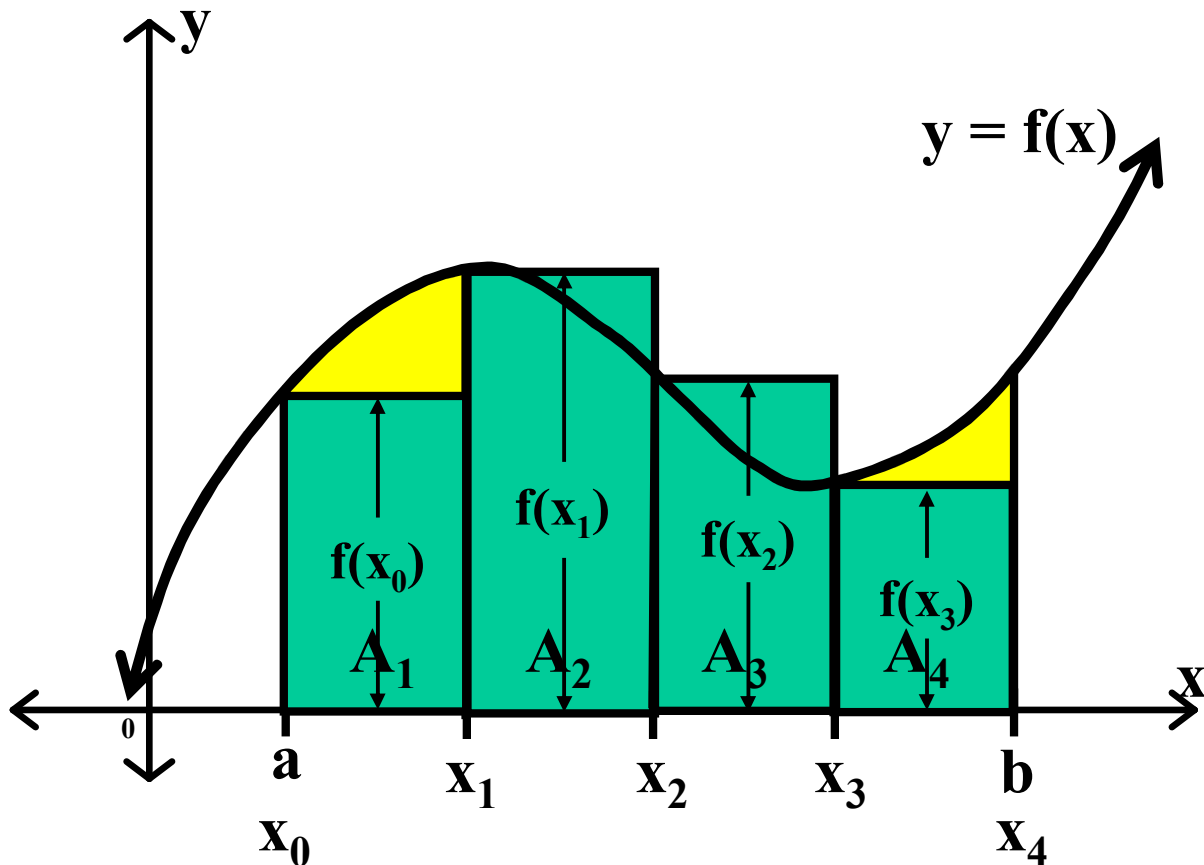
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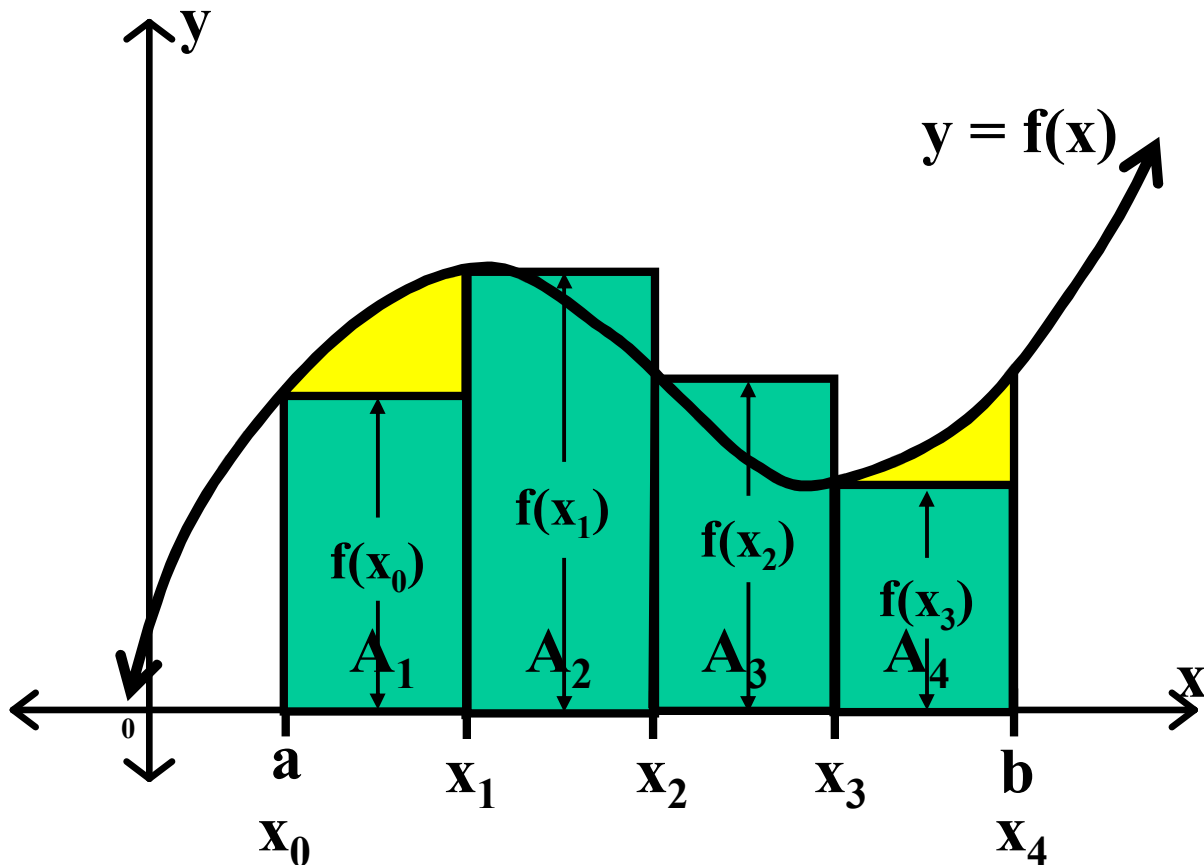


The width of each rectangle is Δx .

$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx f(x_3)\Delta x$$

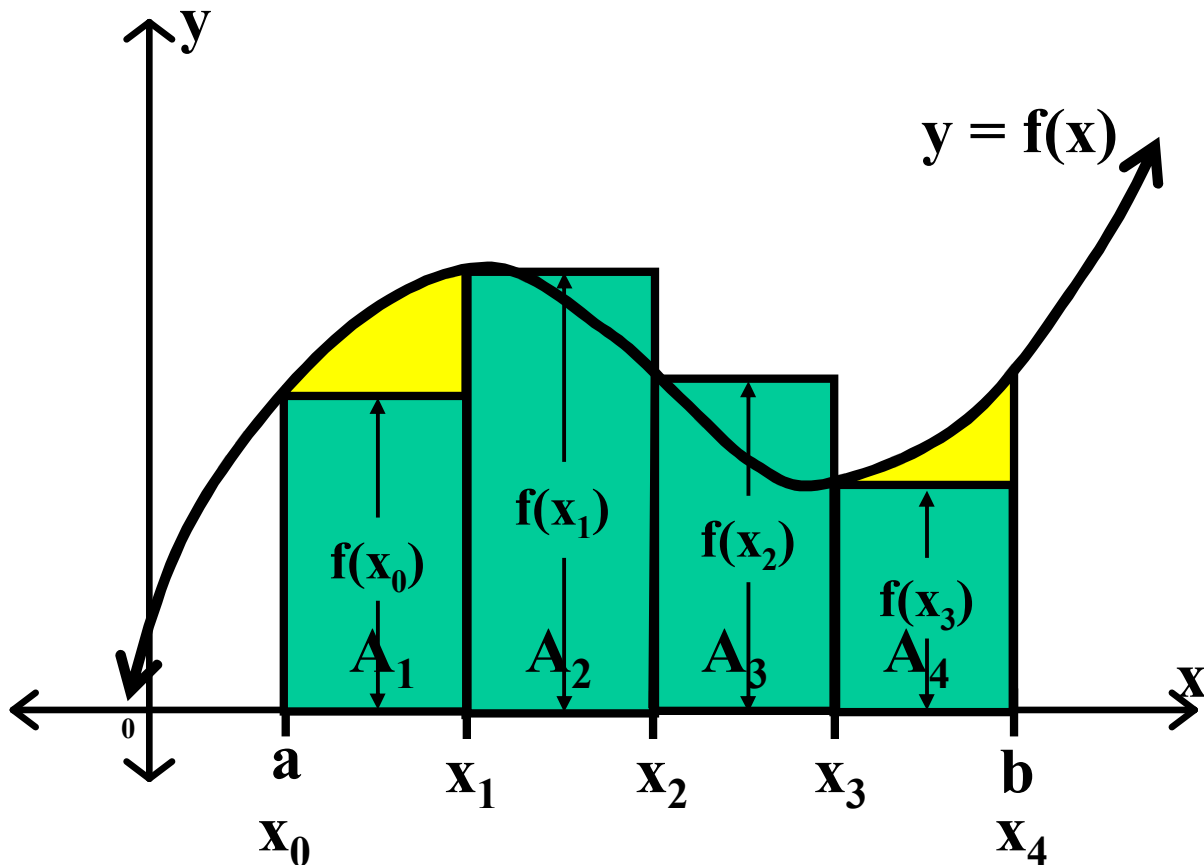


$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx f(x_3)\Delta x$$



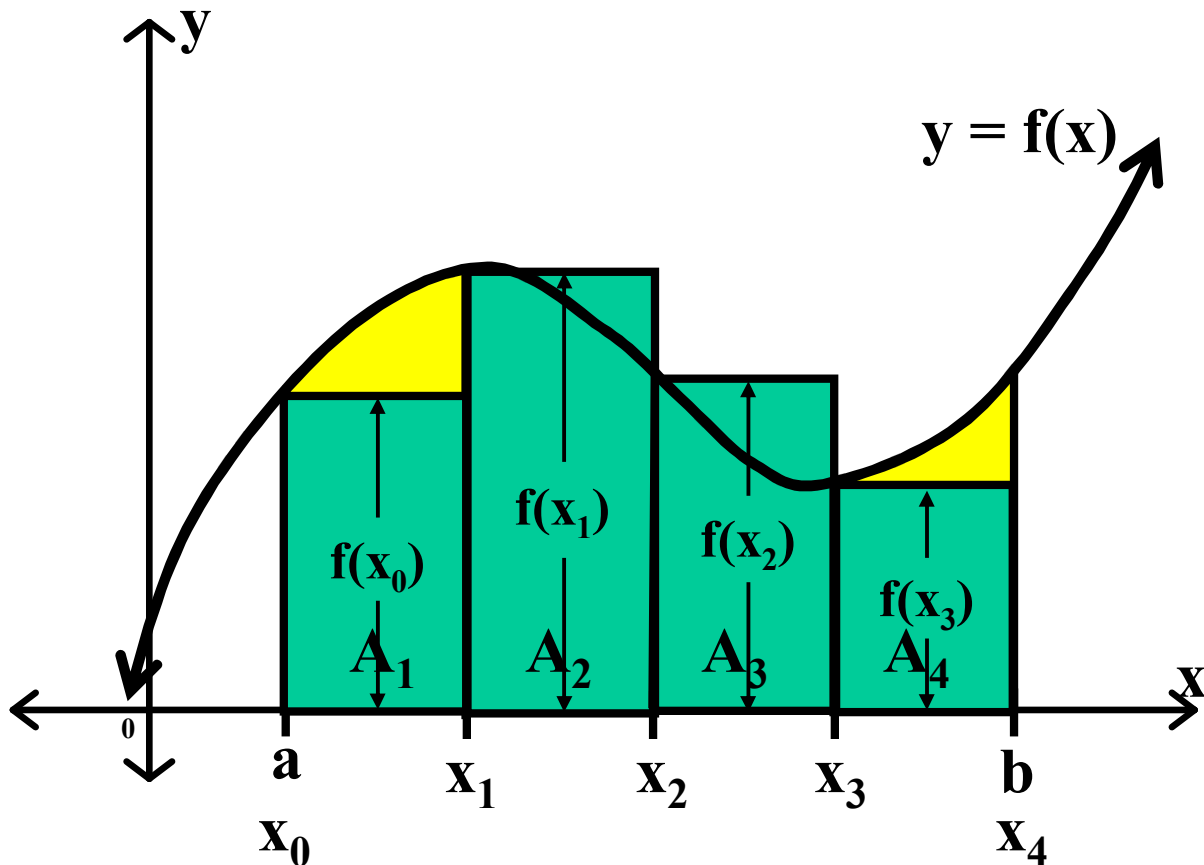
$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx f(x_3)\Delta x$$

Notice that, in general, A_i



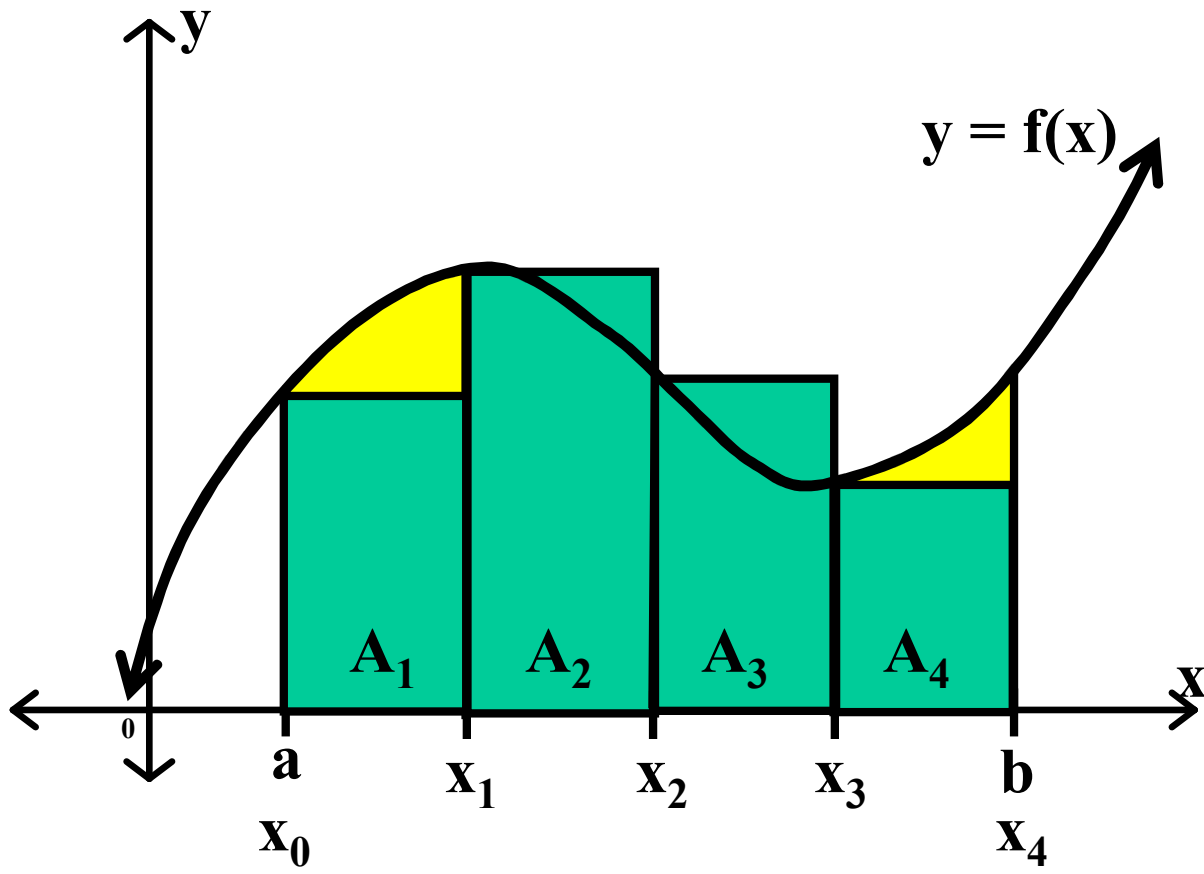
$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx f(x_3)\Delta x$$

Notice that, in general, $A_i \approx f(x_{i-1})$

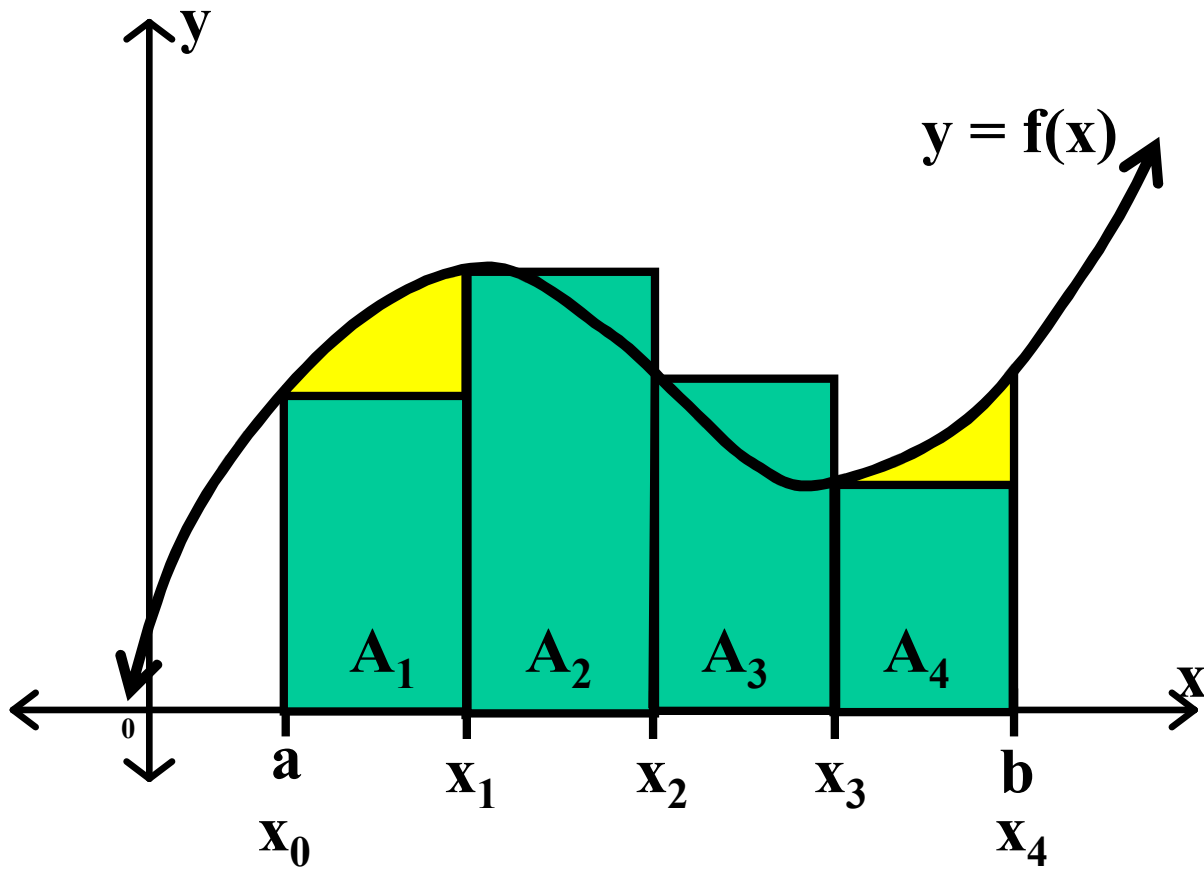


$$A_1 \approx f(x_0)\Delta x \quad A_2 \approx f(x_1)\Delta x \quad A_3 \approx f(x_2)\Delta x \quad A_4 \approx f(x_3)\Delta x$$

Notice that, in general, $A_i \approx f(x_{i-1})\Delta x$.

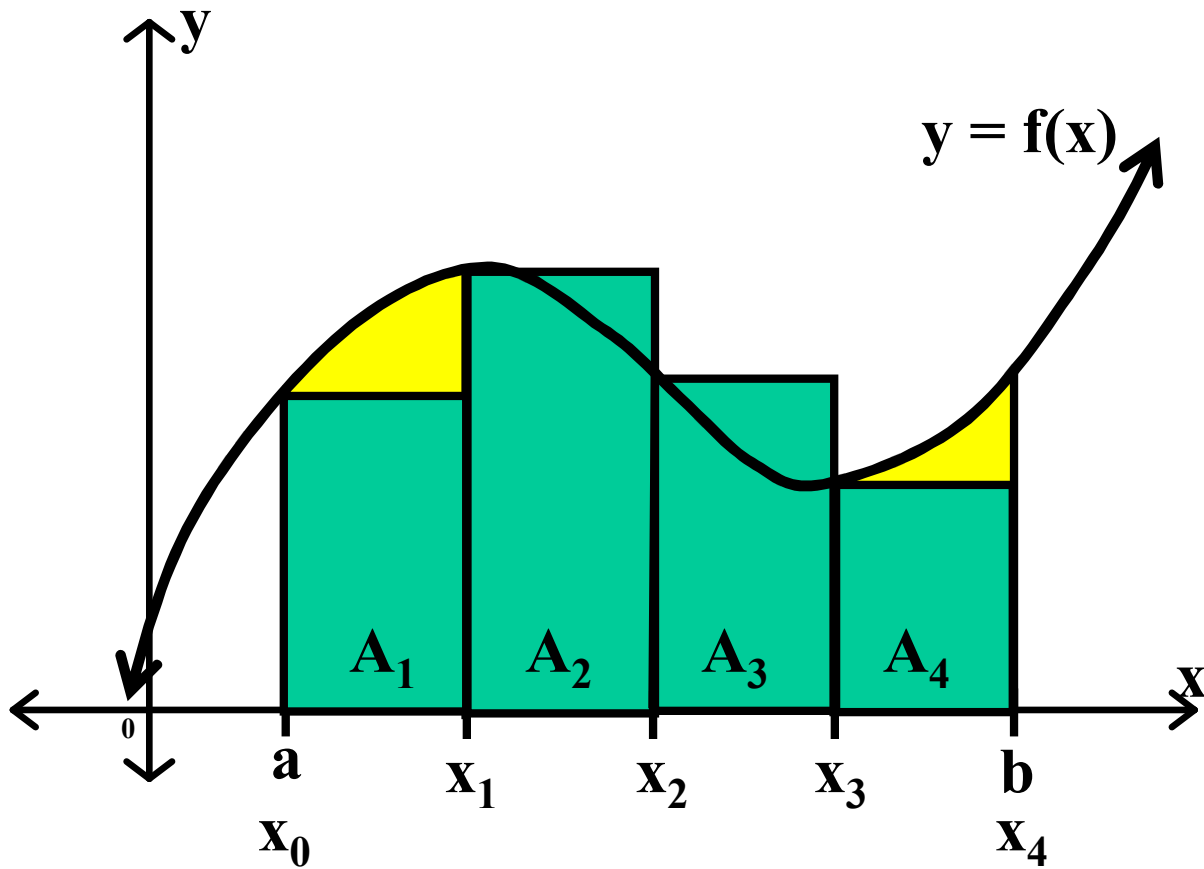


$$A_i \approx f(x_{i-1})\Delta x$$



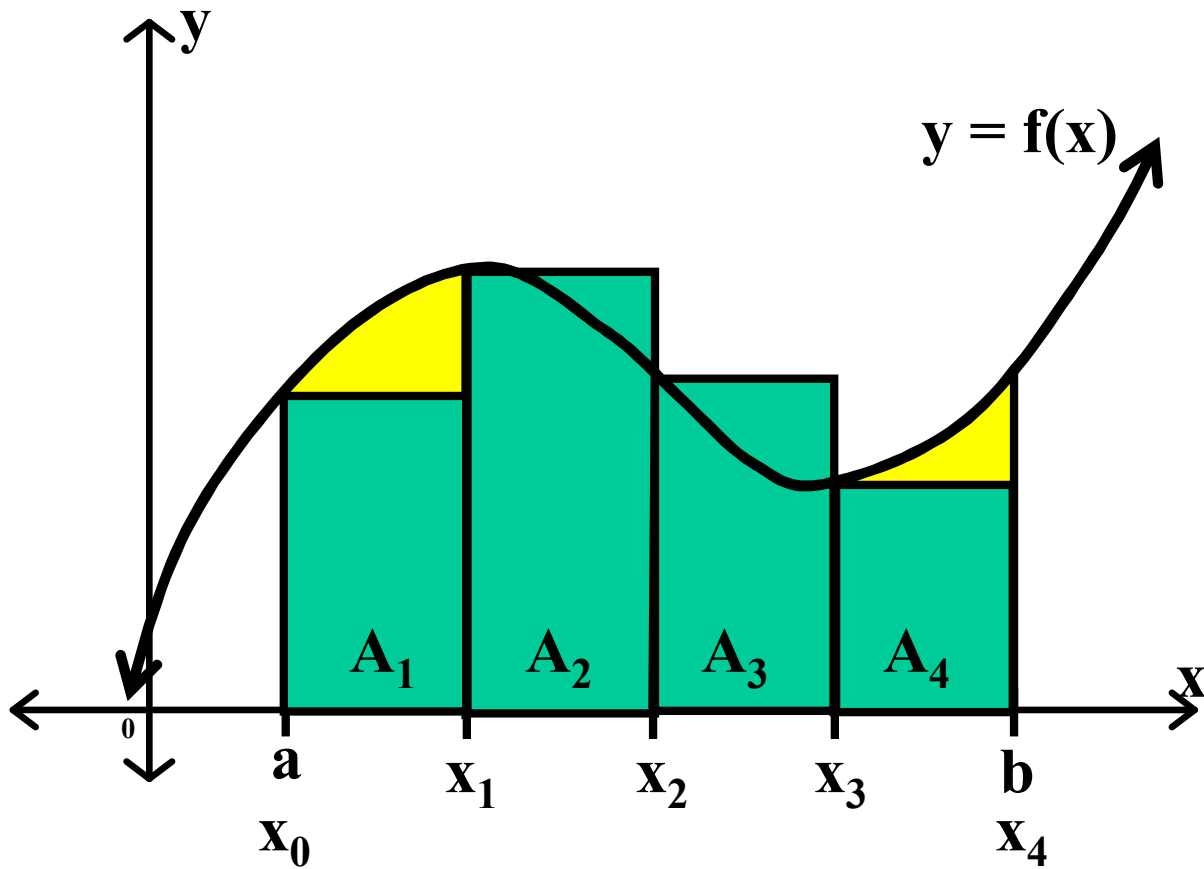
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx$$



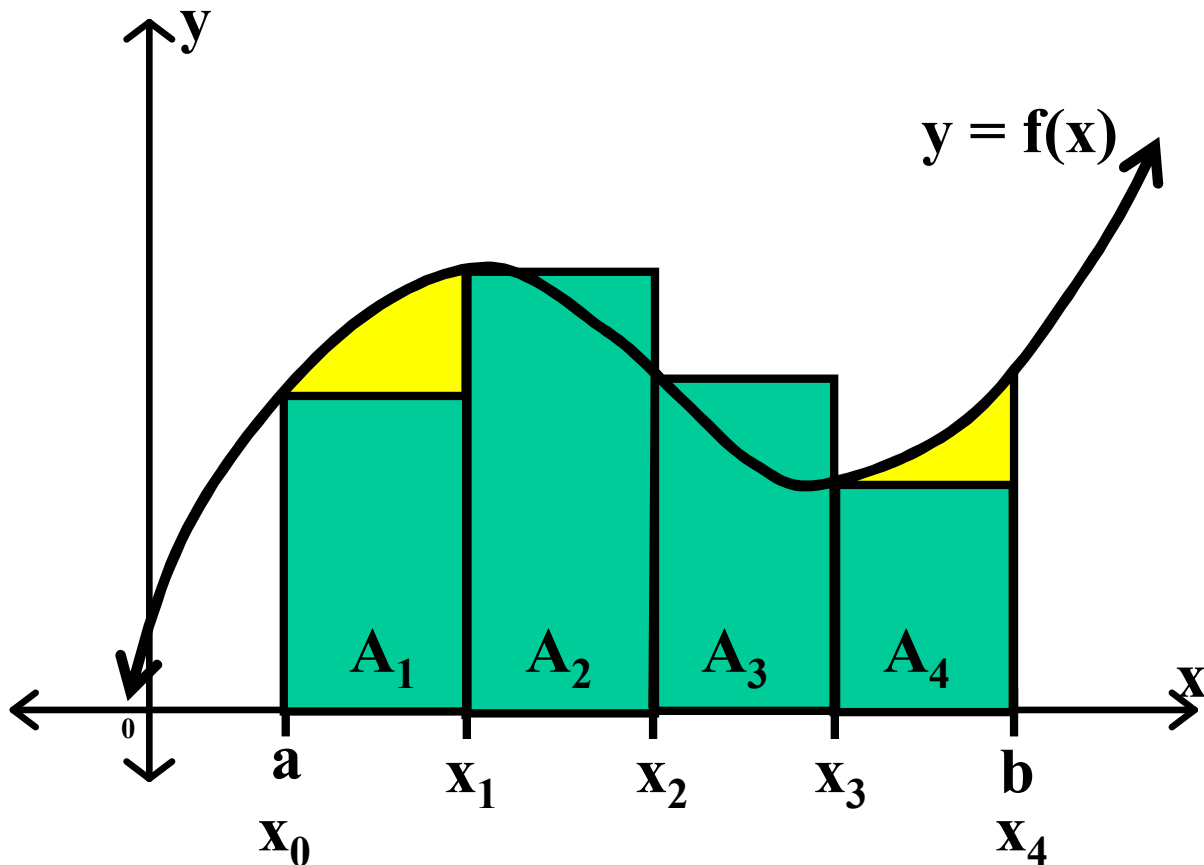
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4$$



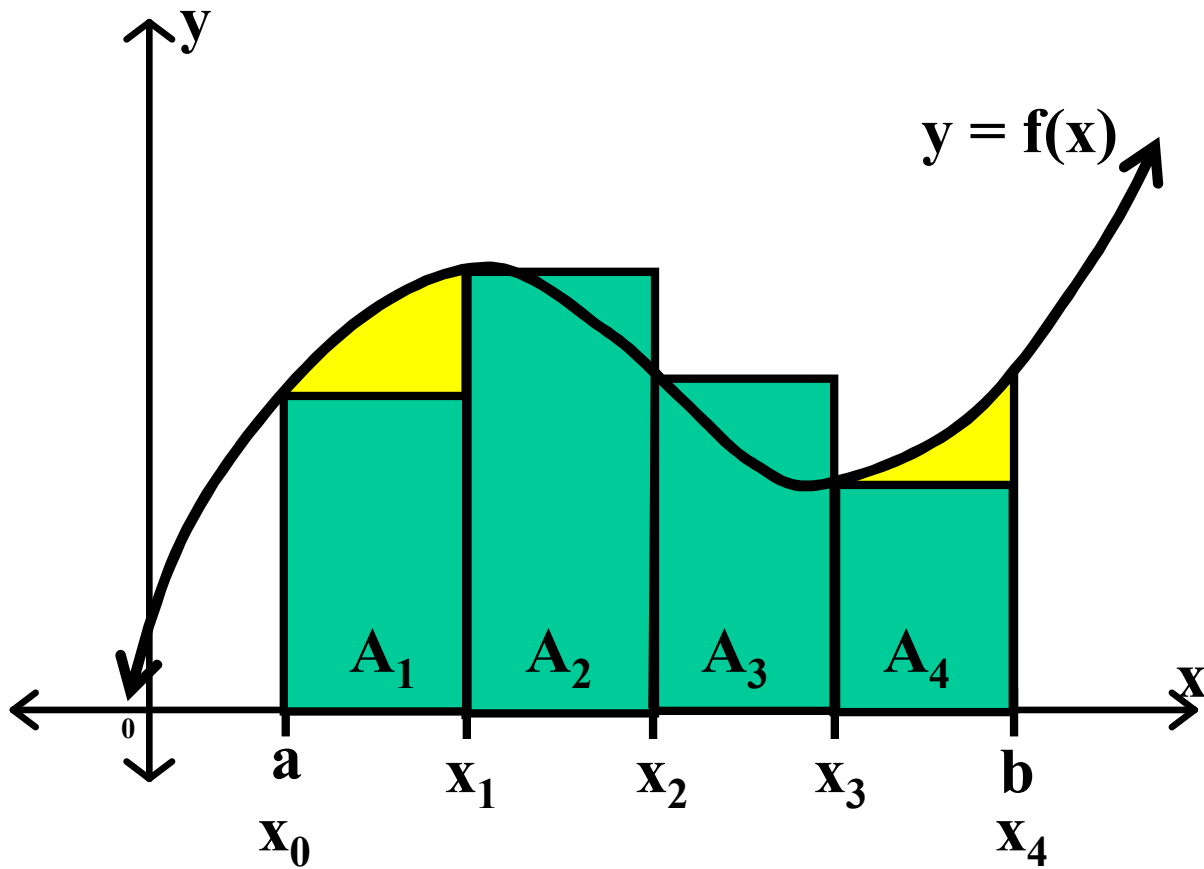
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i$$



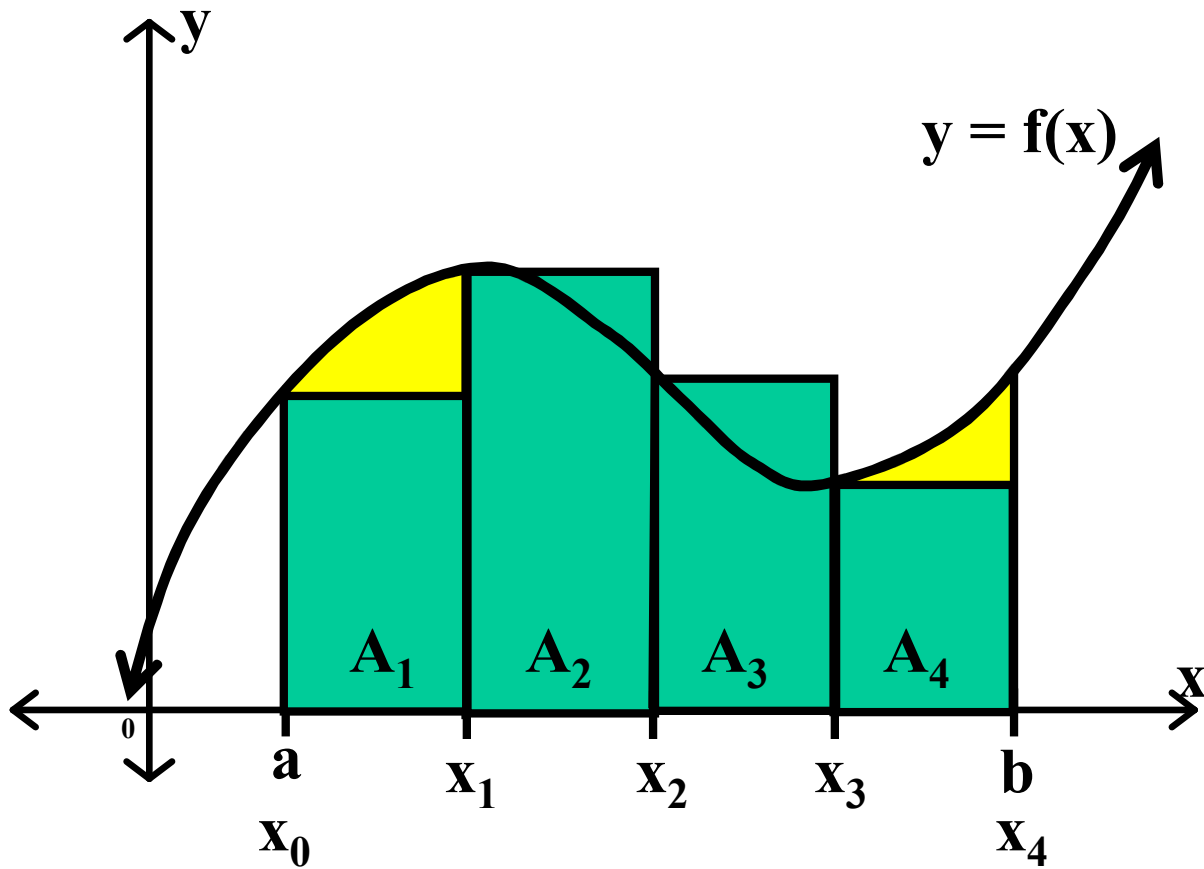
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i \quad (\text{In this case, } n = 4.)$$



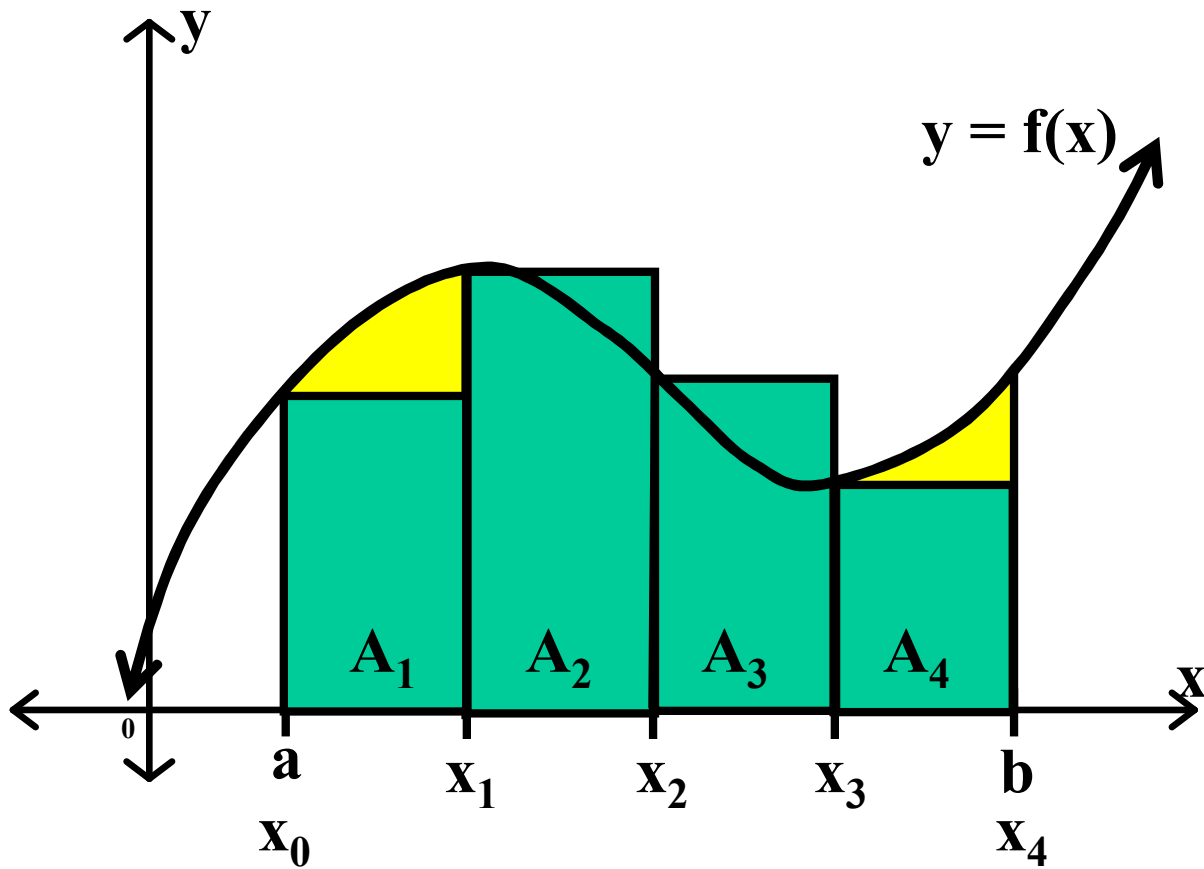
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i$$



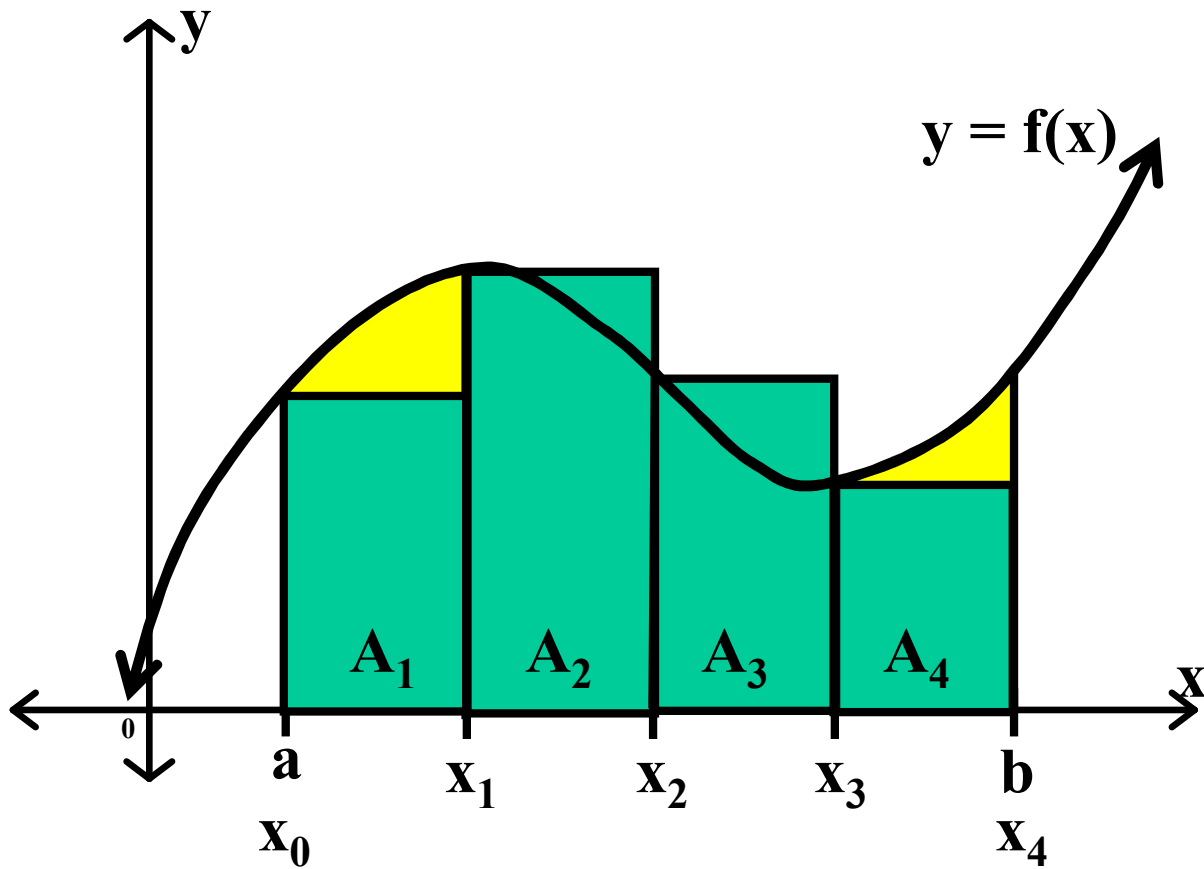
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx$$



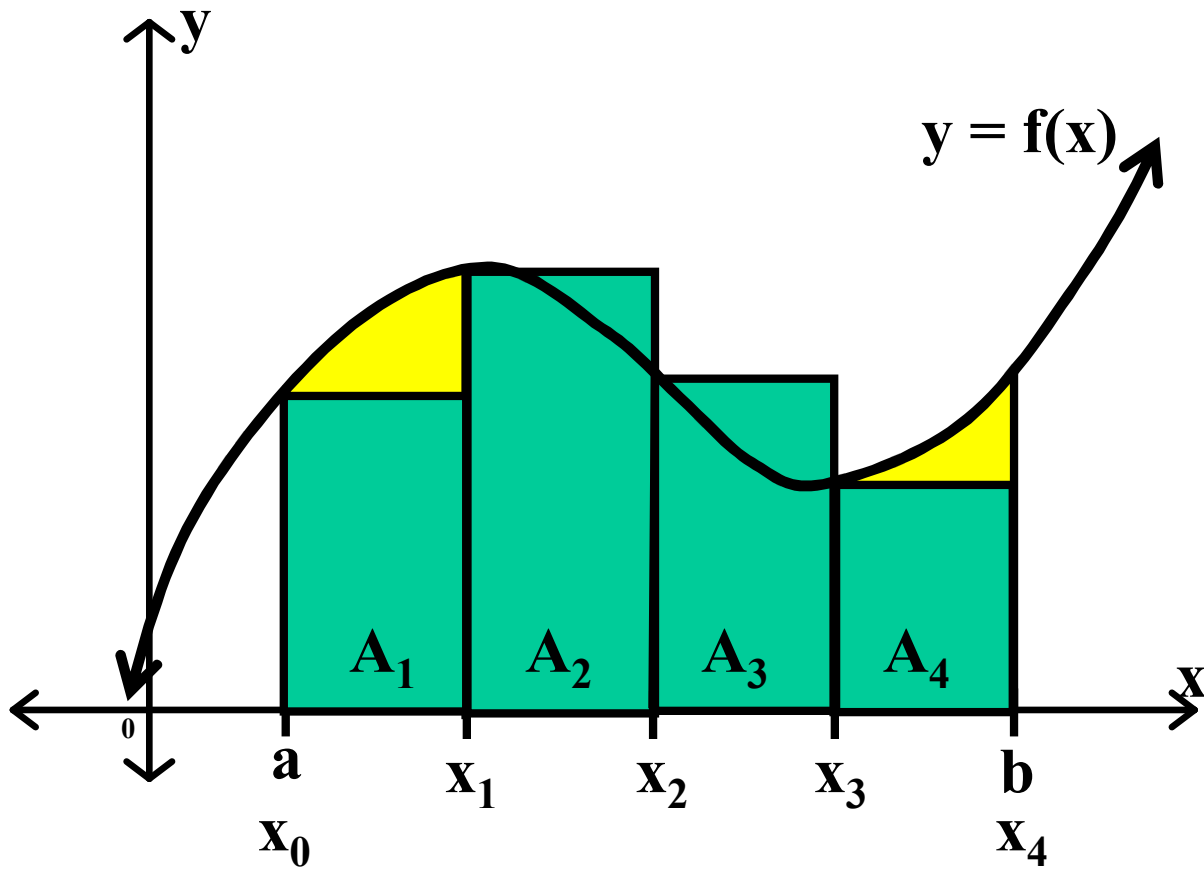
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n$$



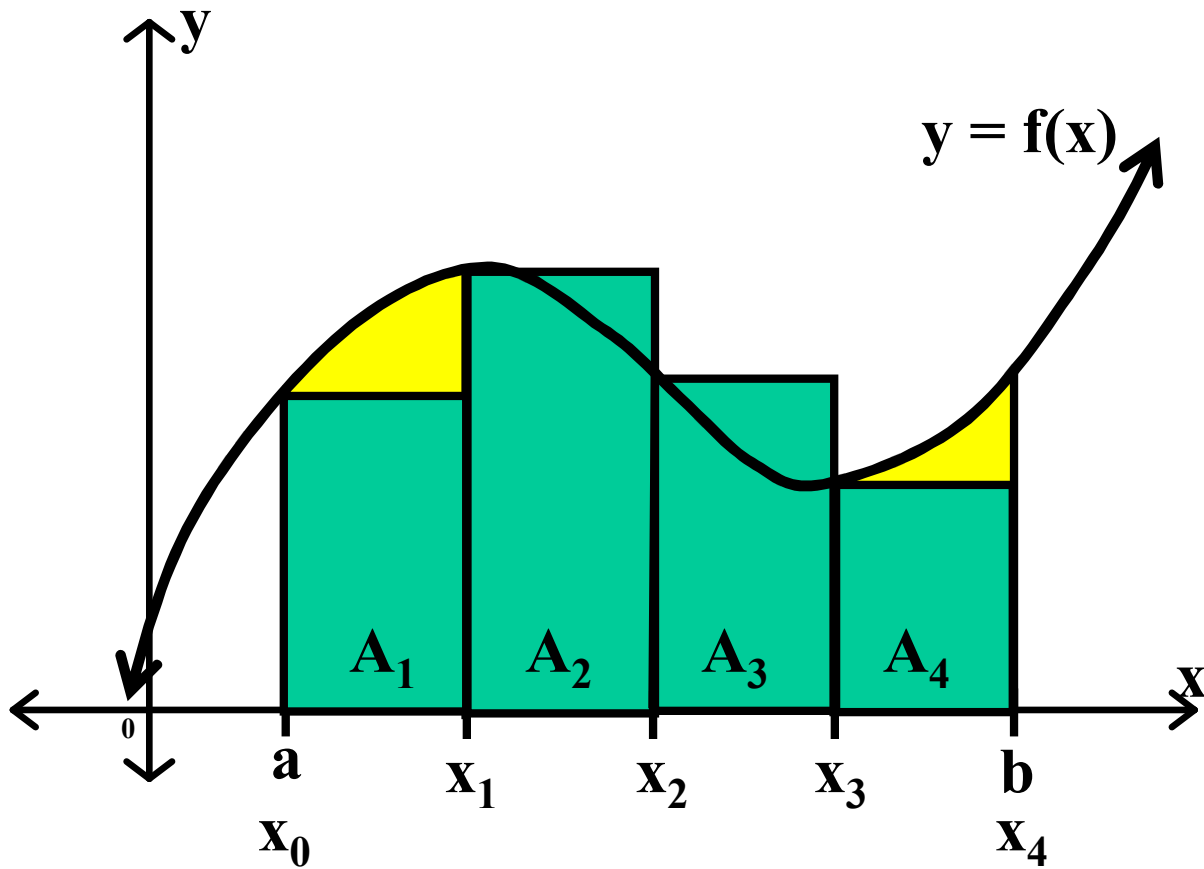
$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_{i-1})\Delta x$$



$$A_i \approx f(x_{i-1})\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_{i-1})\Delta x = S_L$$



$$S_L = \sum_{i=1}^n f(x_{i-1})\Delta x$$

The Left Rectangular Approximation

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx$$

Step 1: Find Δx .

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n}$$

Step 1: Find Δx .

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6}$$

Step 1: Find Δx .

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

Step 1: Find Δx .

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11


Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

 Add Δx .

$$x_1 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$\begin{aligned} x_0 &= a = 2 \\ x_1 &= 2.5 \end{aligned} \quad \begin{array}{l} \text{Add } \Delta x. \\ \leftarrow \end{array}$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5 \quad \text{Add } \Delta x.$$

$$x_2 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5 \quad \text{Add } \Delta x.$$

$$x_2 = 3$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 =$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 2: Calculate the x_i 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) =$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) =$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) =$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) =$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) =$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) =$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{aligned} x_0 &= a = 2 & f(x_0) &= f(a) = f(2) = \sqrt{5} \\ x_1 &= 2.5 & f(x_1) &= f(2.5) = \sqrt{12.625} \\ x_2 &= 3 & f(x_2) &= \\ x_3 &= 3.5 & & \\ x_4 &= 4 & & \\ x_5 &= 4.5 & & \\ x_6 &= b = 5 & & \end{aligned}$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) =$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

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$$x_3 = 3.5 \quad f(x_3) =$$

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$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

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$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) =$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{aligned} x_0 &= a = 2 & f(x_0) &= f(a) = f(2) = \sqrt{5} \\ x_1 &= 2.5 & f(x_1) &= f(2.5) = \sqrt{12.625} \\ x_2 &= 3 & f(x_2) &= f(3) = \sqrt{24} \\ x_3 &= 3.5 & f(x_3) &= f(3.5) = \sqrt{39.875} \\ x_4 &= 4 \\ x_5 &= 4.5 \\ x_6 &= b = 5 \end{aligned}$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

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Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

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Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

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Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{aligned} x_0 = a = 2 & & f(x_0) = f(a) = f(2) &= \sqrt{5} \\ x_1 = 2.5 & & f(x_1) = f(2.5) &= \sqrt{12.625} \\ x_2 = 3 & & f(x_2) = f(3) &= \sqrt{24} \\ x_3 = 3.5 & & f(x_3) = f(3.5) &= \sqrt{39.875} \\ x_4 = 4 & & f(x_4) = f(4) &= \sqrt{61} \\ x_5 = 4.5 & & f(x_5) &= \\ x_6 = b = 5 & & & \end{aligned}$$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

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Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

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$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) =$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

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$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) =$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
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$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) =$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
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$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) = \sqrt{122}$

Step 3: Calculate the $f(x_i)$'s.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) = \sqrt{122}$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{array}{l} x_0 = a = 2 \\ x_1 = 2.5 \\ x_2 = 3 \\ x_3 = 3.5 \\ x_4 = 4 \\ x_5 = 4.5 \\ x_6 = b = 5 \end{array} \quad \begin{array}{l} f(x_0) = f(a) = f(2) = \sqrt{5} \\ f(x_1) = f(2.5) = \sqrt{12.625} \\ f(x_2) = f(3) = \sqrt{24} \\ f(x_3) = f(3.5) = \sqrt{39.875} \\ f(x_4) = f(4) = \sqrt{61} \\ f(x_5) = f(4.5) = \sqrt{88.125} \\ f(x_6) = f(b) = f(5) = \sqrt{122} \end{array} \quad S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{array}{l} x_0 = a = 2 \\ x_1 = 2.5 \\ x_2 = 3 \\ x_3 = 3.5 \\ x_4 = 4 \\ x_5 = 4.5 \\ x_6 = b = 5 \end{array} \quad \begin{array}{l} f(x_0) = f(a) = f(2) = \sqrt{5} \\ f(x_1) = f(2.5) = \sqrt{12.625} \\ f(x_2) = f(3) = \sqrt{24} \\ f(x_3) = f(3.5) = \sqrt{39.875} \\ f(x_4) = f(4) = \sqrt{61} \\ f(x_5) = f(4.5) = \sqrt{88.125} \\ f(x_6) = f(b) = f(5) = \sqrt{122} \end{array} \quad S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

Class Worksheet #5 Unit 11

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

$$x_4 = 4 \quad f(x_4) = f(4) = \sqrt{61}$$

$$x_5 = 4.5 \quad f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$S_L = \sum_{i=1}^6 f(x_{i-1}) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

$$x_4 = 4 \quad f(x_4) = f(4) = \sqrt{61}$$

$$x_5 = 4.5 \quad f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$S_L = \sum_{i=1}^6 f(x_{i-1}) \Delta x$$

$$S_L = f(x_0) \Delta x + f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5$$

$$f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3$$

$$f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5$$

$$f(x_3) = f(3.5) = \sqrt{39.875}$$

$$x_4 = 4$$

$$f(x_4) = f(4) = \sqrt{61}$$

$$x_5 = 4.5$$

$$f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5$$

$$f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$S_L = \sum_{i=1}^6 f(x_{i-1}) \Delta x$$

$$S_L = f(x_0) \Delta x + f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$S_L = \sum_{i=1}^6 f(x_{i-1}) \Delta x$$

$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$$

$$S_L = (\sqrt{5} + \sqrt{12.625} + \sqrt{24} + \sqrt{39.875} + \sqrt{61} + \sqrt{88.125})(.5)$$

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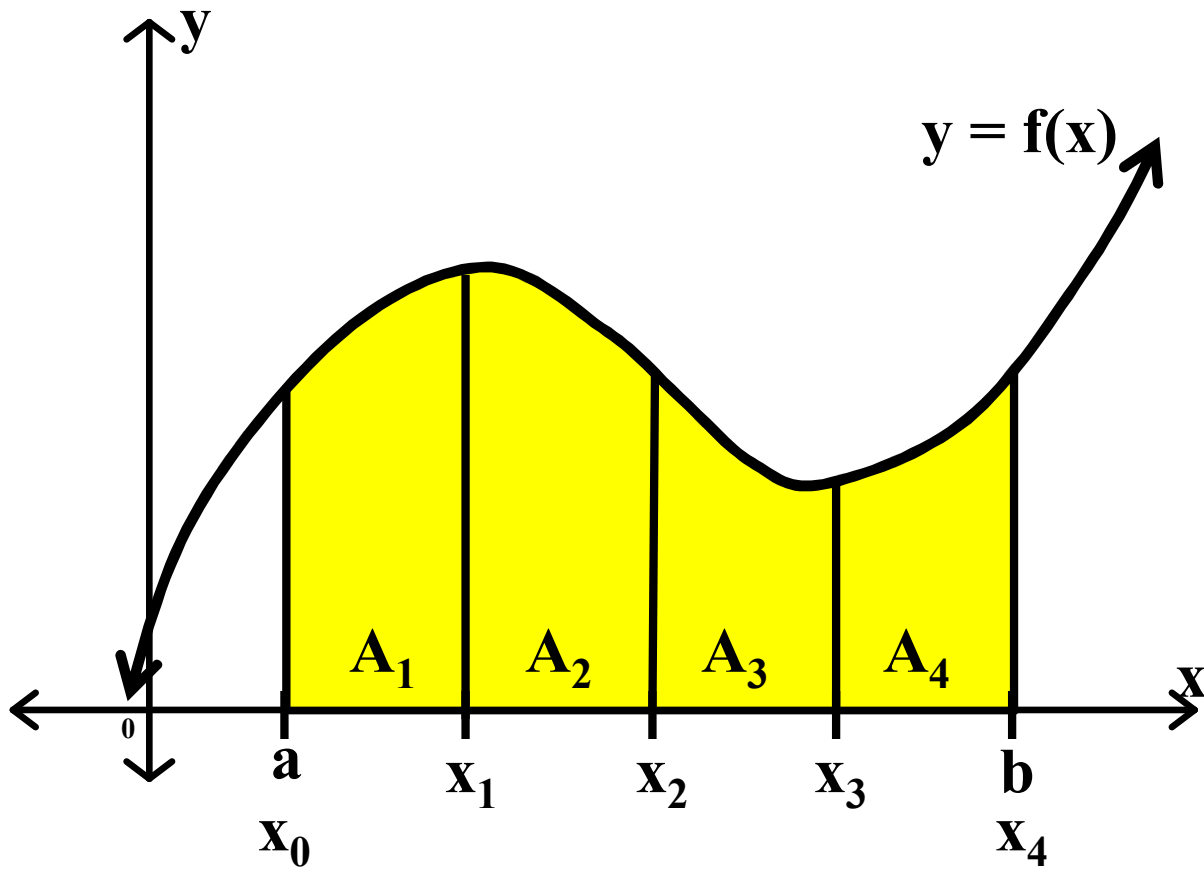
$$S_L = \sum_{i=1}^n f(x_{i-1}) \Delta x$$

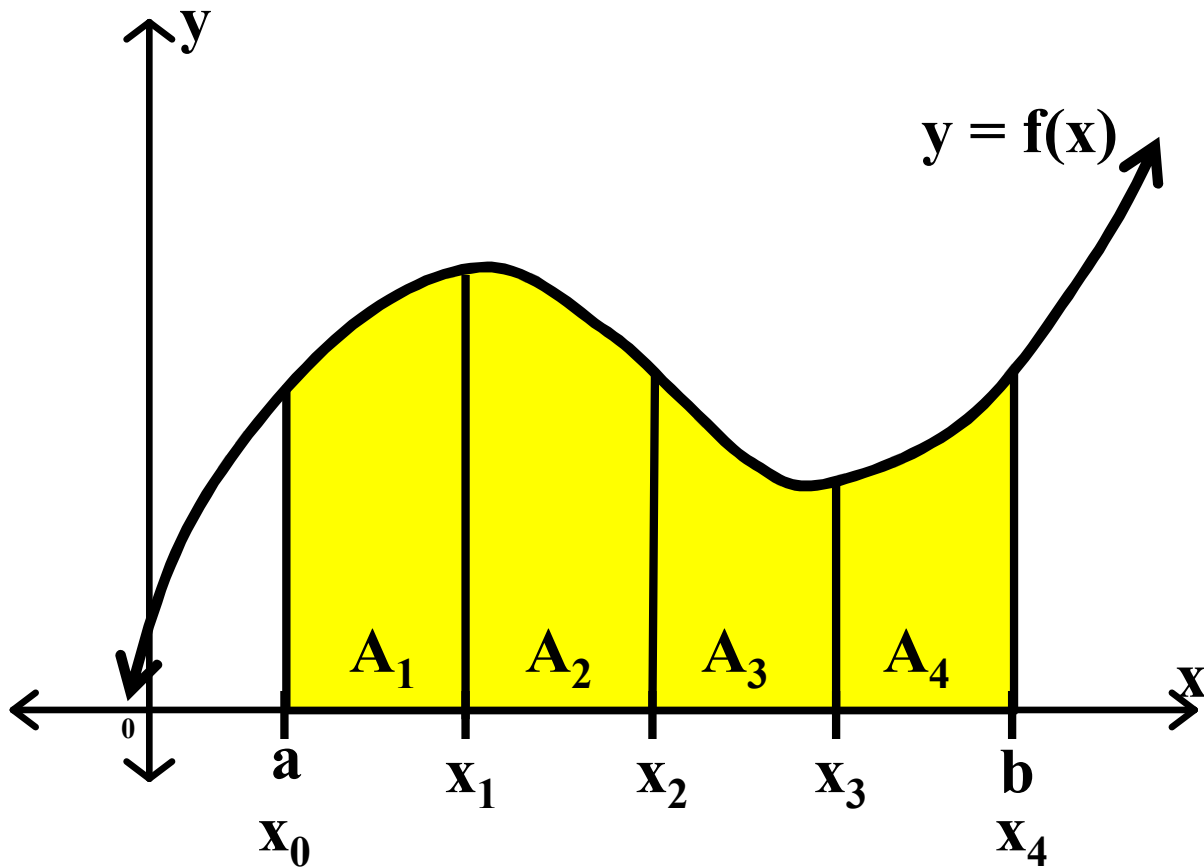
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$$S_L = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$$

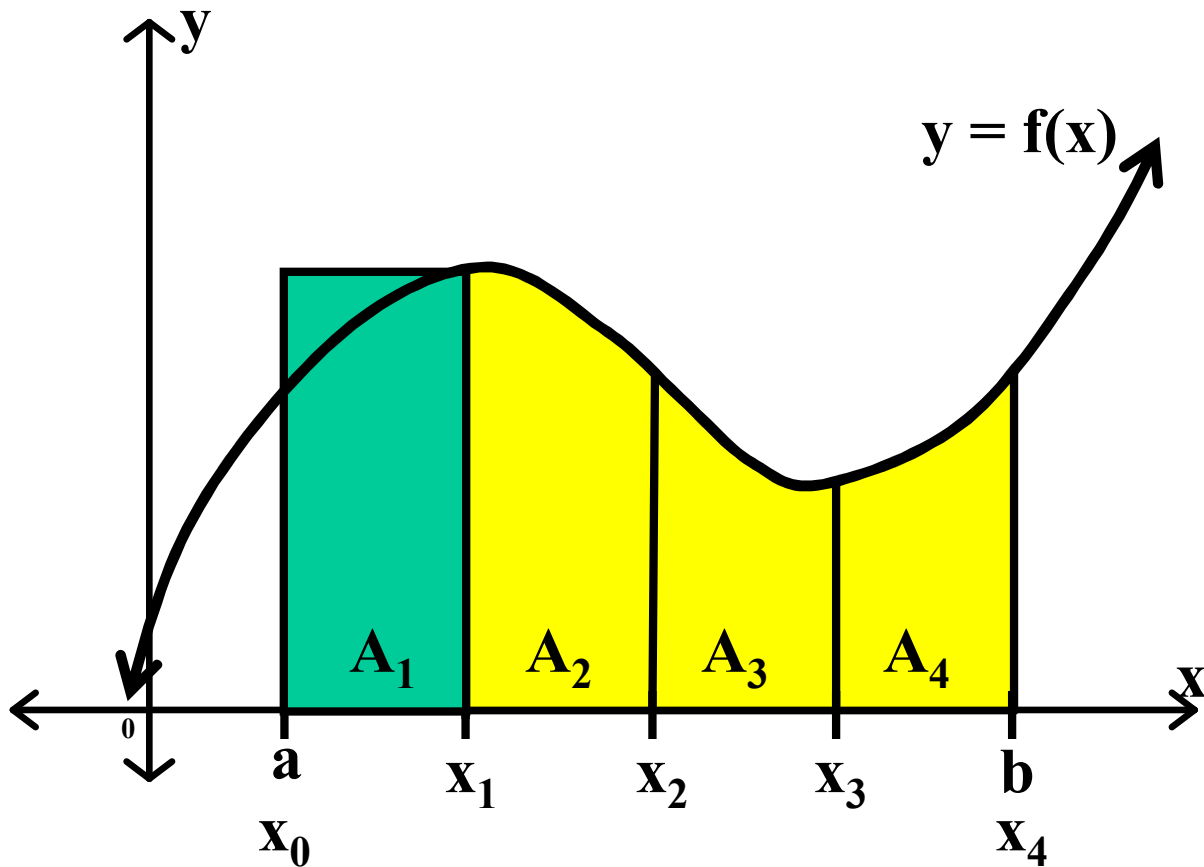
$$S_L = (\sqrt{5} + \sqrt{12.625} + \sqrt{24} + \sqrt{39.875} + \sqrt{61} + \sqrt{88.125})(.5)$$

$$S_L \approx 17.10$$

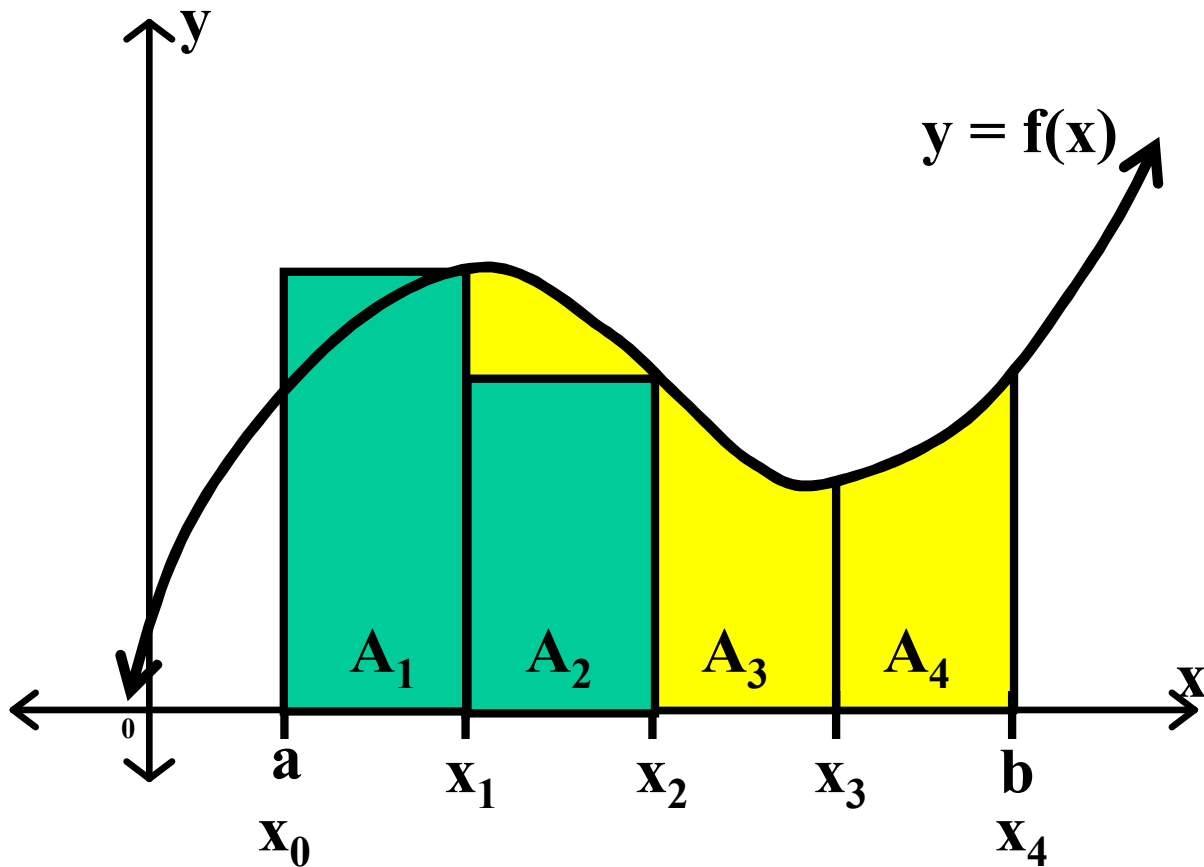




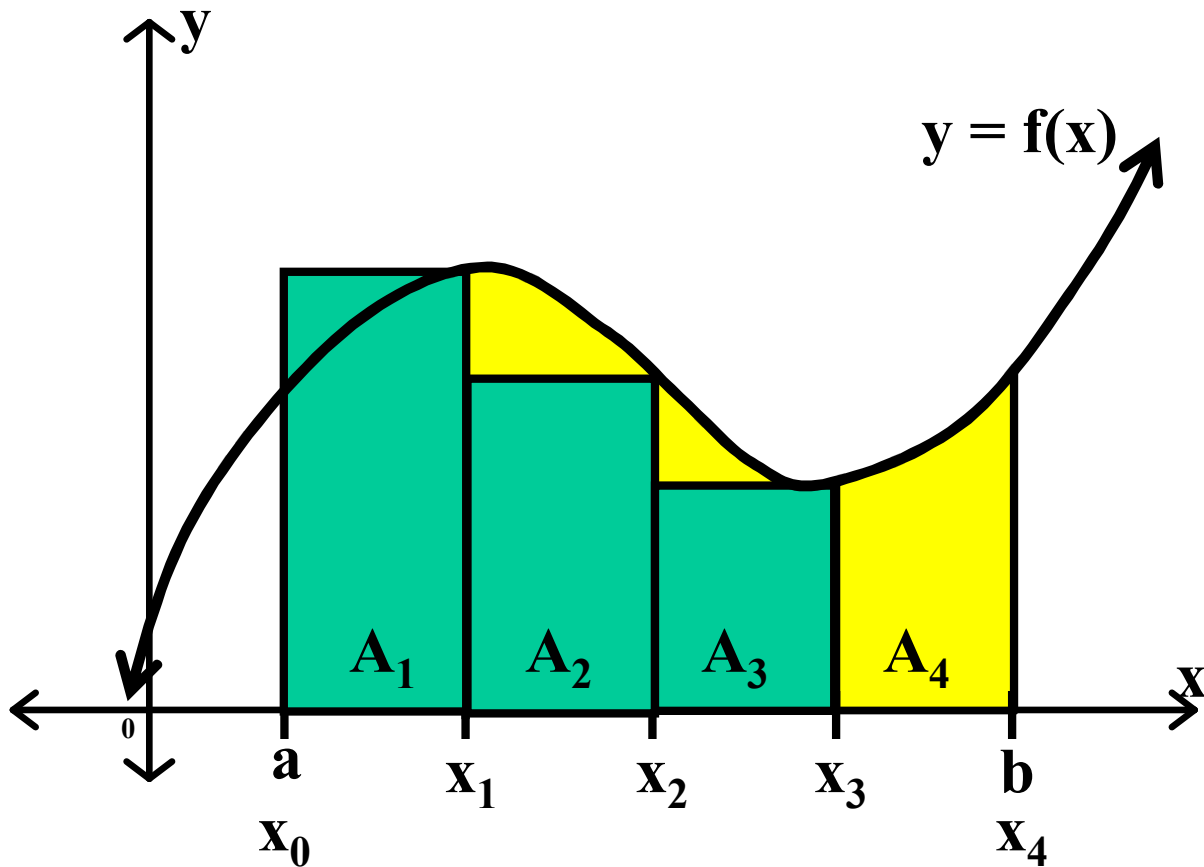
The next 'rectangular' approximation uses the length of the right hand side of each strip as the length of the rectangle.



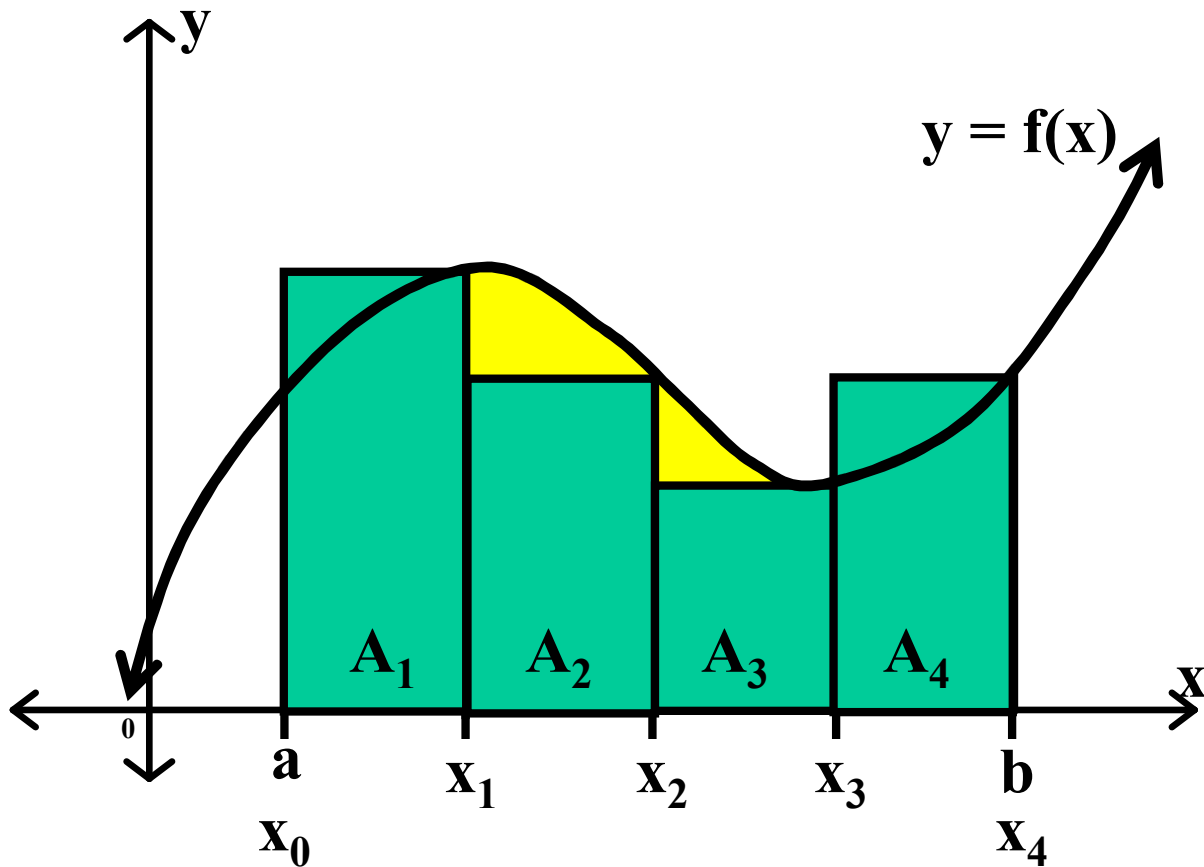
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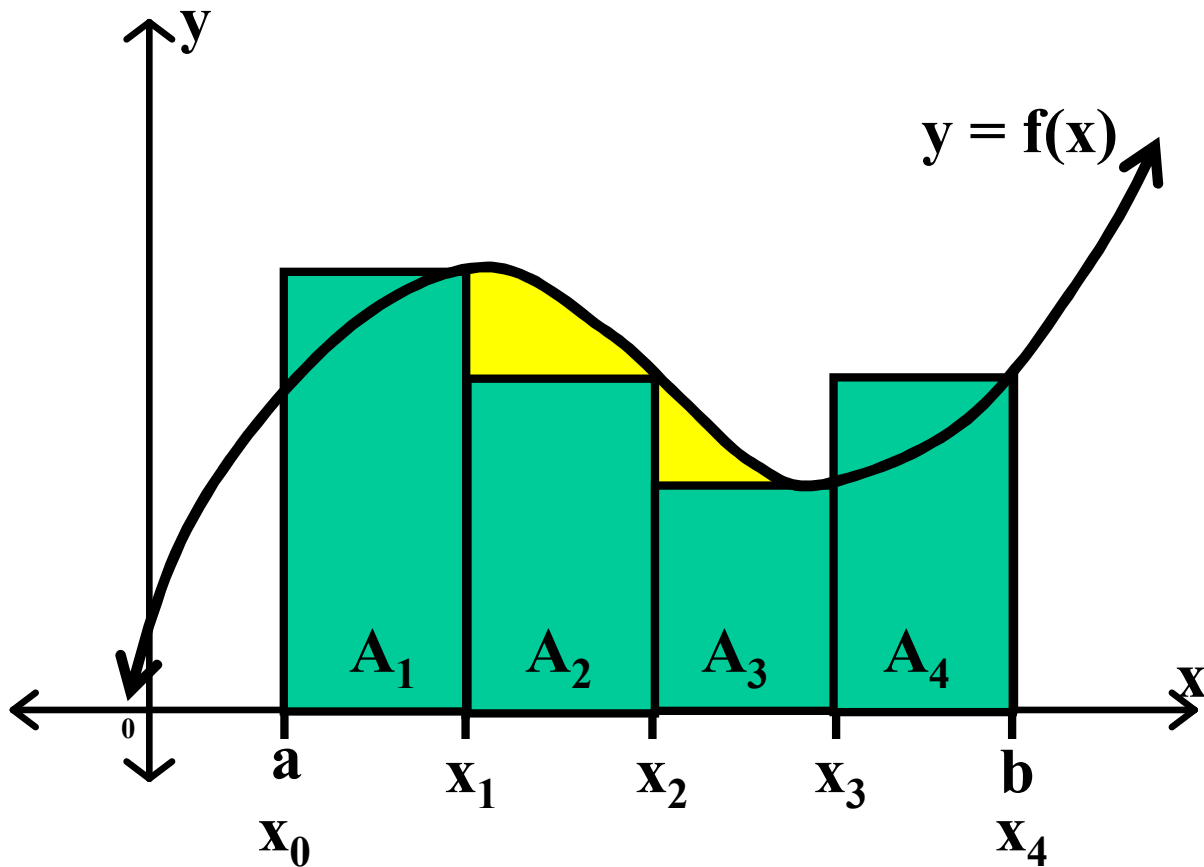
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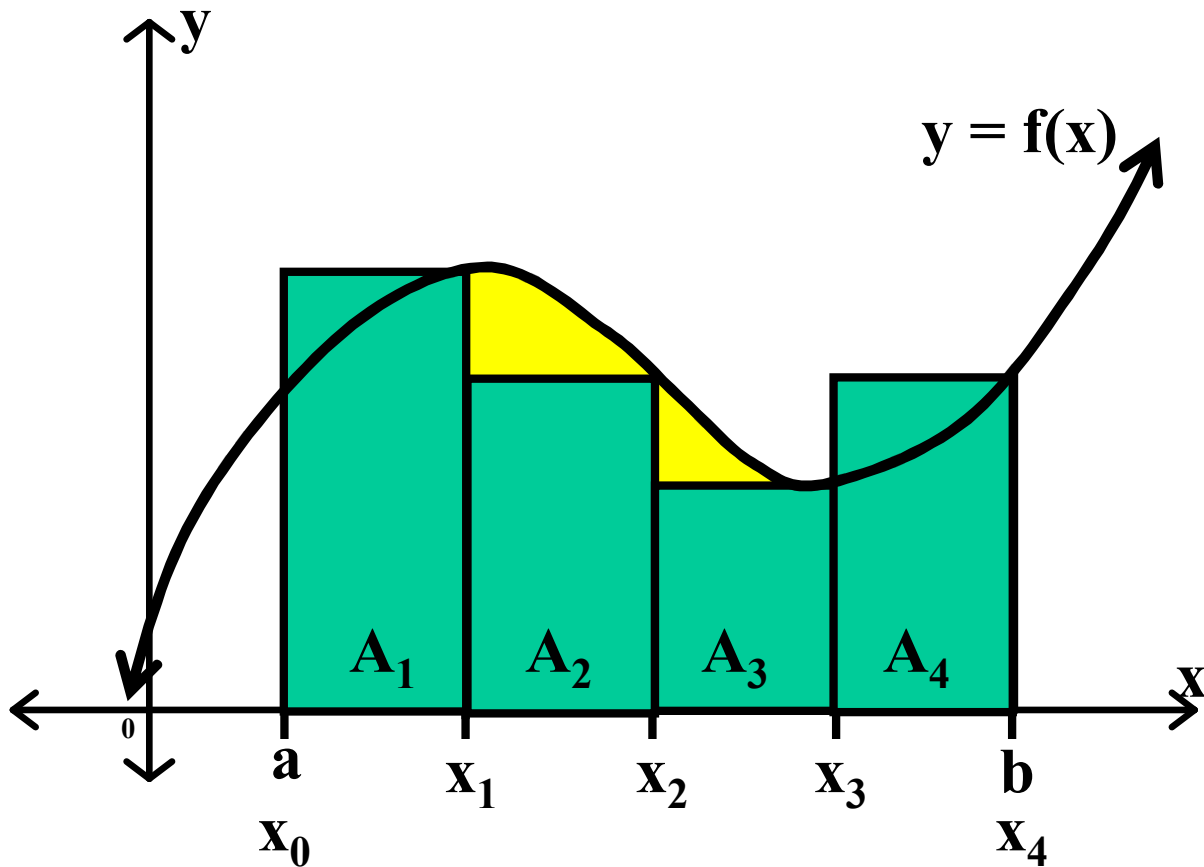
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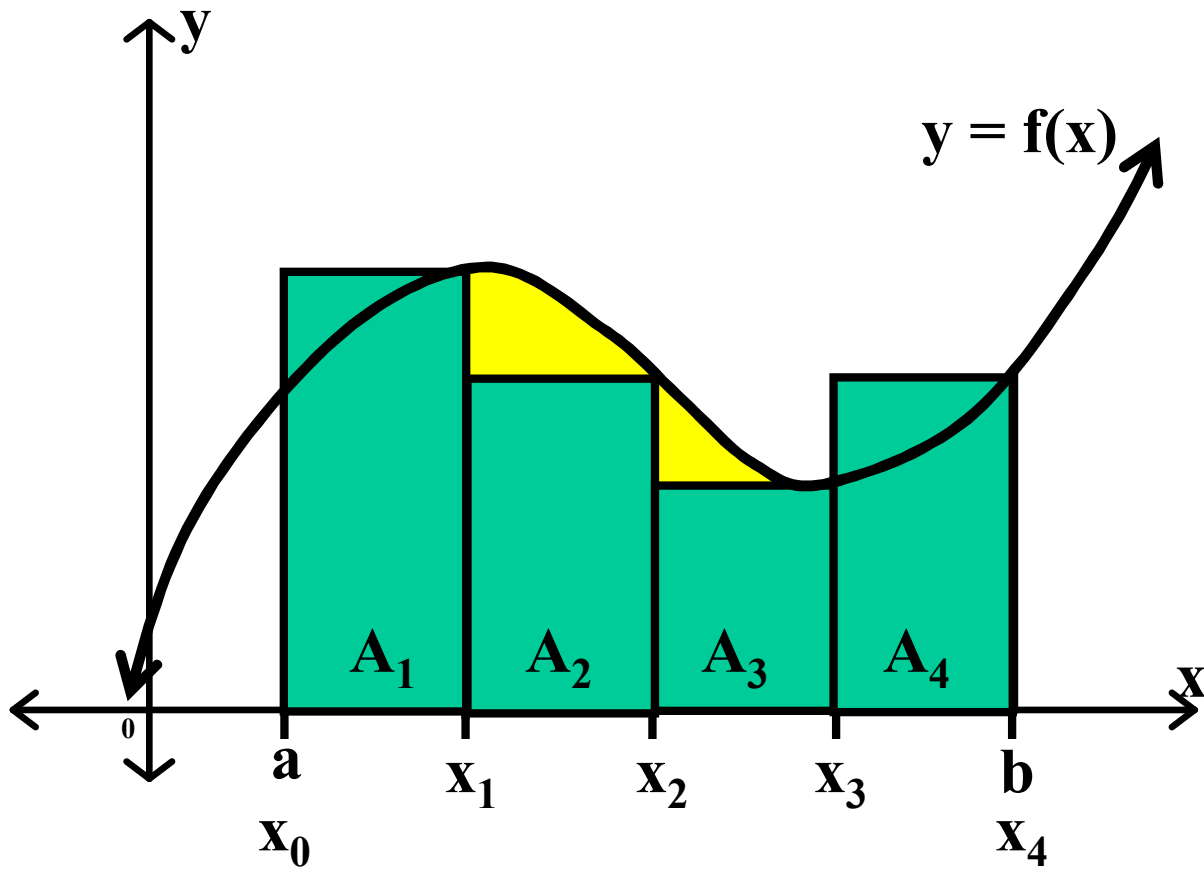
The next ‘rectangular’ approximation uses the length of the right hand side of each strip as the length of the rectangle.

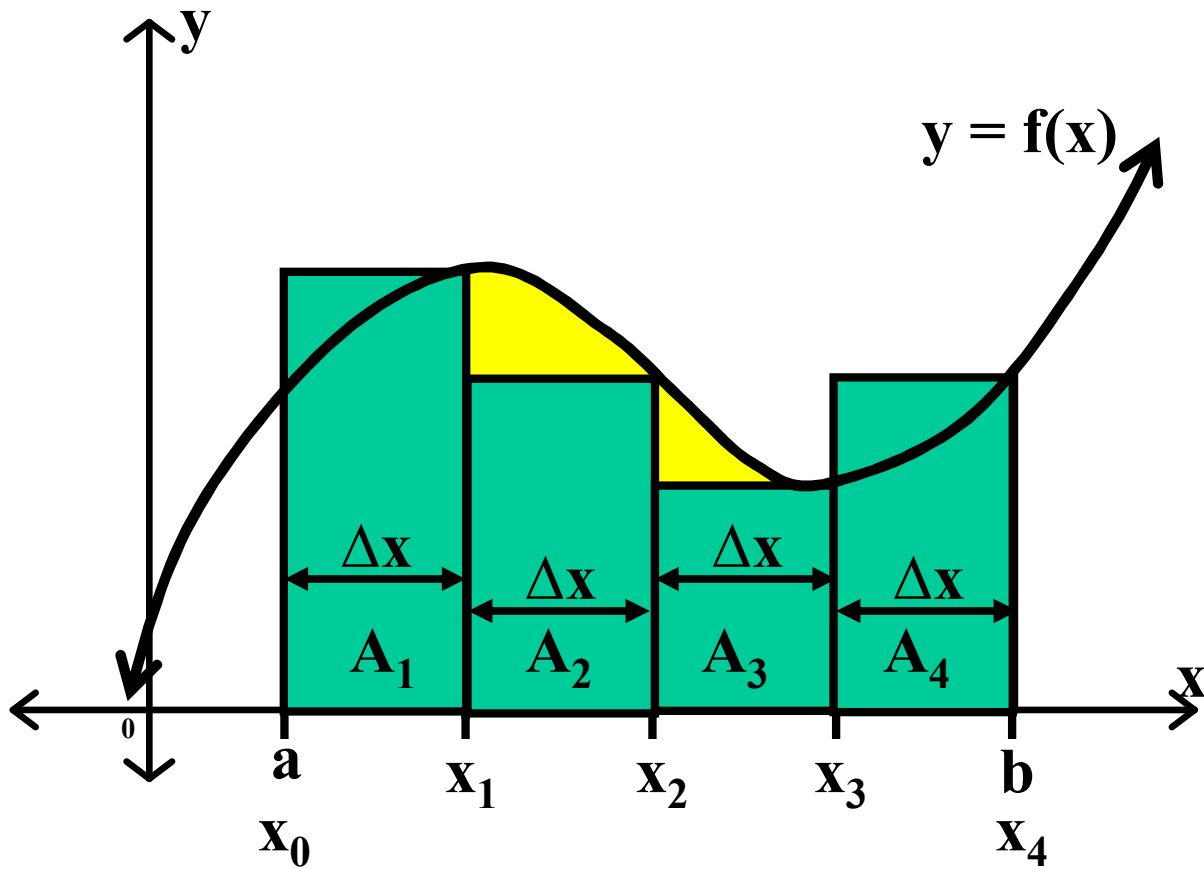


The next ‘rectangular’ approximation uses the length of the right hand side of each strip as the length of the rectangle. This is called the ‘right rectangular’ approximation.

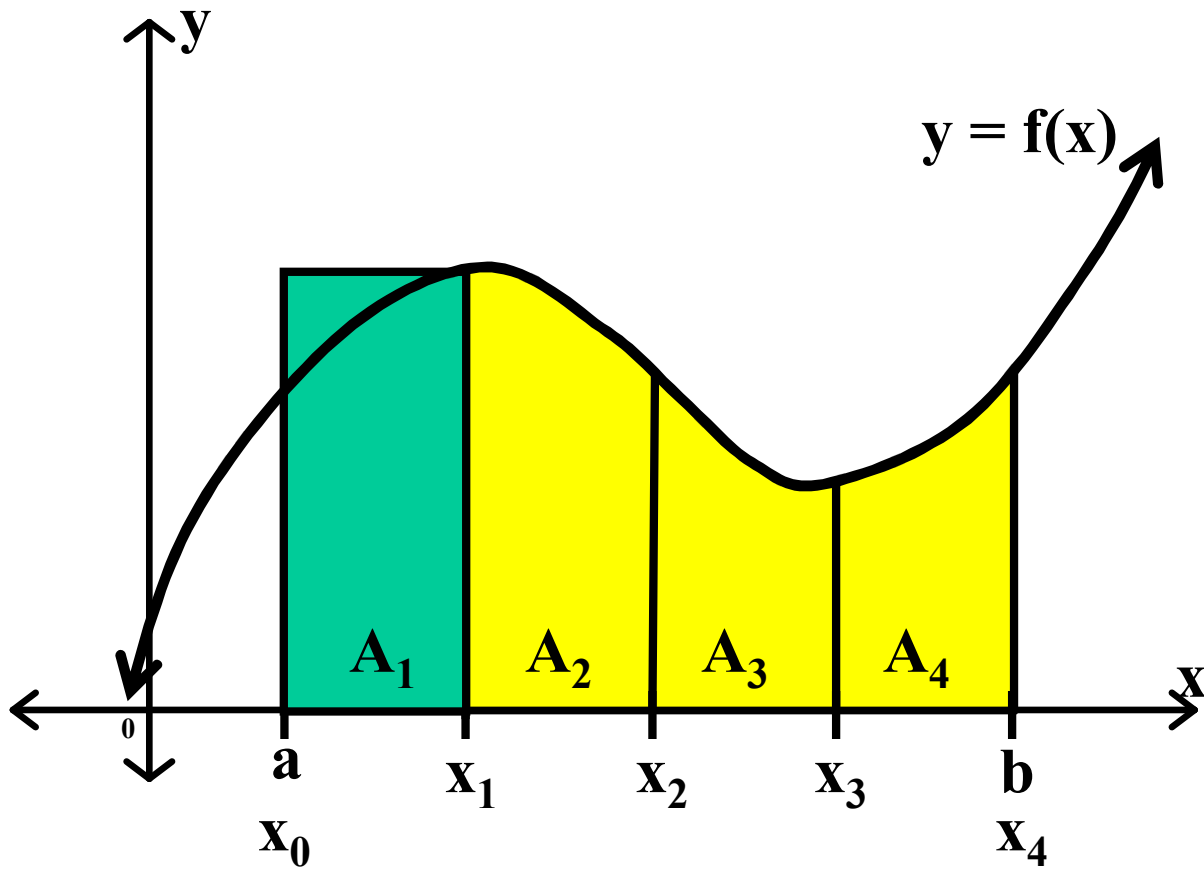


The next ‘rectangular’ approximation uses the length of the right hand side of each strip as the length of the rectangle. This is called the ‘right rectangular’ approximation, S_R .

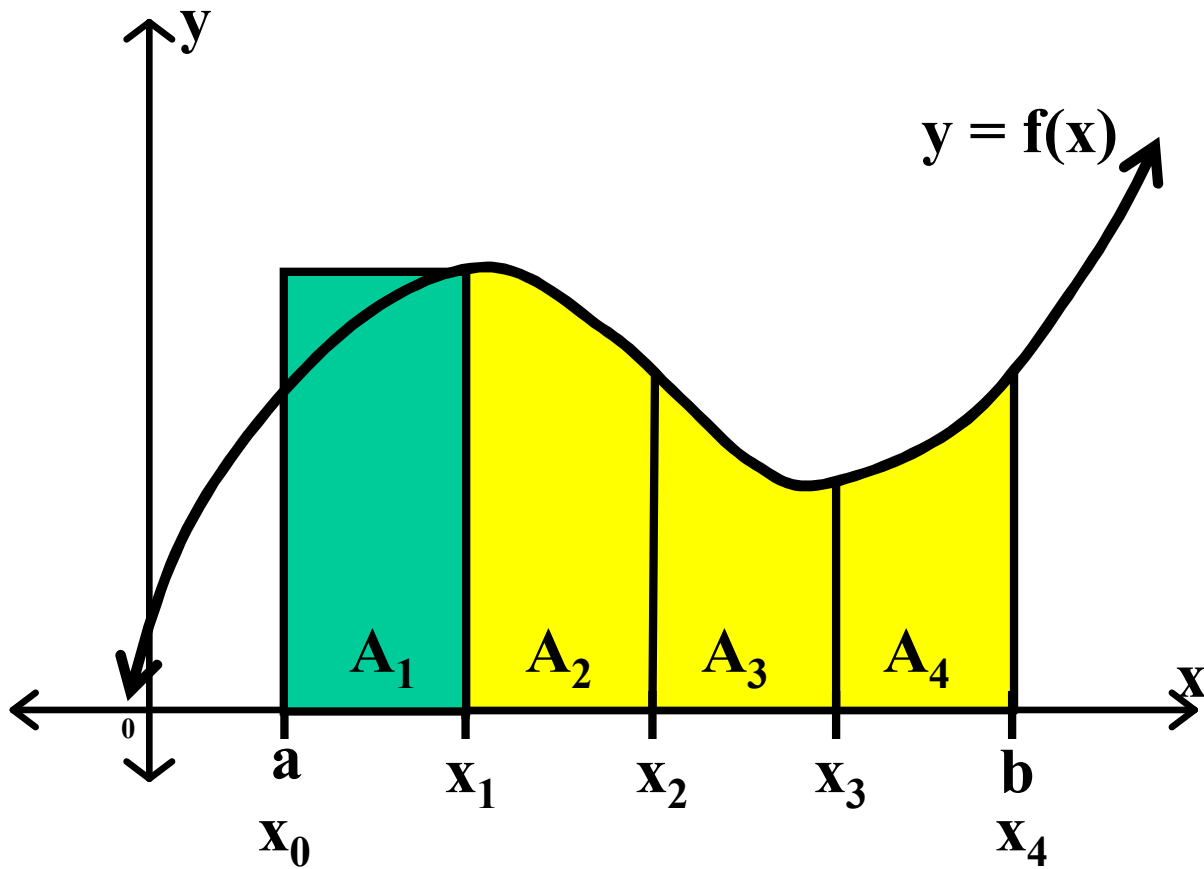




The width of each rectangle is Δx .

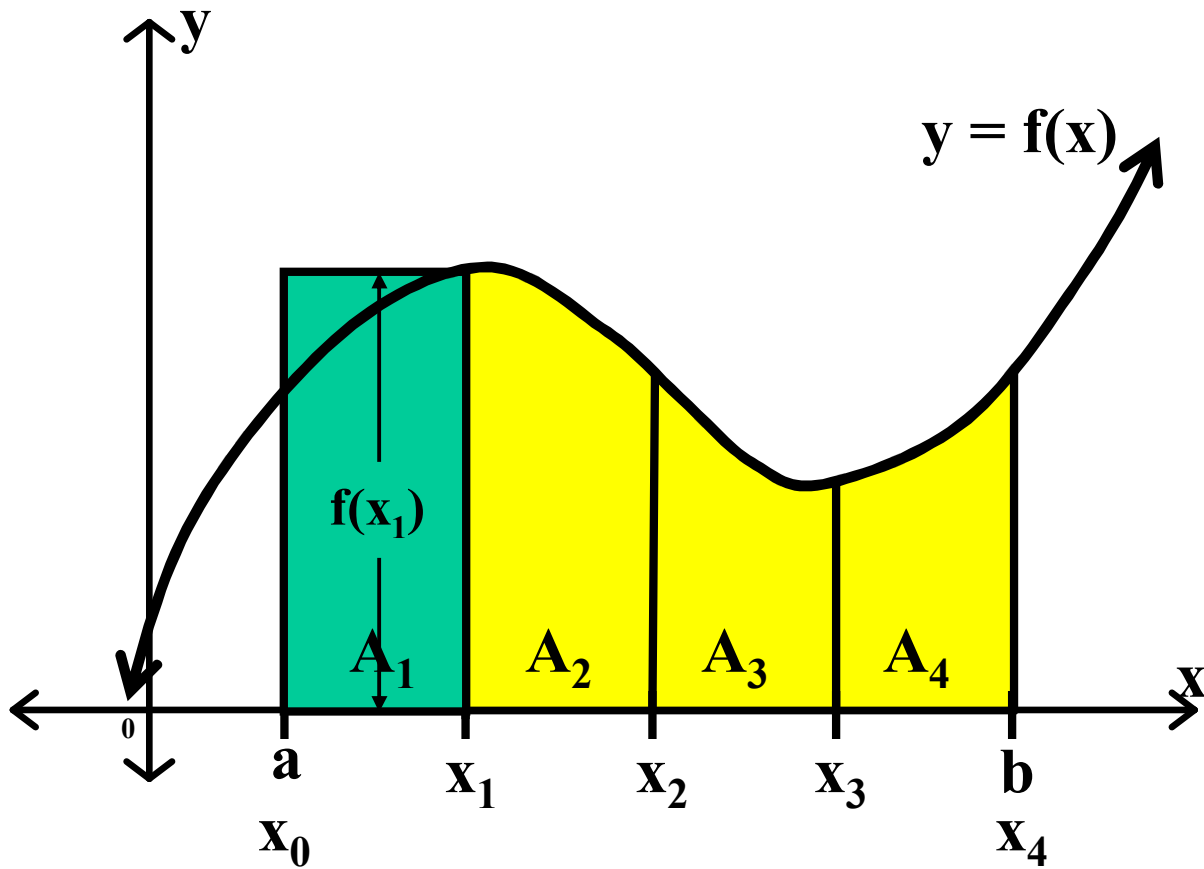


The width of each rectangle is Δx .



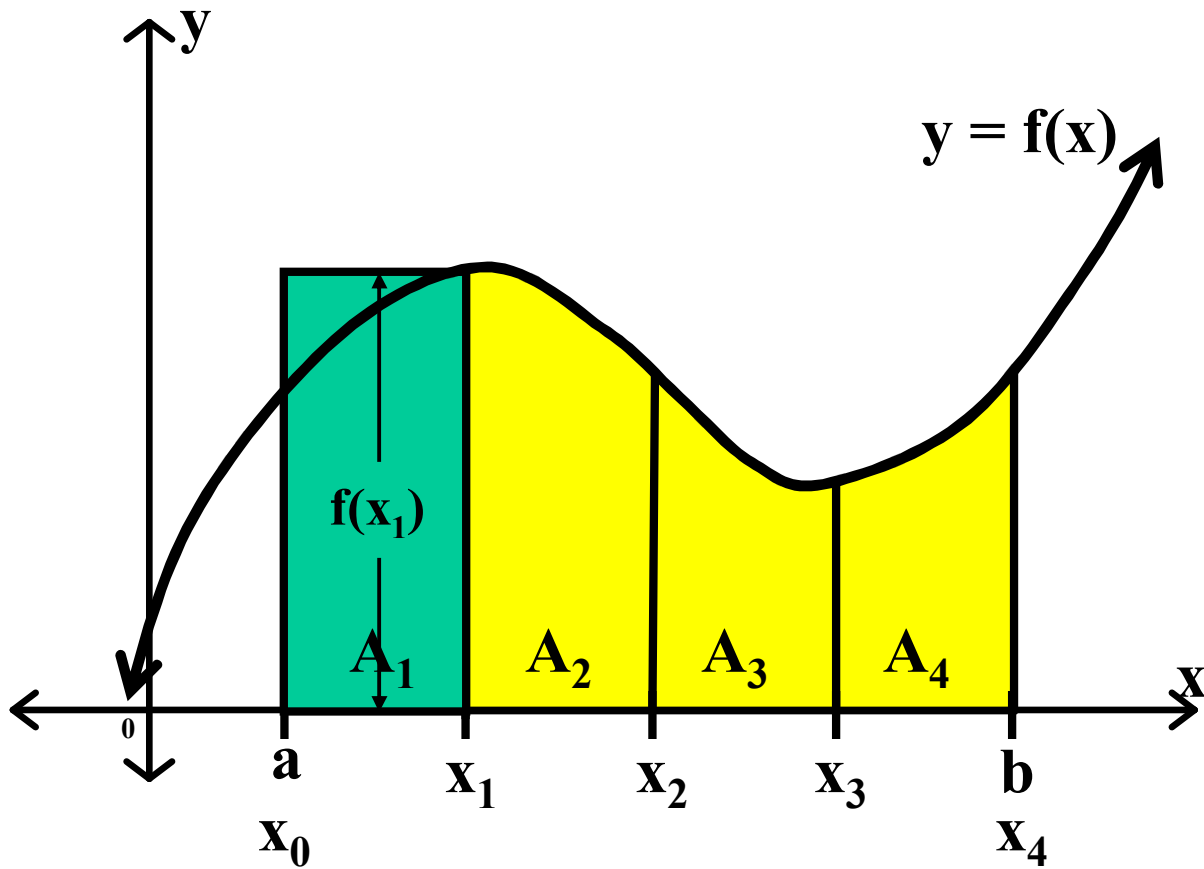
The width of each rectangle is Δx .

$$A_1 \approx$$



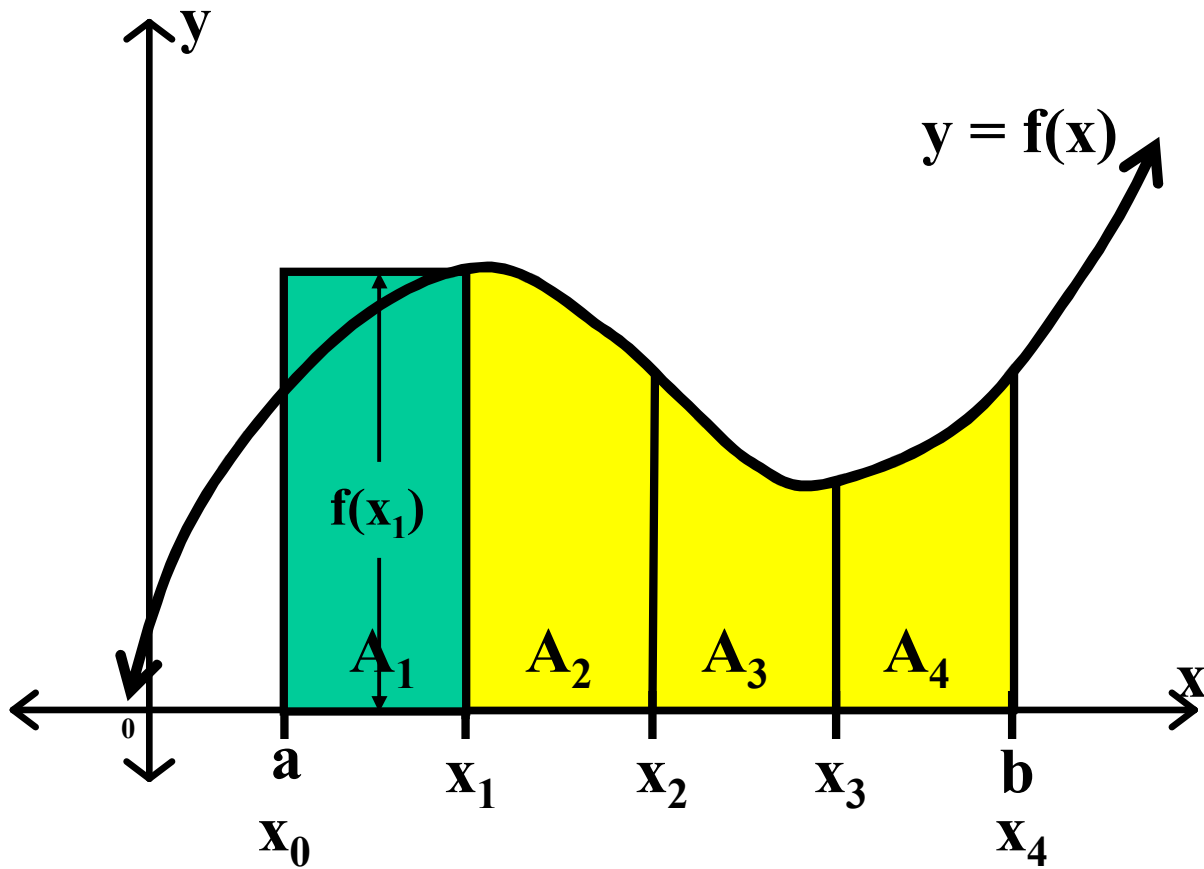
The width of each rectangle is Δx .

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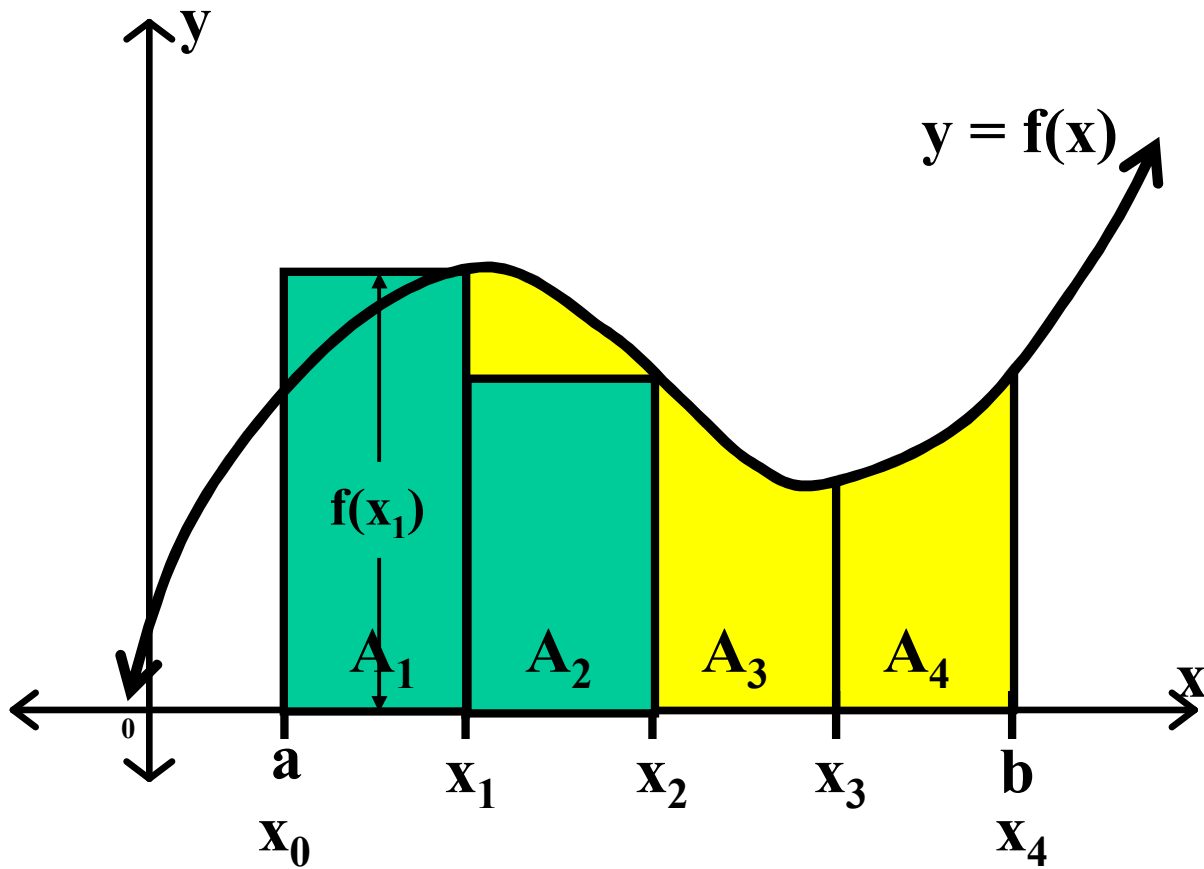
The width of each rectangle is Δx .

$$A_1 \approx f(x_1) \Delta x$$



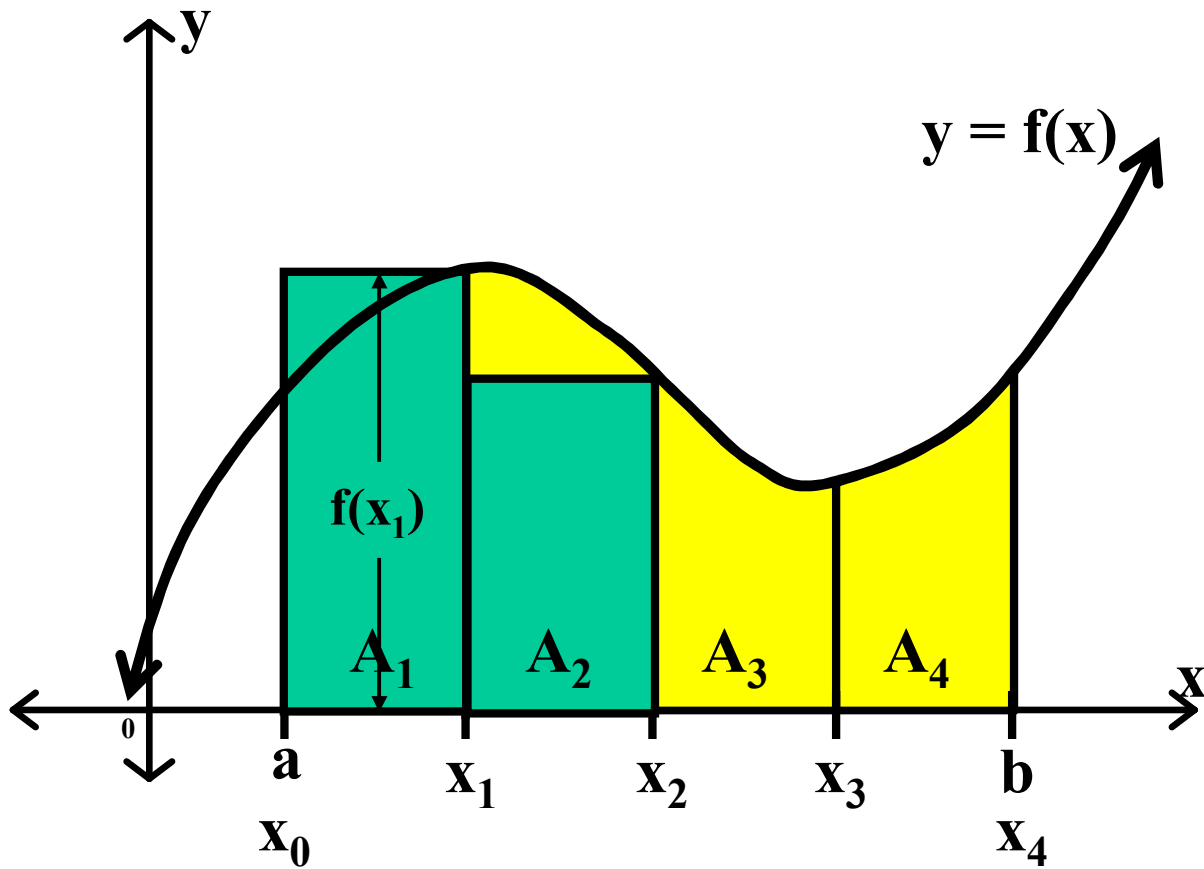
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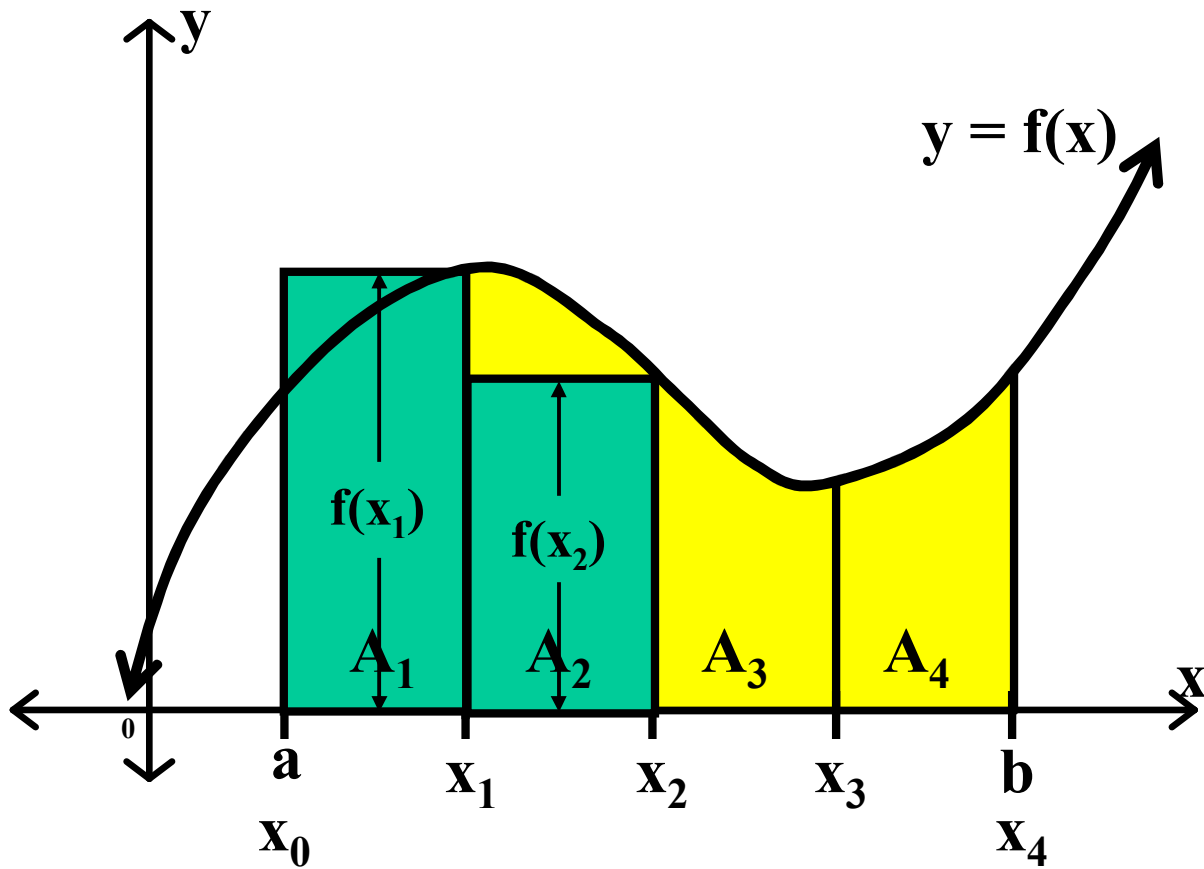
The width of each rectangle is Δx .

$$A_1 \approx f(x_1)\Delta x$$



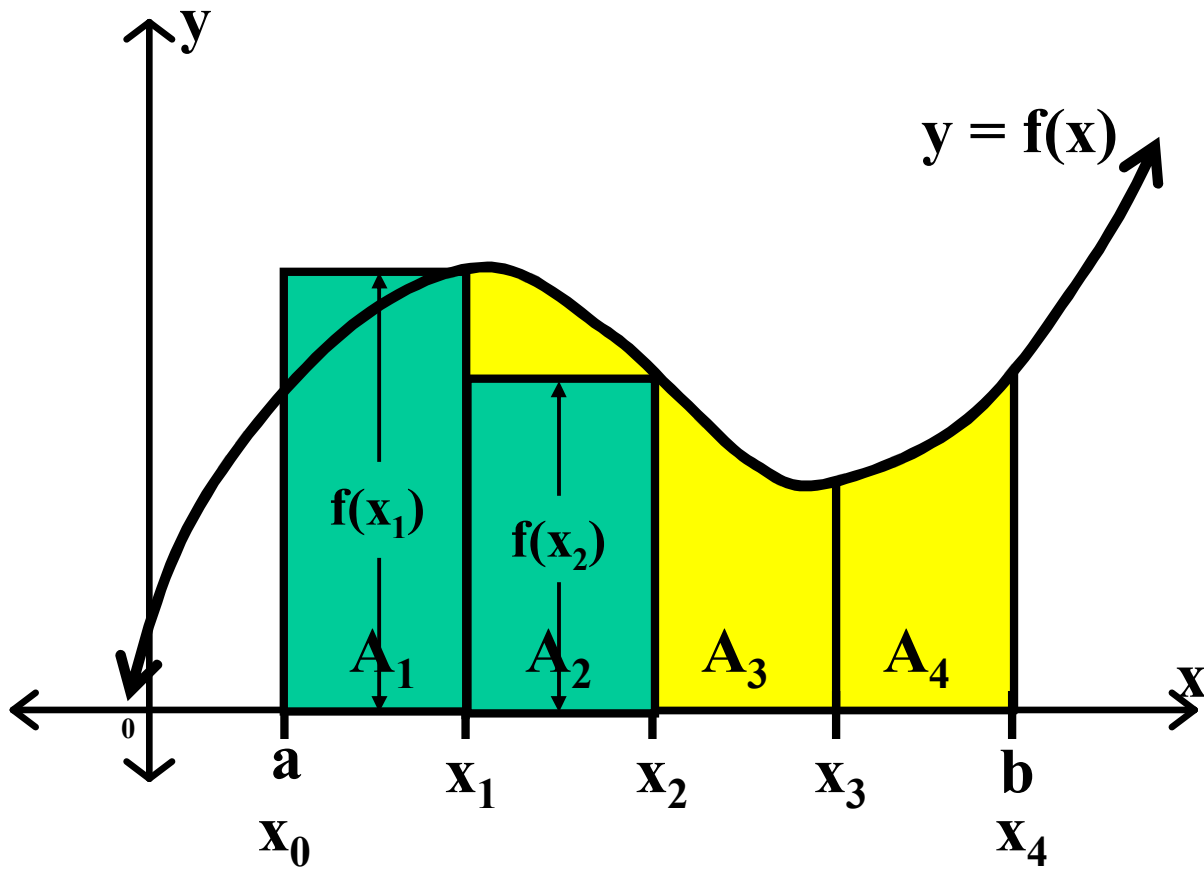
The width of each rectangle is Δx .

$$A_1 \approx f(x_1)\Delta x \quad A_2 \approx$$



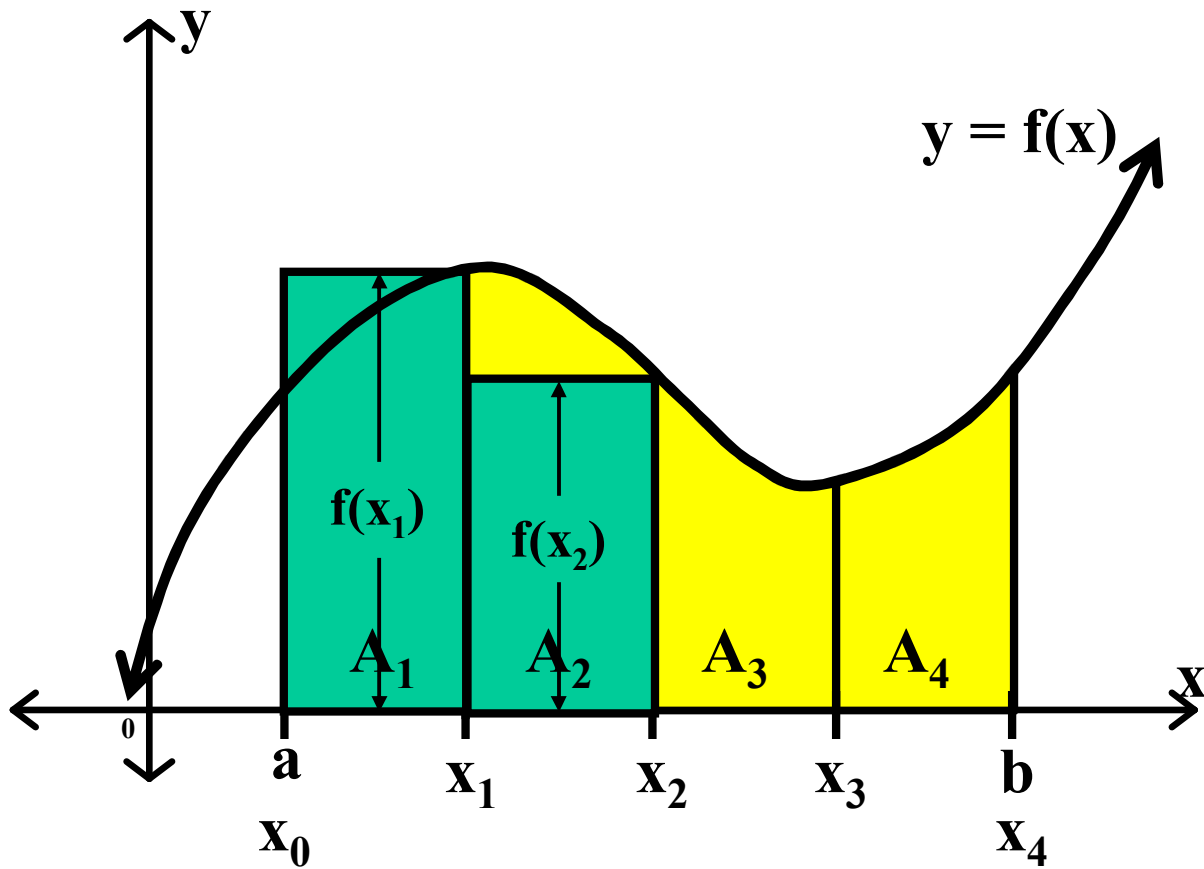
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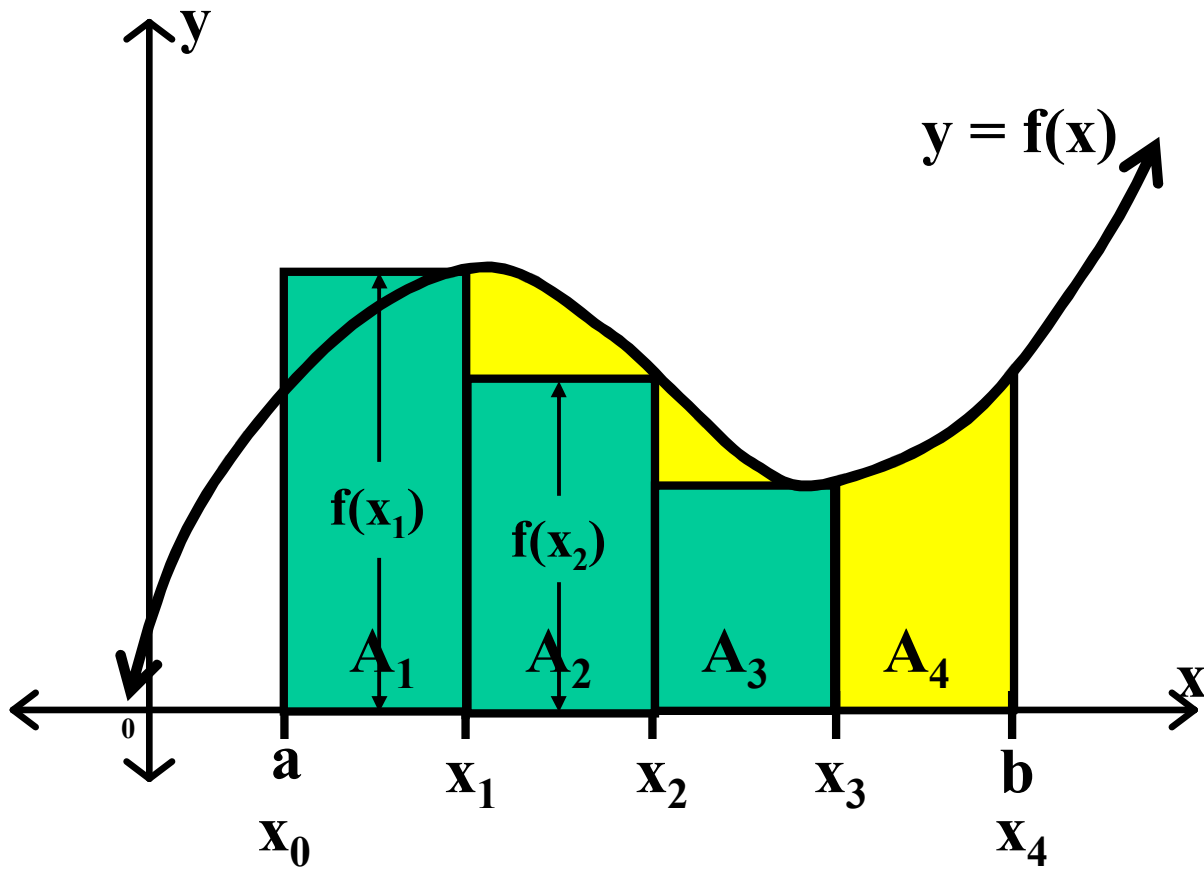
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$$A_1 \approx f(x_1)\Delta x \quad A_2 \approx f(x_2)\Delta x$$



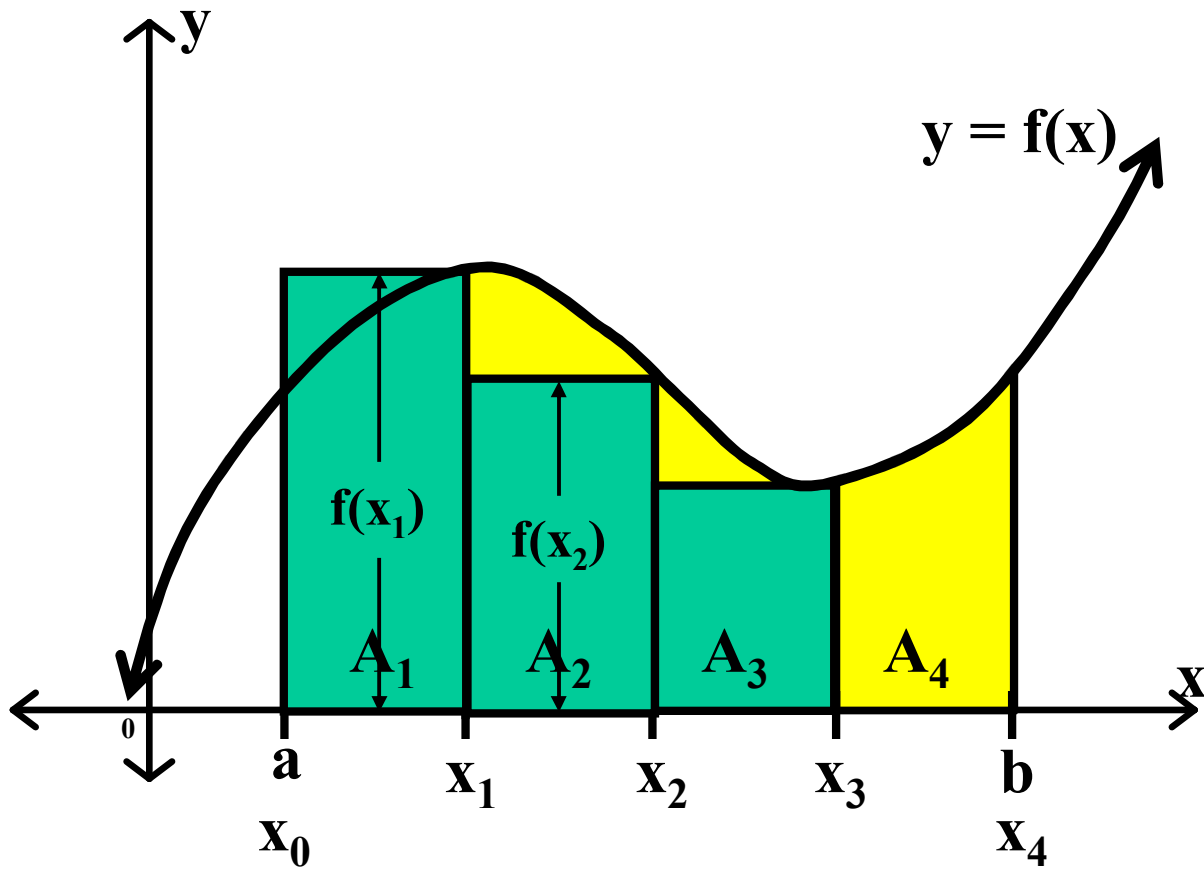
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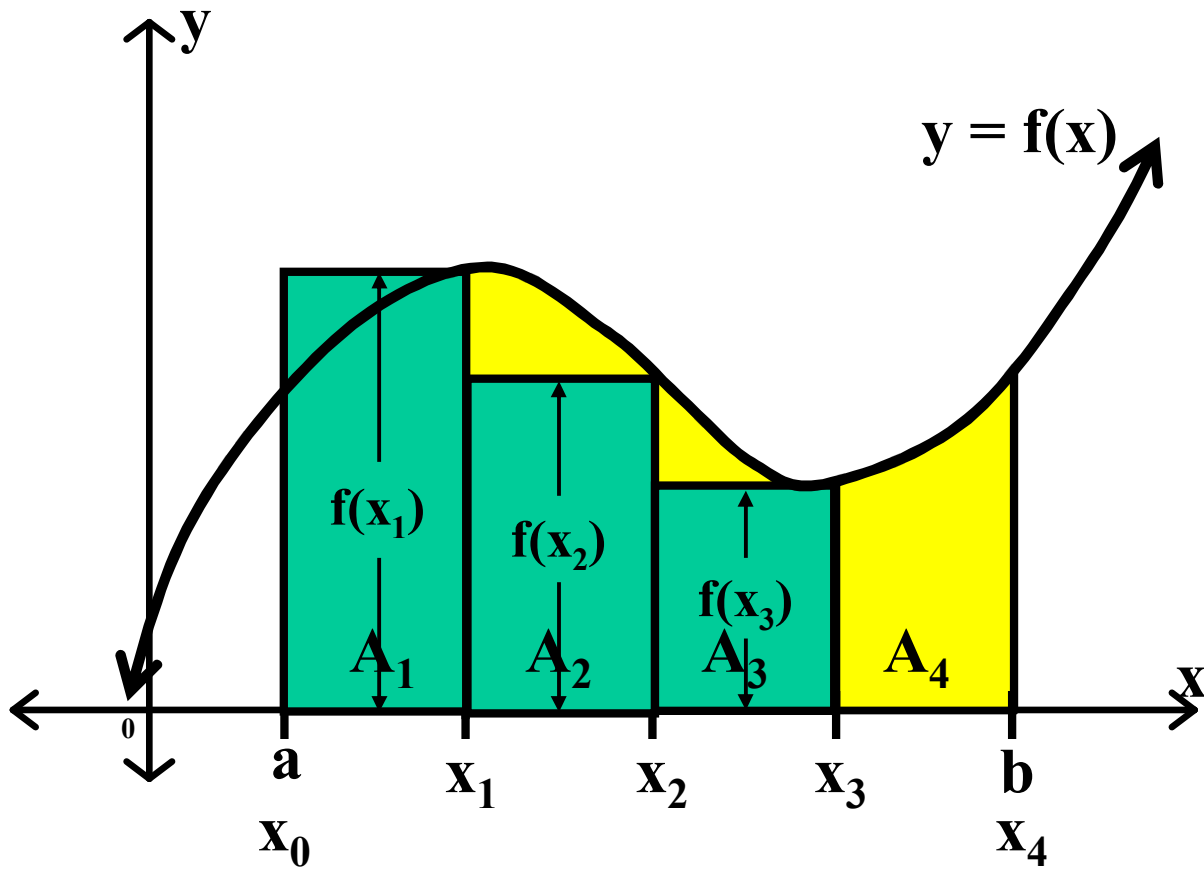
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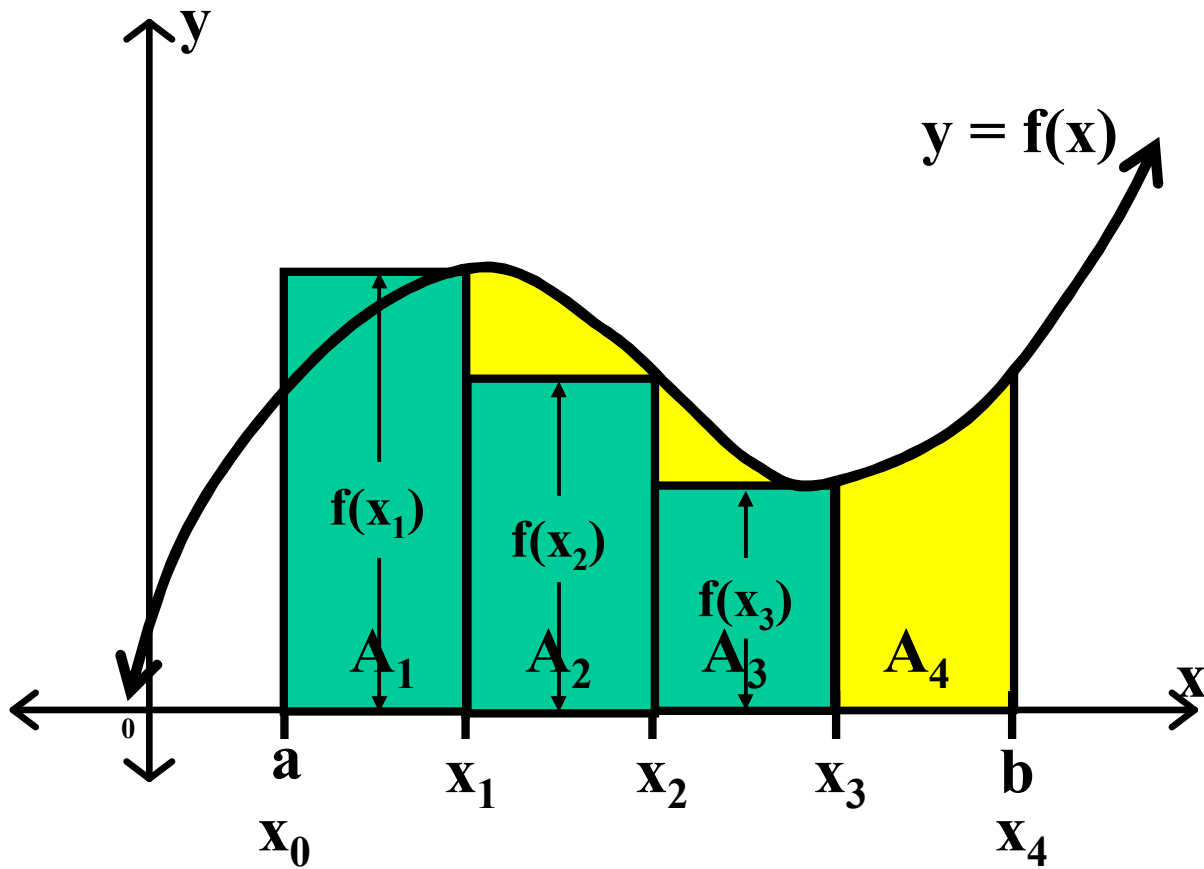
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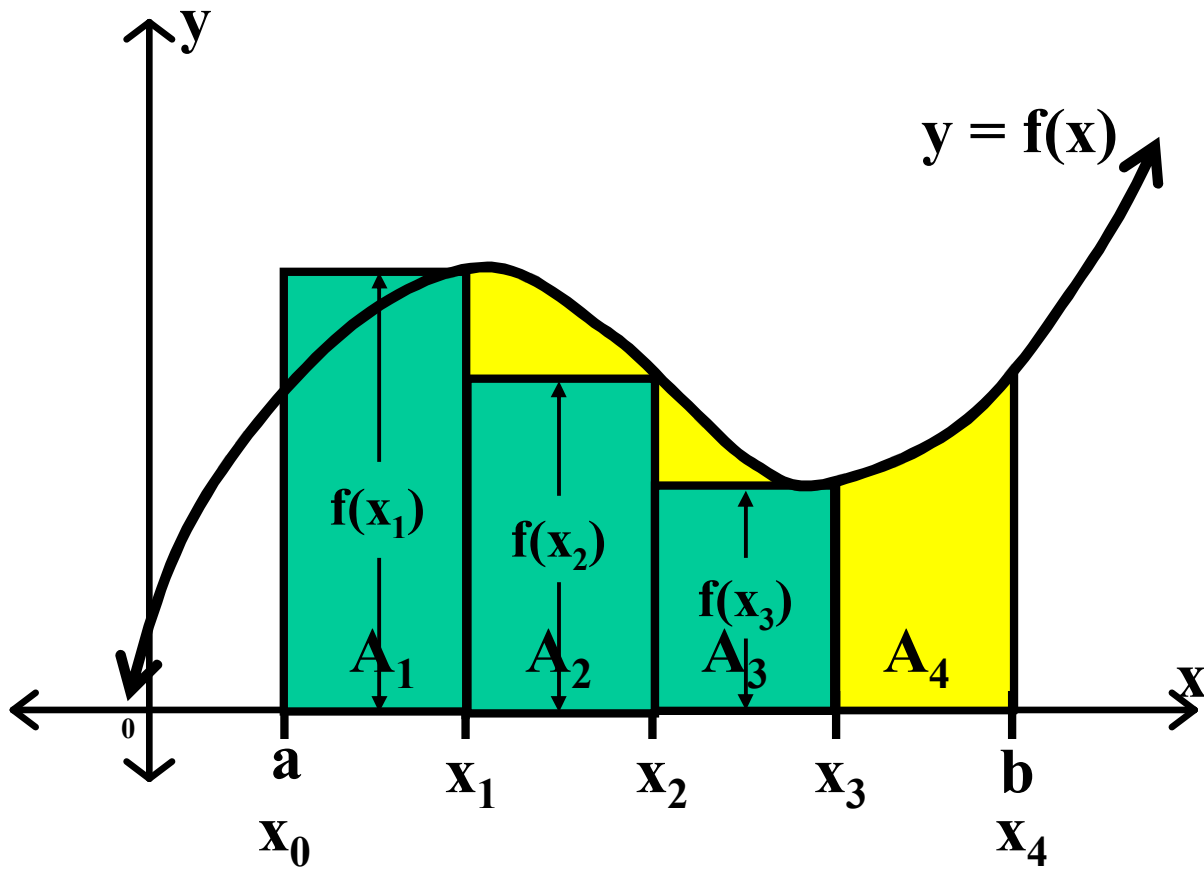
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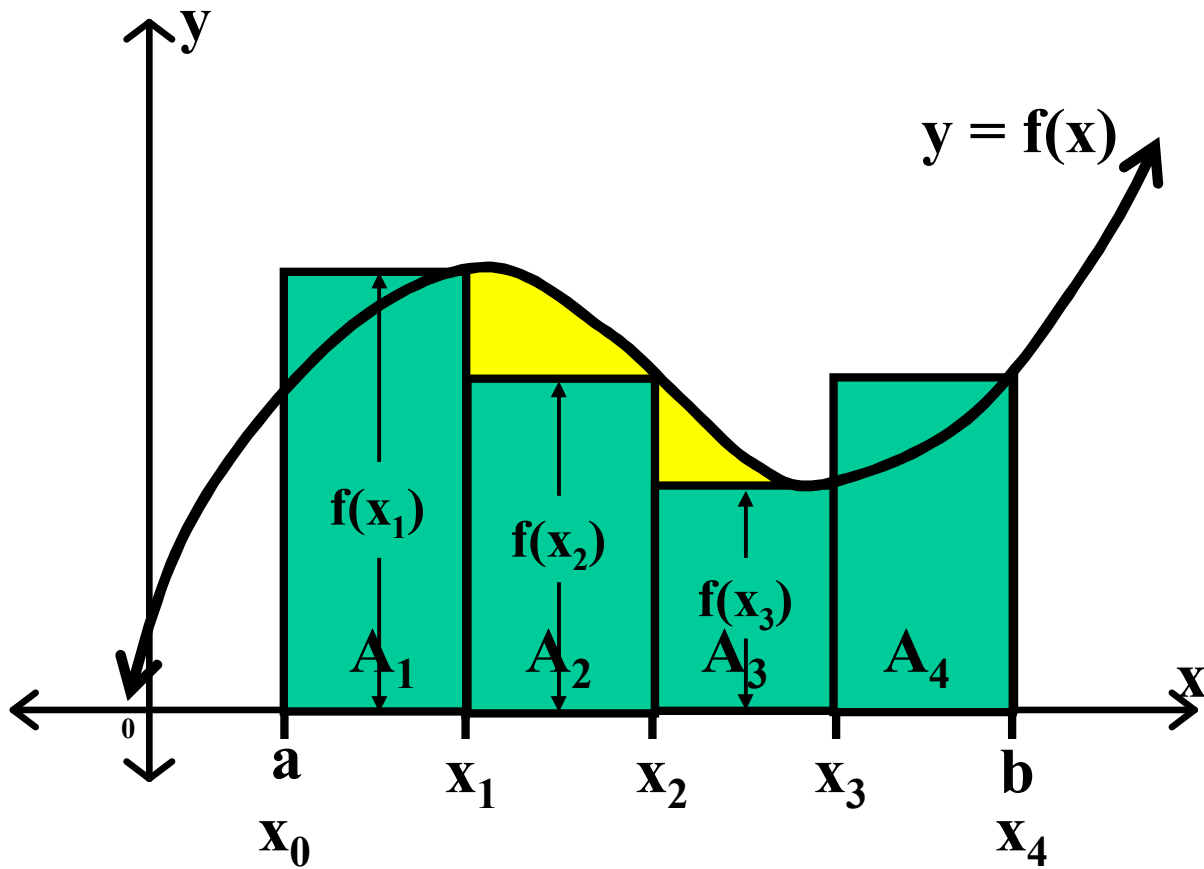
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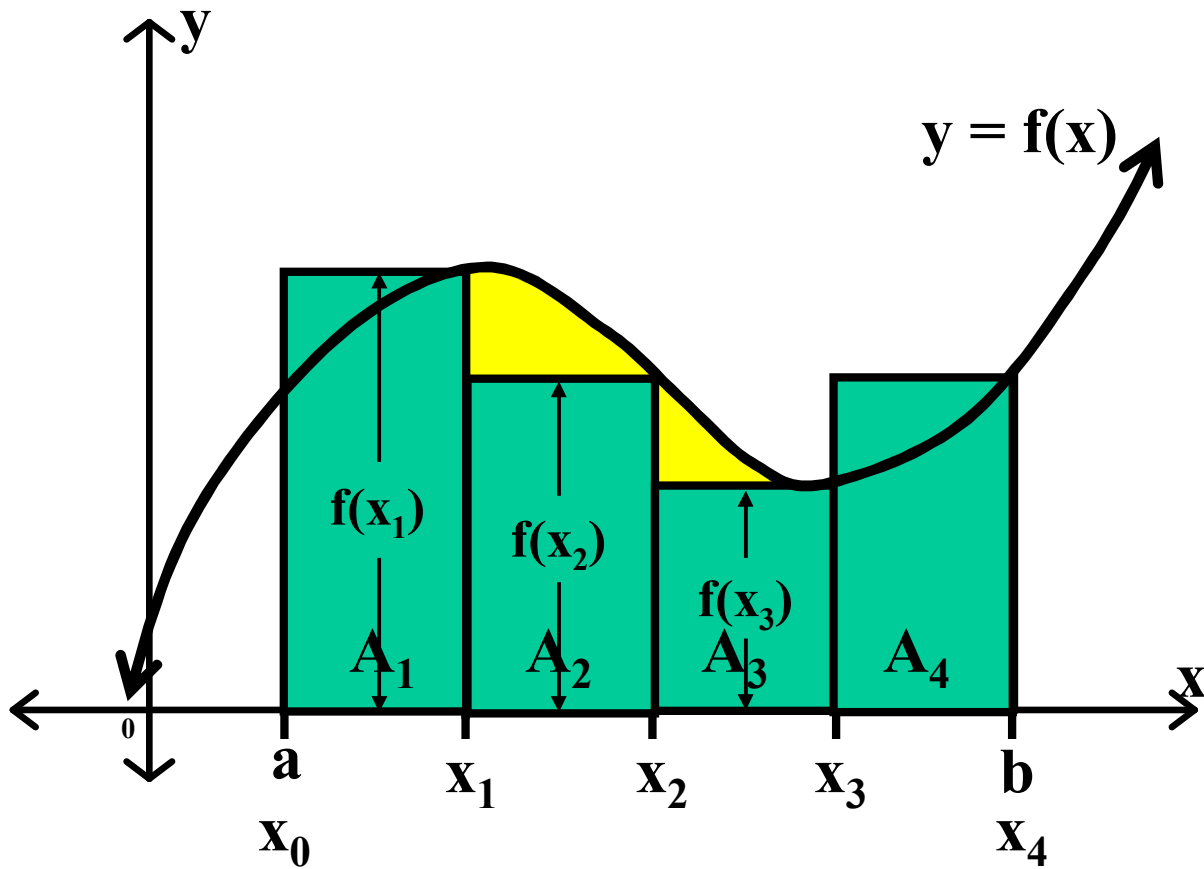
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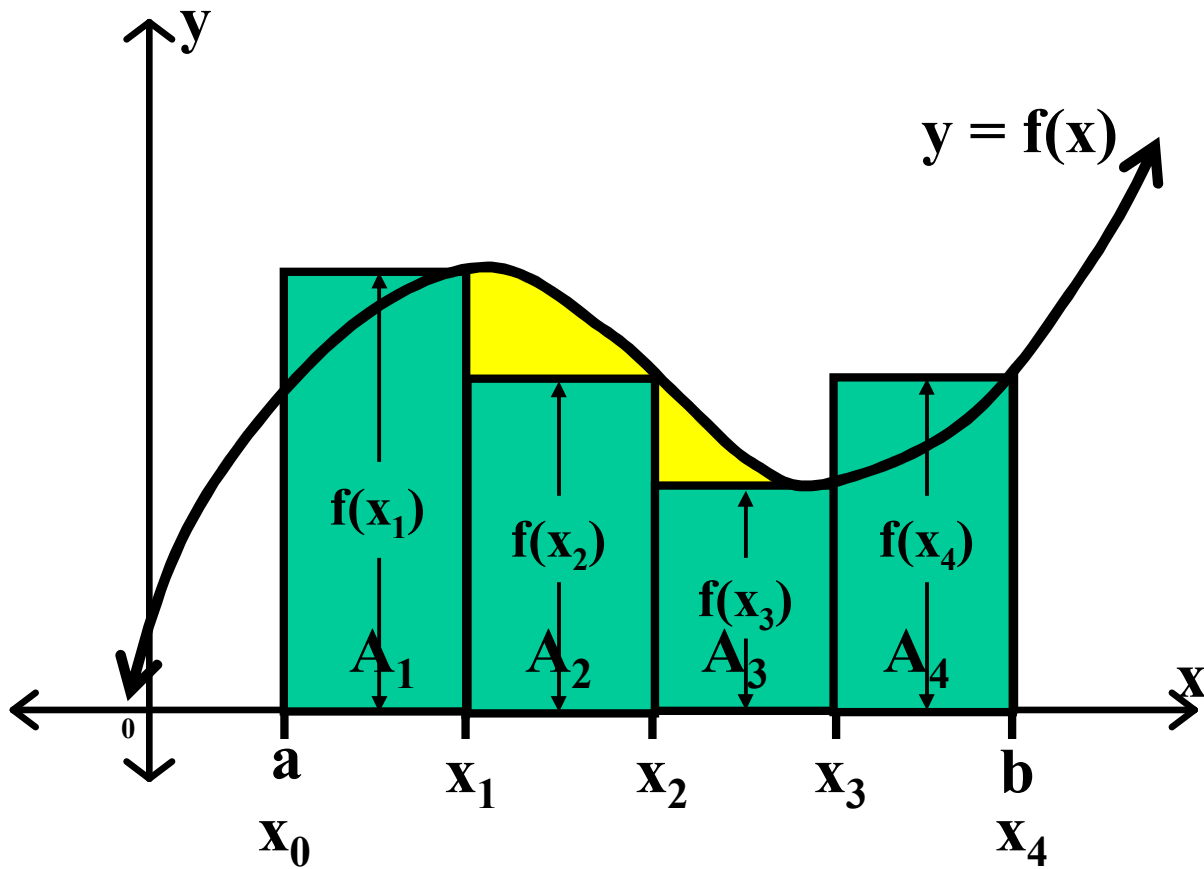
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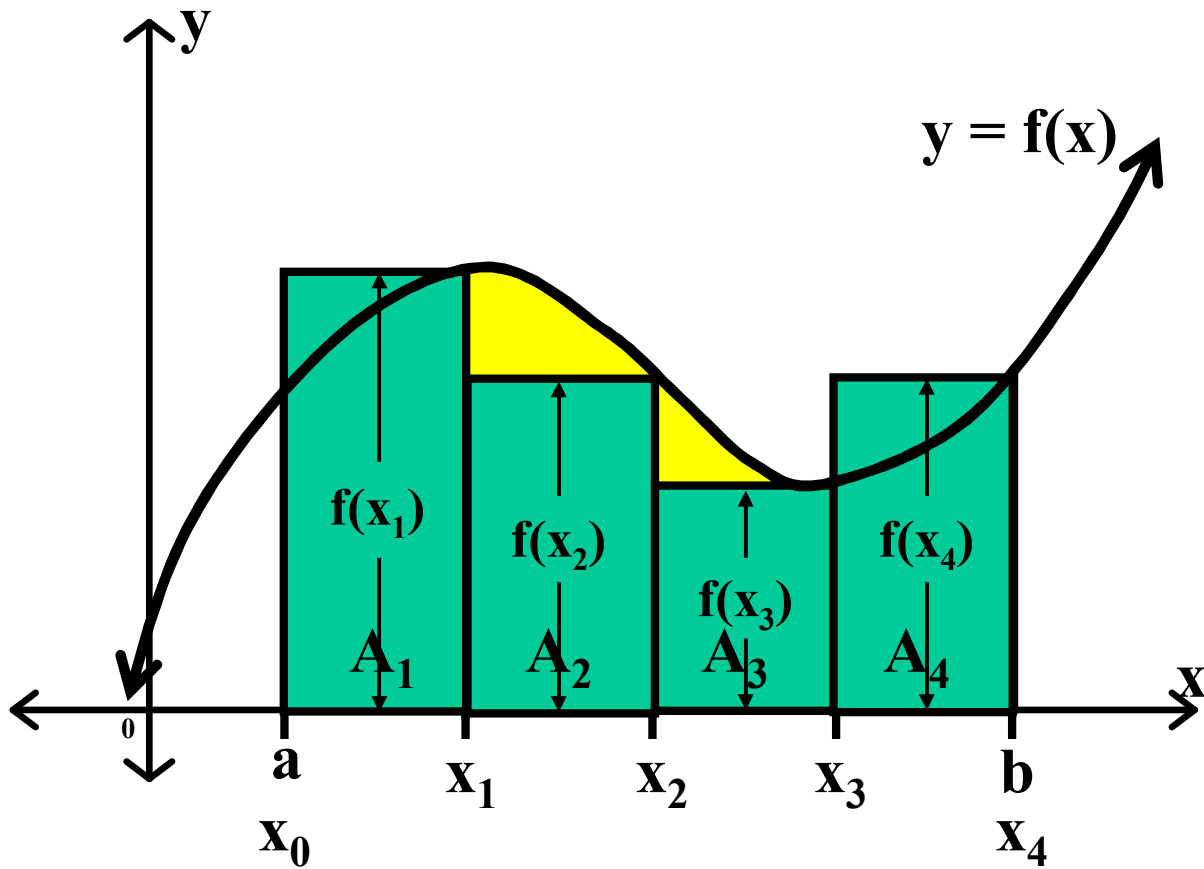
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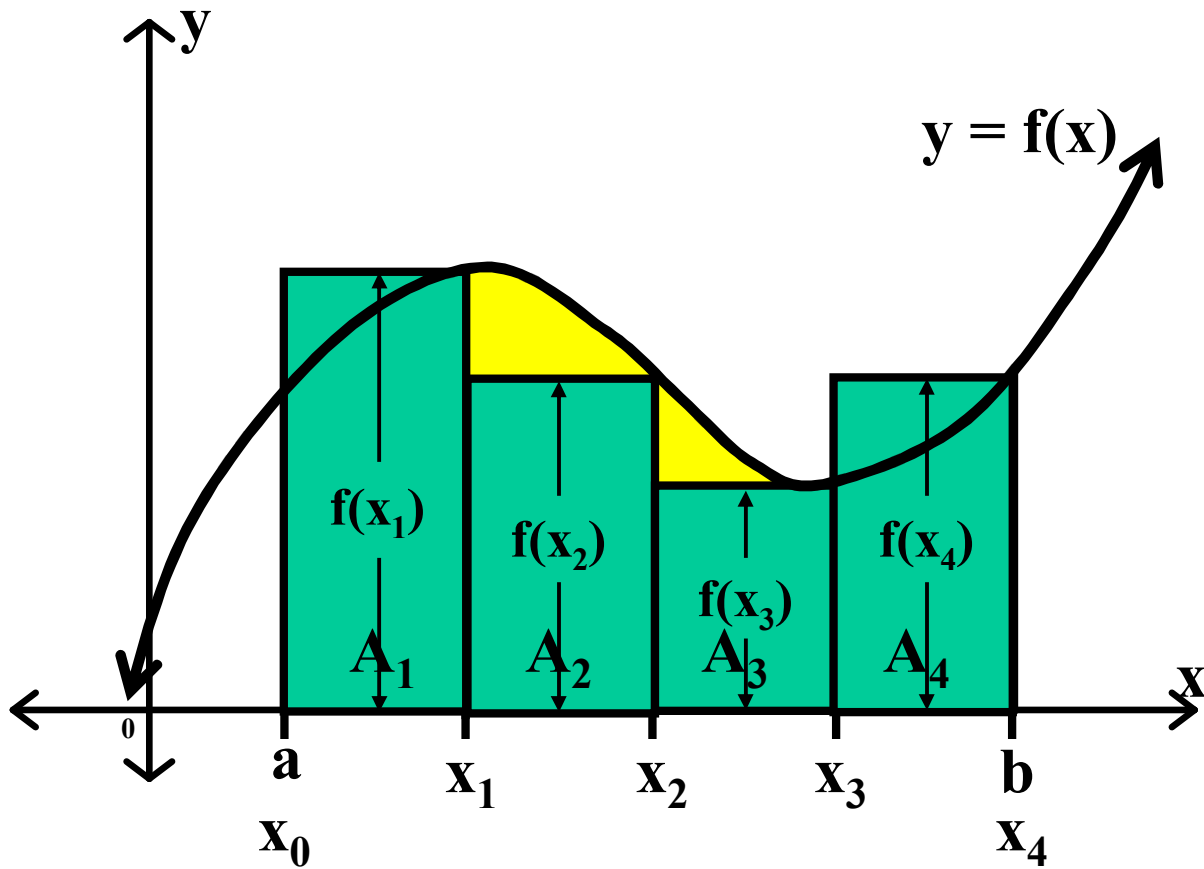
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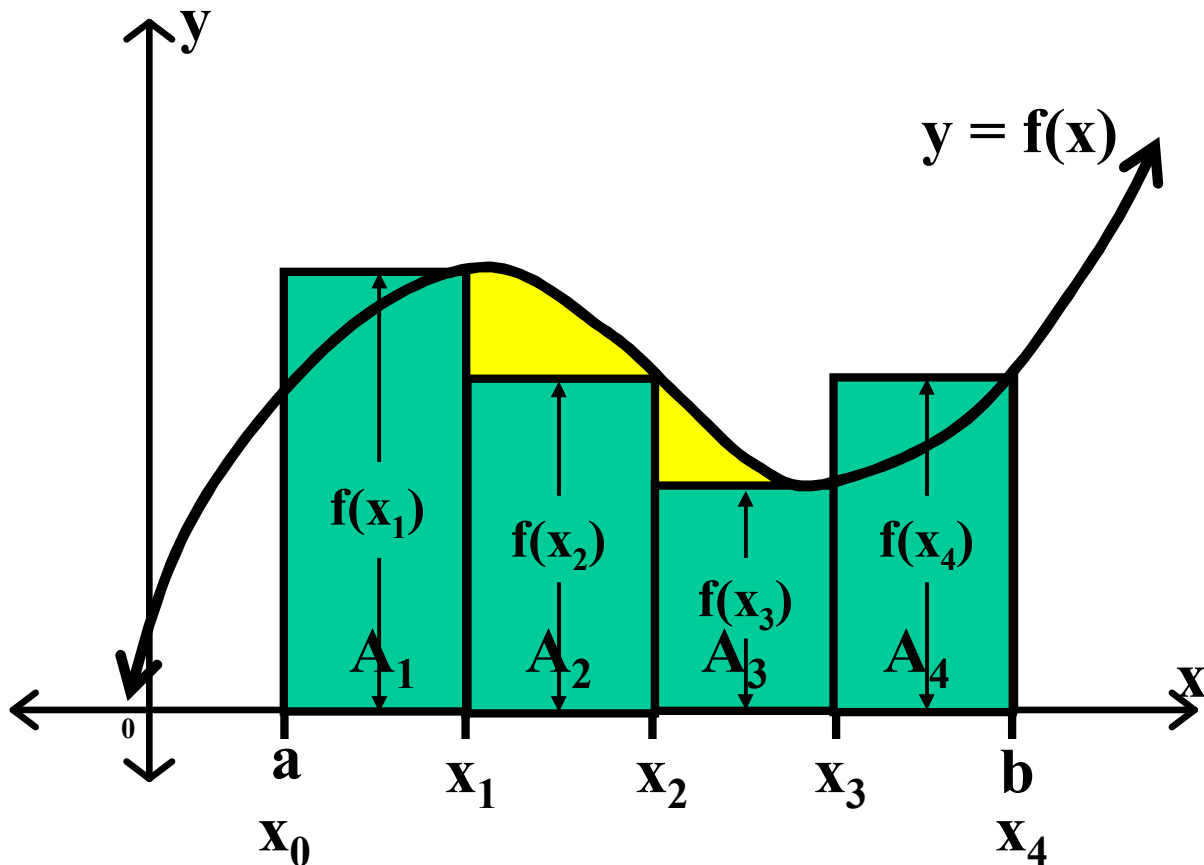
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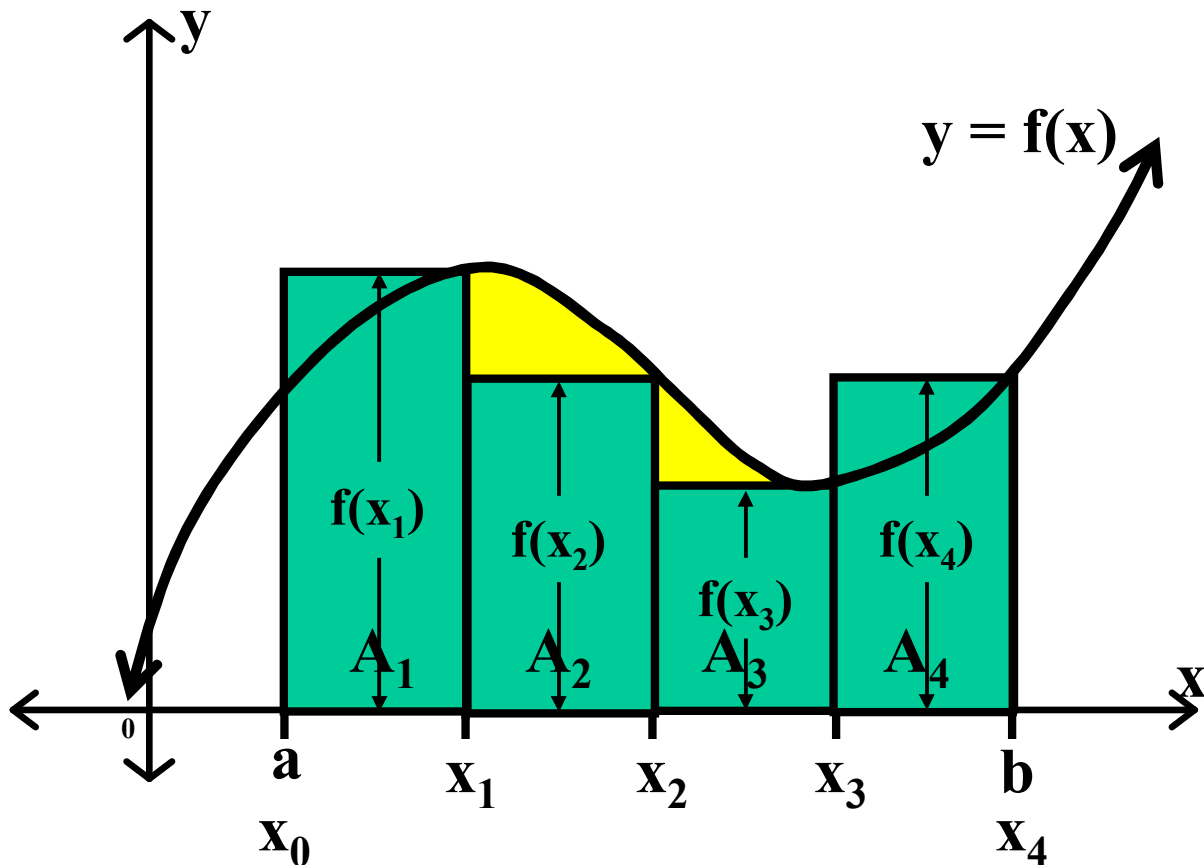


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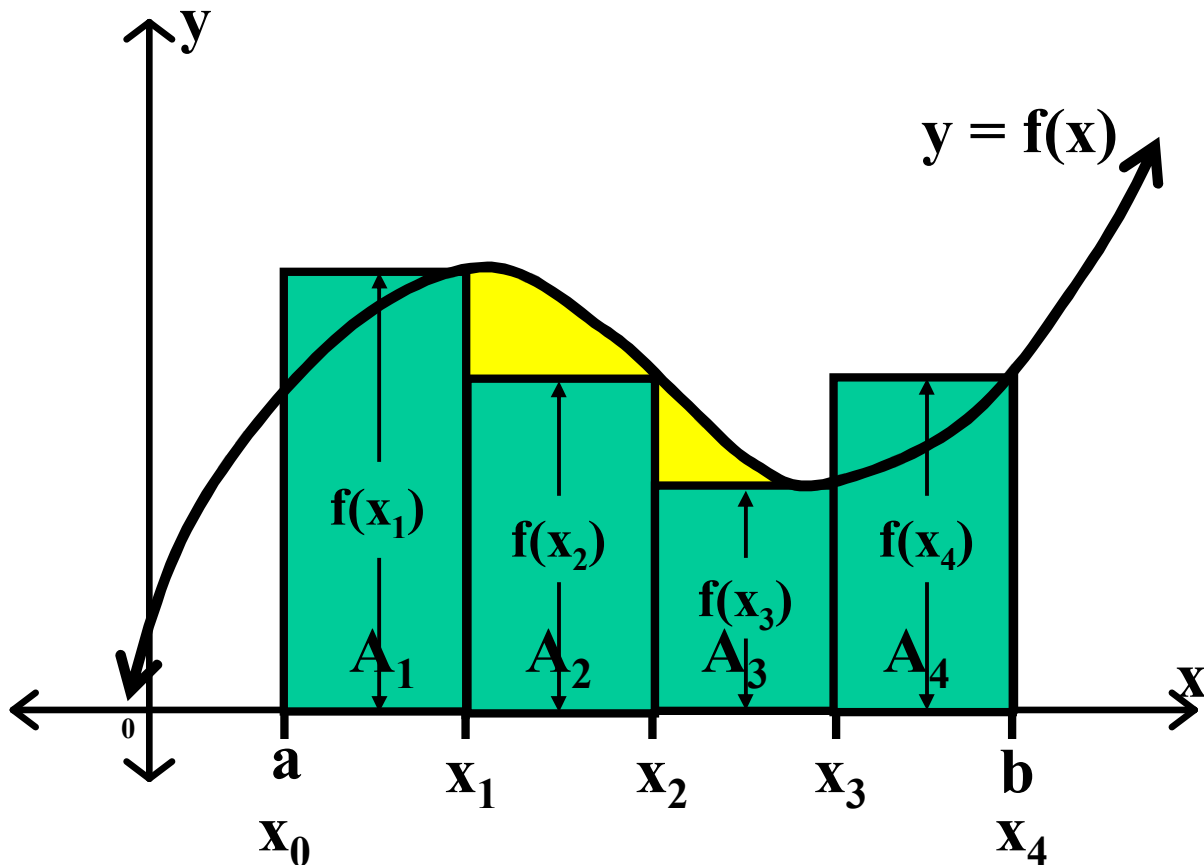


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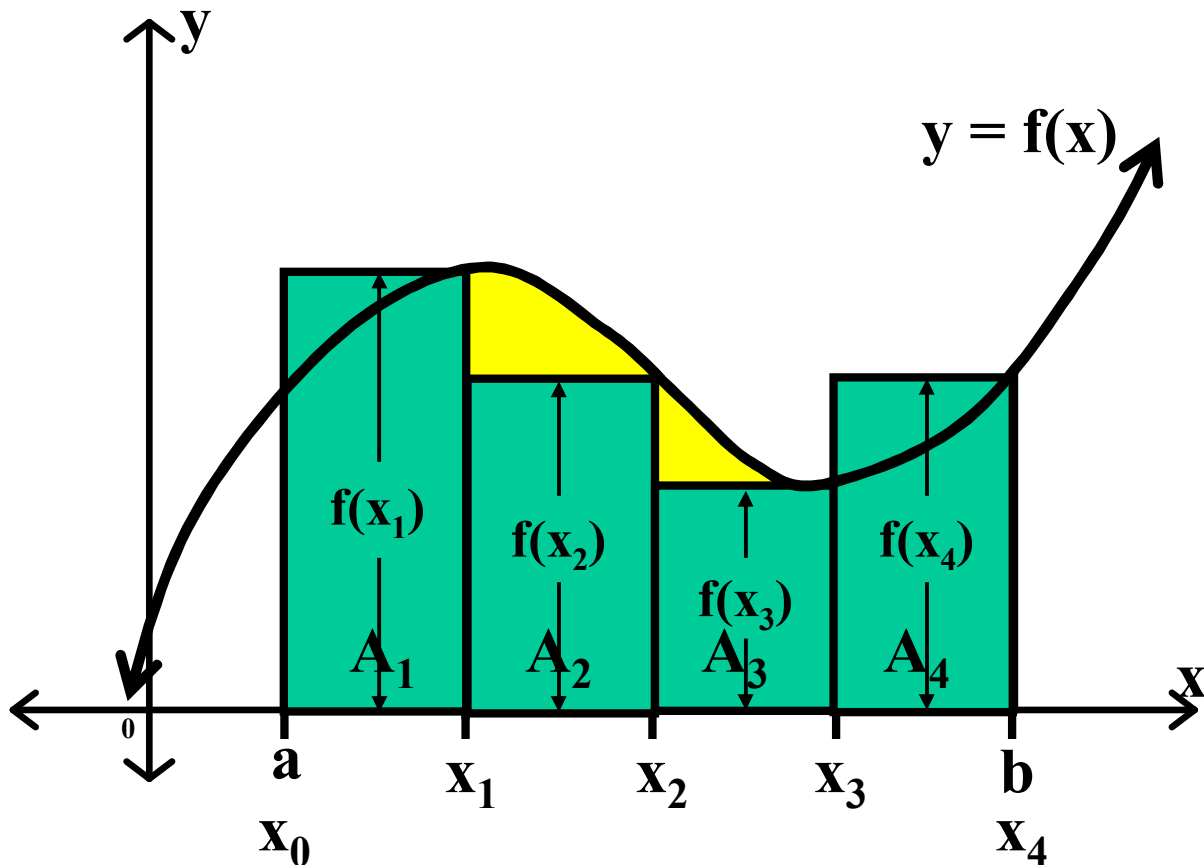
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Notice that, in general,



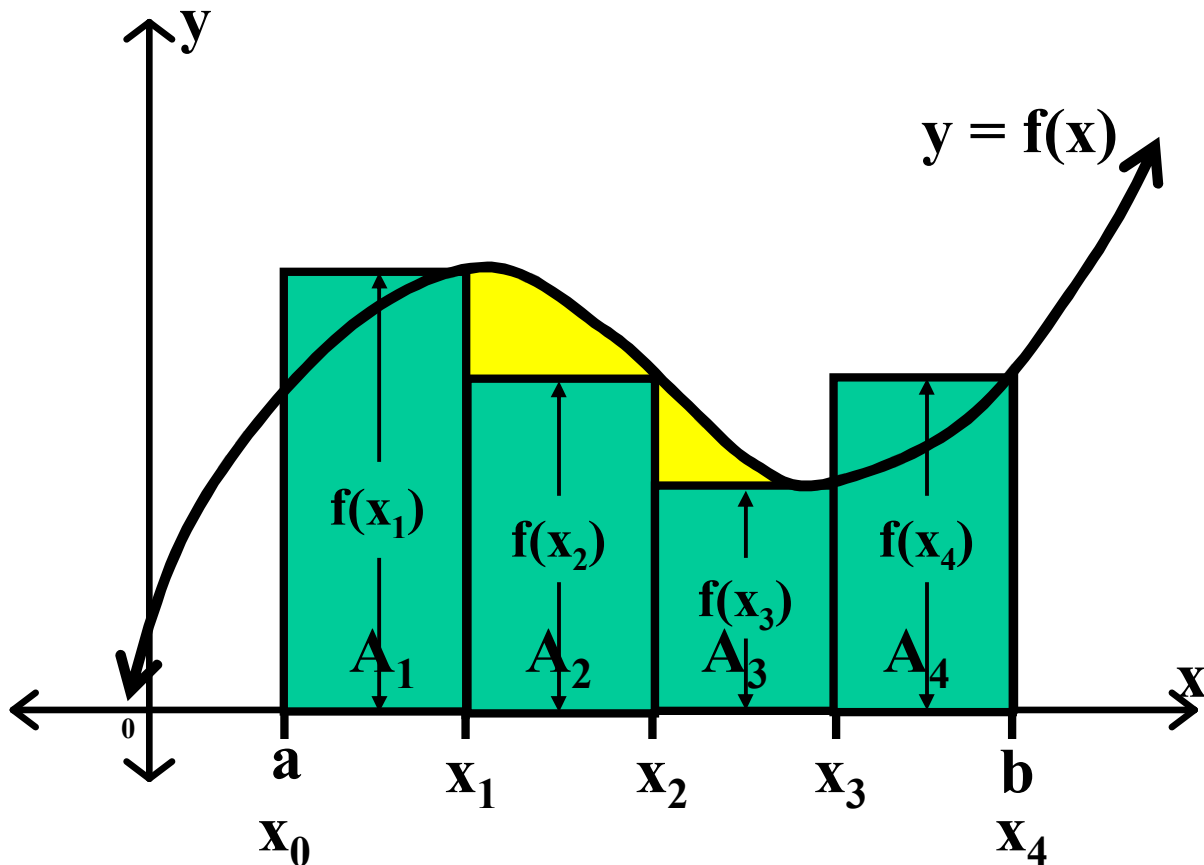
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Notice that, in general, $A_i \approx$



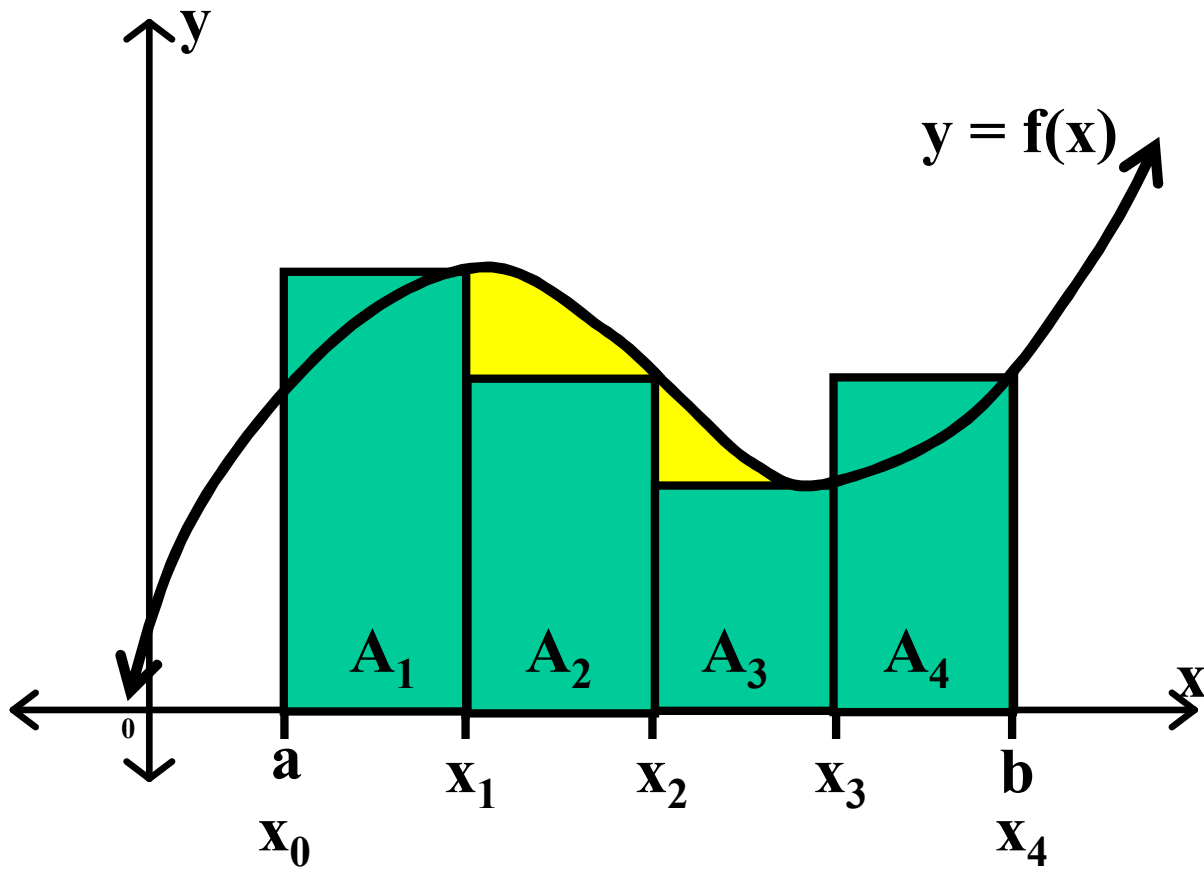
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Notice that, in general, $A_i \approx f(x_i)$

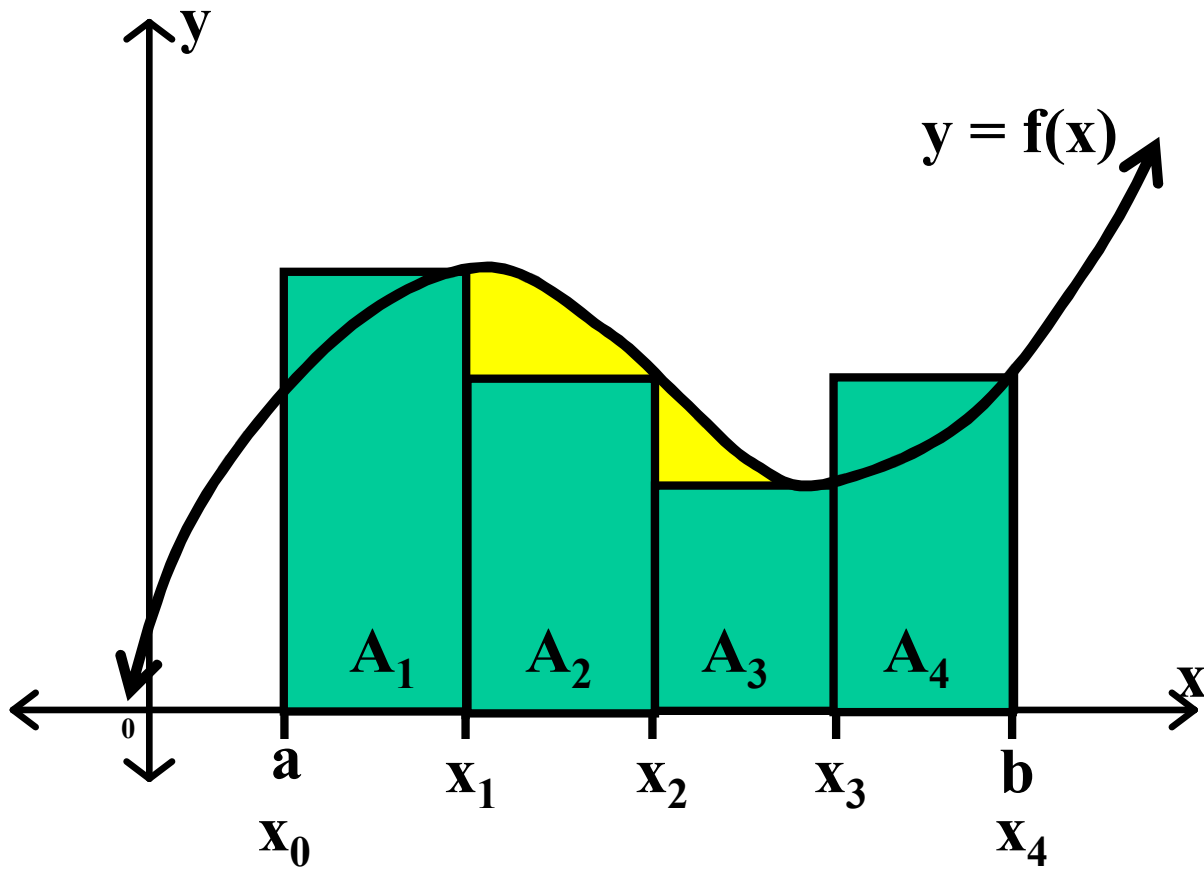


$$A_1 \approx f(x_1)\Delta x \quad A_2 \approx f(x_2)\Delta x \quad A_3 \approx f(x_3)\Delta x \quad A_4 \approx f(x_4)\Delta x$$

Notice that, in general, $A_i \approx f(x_i)\Delta x$.

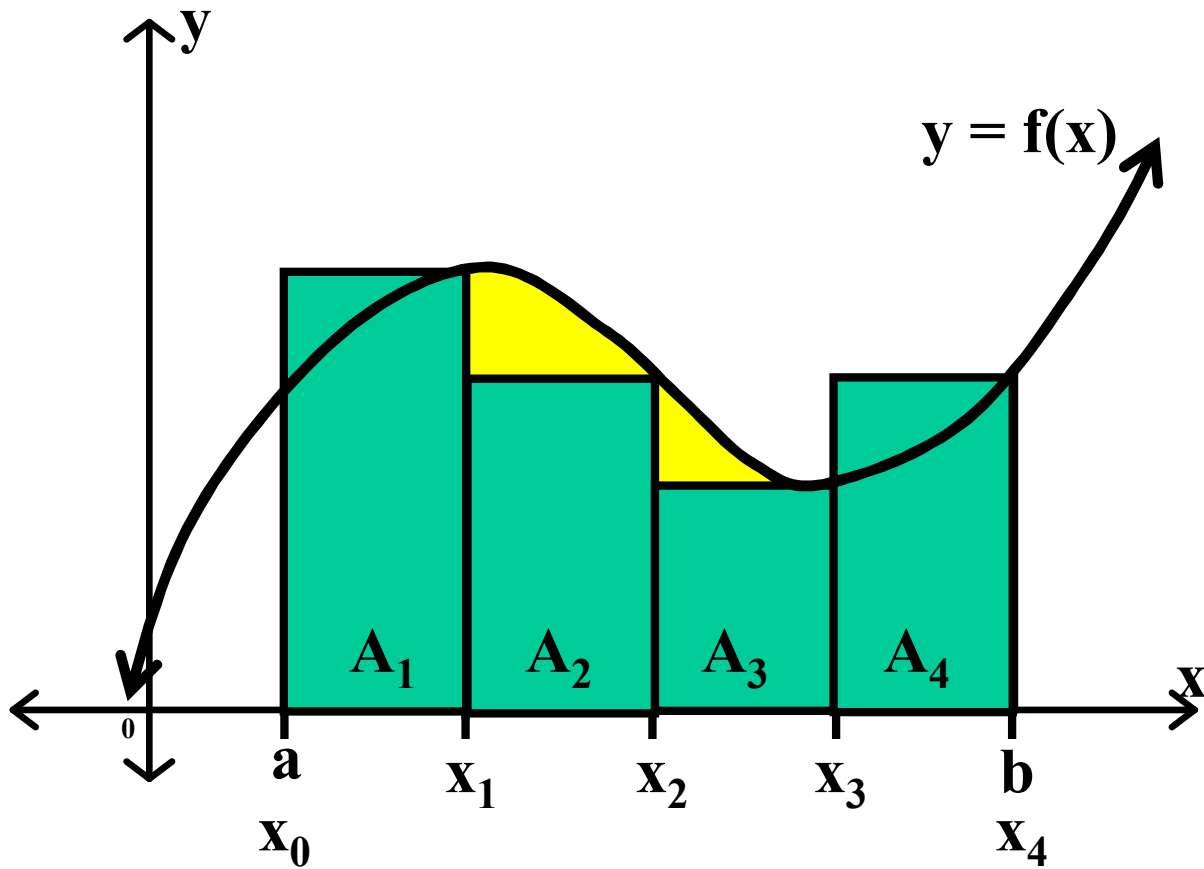


$$A_i \approx f(x_i) \Delta x$$



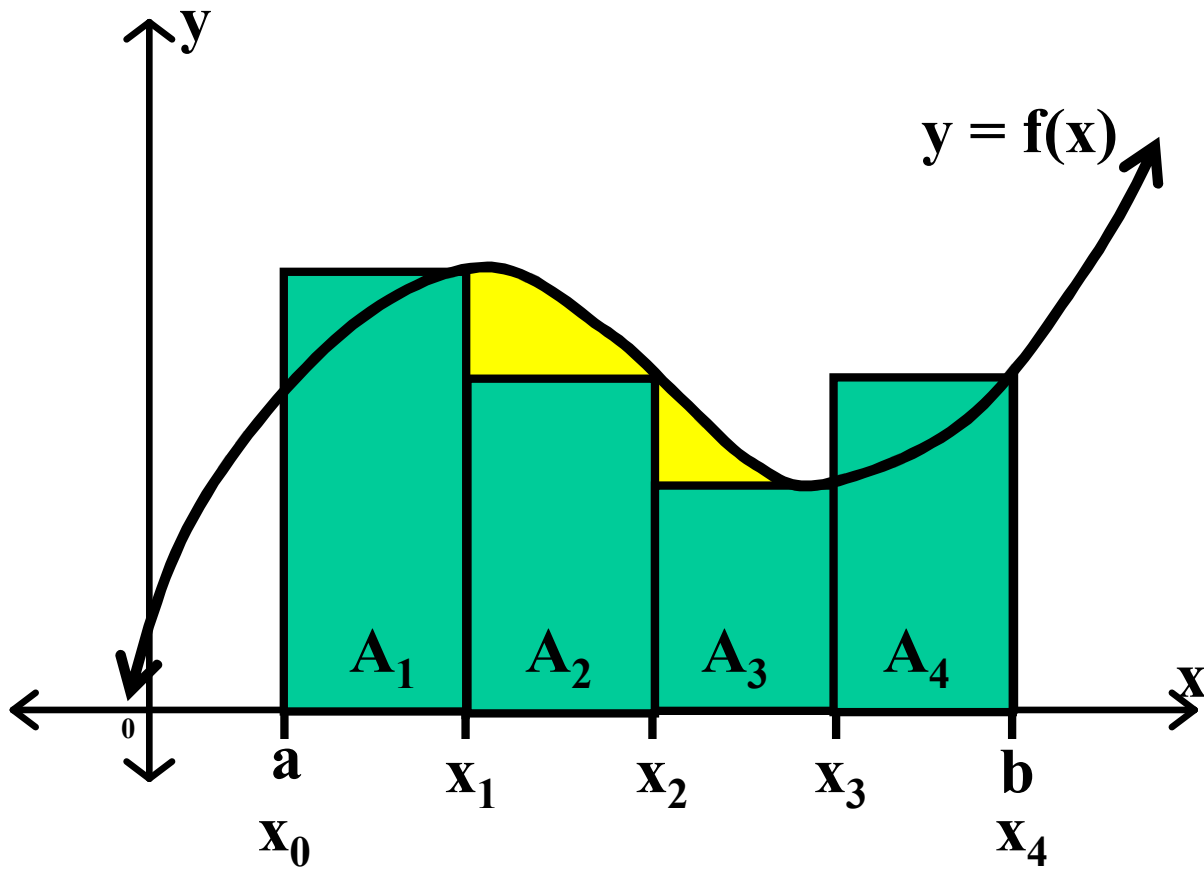
$$A_i \approx f(x_i)\Delta x$$

$$A = \int_a^b f(x)dx$$



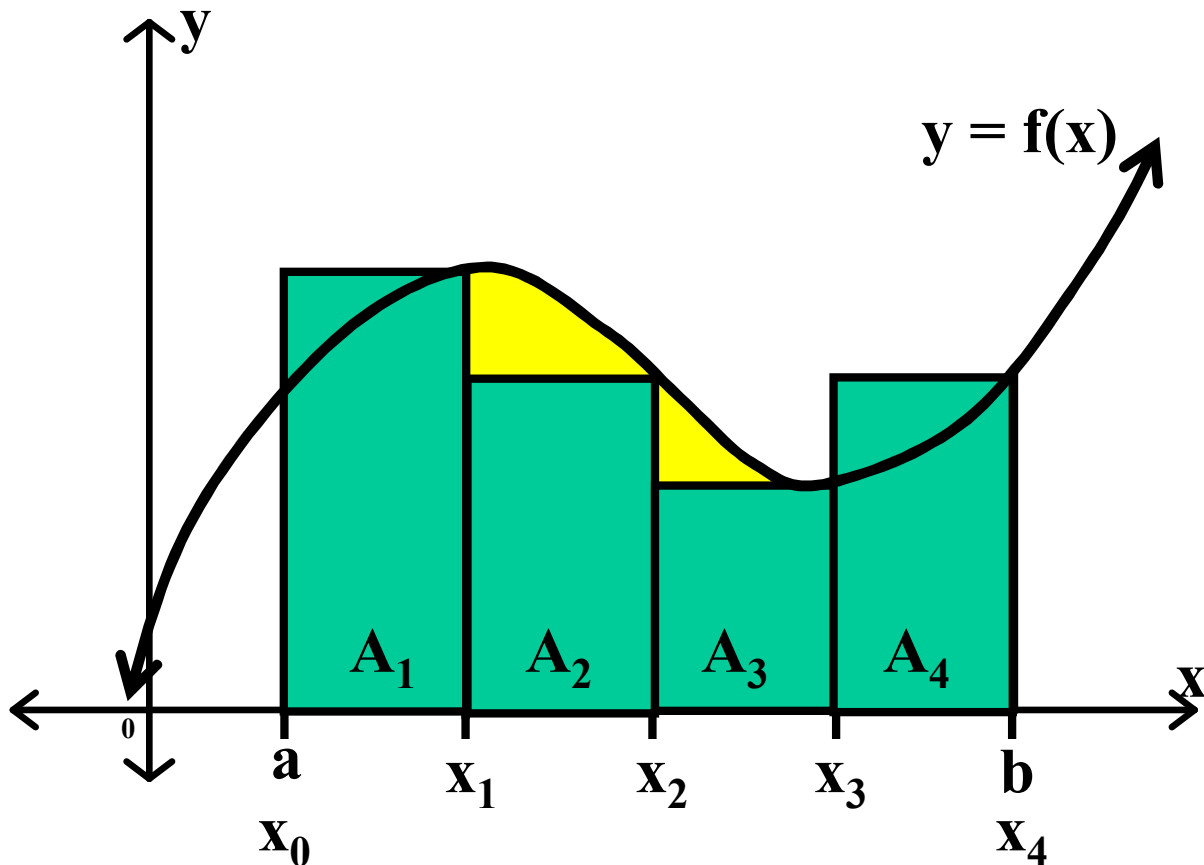
$$A_i \approx f(x_i)\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4$$



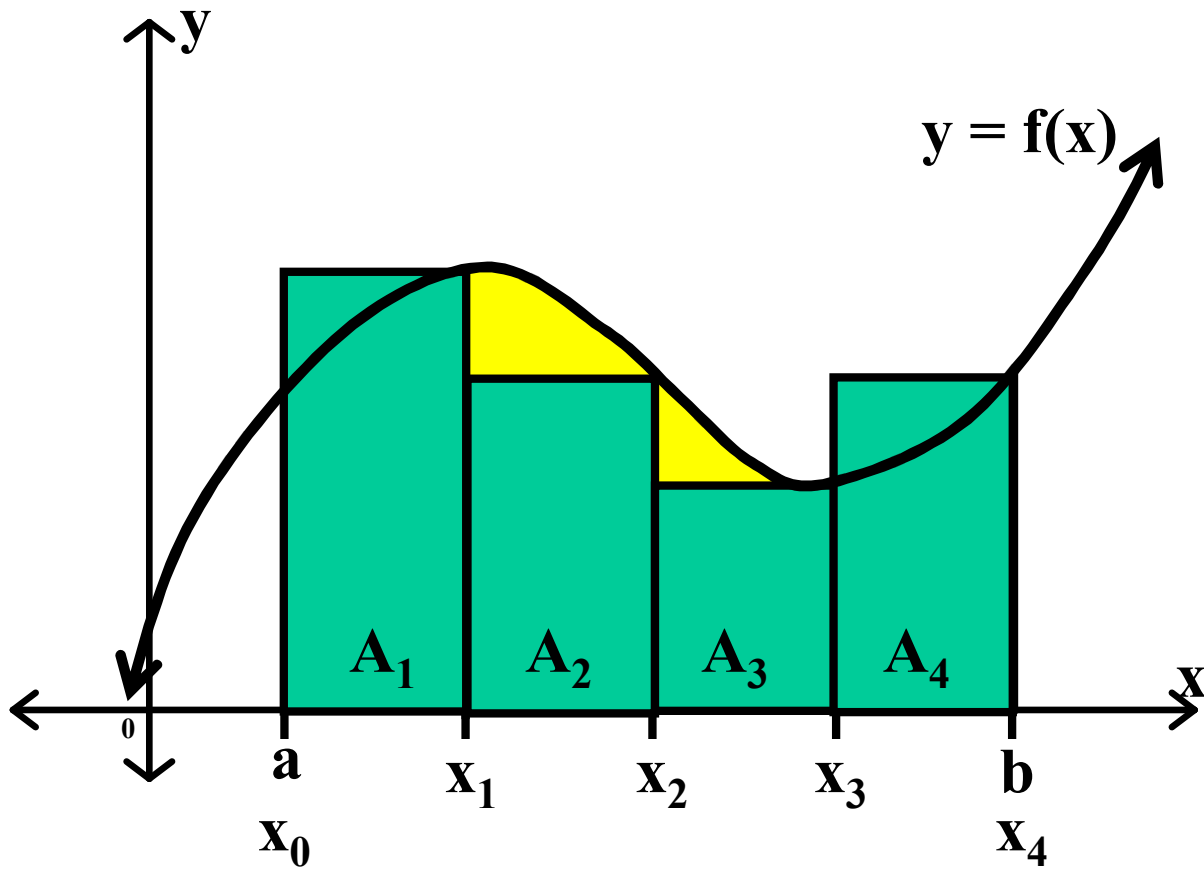
$$A_i \approx f(x_i) \Delta x$$

$$A = \int_a^b f(x) dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i$$



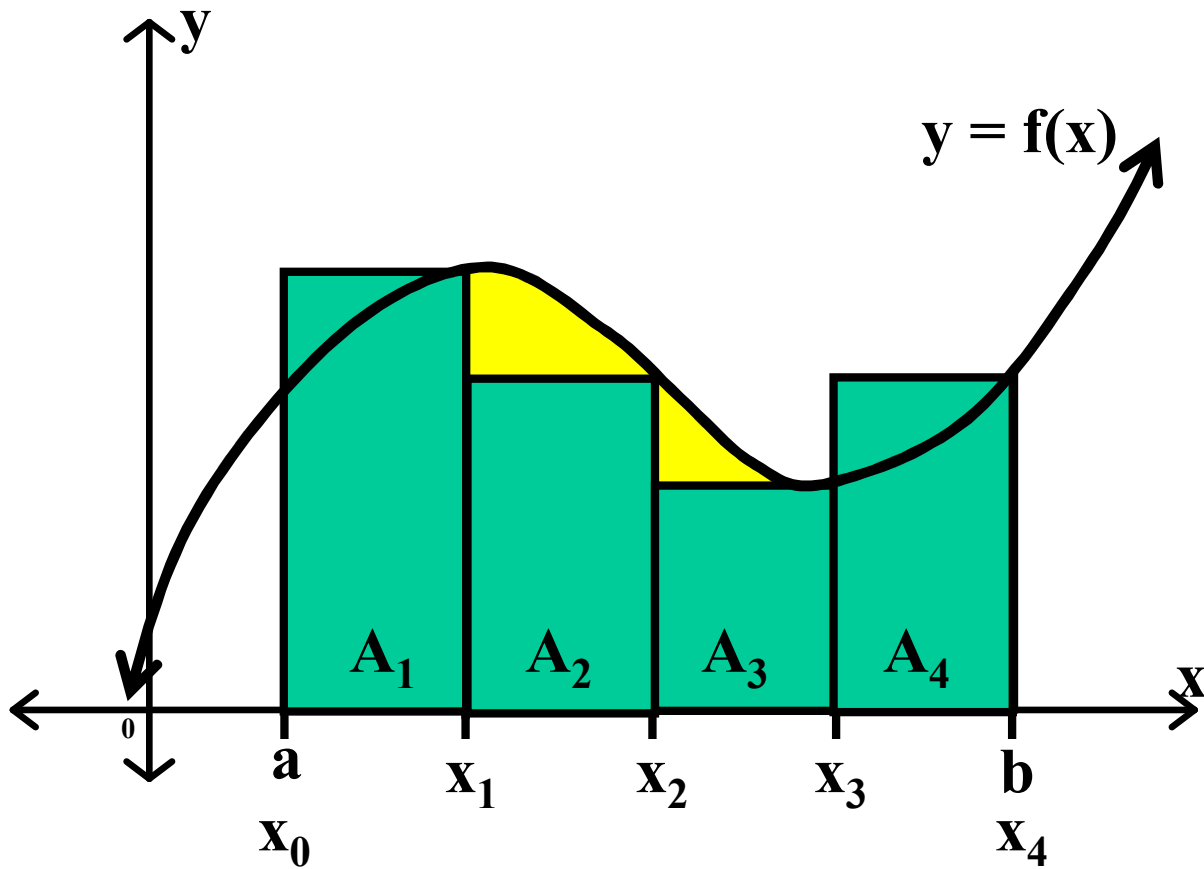
$$A_i \approx f(x_i)\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i \quad (\text{In this case, } n = 4.)$$



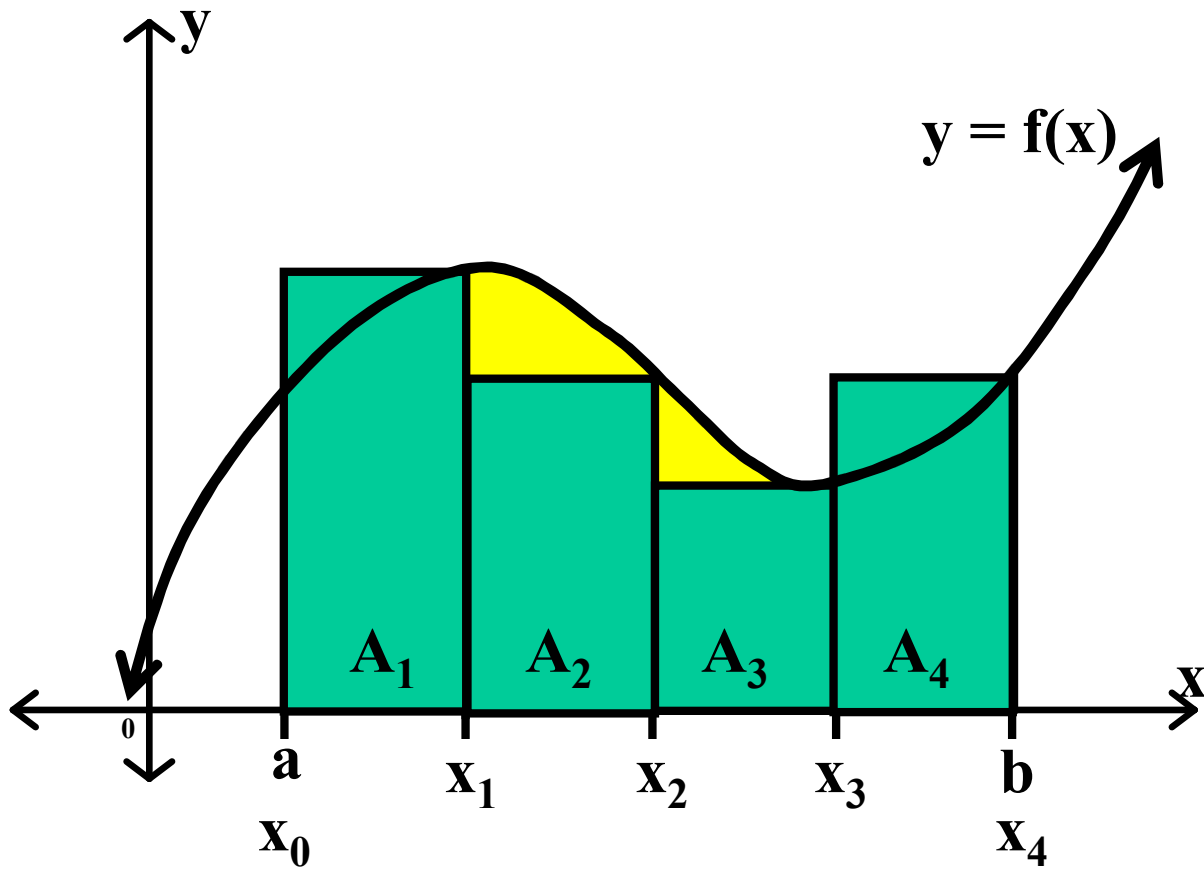
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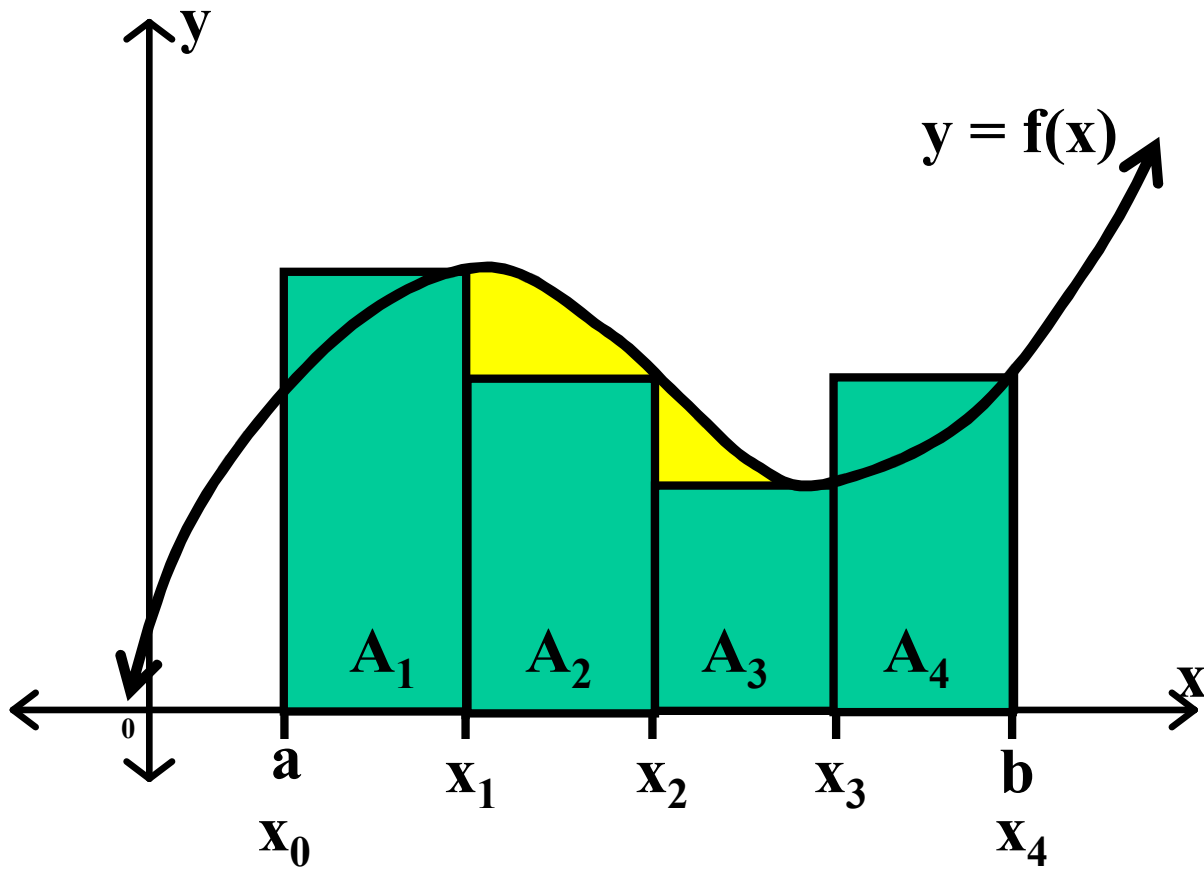
$$A_i \approx f(x_i) \Delta x$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i \approx$$



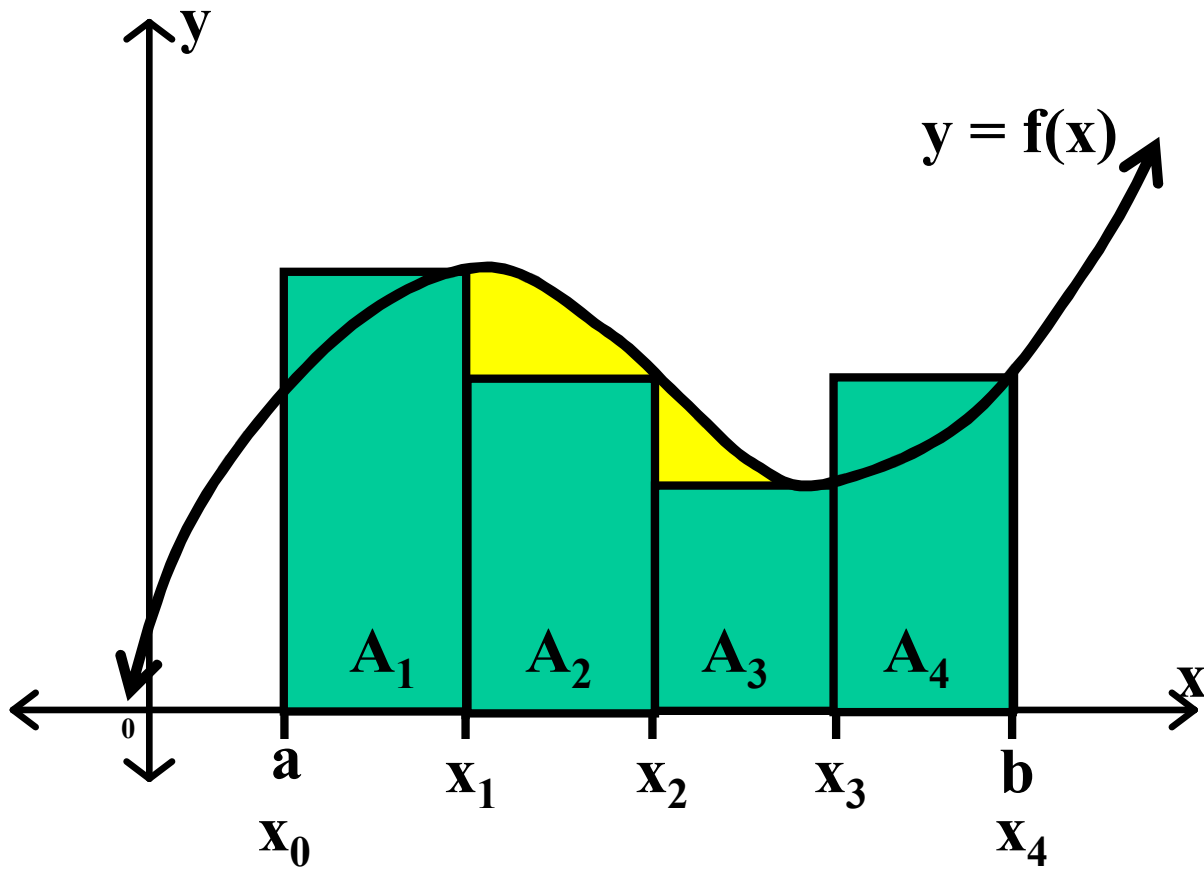
$$A_i \approx f(x_i) \Delta x$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n$$



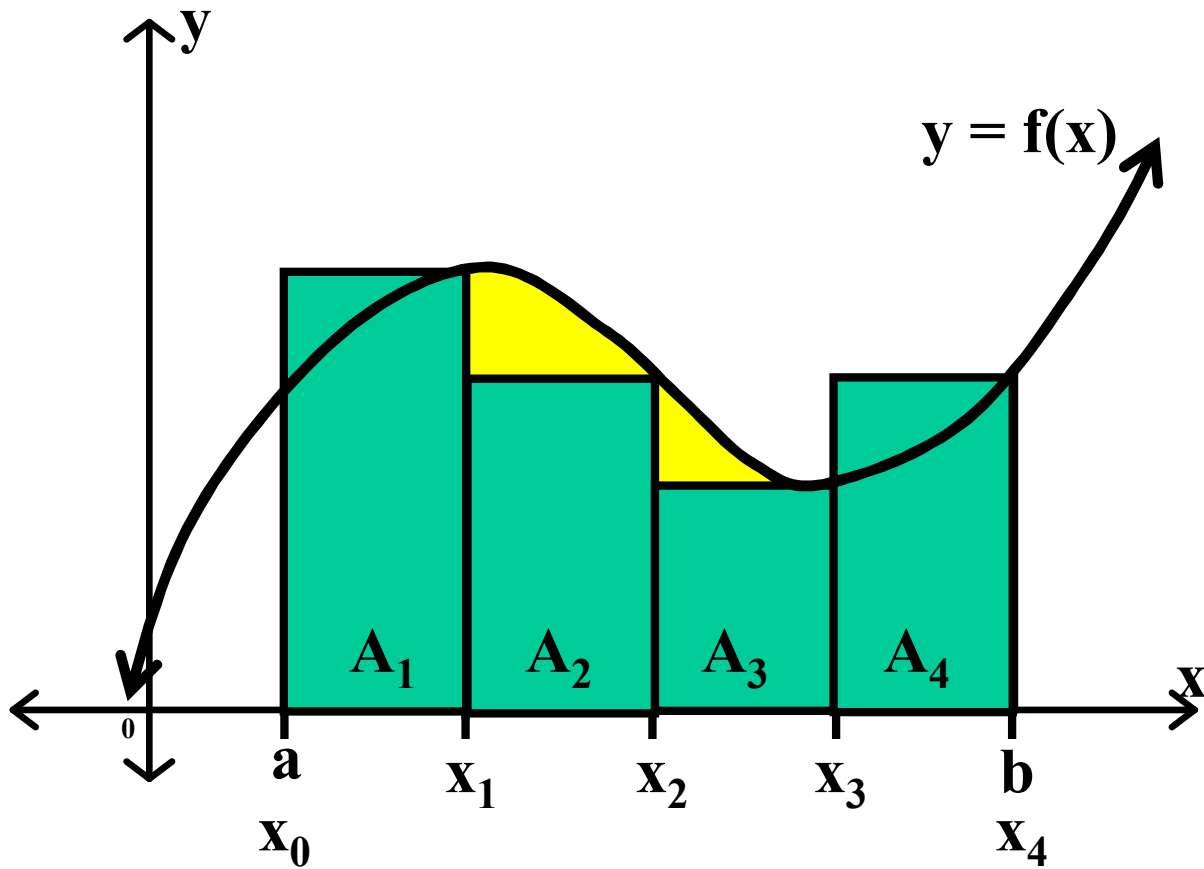
$$A_i \approx f(x_i)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_i)\Delta x$$



$$A_i \approx f(x_i)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_i)\Delta x = S_R$$



$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

The Right Rectangular Approximation

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) = \sqrt{122}$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{array}{l} x_0 = a = 2 \\ x_1 = 2.5 \\ x_2 = 3 \\ x_3 = 3.5 \\ x_4 = 4 \\ x_5 = 4.5 \\ x_6 = b = 5 \end{array} \quad \begin{array}{l} f(x_0) = f(a) = f(2) = \sqrt{5} \\ f(x_1) = f(2.5) = \sqrt{12.625} \\ f(x_2) = f(3) = \sqrt{24} \\ f(x_3) = f(3.5) = \sqrt{39.875} \\ f(x_4) = f(4) = \sqrt{61} \\ f(x_5) = f(4.5) = \sqrt{88.125} \\ f(x_6) = f(b) = f(5) = \sqrt{122} \end{array} \quad S_R = \sum_{i=1}^n f(x_i) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{array}{ll} x_0 = a = 2 & f(x_0) = f(a) = f(2) = \sqrt{5} \\ x_1 = 2.5 & f(x_1) = f(2.5) = \sqrt{12.625} \\ x_2 = 3 & f(x_2) = f(3) = \sqrt{24} \\ x_3 = 3.5 & f(x_3) = f(3.5) = \sqrt{39.875} \\ x_4 = 4 & f(x_4) = f(4) = \sqrt{61} \\ x_5 = 4.5 & f(x_5) = f(4.5) = \sqrt{88.125} \\ x_6 = b = 5 & f(x_6) = f(b) = f(5) = \sqrt{122} \end{array} \quad S_R = \sum_{i=1}^n f(x_i) \Delta x$$

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$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

$$x_4 = 4 \quad f(x_4) = f(4) = \sqrt{61}$$

$$x_5 = 4.5 \quad f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

$$S_R = \sum_{i=1}^6 f(x_i) \Delta x$$

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$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

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$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

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$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

$$S_R = \sum_{i=1}^6 f(x_i) \Delta x$$

$$S_L = f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x + f(x_6) \Delta x$$

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Approximate the following definite integral using each of the following approximation methods.

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Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

$$x_1 = 2.5 \quad f(x_1) = f(2.5) = \sqrt{12.625}$$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

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$$x_5 = 4.5 \quad f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

$$S_R = \sum_{i=1}^6 f(x_i) \Delta x$$

$$S_L = f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x + f(x_6) \Delta x$$

Class Worksheet #5 Unit 11

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$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

$$S_R = \sum_{i=1}^6 f(x_i) \Delta x$$

$$S_L = f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x + f(x_6) \Delta x$$

$$S_R = (\sqrt{12.625} + \sqrt{24} + \sqrt{39.875} + \sqrt{61} + \sqrt{88.125} + \sqrt{122}) (.5)$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad f(x_0) = f(a) = f(2) = \sqrt{5}$$

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$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

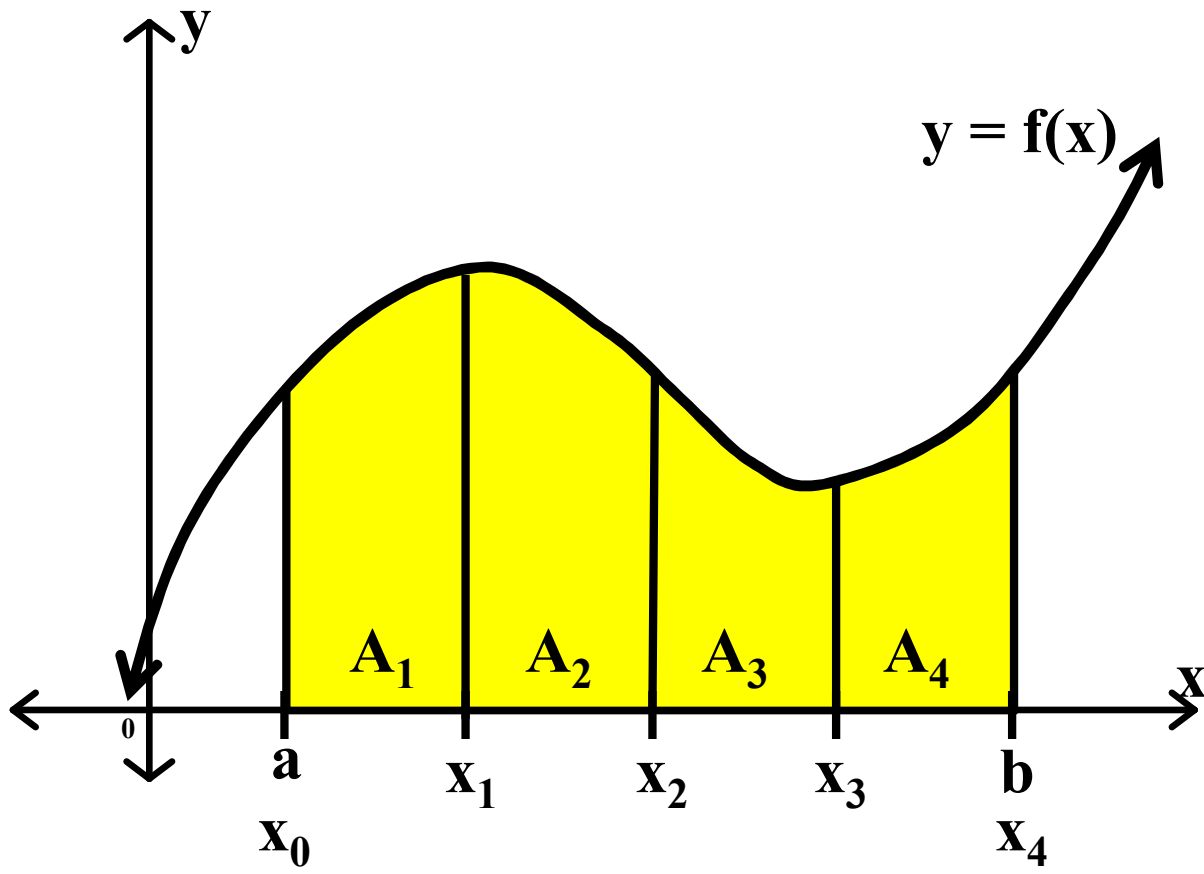
$$S_R = \sum_{i=1}^n f(x_i) \Delta x$$

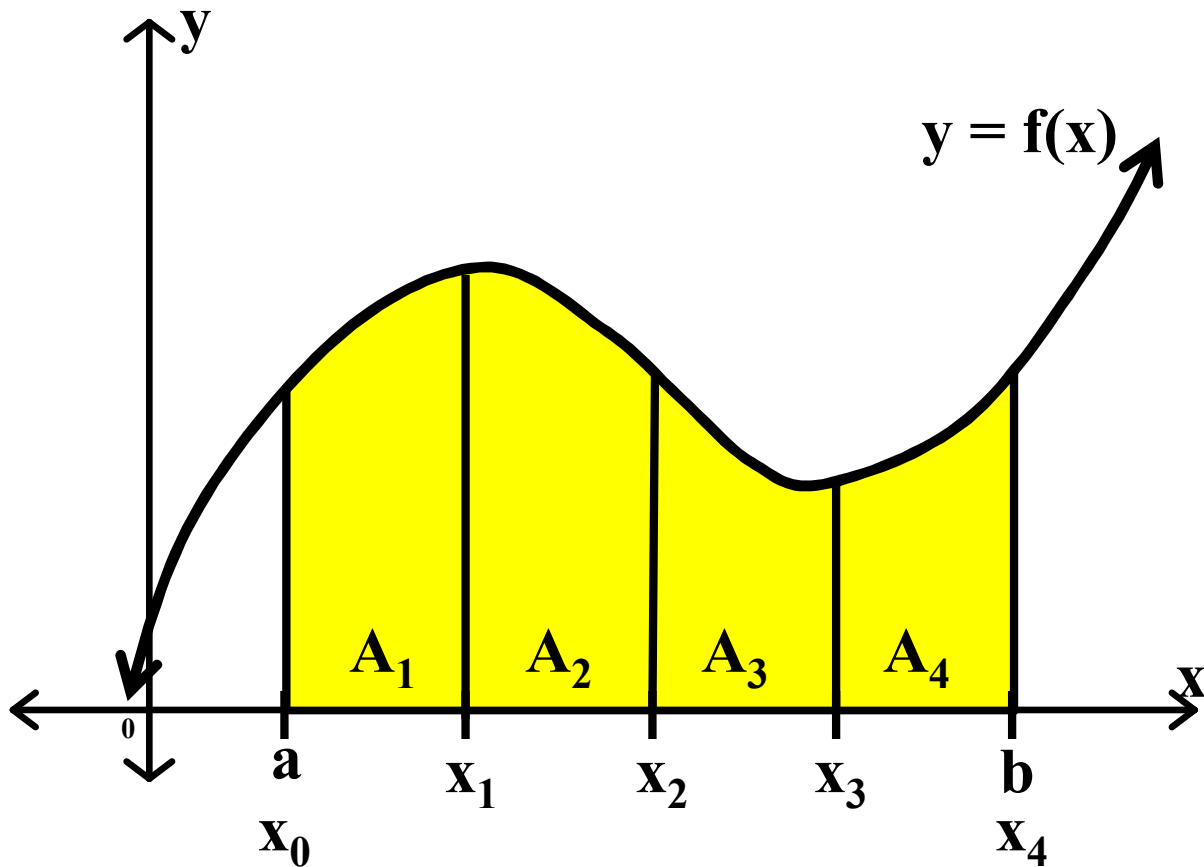
$$S_R = \sum_{i=1}^6 f(x_i) \Delta x$$

$$S_L = f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x + f(x_6) \Delta x$$

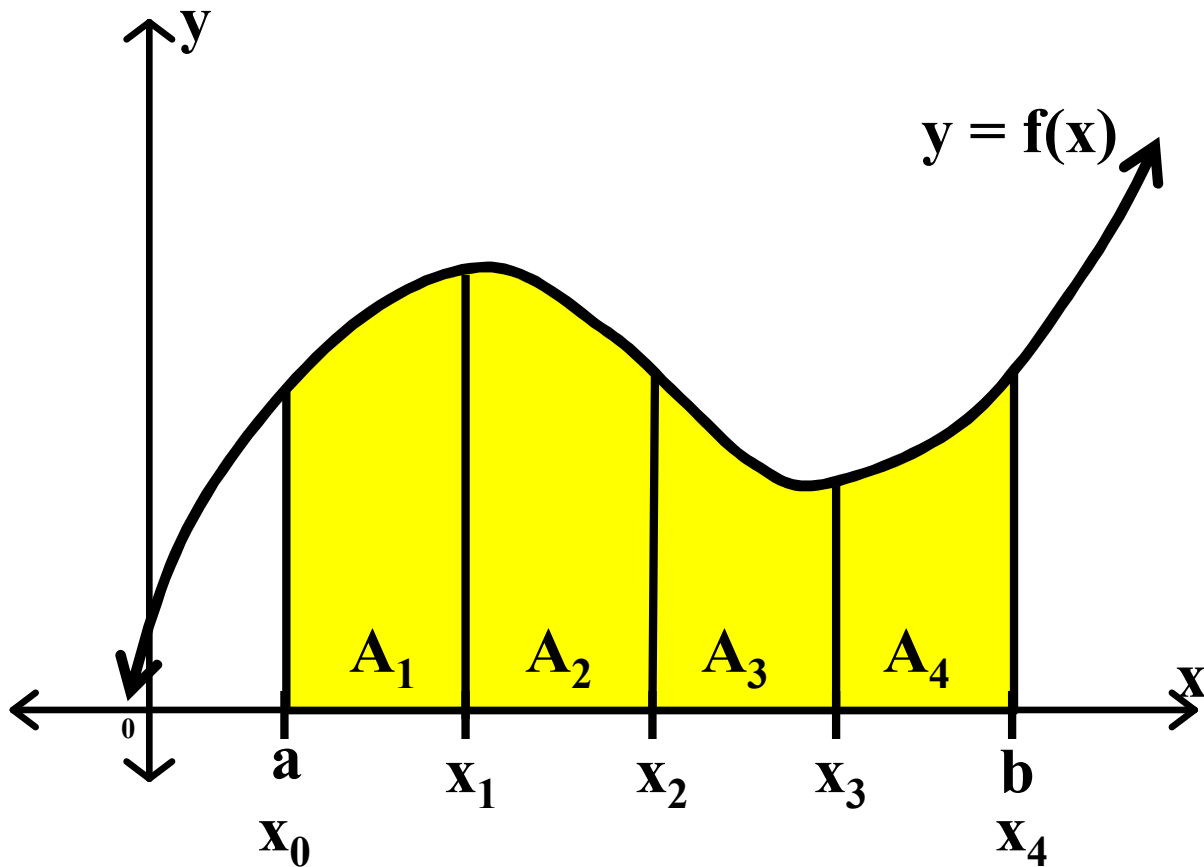
$$S_R = (\sqrt{12.625} + \sqrt{24} + \sqrt{39.875} + \sqrt{61} + \sqrt{88.125} + \sqrt{122}) (.5)$$

$$S_R \approx 21.50$$

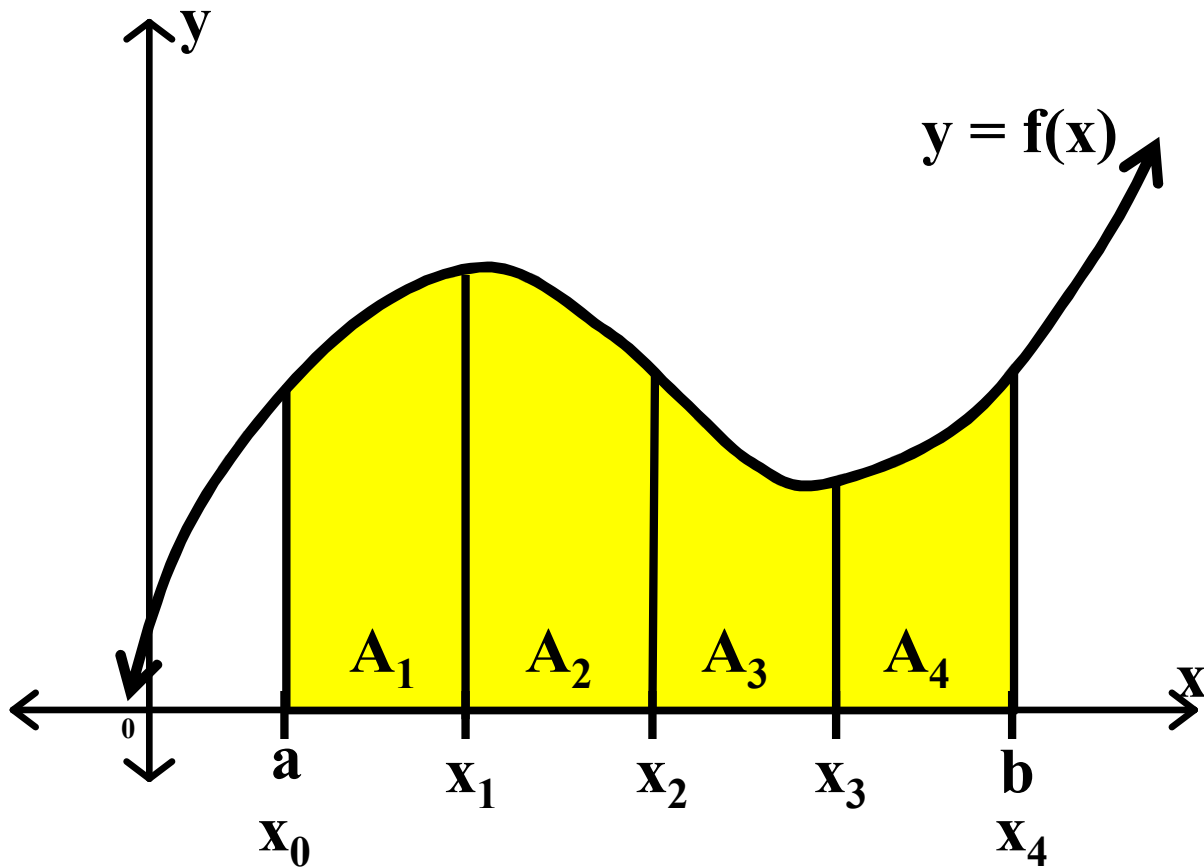




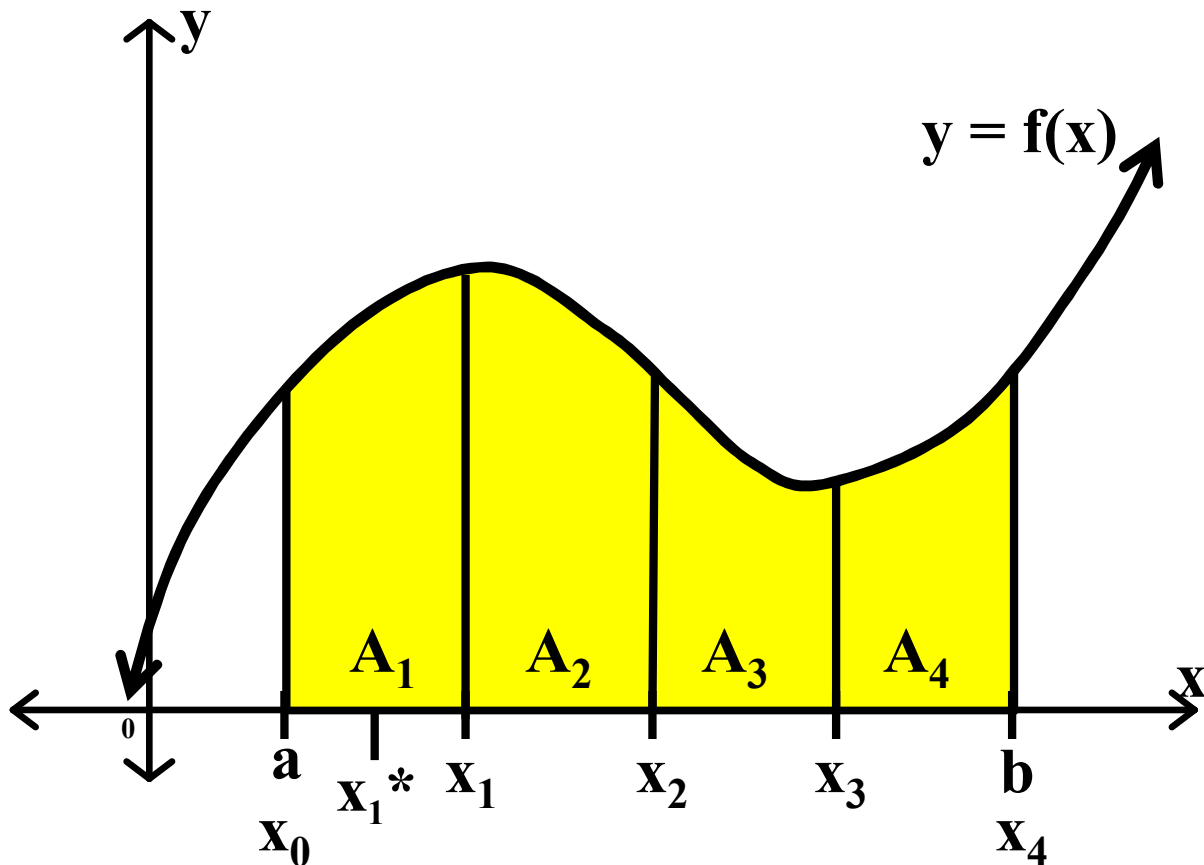
The last Rectangular Approximation is called the Mid-Rectangular Approximation.



The last Rectangular Approximation is called the Mid-Rectangular Approximation, S_M .



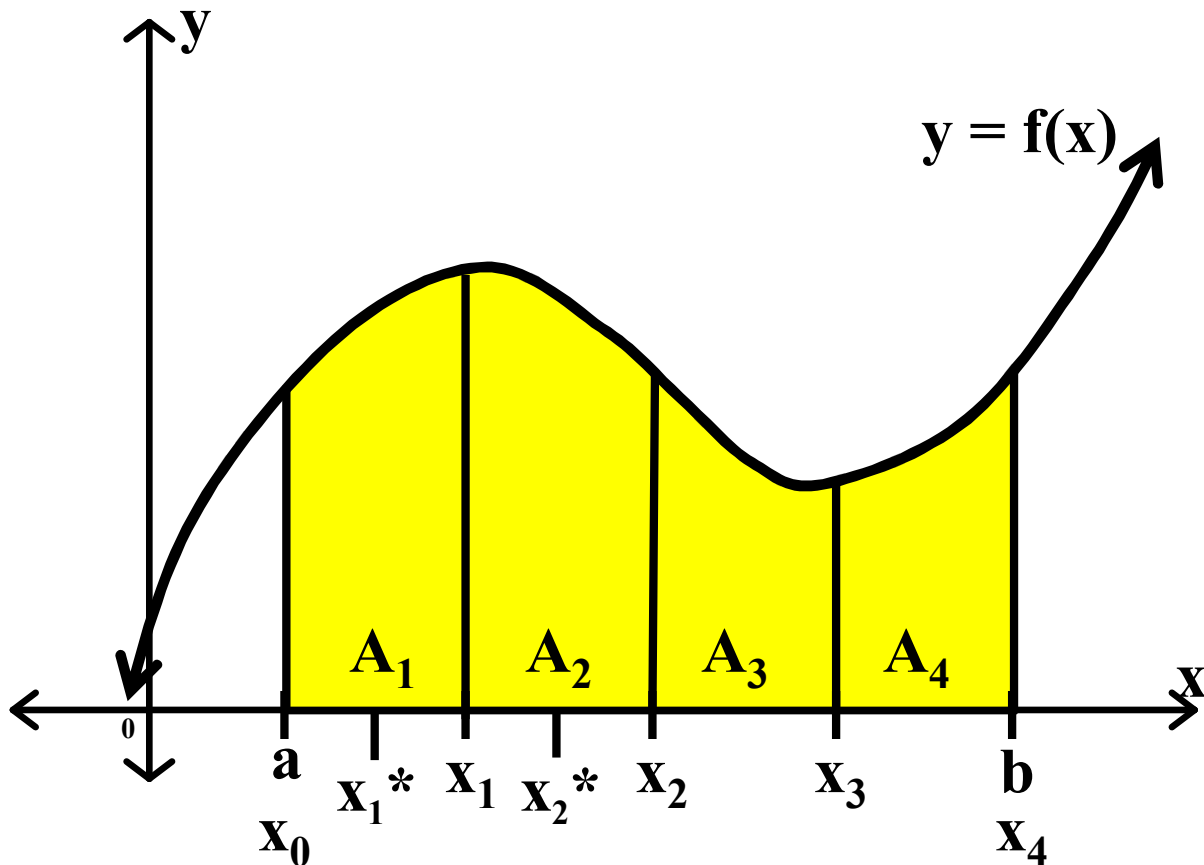
The last Rectangular Approximation is called the Mid-Rectangular Approximation, S_M .
Let x_i^* represent the midpoint of the i^{th} subinterval.



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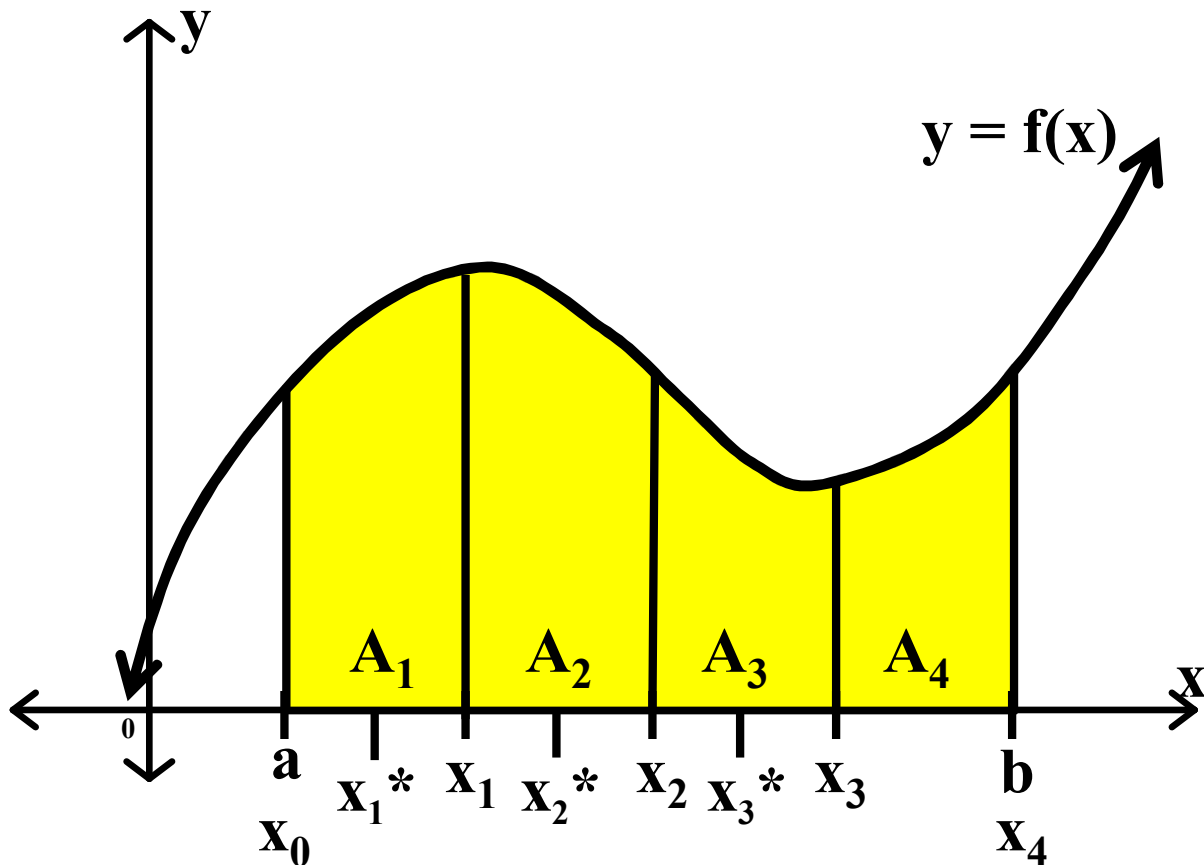
x_1^* is the midpoint of the 1st subinterval.



The last Rectangular Approximation is called the Mid-Rectangular Approximation, S_M .

Let x_i^* represent the midpoint of the i^{th} subinterval.

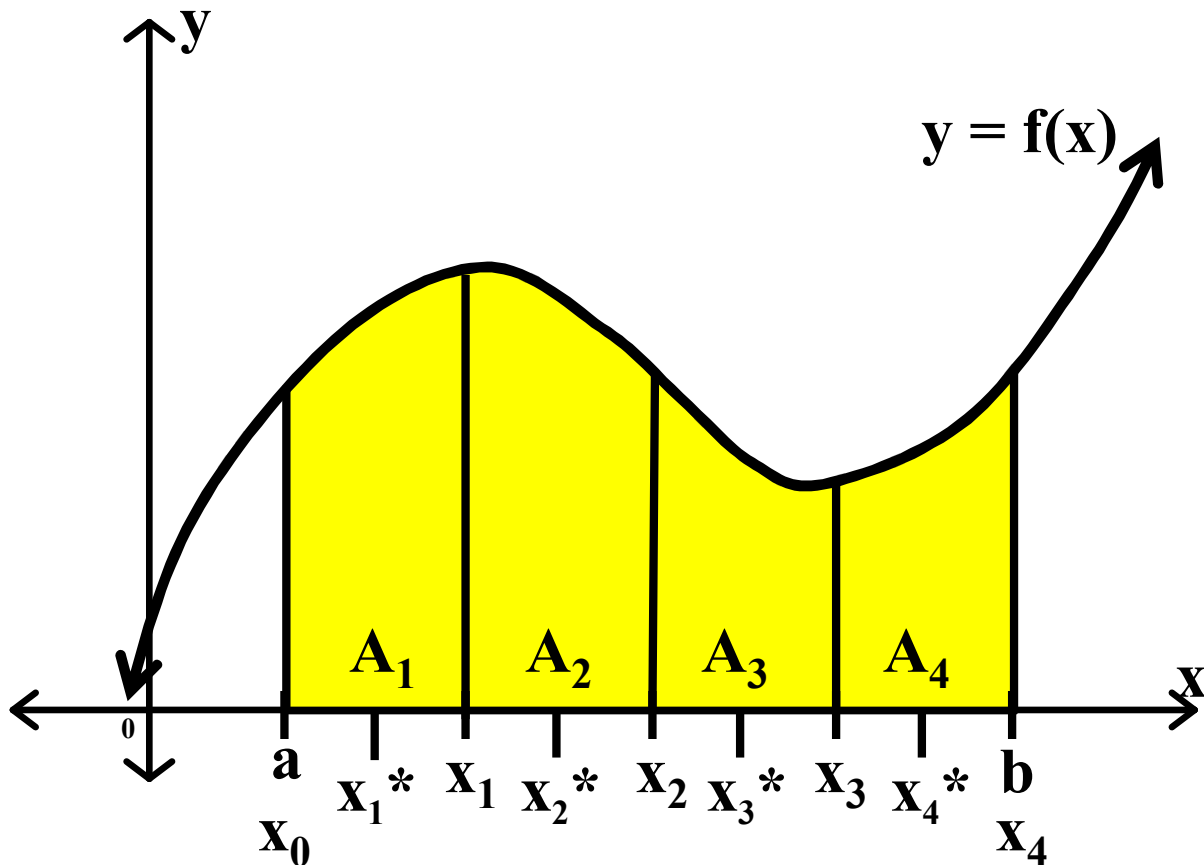
x_2^* is the midpoint of the 2nd subinterval.



The last Rectangular Approximation is called the Mid-Rectangular Approximation, S_M .

Let x_i^* represent the midpoint of the i^{th} subinterval.

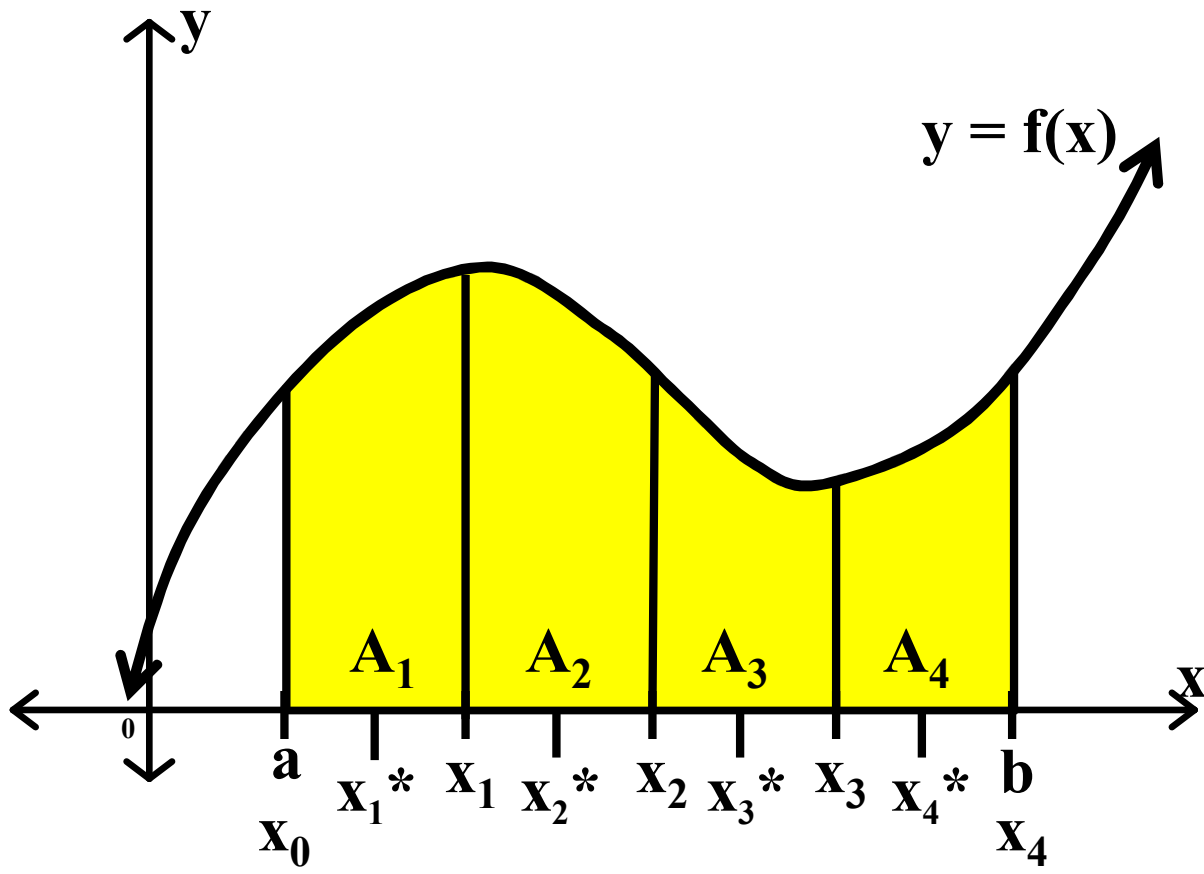
x_3^* is the midpoint of the 3rd subinterval.

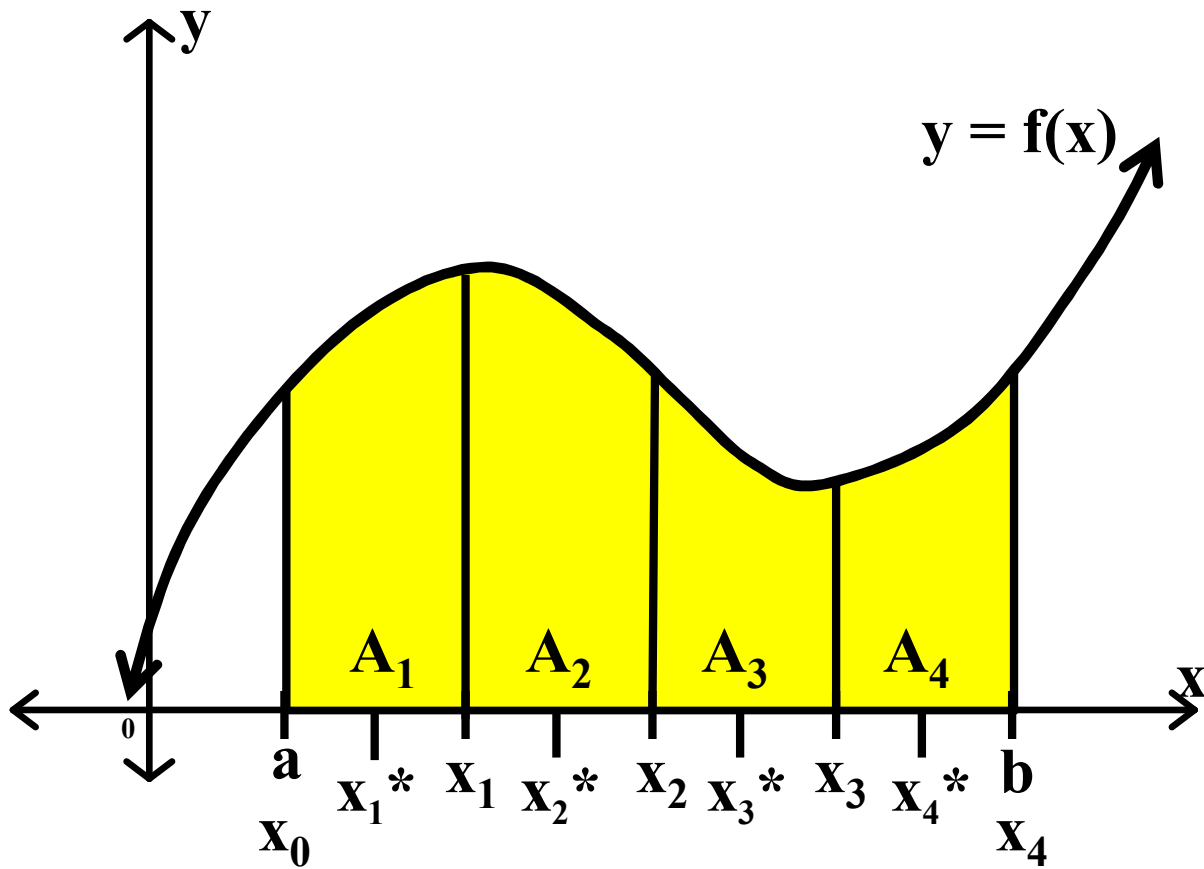


The last Rectangular Approximation is called the Mid-Rectangular Approximation, S_M .

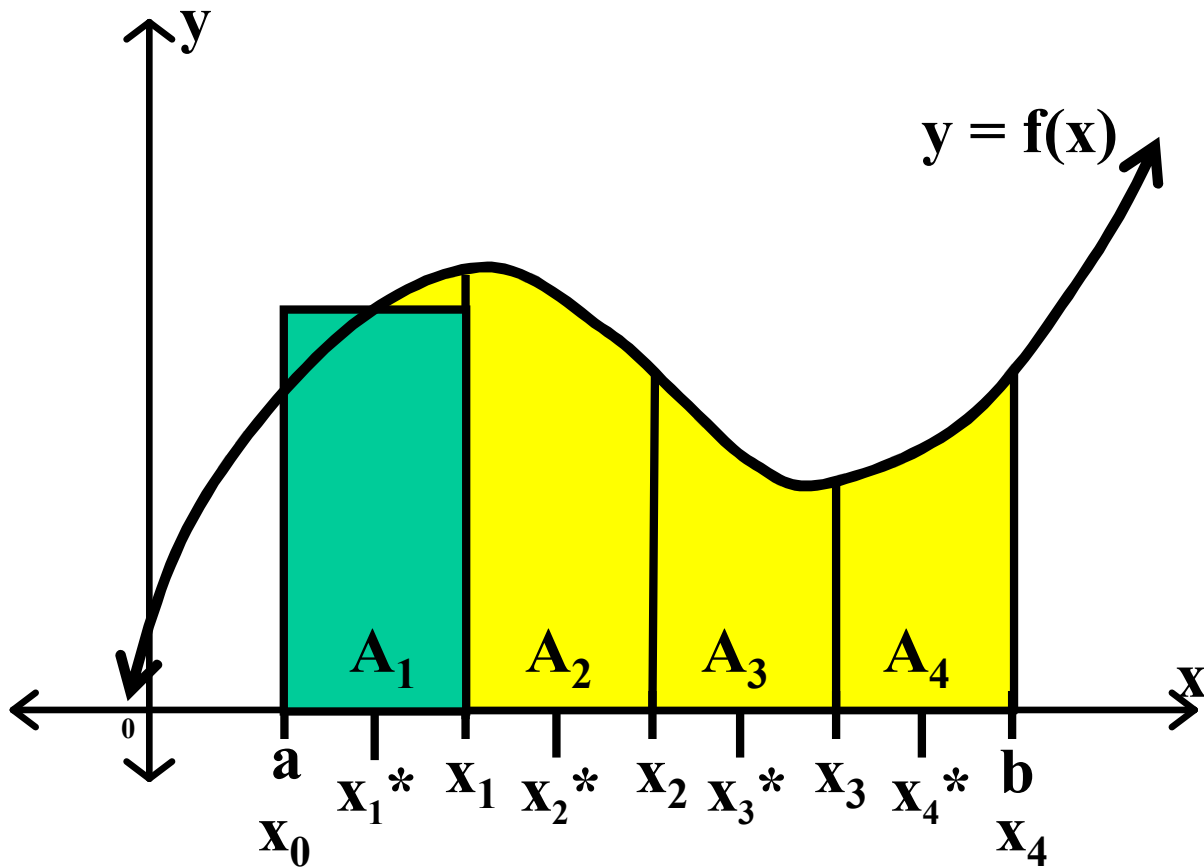
Let x_i^* represent the midpoint of the i^{th} subinterval.

x_4^* is the midpoint of the 4th subinterval.

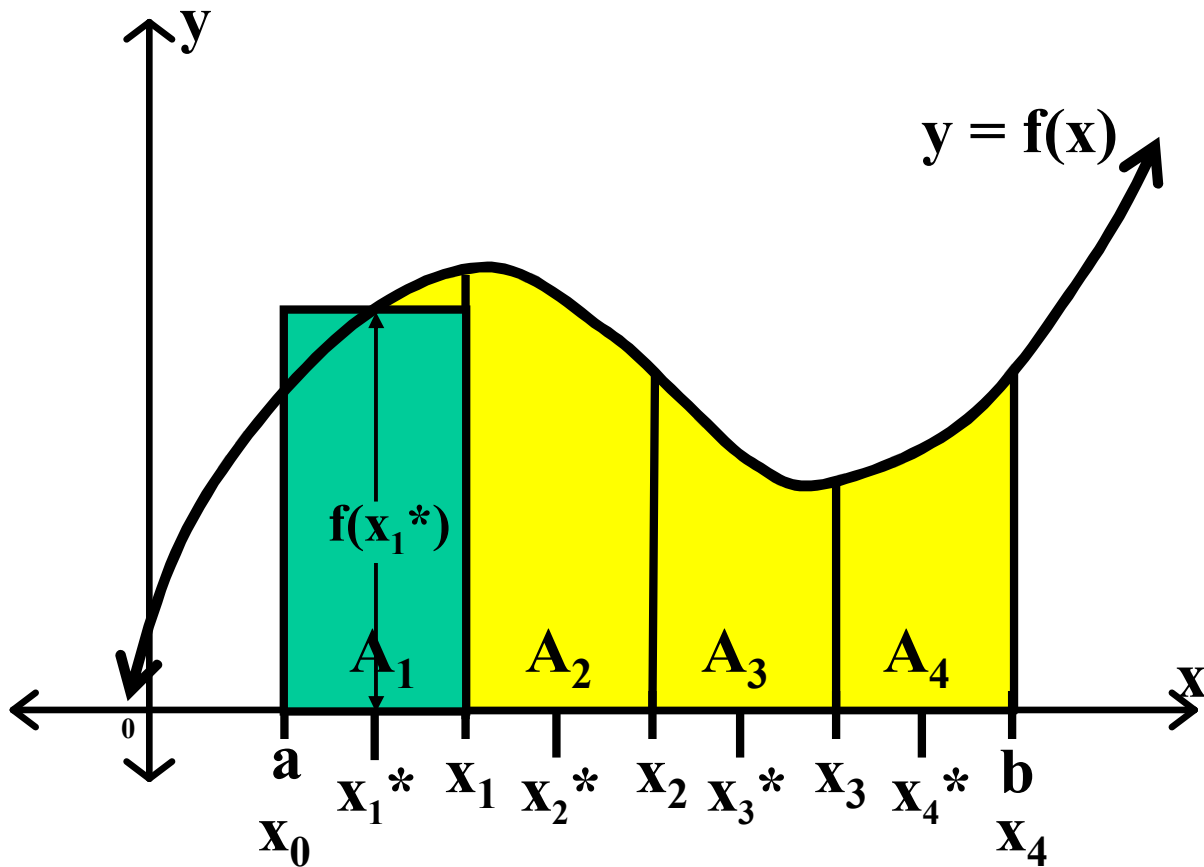




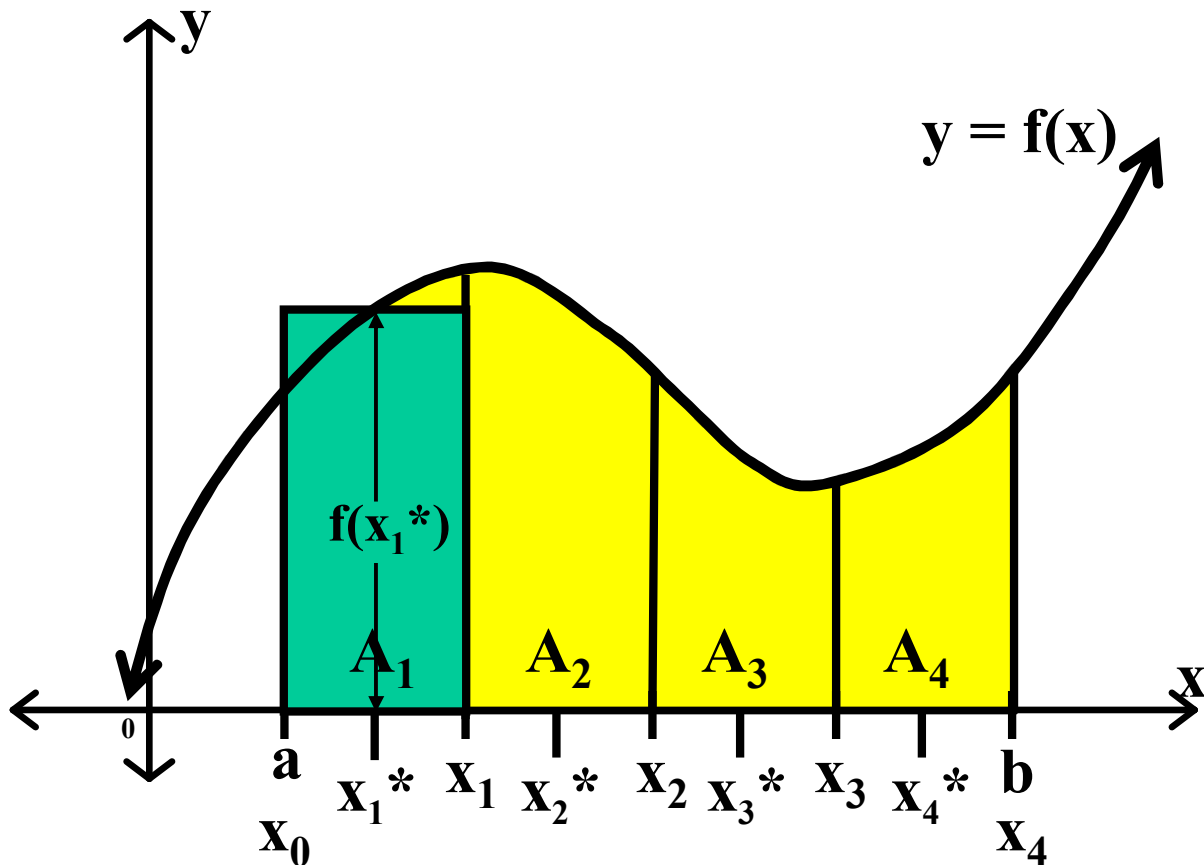
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.



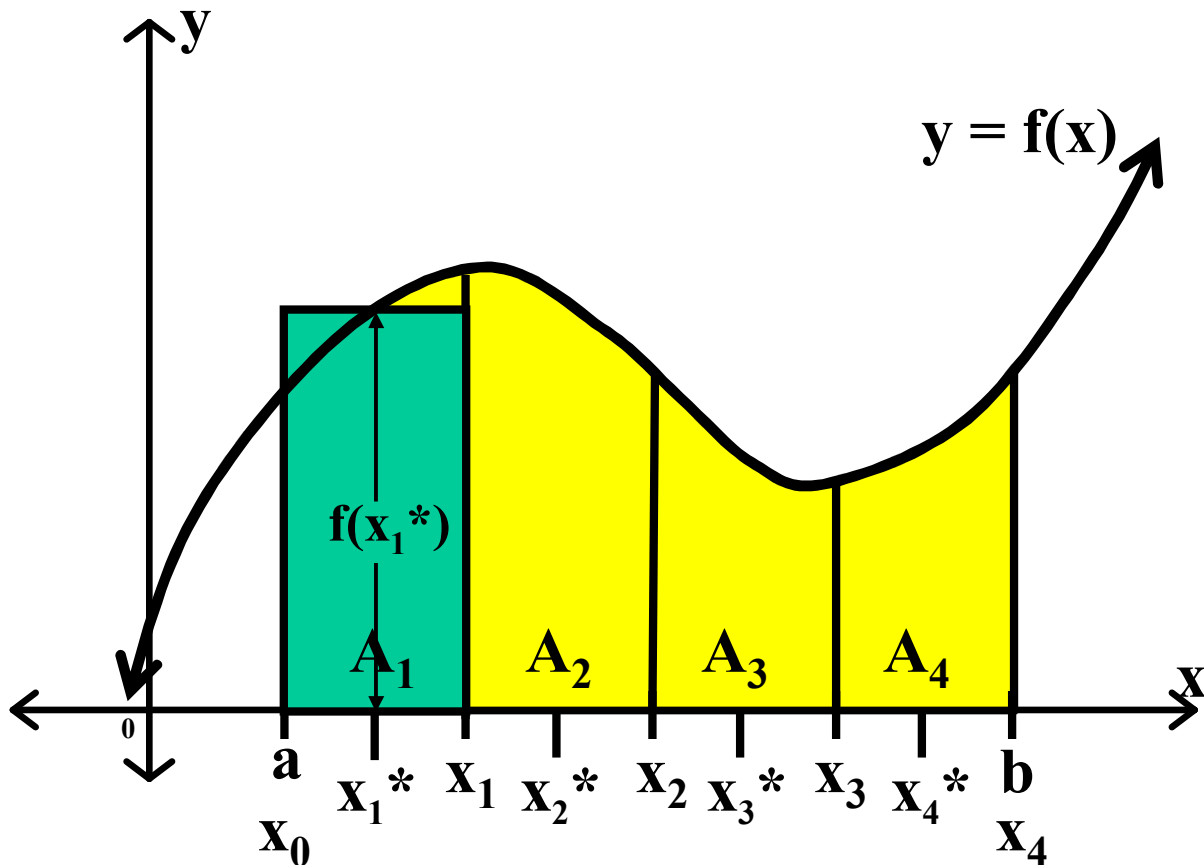
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
The length of the 1st Mid-Rectangle is $f(x_1^*)$.



The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
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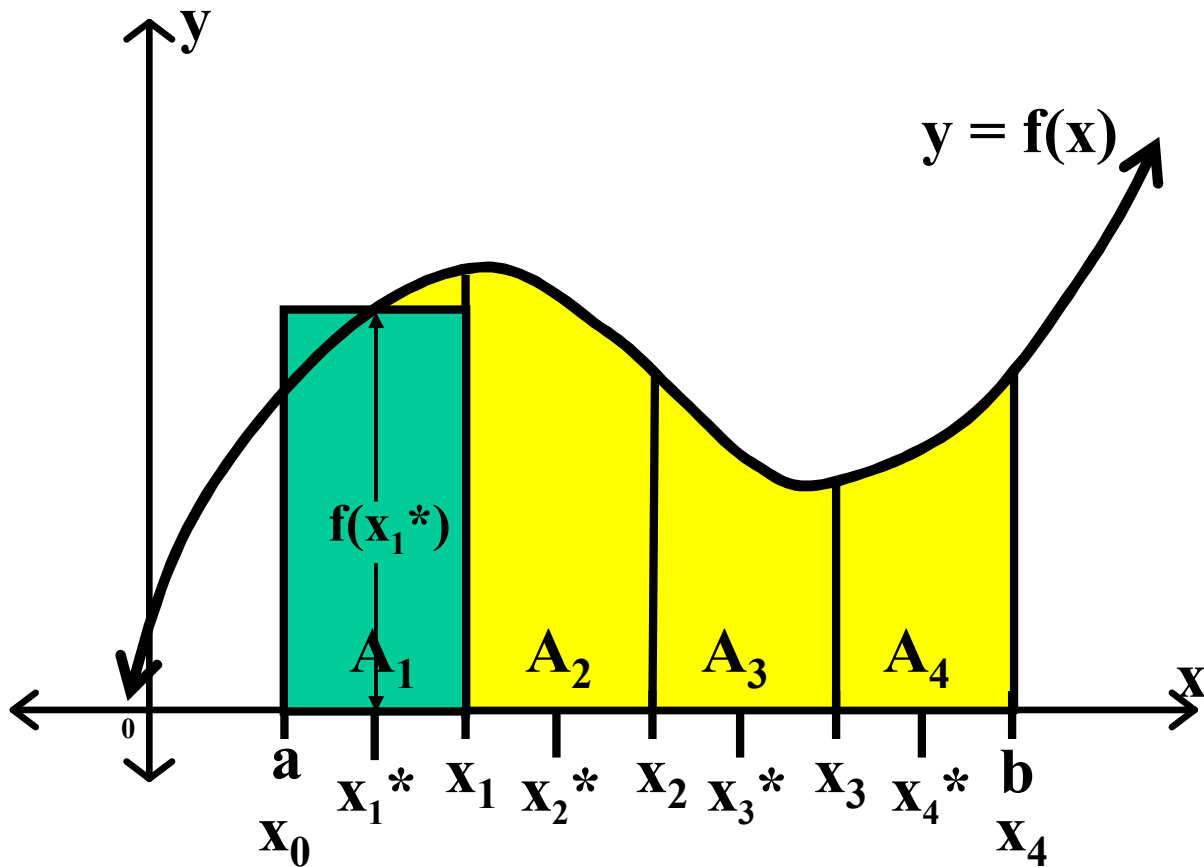
**The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
 The length of the 1st Mid-Rectangle is $f(x_1^*)$. Its width is Δx .**



$$A_1 \approx$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

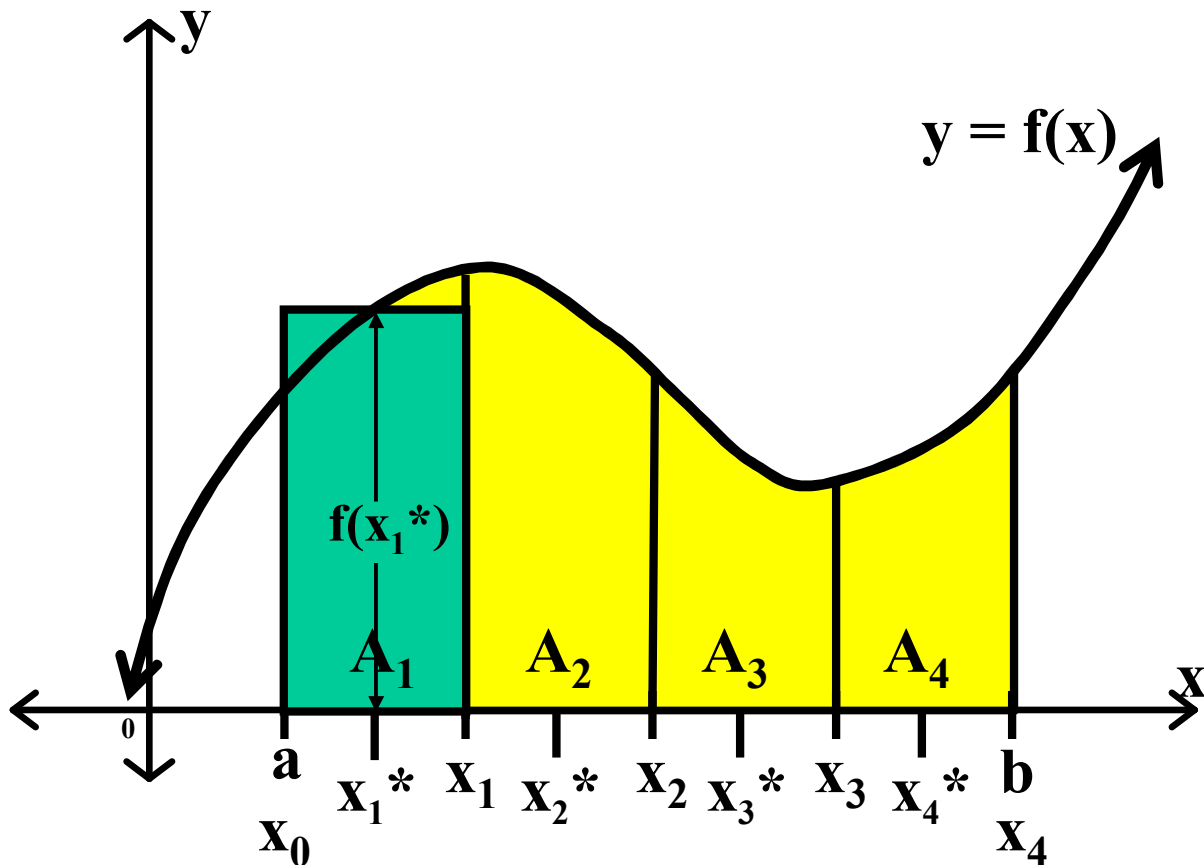
The length of the 1st Mid-Rectangle is $f(x_1^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

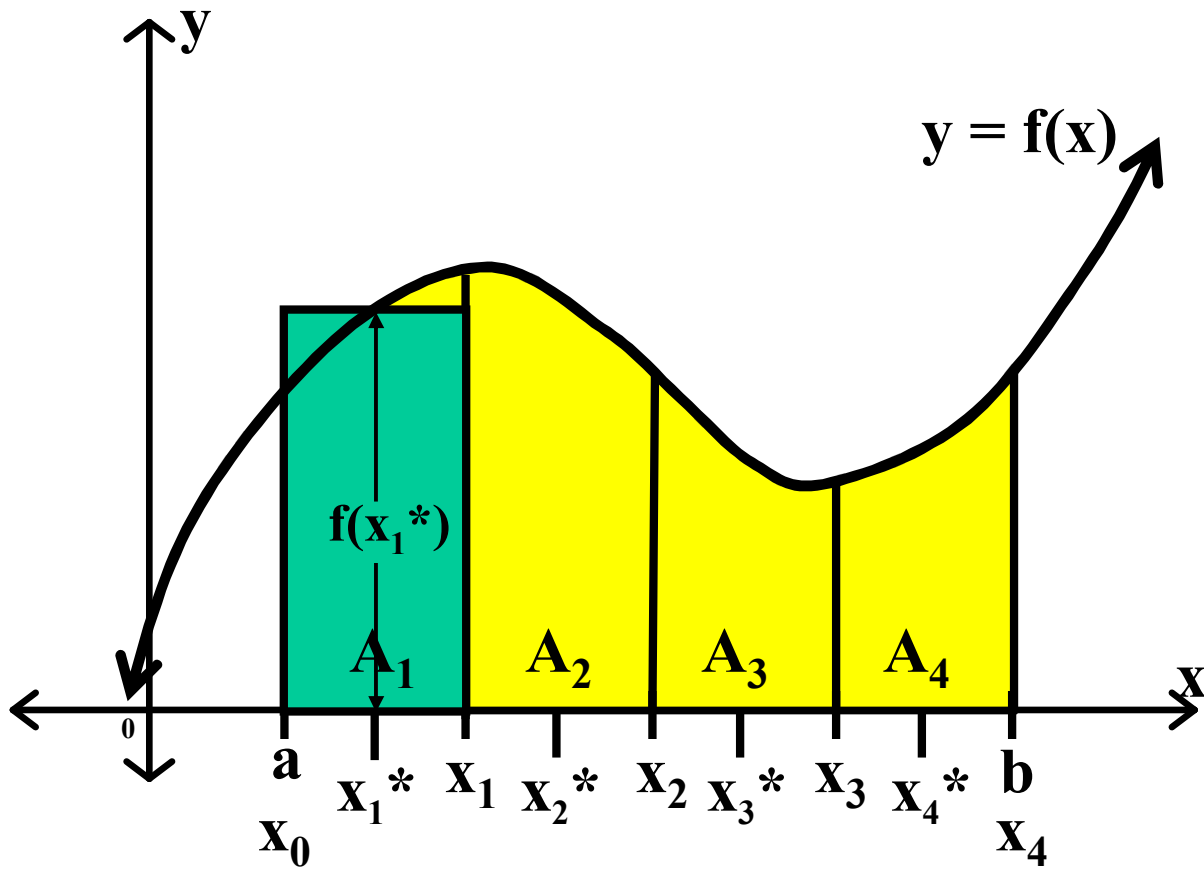
The length of the 1st Mid-Rectangle is $f(x_1^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x$$

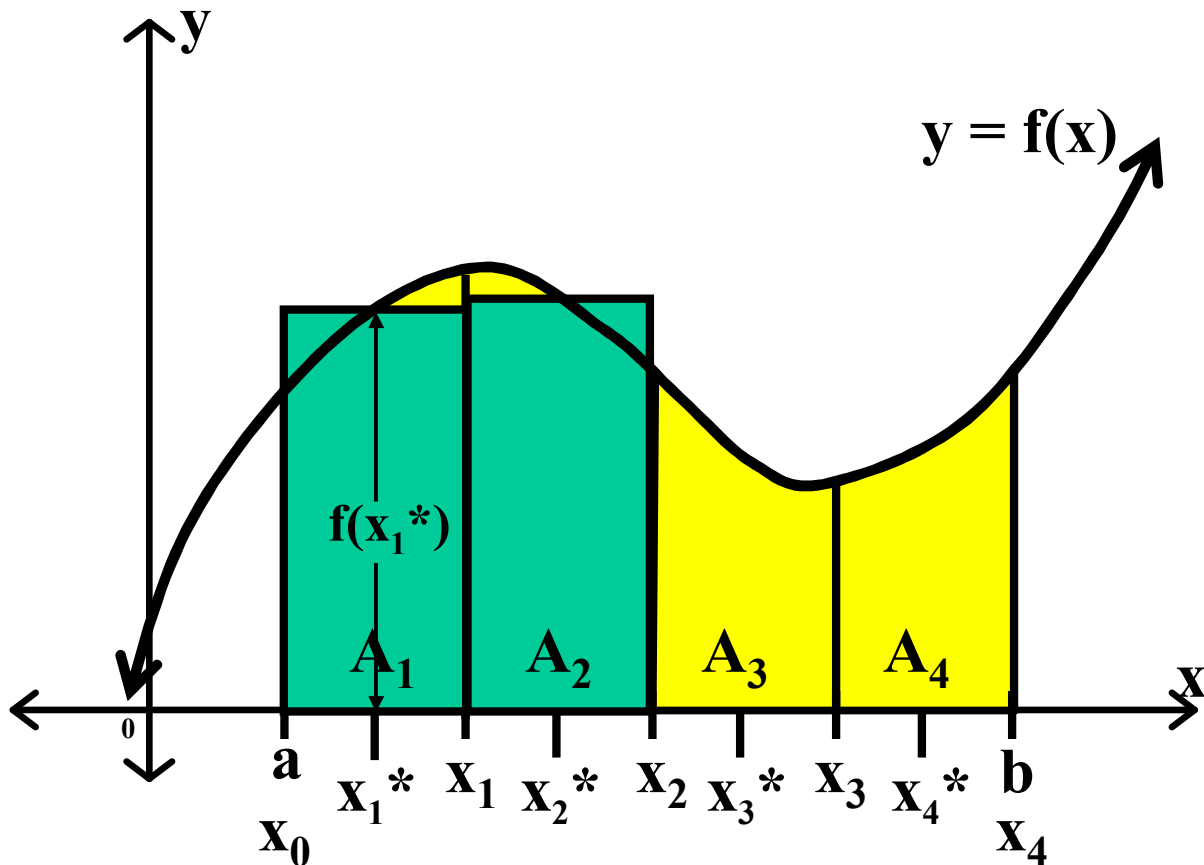
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

The length of the 1st Mid-Rectangle is $f(x_1^*)$. Its width is Δx .



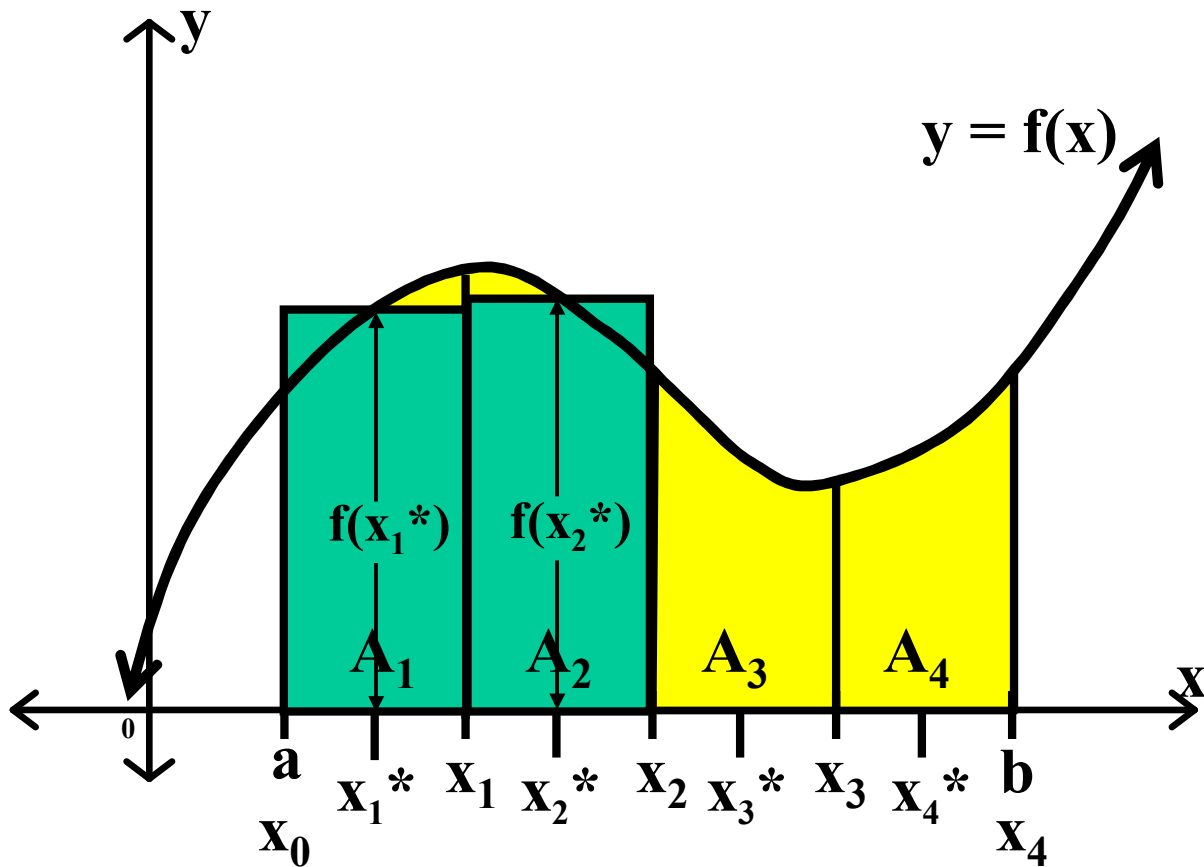
$$A_1 \approx f(x_1^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.



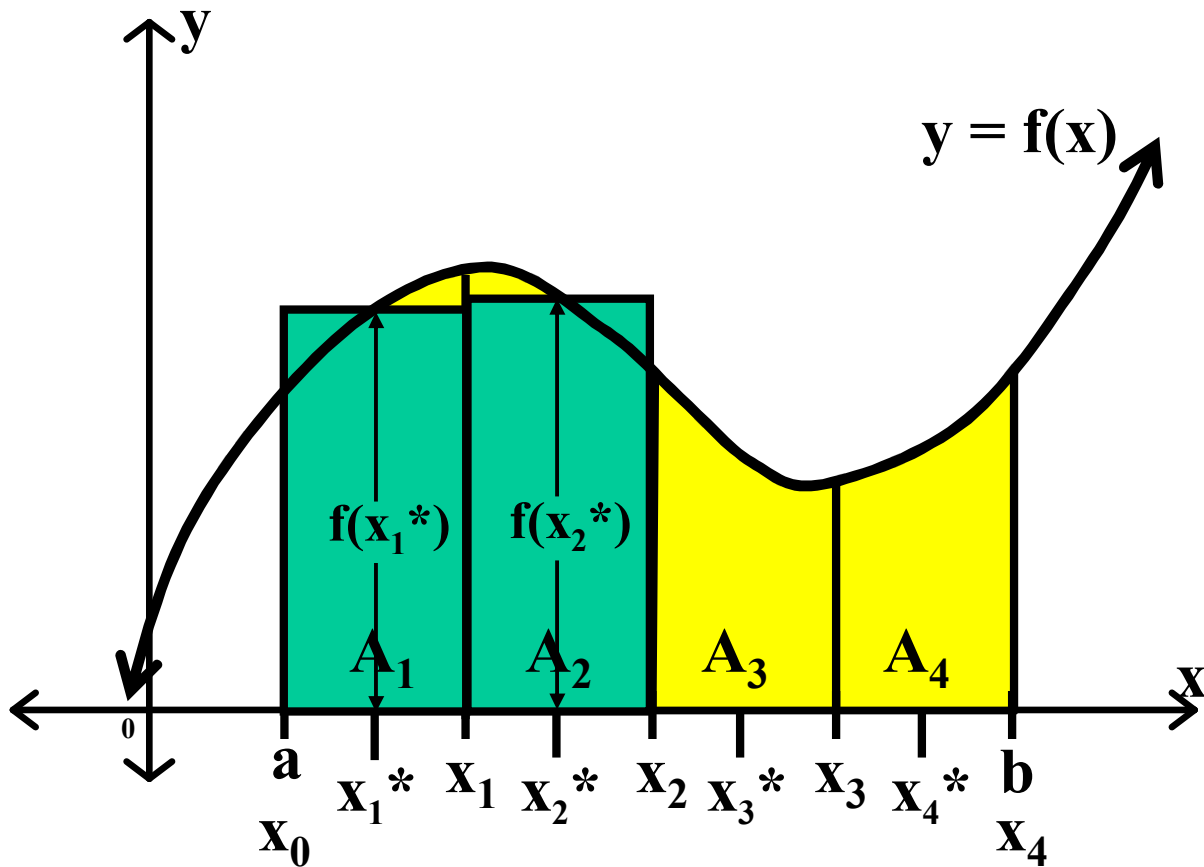
$$A_1 \approx f(x_1^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
 The length of the 2nd Mid-Rectangle is $f(x_2^*)$.



$$A_1 \approx f(x_1^*)\Delta x$$

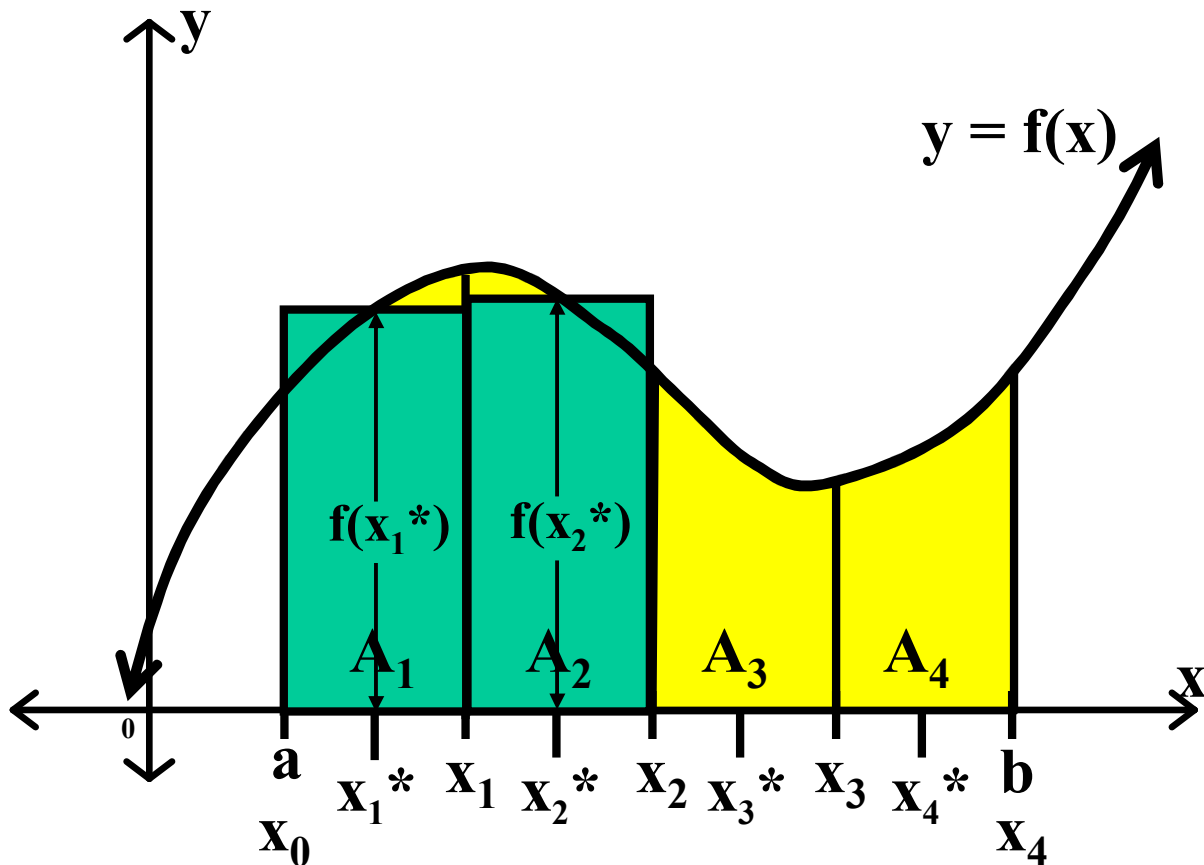
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
 The length of the 2^{nd} Mid-Rectangle is $f(x_2^*)$.



$$A_1 \approx f(x_1^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

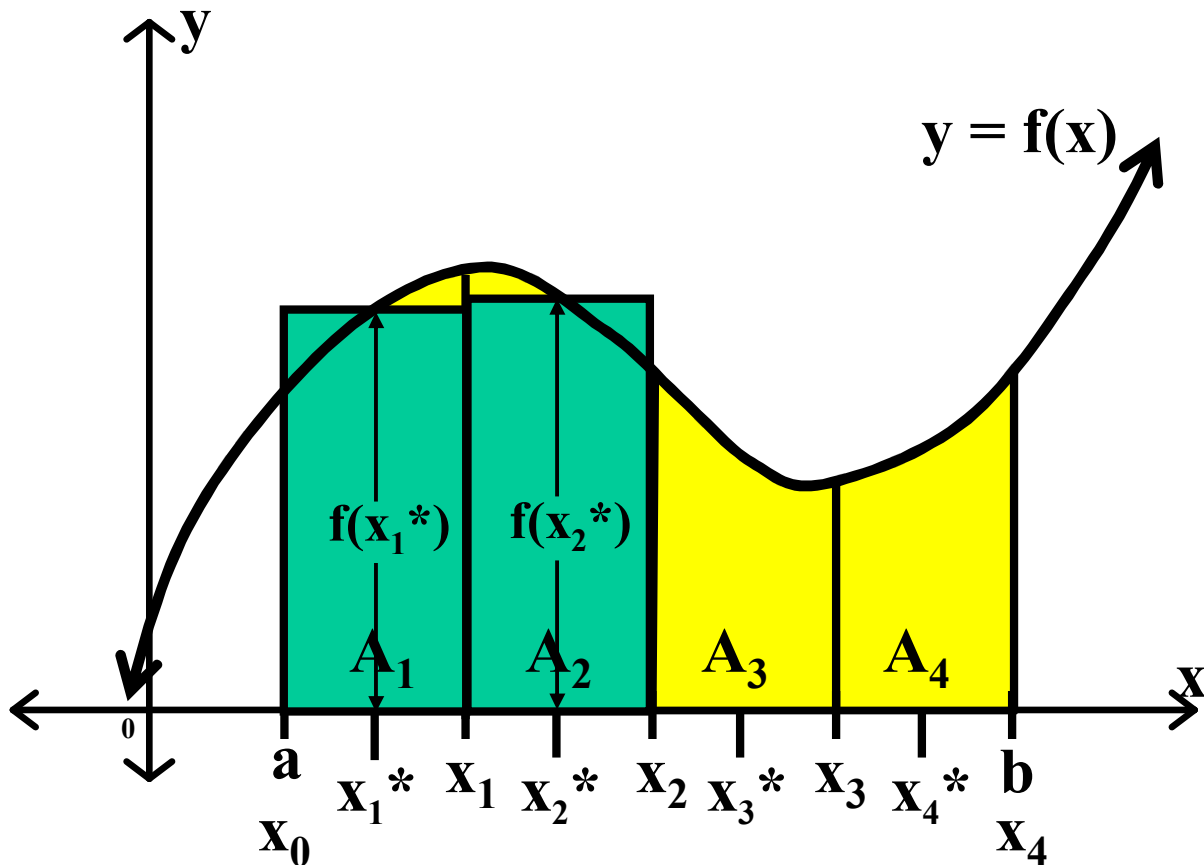
The length of the 2nd Mid-Rectangle is $f(x_2^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

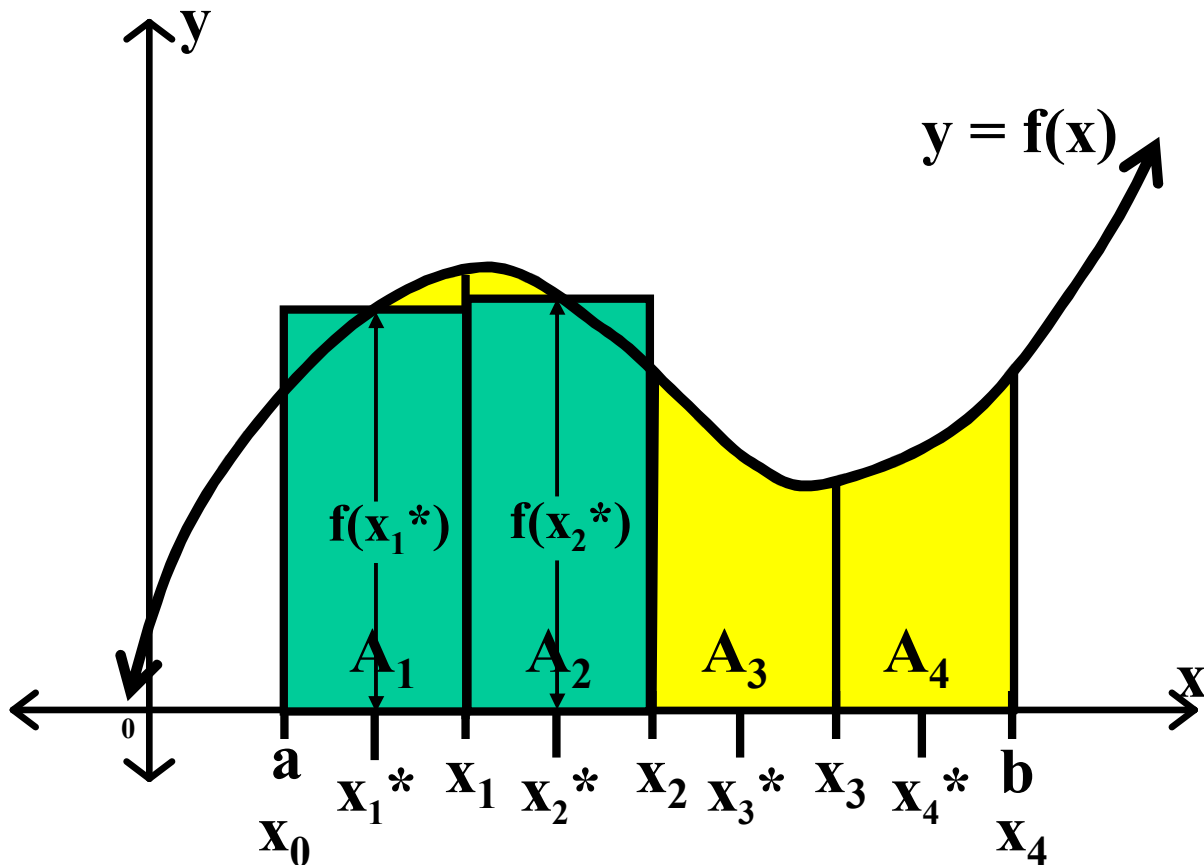
The length of the 2nd Mid-Rectangle is $f(x_2^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

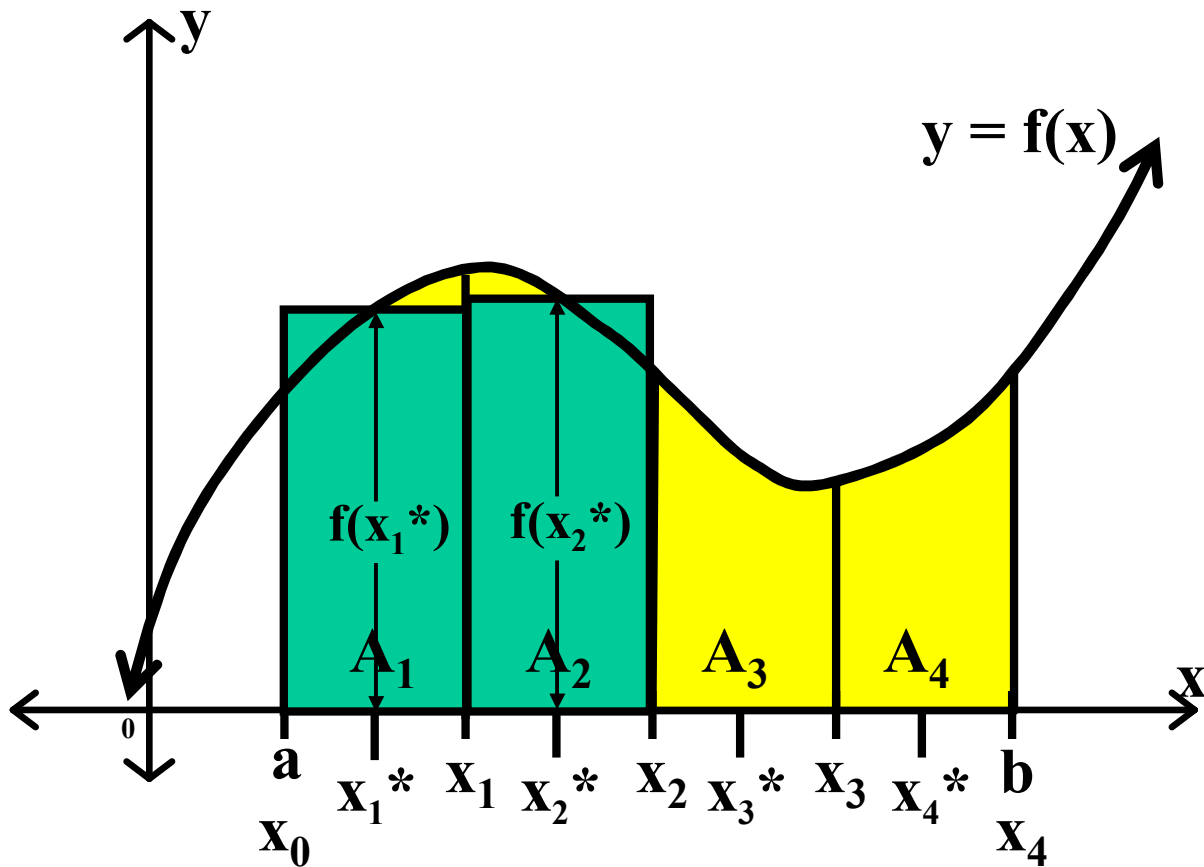
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$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x$$

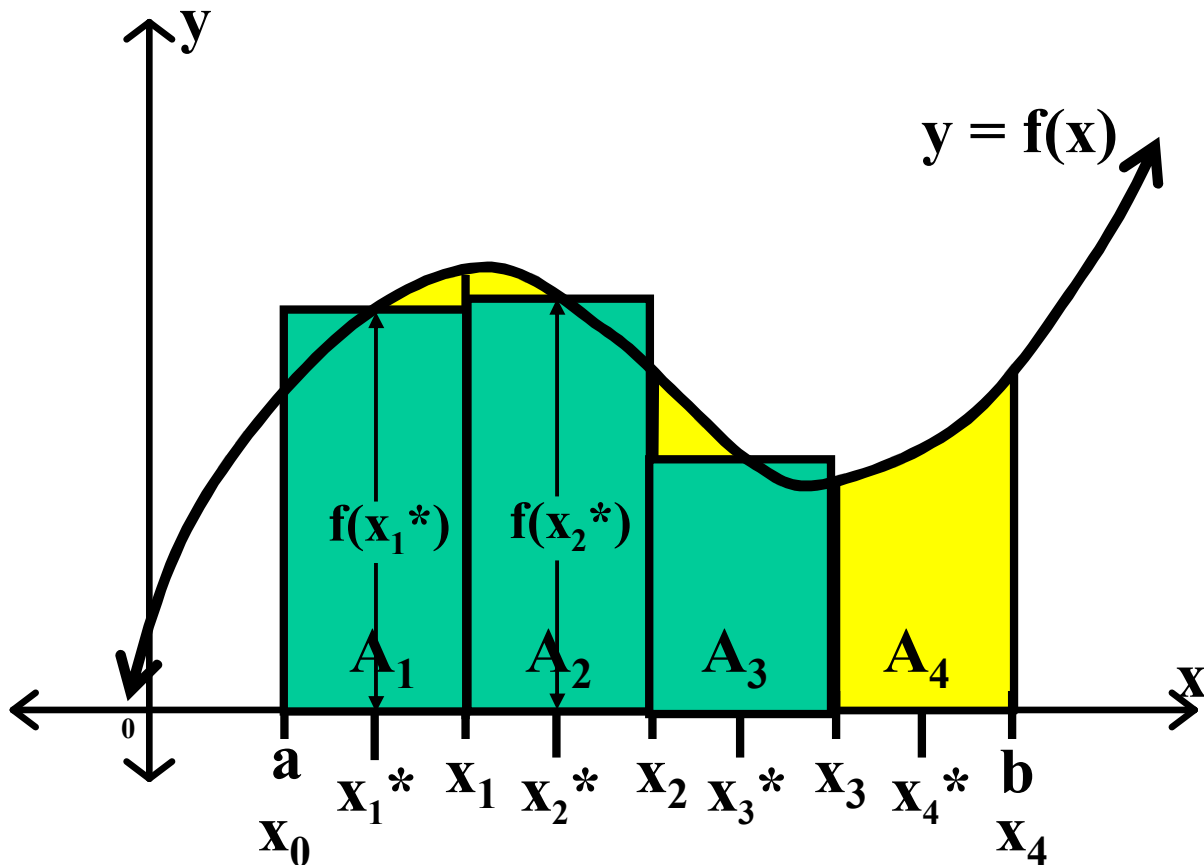
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

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$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x$$

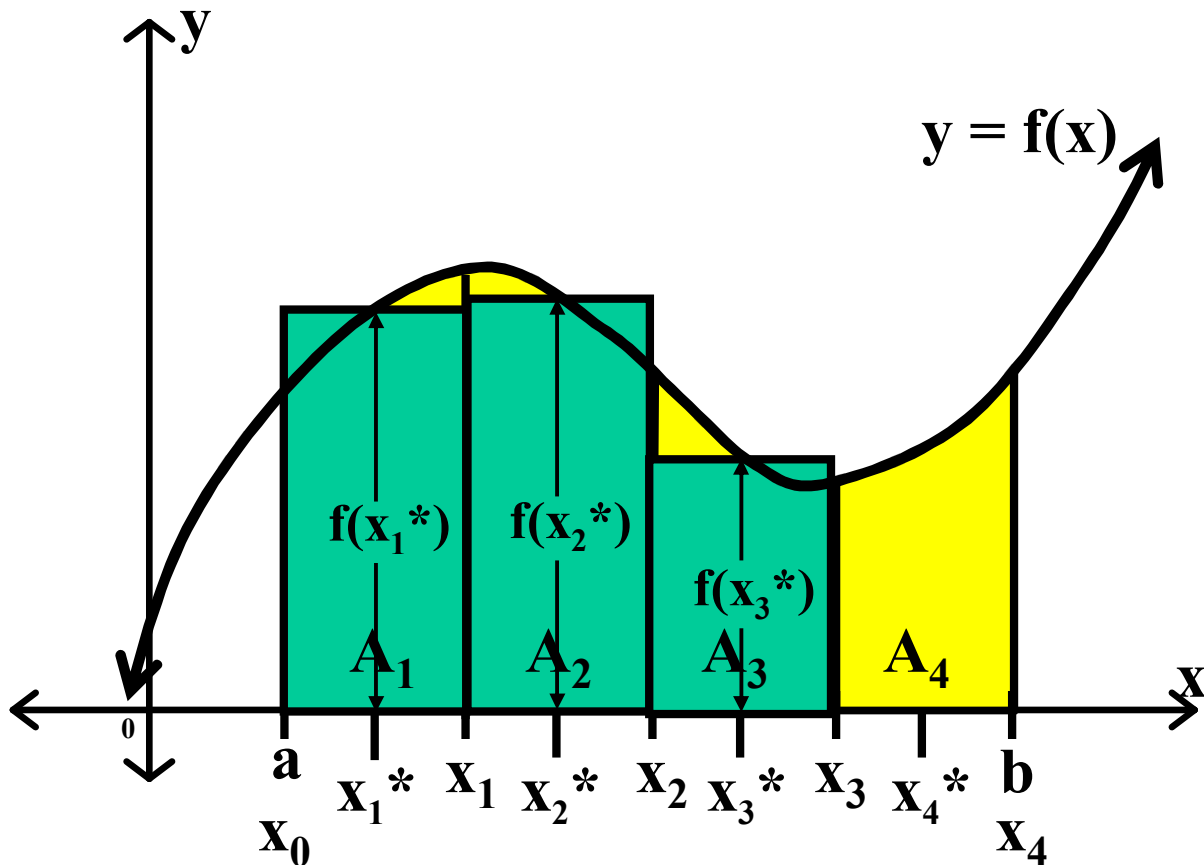
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

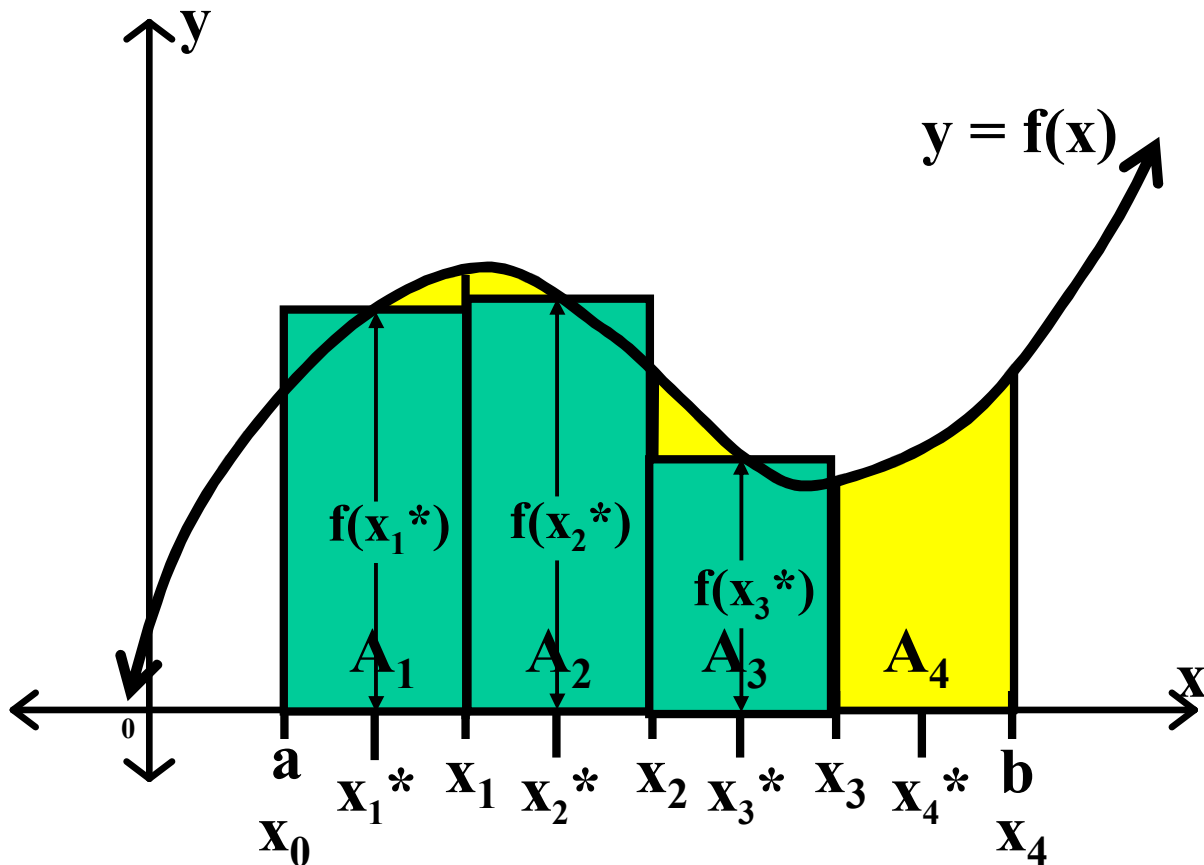
The length of the 3rd Mid-Rectangle is $f(x_3^*)$.



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

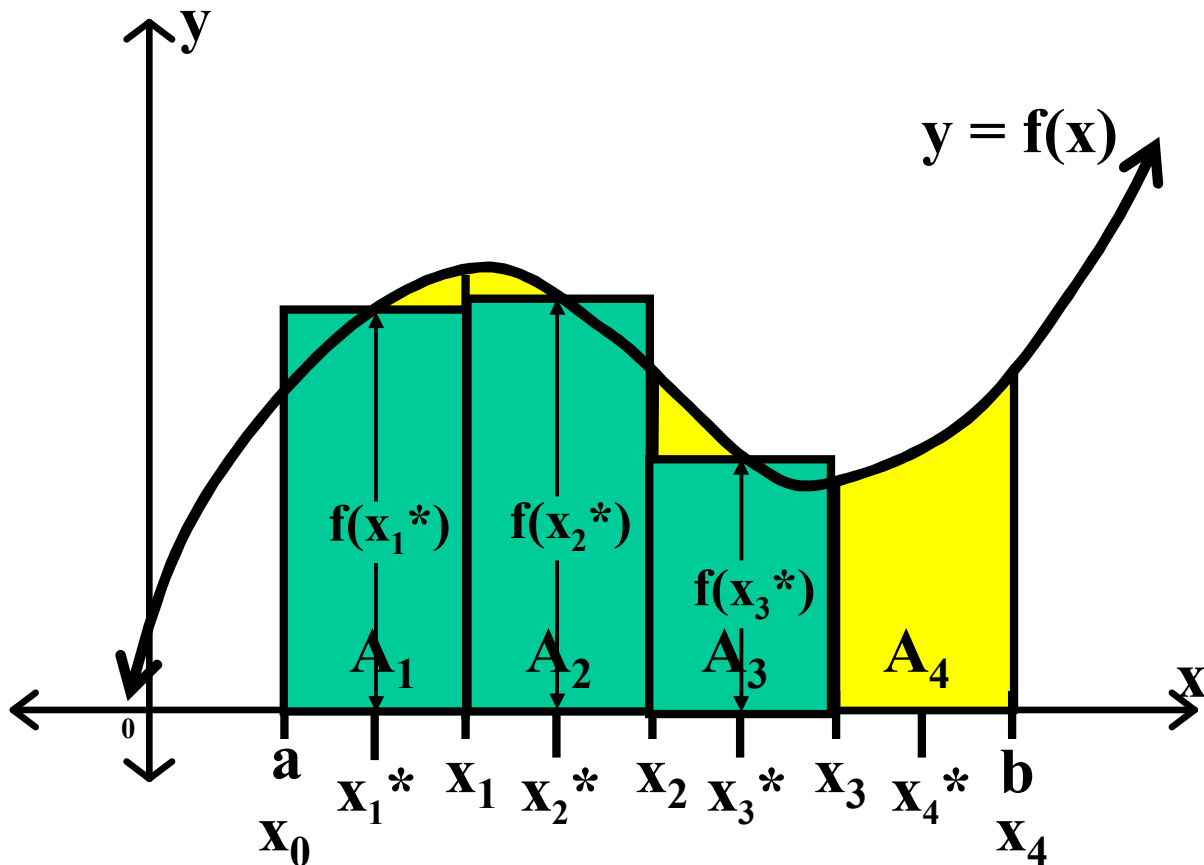
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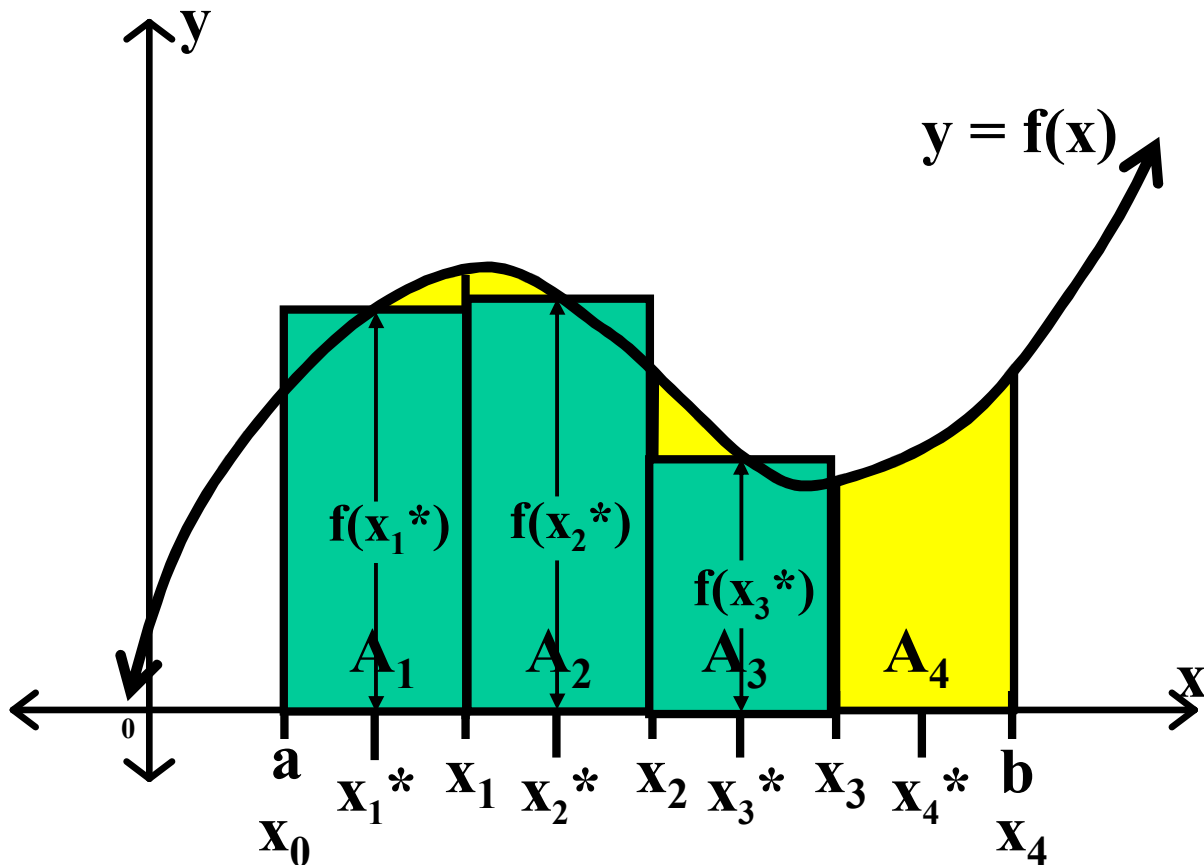
The length of the 3rd Mid-Rectangle is $f(x_3^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

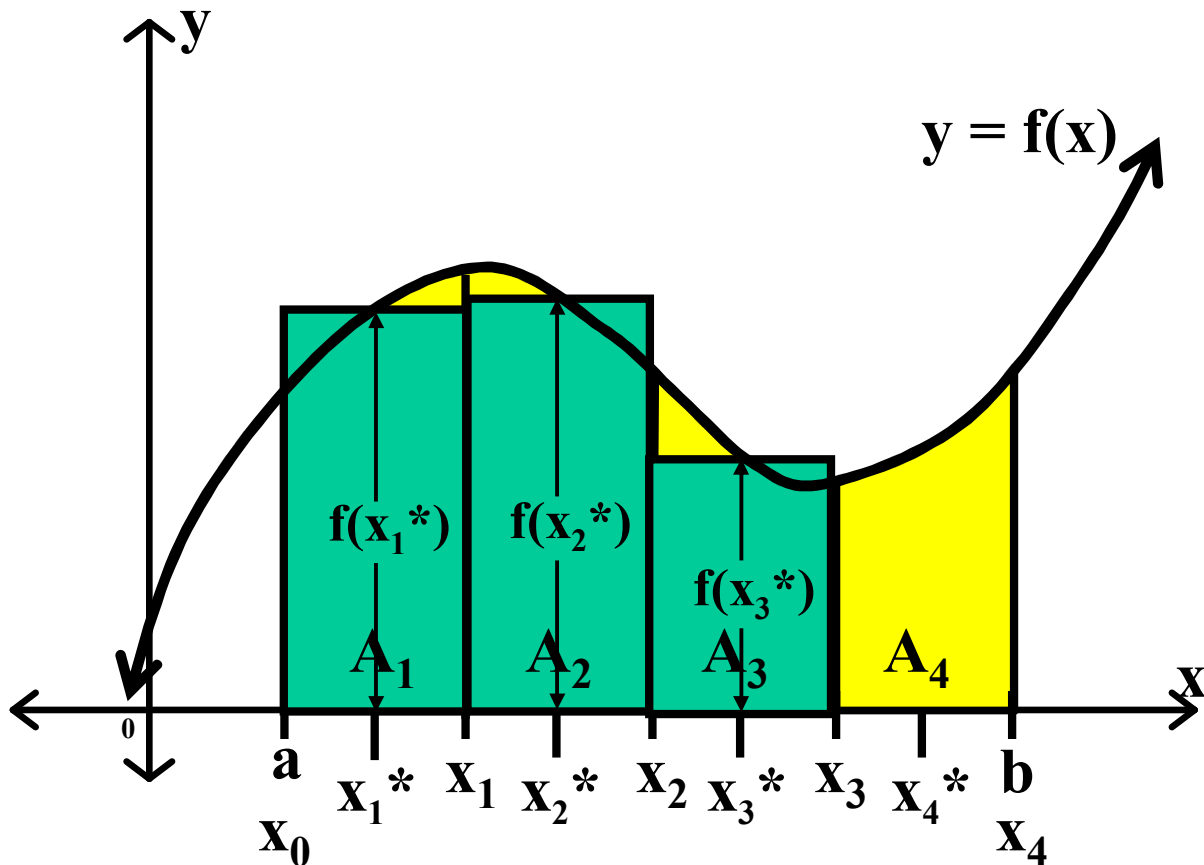
The length of the 3rd Mid-Rectangle is $f(x_3^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

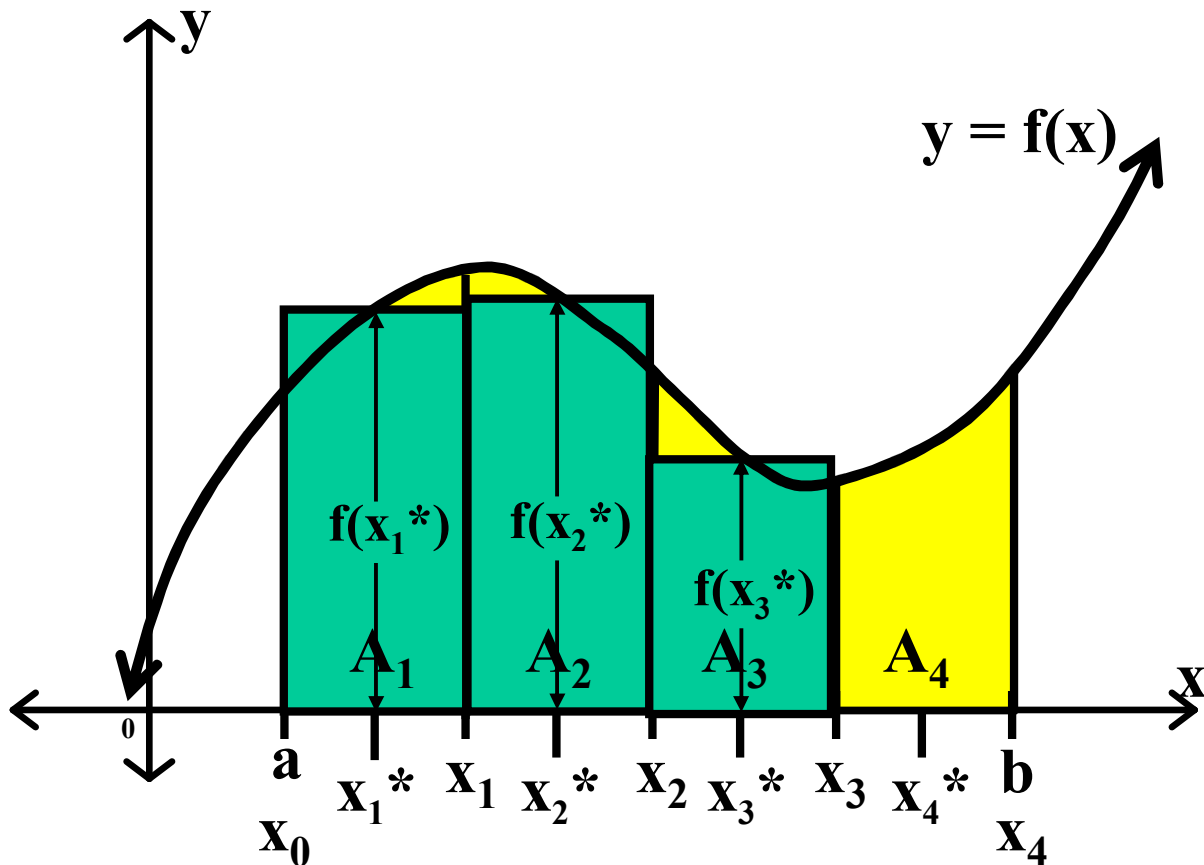
The length of the 3rd Mid-Rectangle is $f(x_3^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

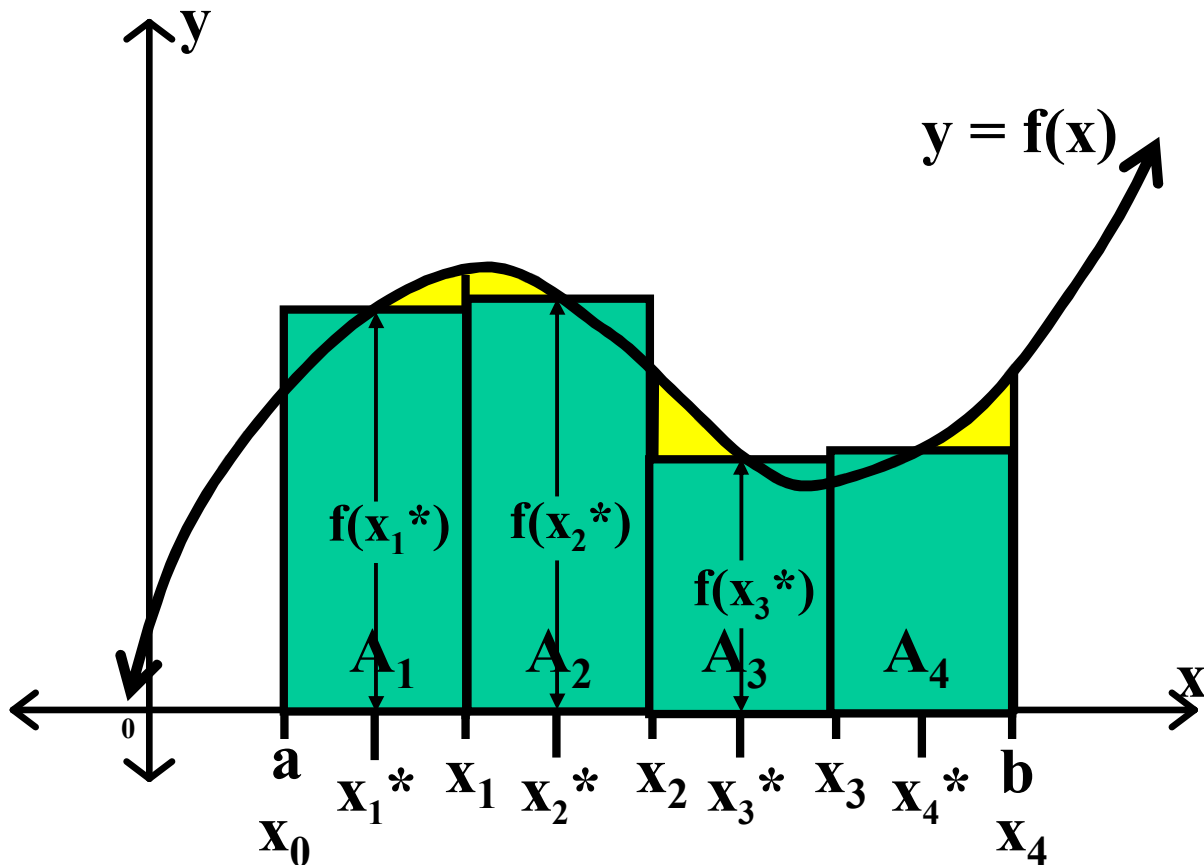
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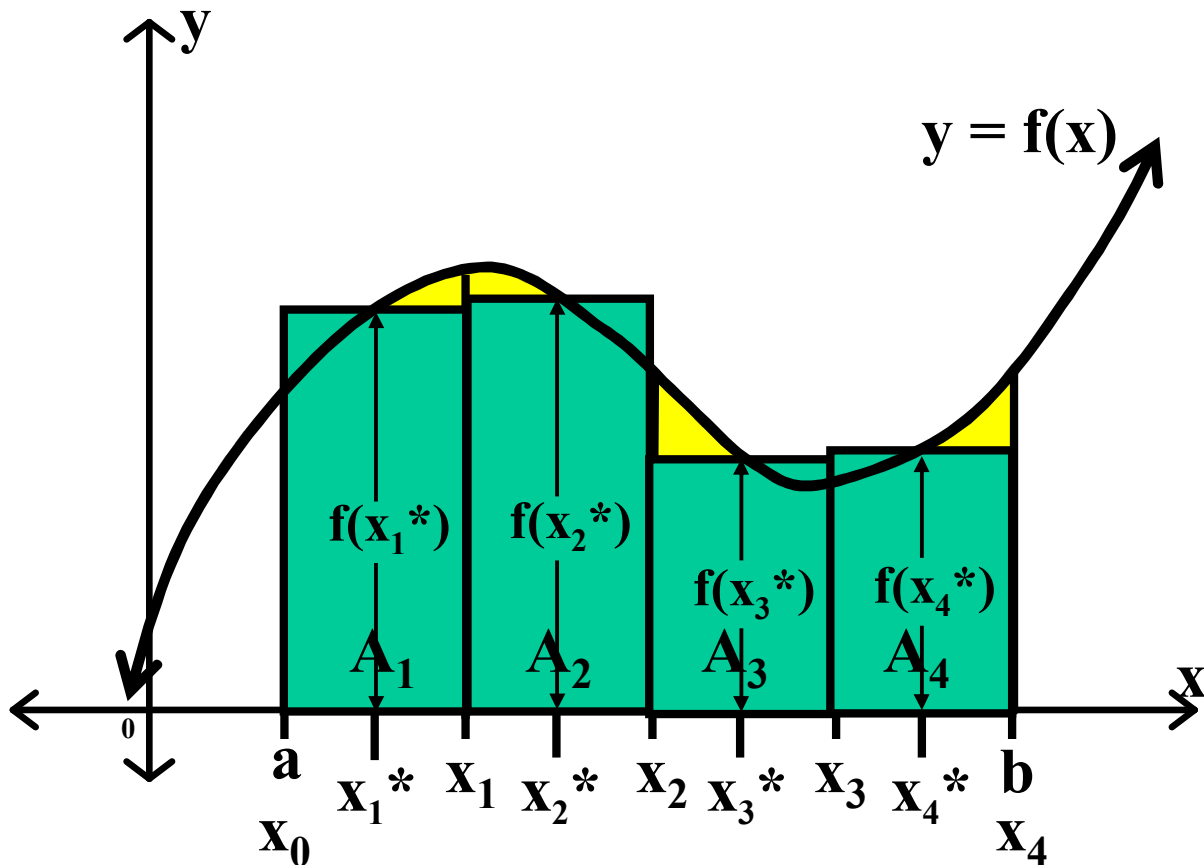
$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

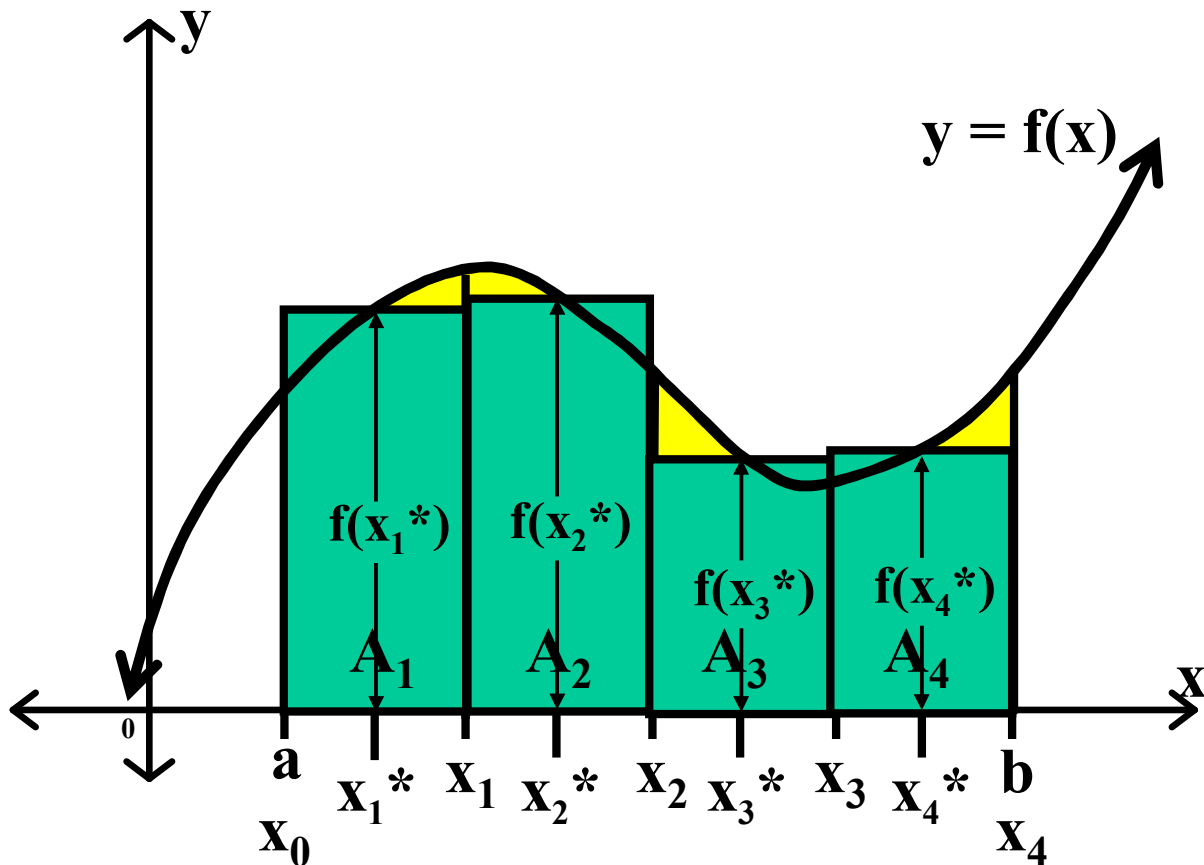
The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.
 The length of the 4^{th} Mid-Rectangle is $f(x_4^*)$.



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

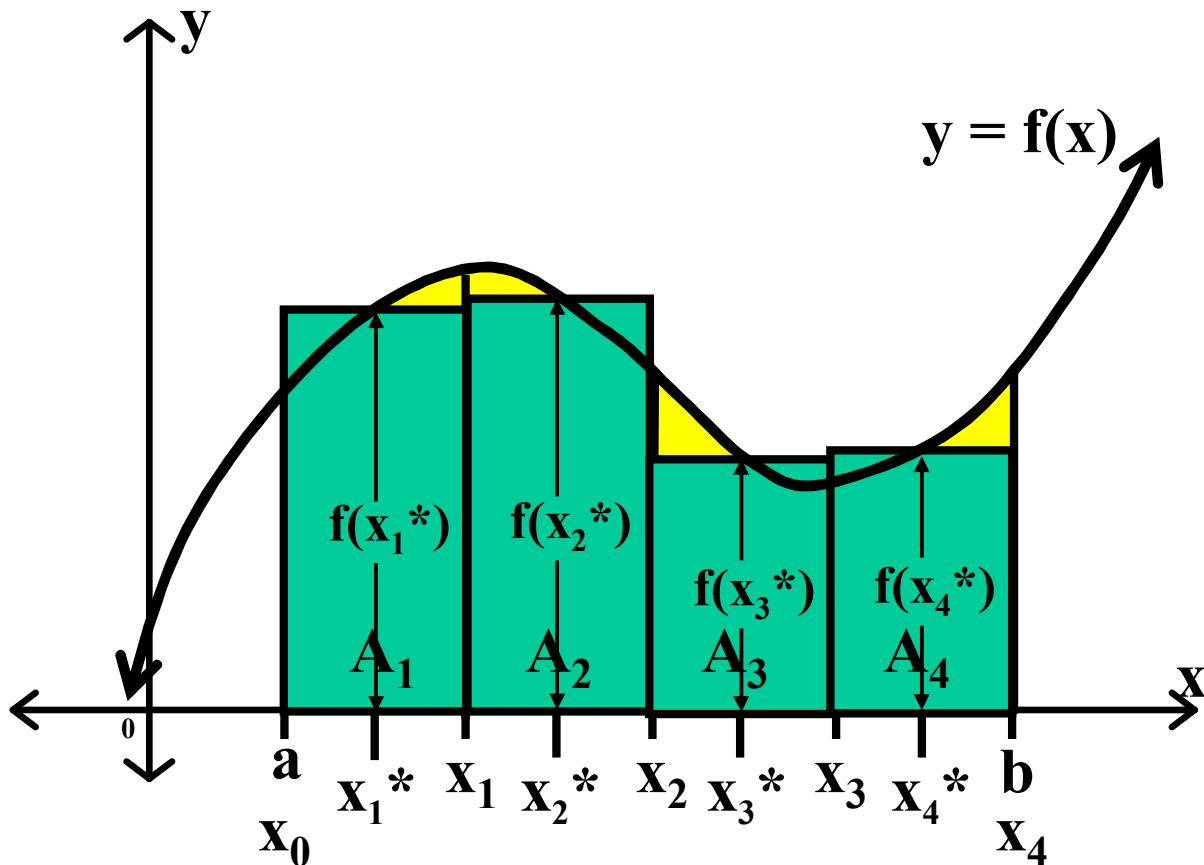
The length of the 4th Mid-Rectangle is $f(x_4^*)$.



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

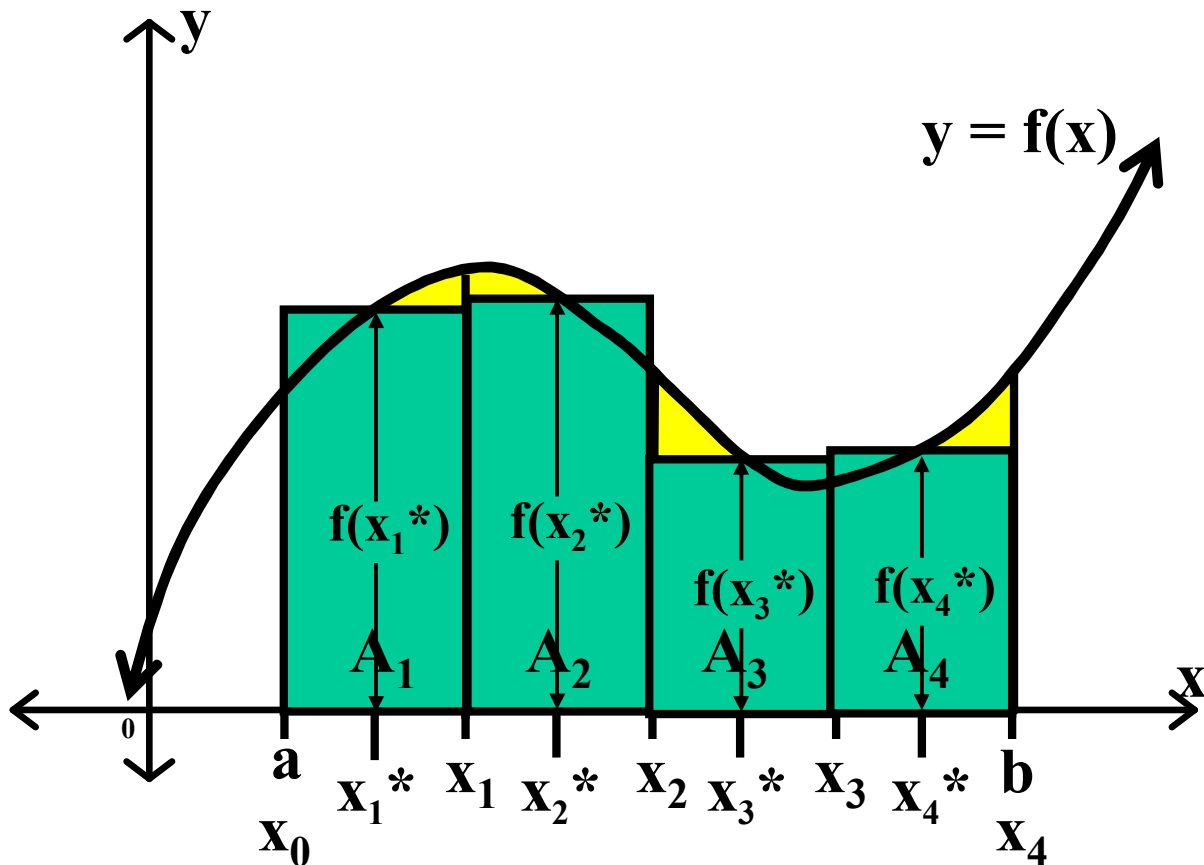
The length of the 4th Mid-Rectangle is $f(x_4^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

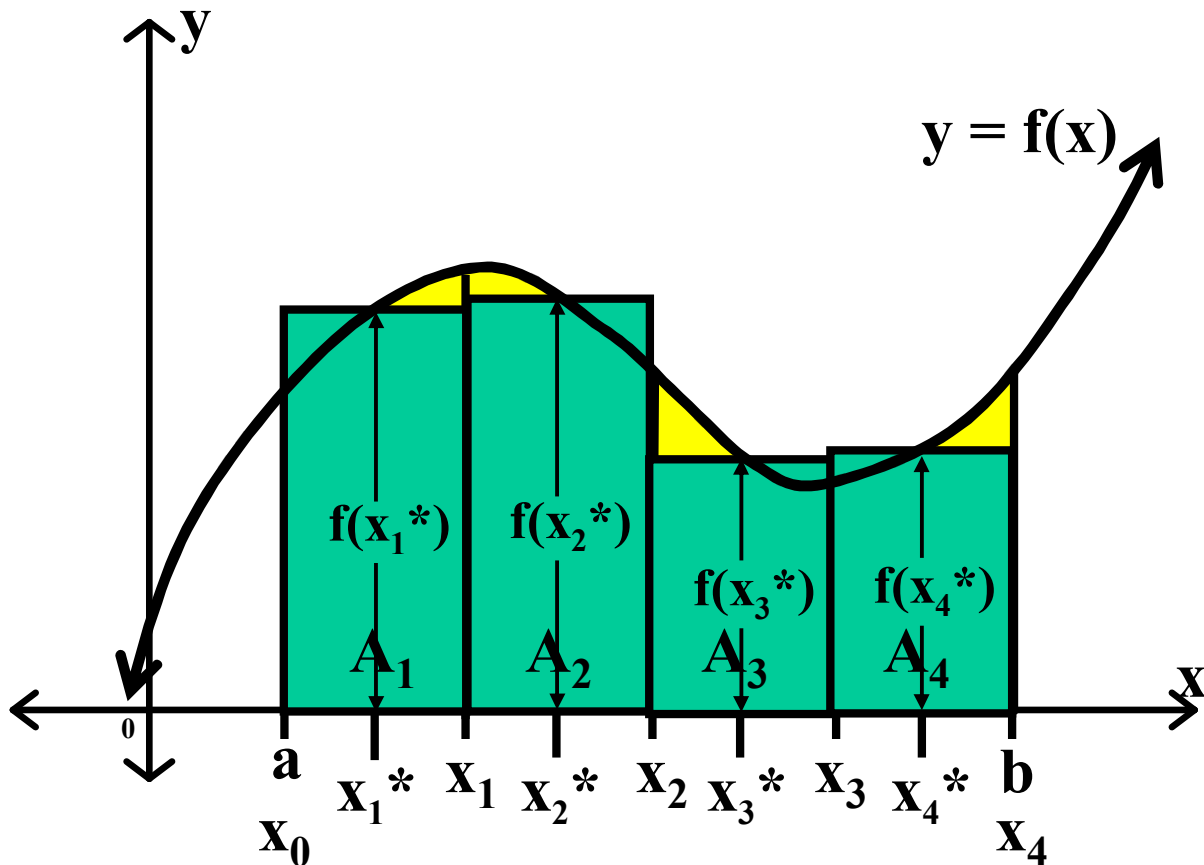
The length of the 4th Mid-Rectangle is $f(x_4^*)$. Its width is Δx .



$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

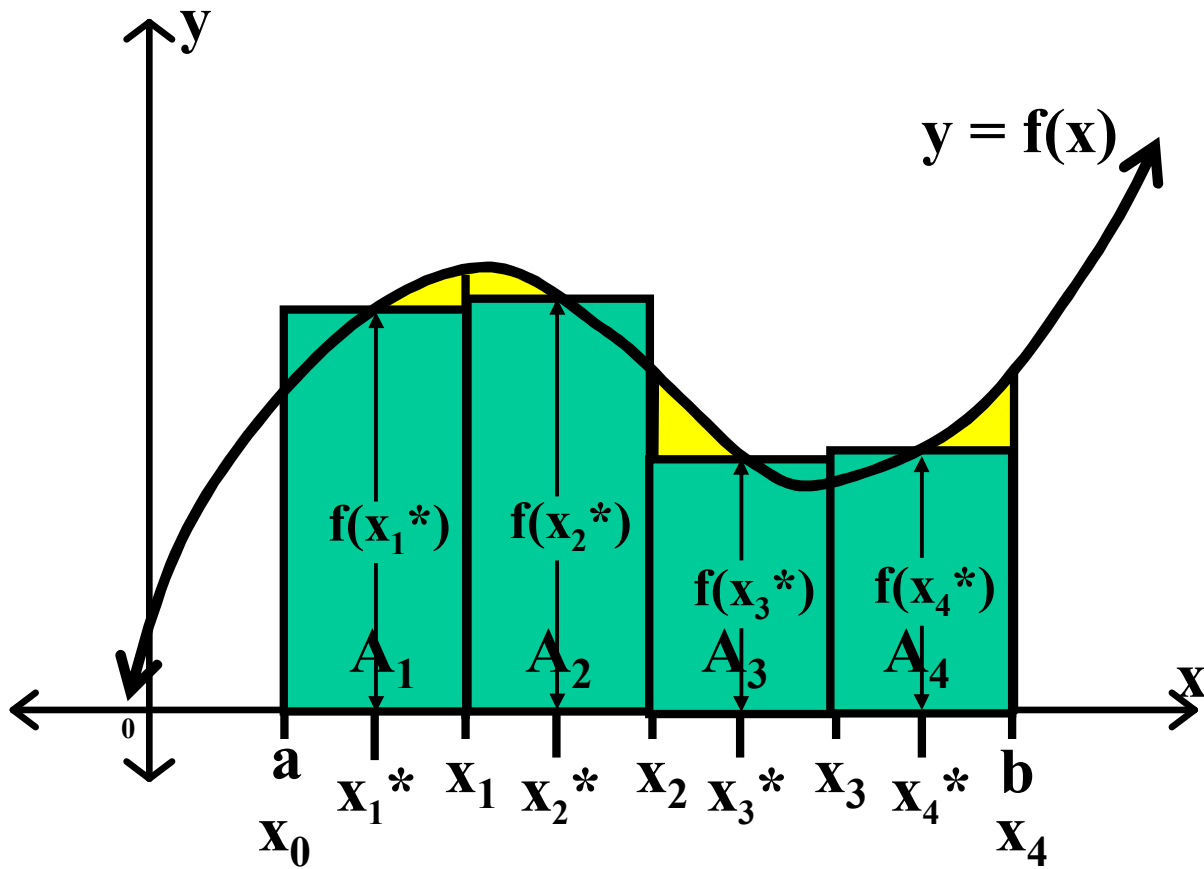
The length of the 4th Mid-Rectangle is $f(x_4^*)$. Its width is Δx .



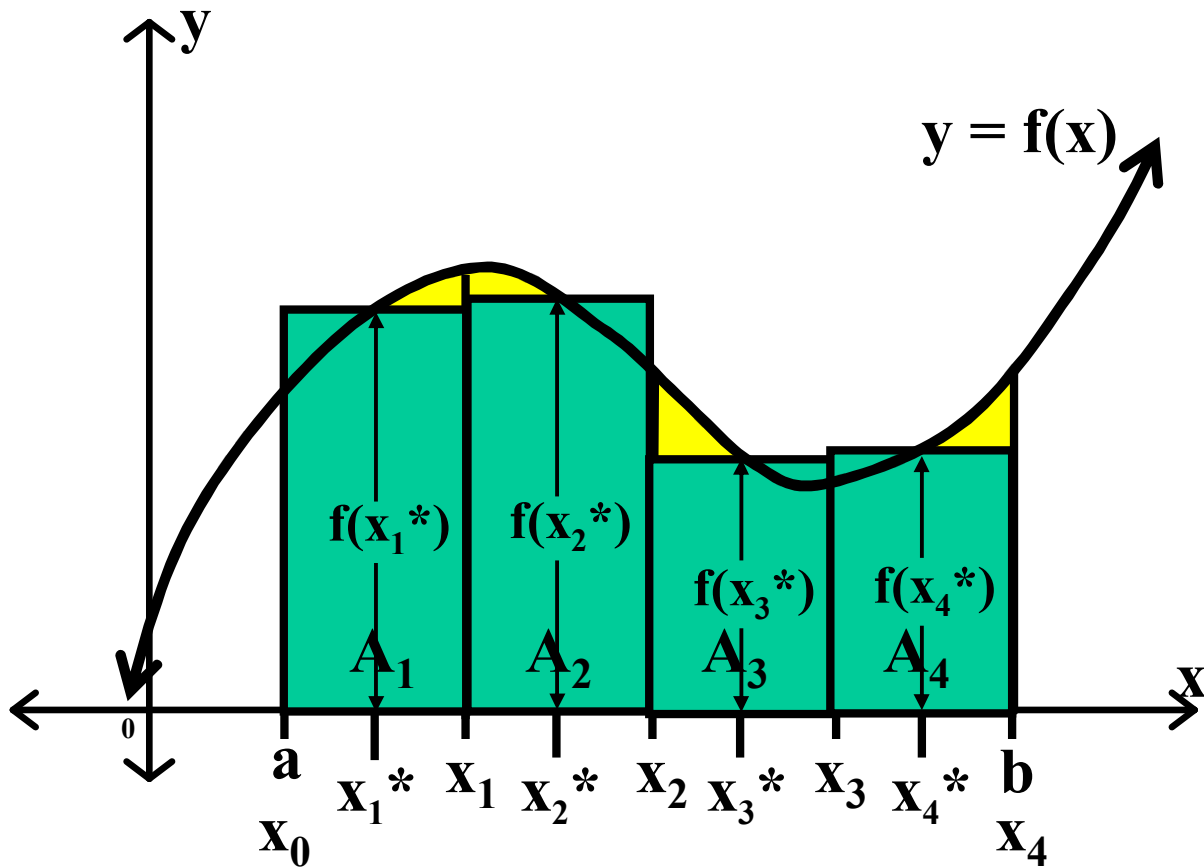
$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

The length of the i^{th} Mid-Rectangle is $f(x_i^*)$.

The length of the 4th Mid-Rectangle is $f(x_4^*)$. Its width is Δx .

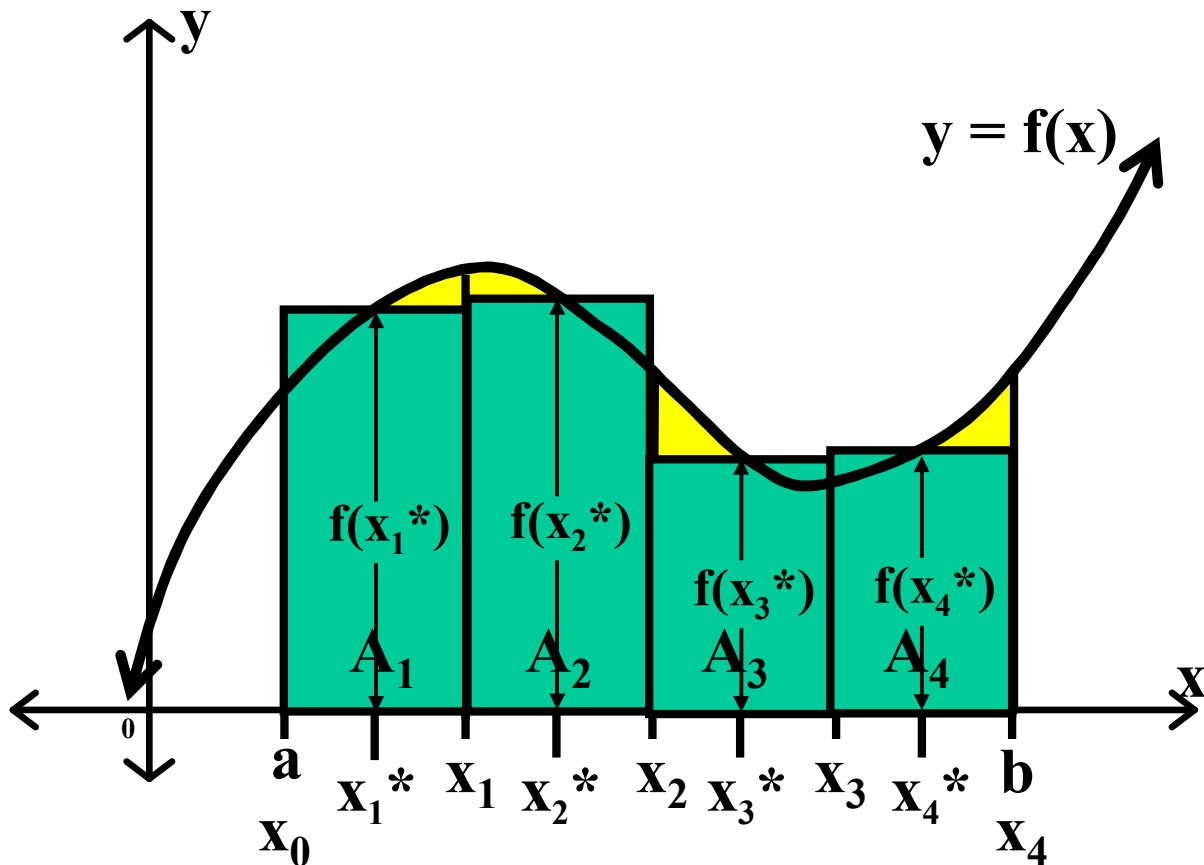


$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$



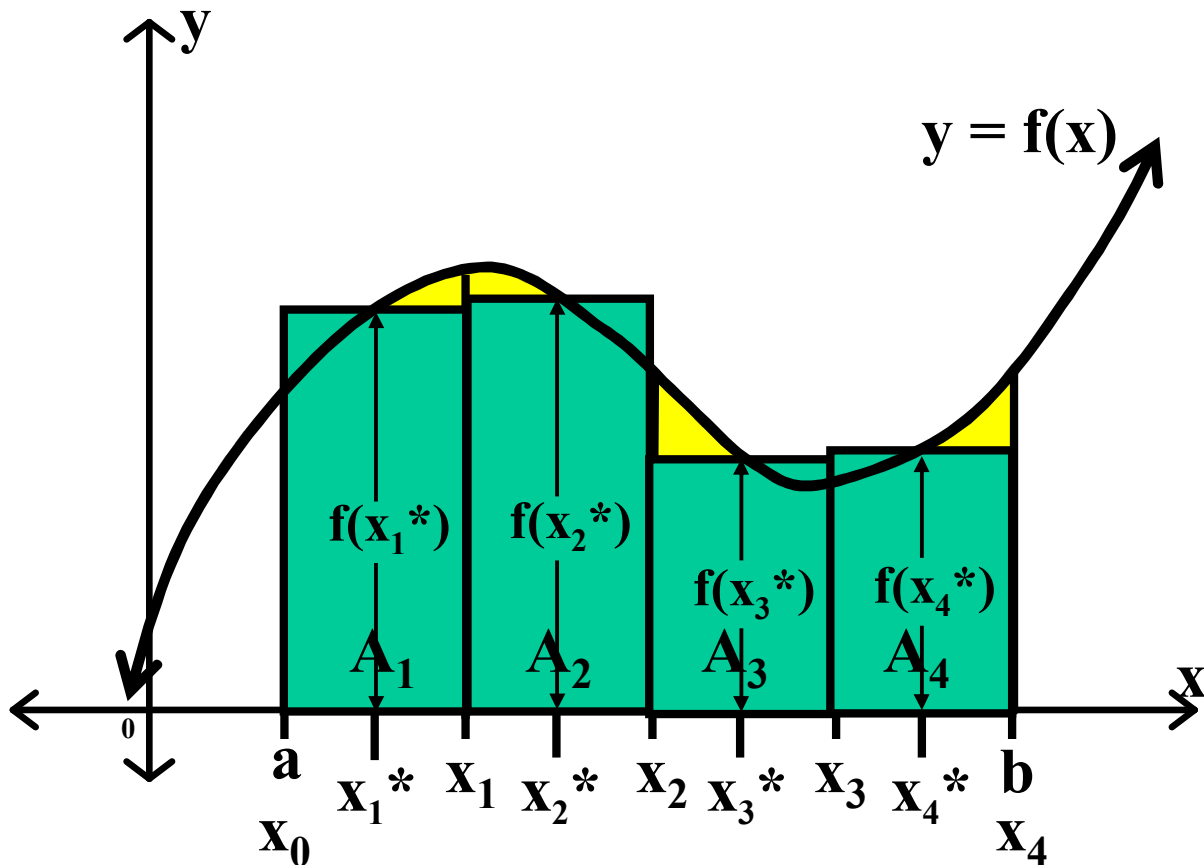
$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

Notice that, in general,



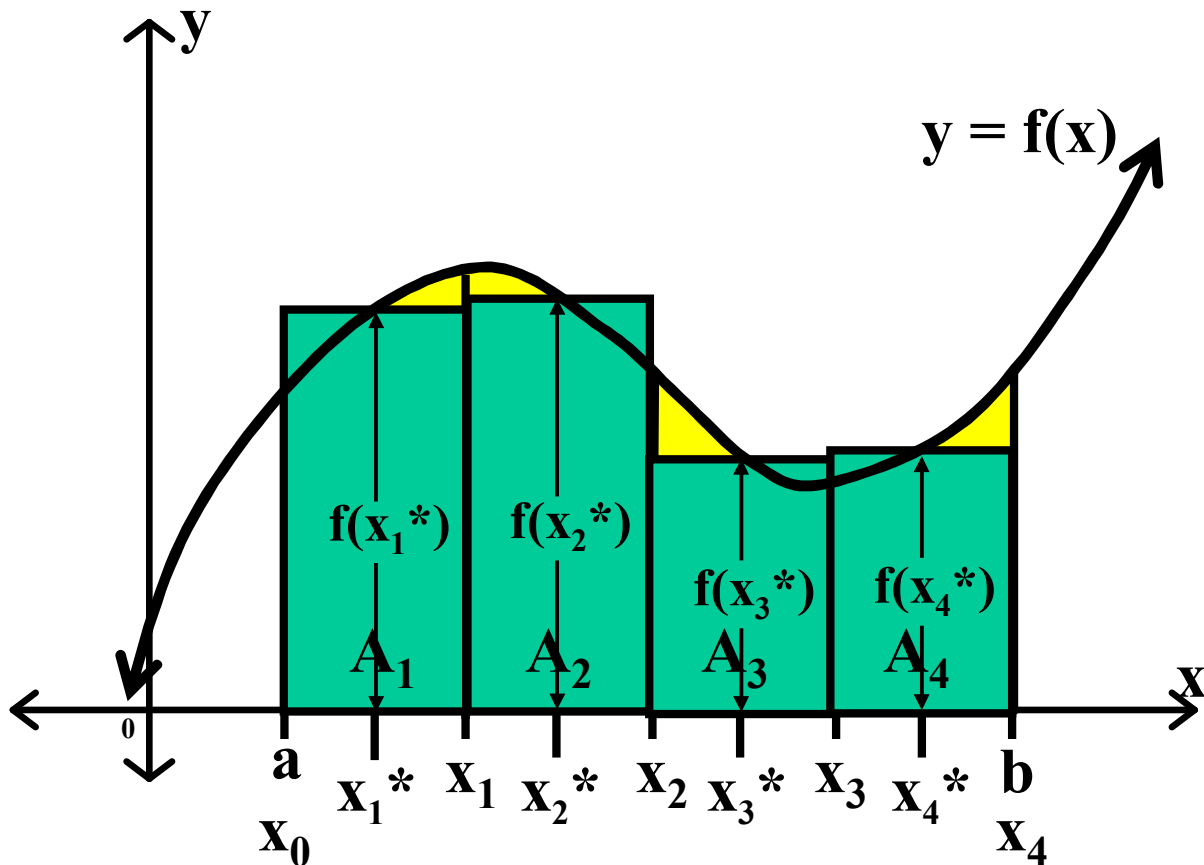
$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

Notice that, in general, $A_i \approx$



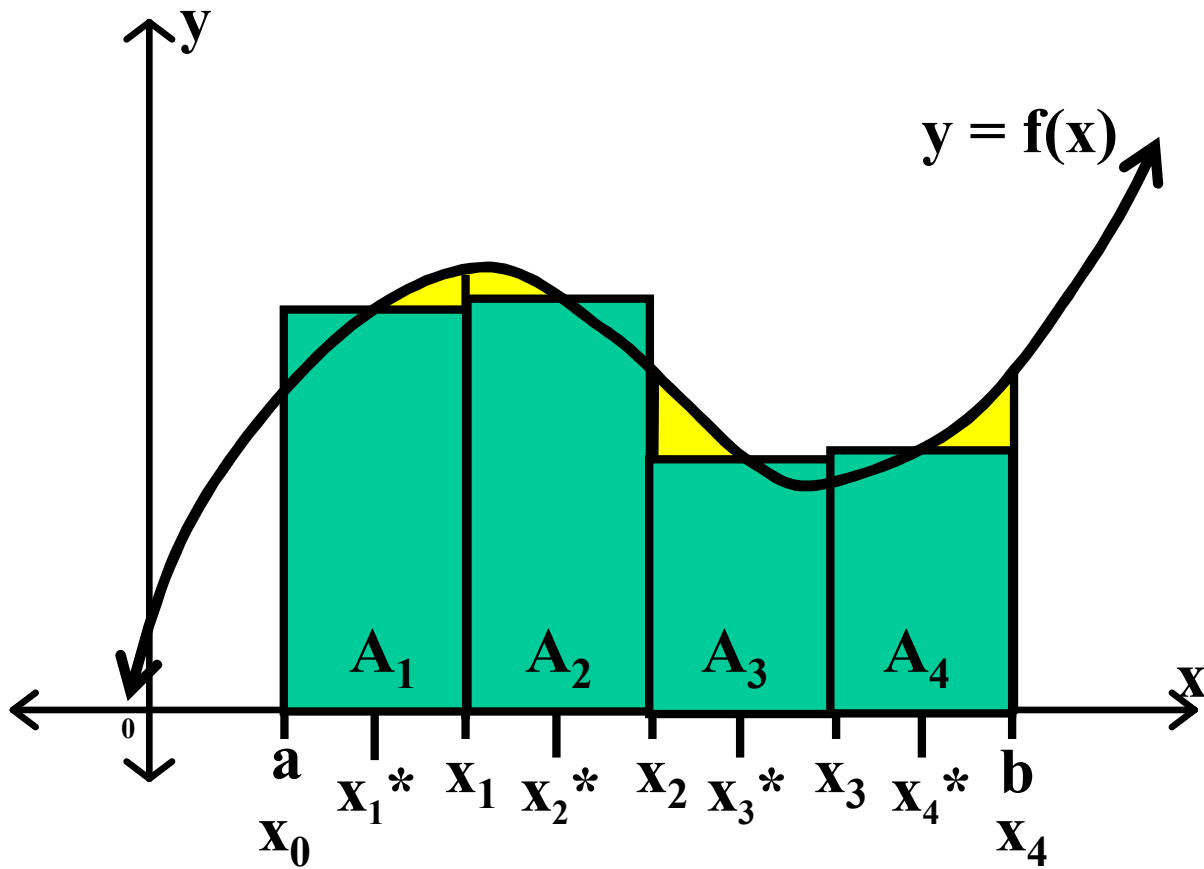
$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

Notice that, in general, $A_i \approx f(x_i^*)$

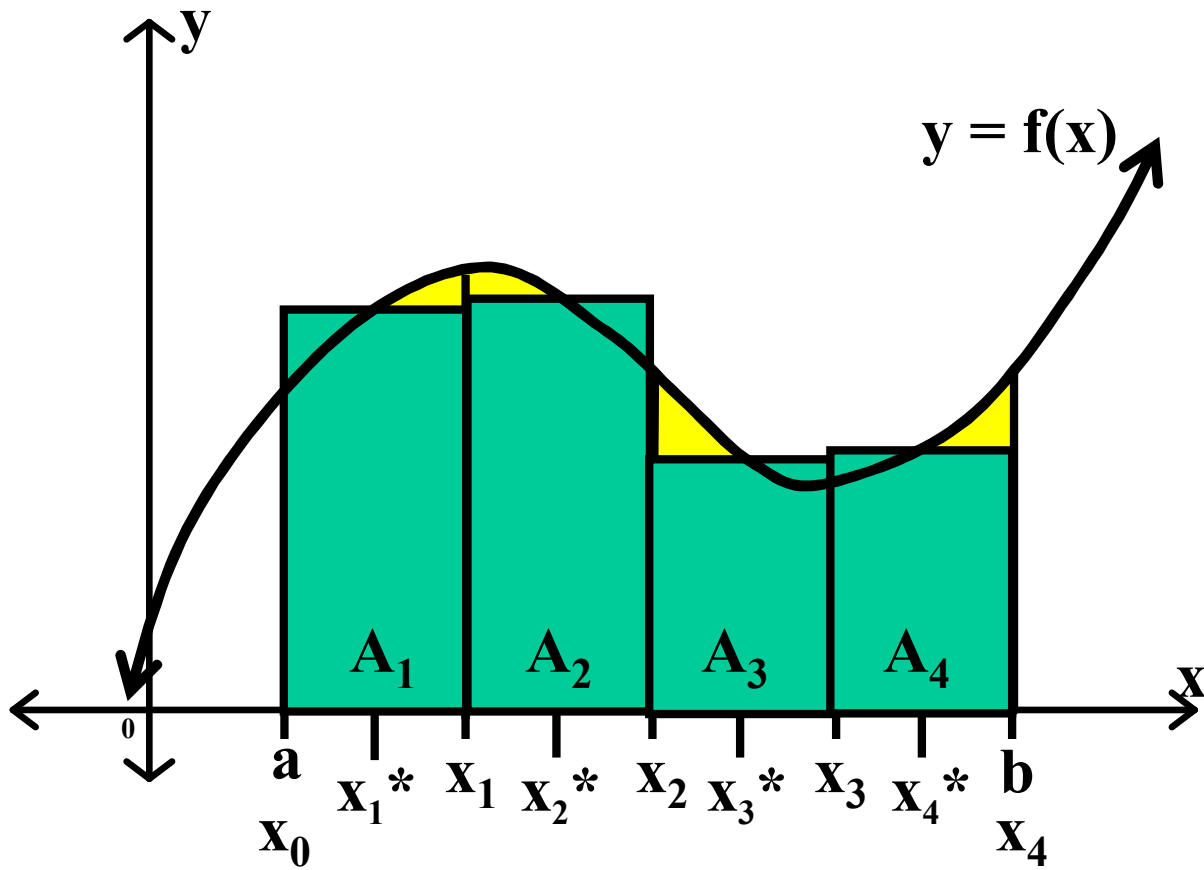


$$A_1 \approx f(x_1^*)\Delta x \quad A_2 \approx f(x_2^*)\Delta x \quad A_3 \approx f(x_3^*)\Delta x \quad A_4 \approx f(x_4^*)\Delta x$$

Notice that, in general, $A_i \approx f(x_i^*)\Delta x$.

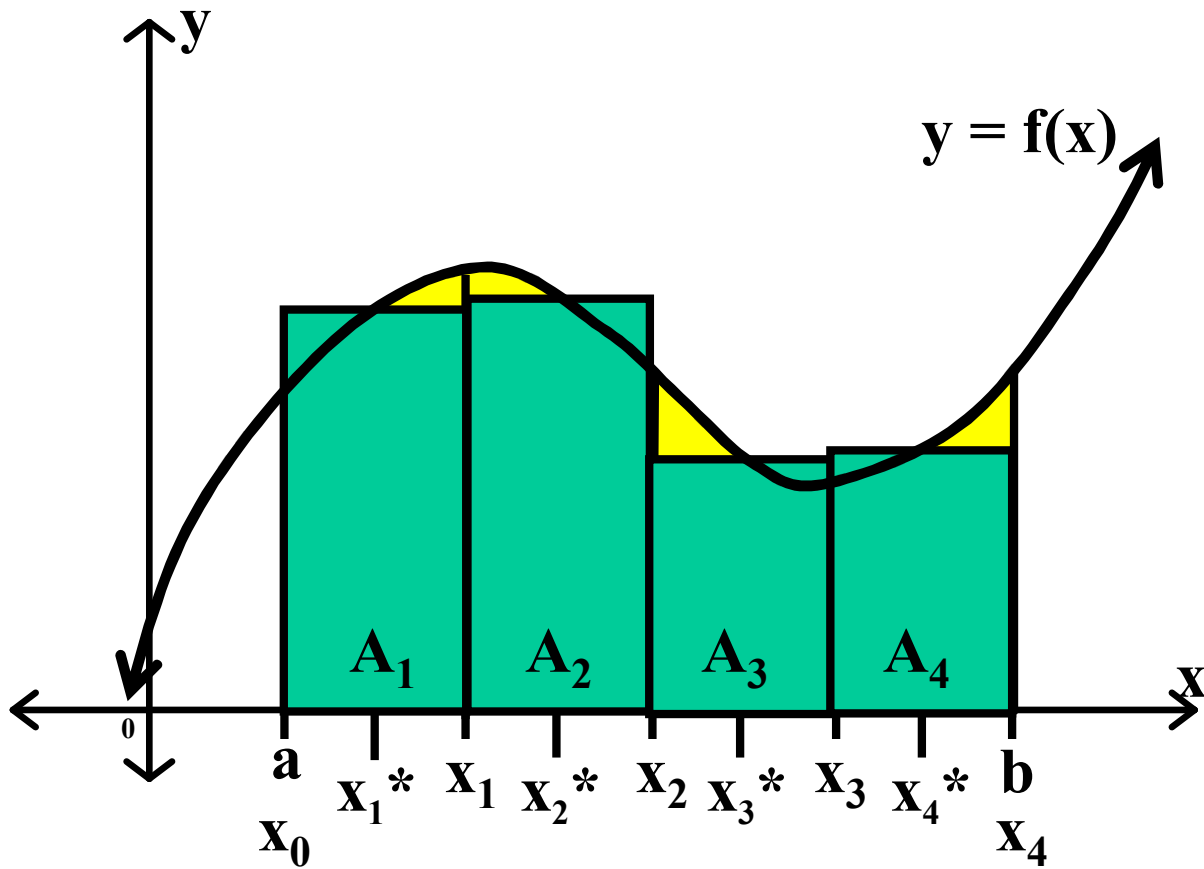


$$A_i \approx f(x_i^*)\Delta x$$



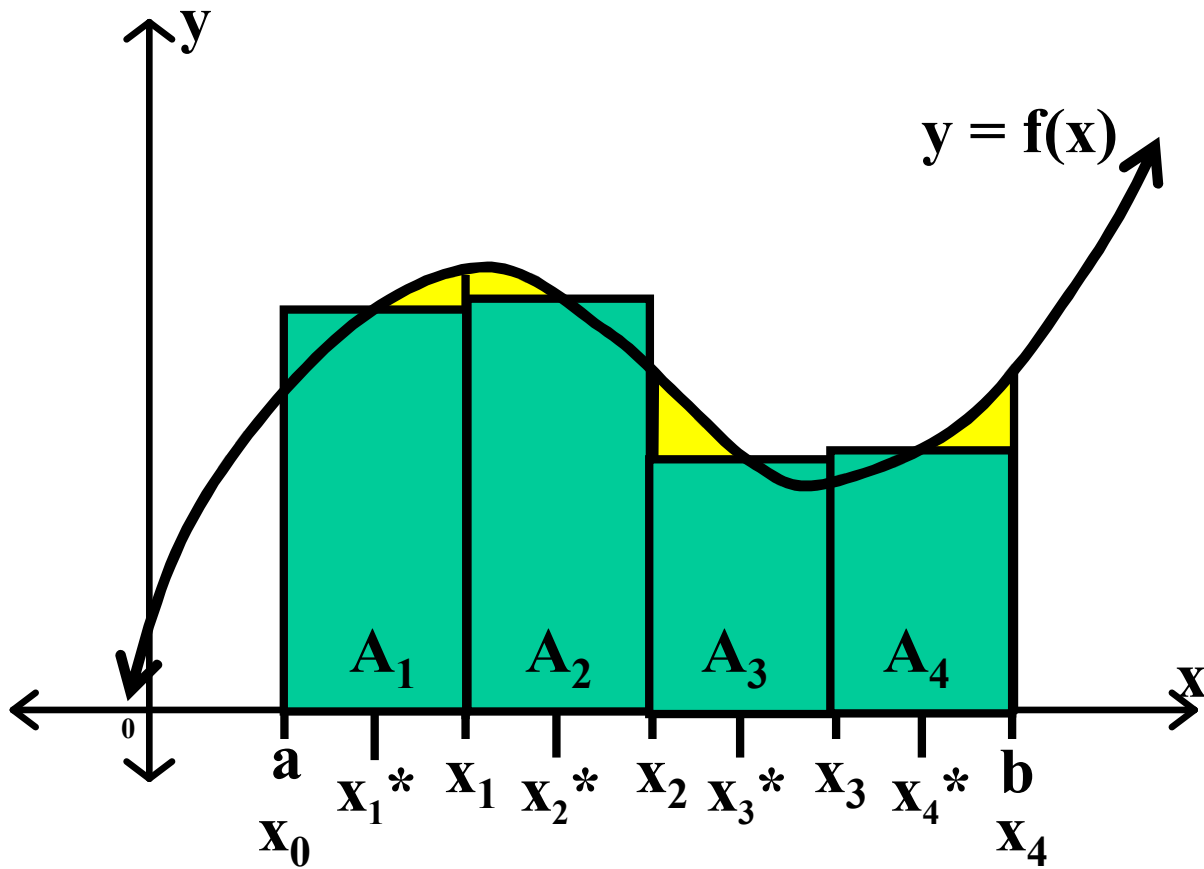
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx$$



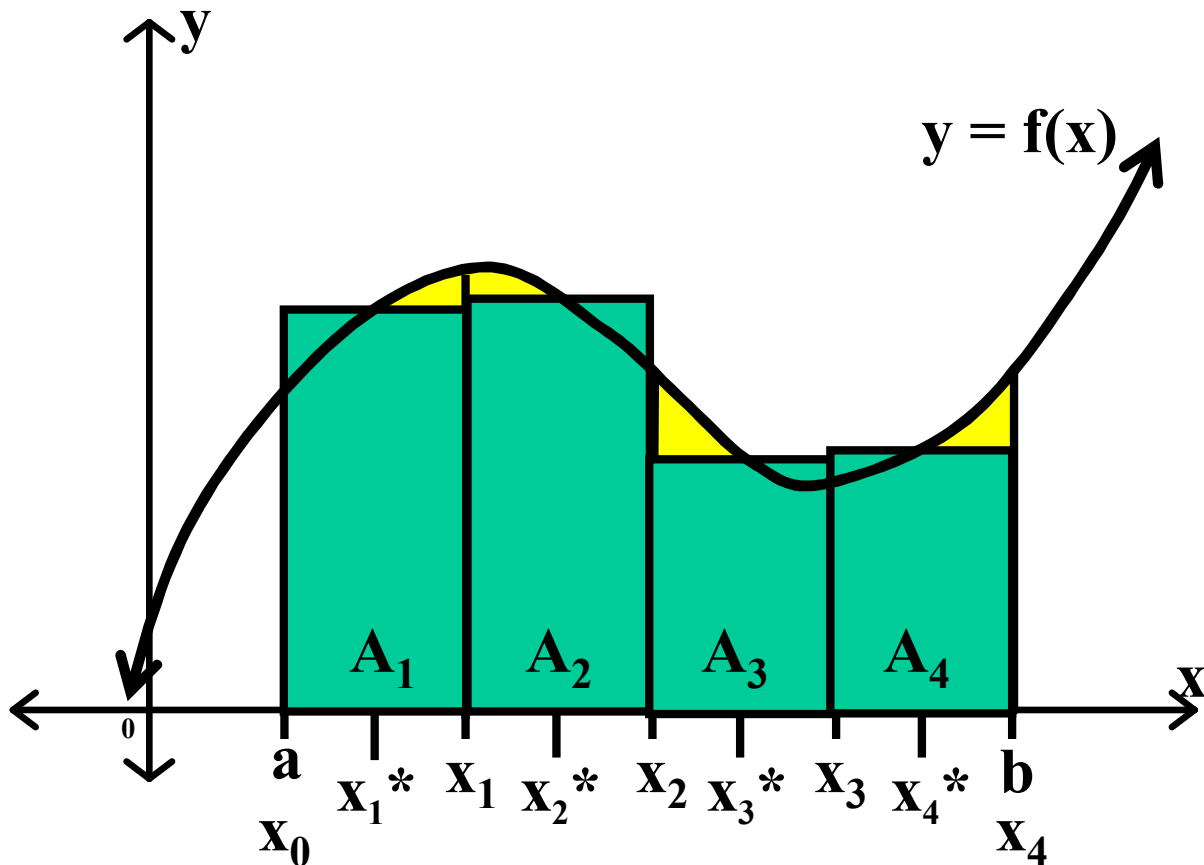
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4$$



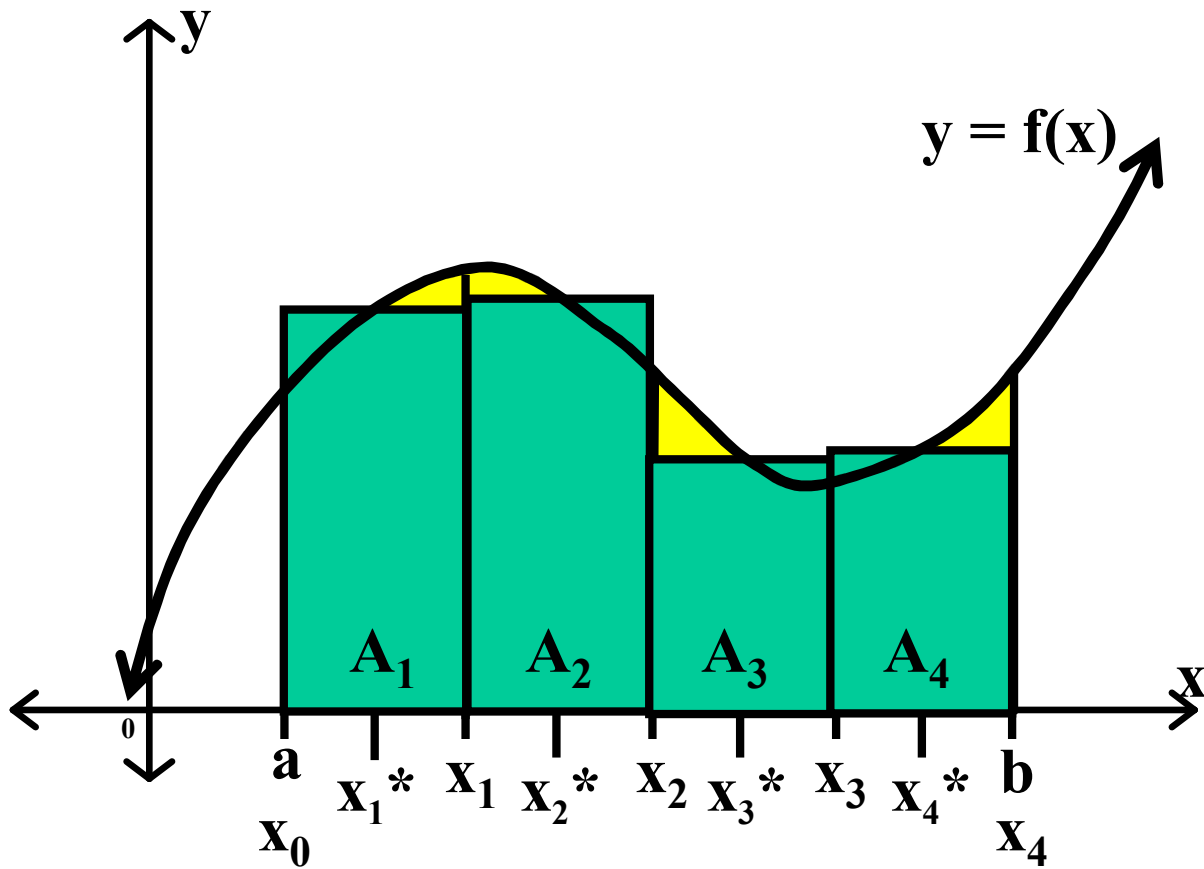
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i$$



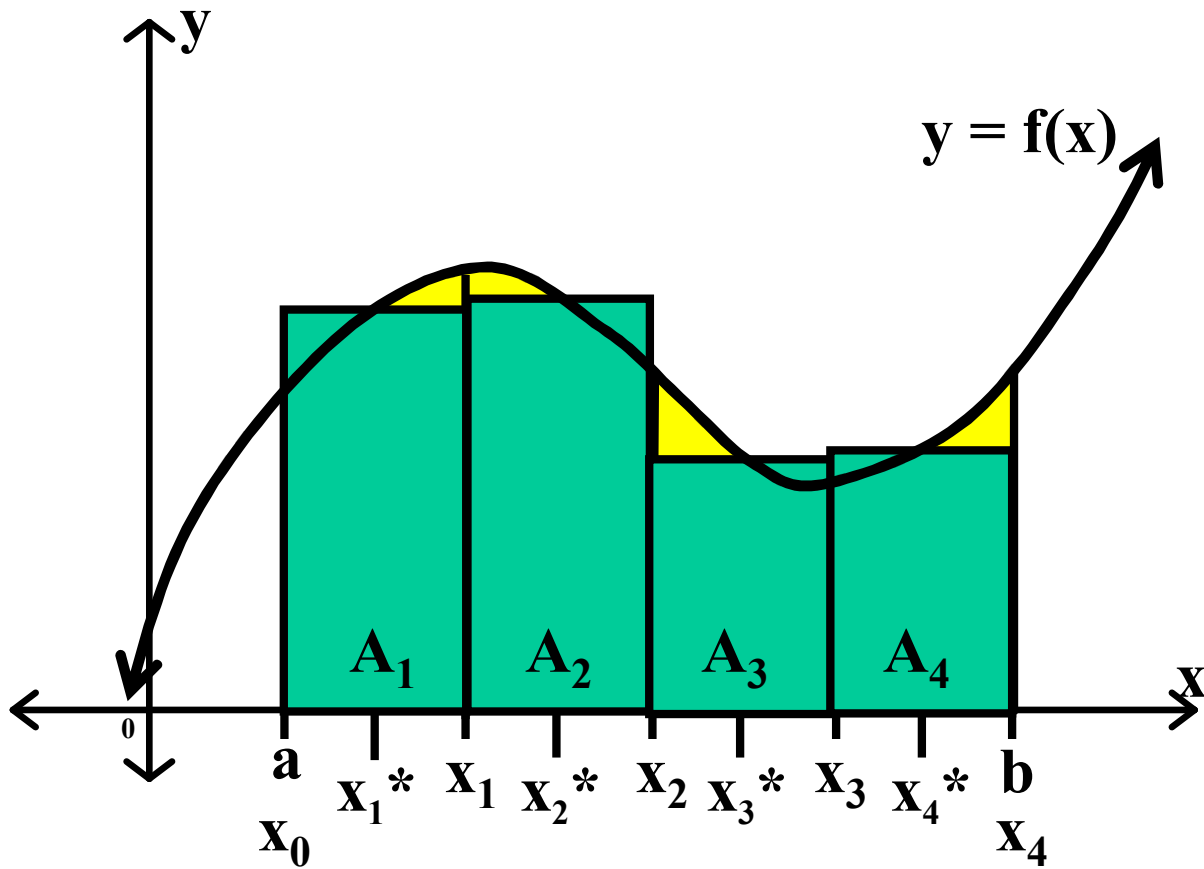
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i \quad (\text{In this case, } n = 4.)$$



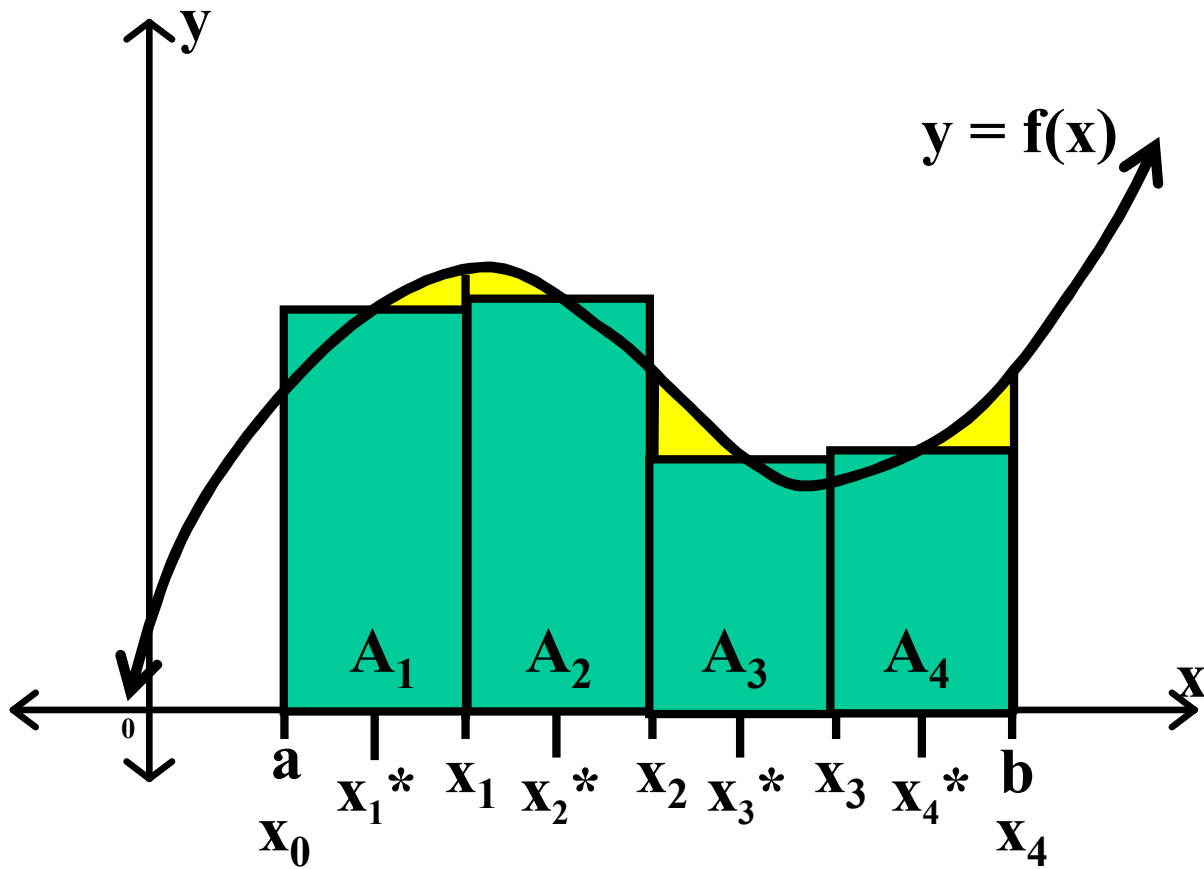
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i$$



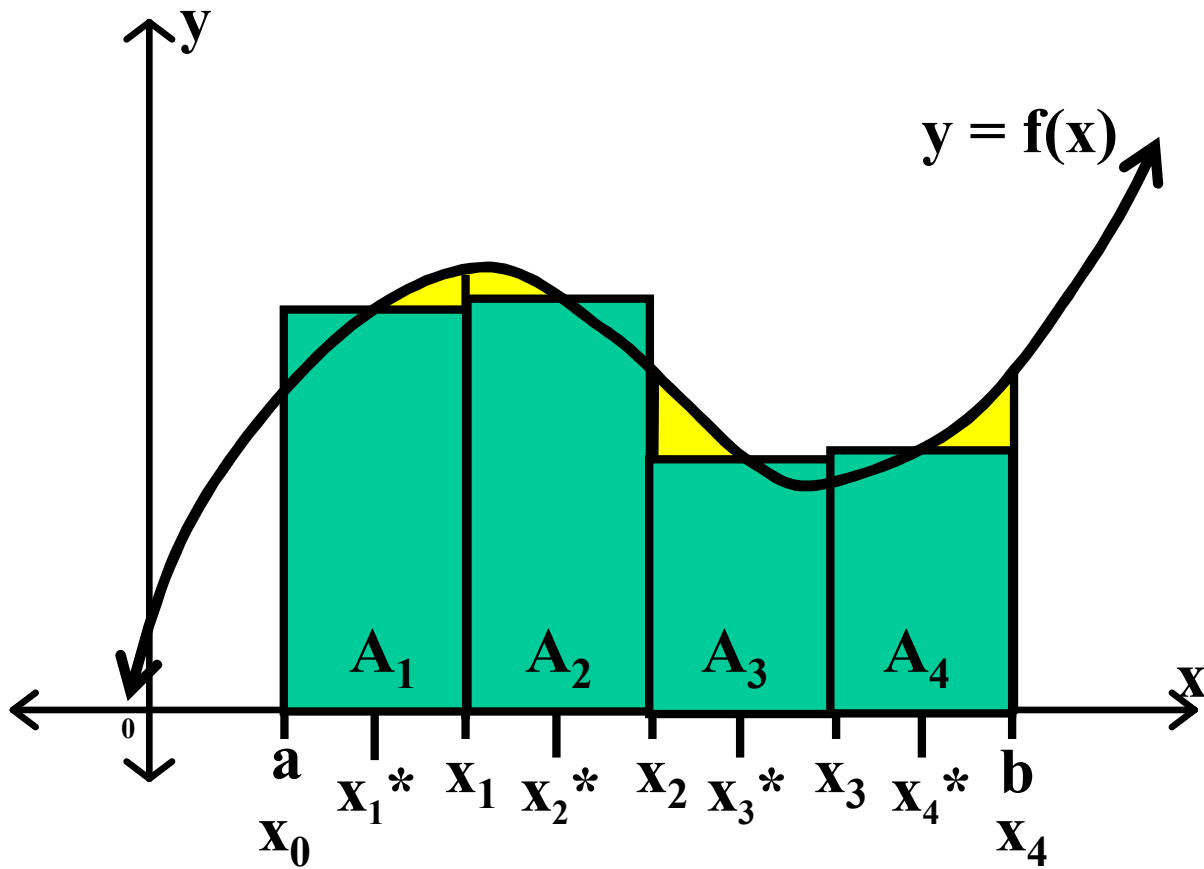
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx$$



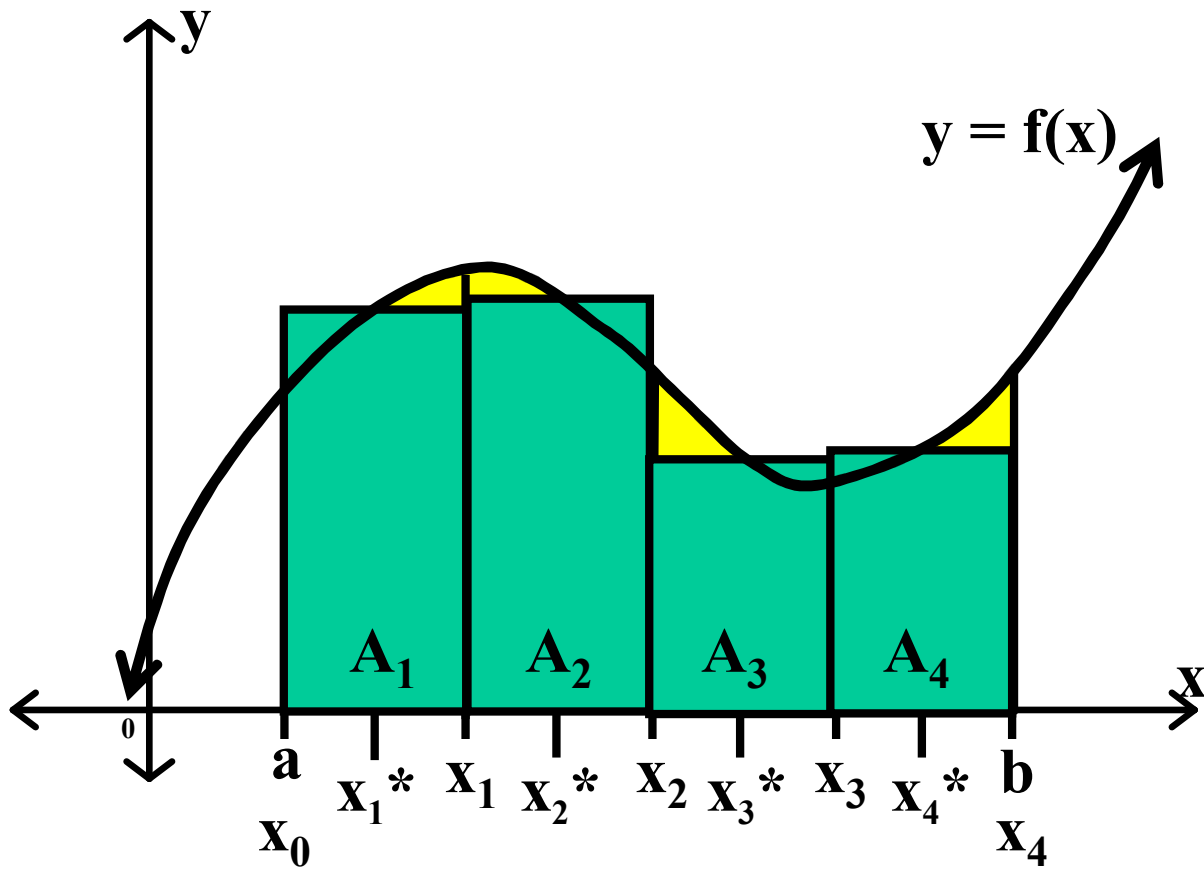
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n$$



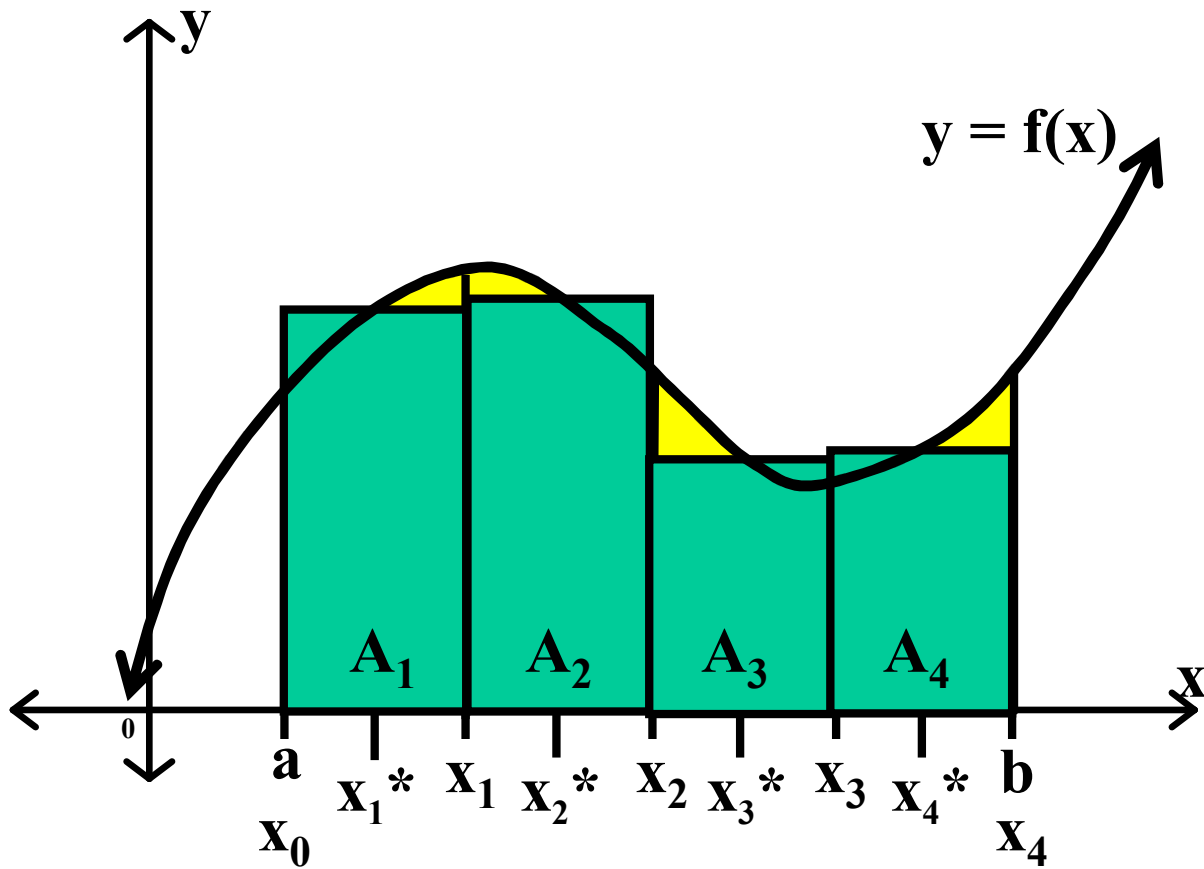
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_i^*)\Delta x$$



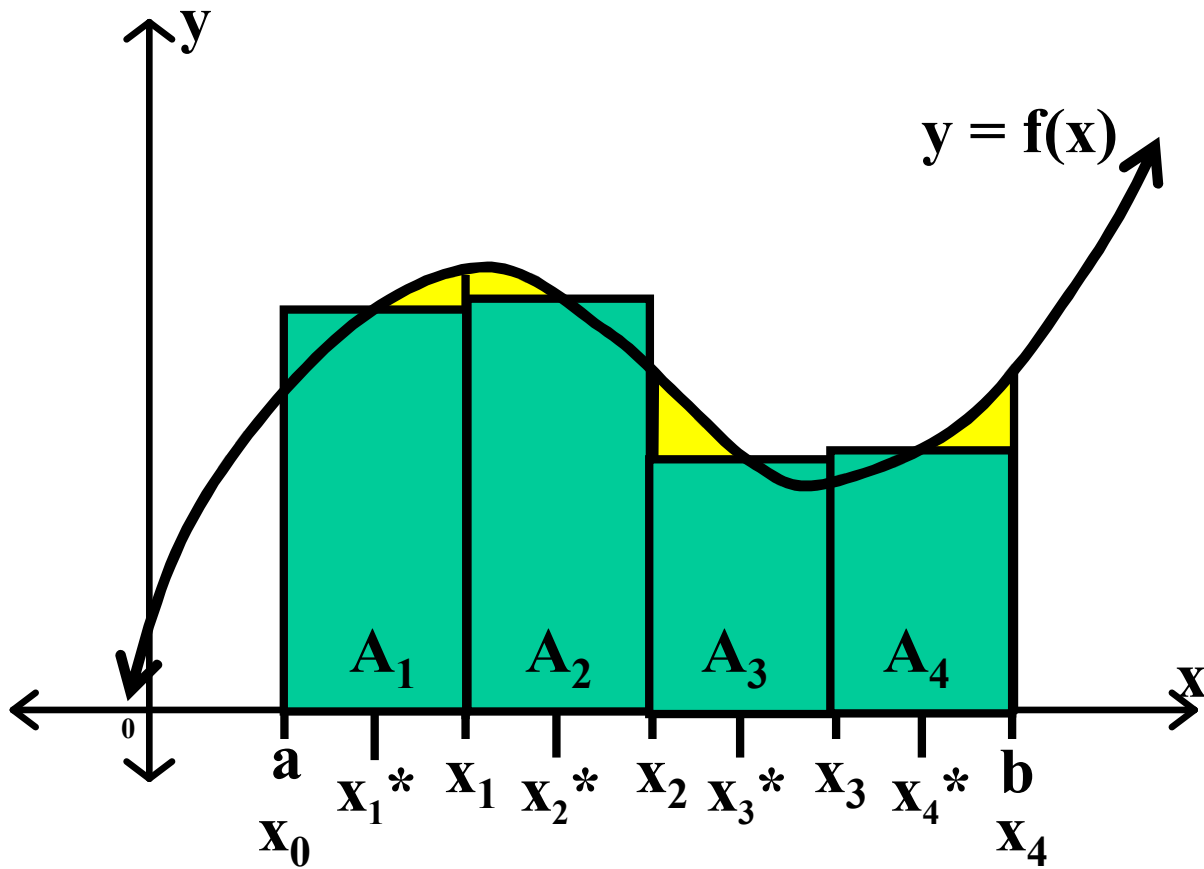
$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_i^*)\Delta x$$



$$A_i \approx f(x_i^*)\Delta x$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \approx \sum_{i=1}^n f(x_i^*)\Delta x = S_M$$



$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

The Mid-Rectangular Approximation

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* =$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* =$$

x_1^* is the midpoint of the 1st sub-interval.

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* =$$

x_1^* is the midpoint of the 1st sub-interval.

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2 \quad x_1^* = 2.25 \quad x_1^* \text{ is the midpoint of the 1}^{\text{st}} \text{ sub-interval.}$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* =$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* =$$

$$x_2 = 3$$

x_2^* is the midpoint of the 2nd sub-interval.

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25$$

$$x_2^* =$$

x_2^* is the midpoint of the 2nd sub-interval.

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* = 2.75$$

$$x_2 = 3$$

x_2^* is the midpoint of the 2nd sub-interval.

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25 \quad \text{Add } \Delta x.$$

$$x_2^* = 2.75$$

x_2^* is the midpoint of the 2nd sub-interval.

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* = 2.75$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* = 2.75$$

$$x_2 = 3$$

$$x_3^* =$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25$$

$$x_2^* = 2.75$$

$$x_3^* =$$

 Add Δx .

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$


$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

 Add Δx .

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1^* = 2.25$$

$$x_1 = 2.5$$

$$x_2^* = 2.75$$

$$x_2 = 3$$

$$x_3^* = 3.25$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{aligned} x_0 &= a = 2 & x_1^* &= 2.25 \\ x_1 &= 2.5 & x_2^* &= 2.75 \\ x_2 &= 3 & x_3^* &= 3.25 \\ x_3 &= 3.5 & x_4^* &= \\ x_4 &= 4 & & \\ x_5 &= 4.5 & & \\ x_6 &= b = 5 & & \end{aligned}$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* =$$

 Add Δx .

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$x_0 = a = 2$$

$$x_1 = 2.5$$

$$x_2 = 3$$

$$x_3 = 3.5$$

$$x_4 = 4$$

$$x_5 = 4.5$$

$$x_6 = b = 5$$

$$x_1^* = 2.25$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

 Add Δx .

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\begin{aligned} x_0 &= a = 2 & x_1^* &= 2.25 \\ x_1 &= 2.5 & x_2^* &= 2.75 \\ x_2 &= 3 & x_3^* &= 3.25 \\ x_3 &= 3.5 & x_4^* &= 3.75 \\ x_4 &= 4 \\ x_5 &= 4.5 \\ x_6 &= b = 5 \end{aligned}$$

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$x_1^* = 2.25$
$x_1 = 2.5$	$x_2^* = 2.75$
$x_2 = 3$	$x_3^* = 3.25$
$x_3 = 3.5$	$x_4^* = 3.75$
$x_4 = 4$	$x_5^* = 4.25$
$x_5 = 4.5$	
$x_6 = b = 5$	

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$x_1^* = 2.25$
$x_1 = 2.5$	$x_2^* = 2.75$
$x_2 = 3$	$x_3^* = 3.25$
$x_3 = 3.5$	$x_4^* = 3.75$
$x_4 = 4$	$x_5^* = 4.25$
$x_5 = 4.5$	$x_6^* = 4.75$
$x_6 = b = 5$	

Calculate the x_i^* 's.

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$x_1^* = 2.25$$

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$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Class Worksheet #5 Unit 11

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) =$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) =$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) =$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

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$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) =$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) =$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

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Approximate the following definite integral using each of the following approximation methods.

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$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

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$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5$$

$$f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75 \quad f(x_6^*) = f(4.75) = \sqrt{4.75^3 - 3}$$

Calculate the $f(x_i^*)$'s).

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

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Class Worksheet #5 Unit 11

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$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75 \quad f(x_6^*) = f(4.75) = \sqrt{4.75^3 - 3}$$

$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75 \quad f(x_6^*) = f(4.75) = \sqrt{4.75^3 - 3}$$

$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75 \quad f(x_6^*) = f(4.75) = \sqrt{4.75^3 - 3}$$

$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

$$S_M = \sum_{i=1}^6 f(x_i^*) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $n = 6$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$x_1^* = 2.25 \quad f(x_1^*) = f(2.25) = \sqrt{2.25^3 - 3}$$

$$x_2^* = 2.75 \quad f(x_2^*) = f(2.75) = \sqrt{2.75^3 - 3}$$

$$x_3^* = 3.25 \quad f(x_3^*) = f(3.25) = \sqrt{3.25^3 - 3}$$

$$x_4^* = 3.75 \quad f(x_4^*) = f(3.75) = \sqrt{3.75^3 - 3}$$

$$x_5^* = 4.25 \quad f(x_5^*) = f(4.25) = \sqrt{4.25^3 - 3}$$

$$x_6^* = 4.75 \quad f(x_6^*) = f(4.75) = \sqrt{4.75^3 - 3}$$

$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

$$S_M = \sum_{i=1}^6 f(x_i^*) \Delta x$$

$$S_L = f(x_1^*) \Delta x + f(x_2^*) \Delta x + f(x_3^*) \Delta x + \\ + f(x_4^*) \Delta x + f(x_5^*) \Delta x + f(x_6^*) \Delta x$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$S_M =$$

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$$S_M =$$

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$$S_M = (\sqrt{2.25^3 - 3} + \sqrt{2.75^3 - 3} + \sqrt{3.25^3 - 3} + \sqrt{3.75^3 - 3} + \sqrt{4.25^3 - 3} + \sqrt{4.75^3 - 3})(.5)$$

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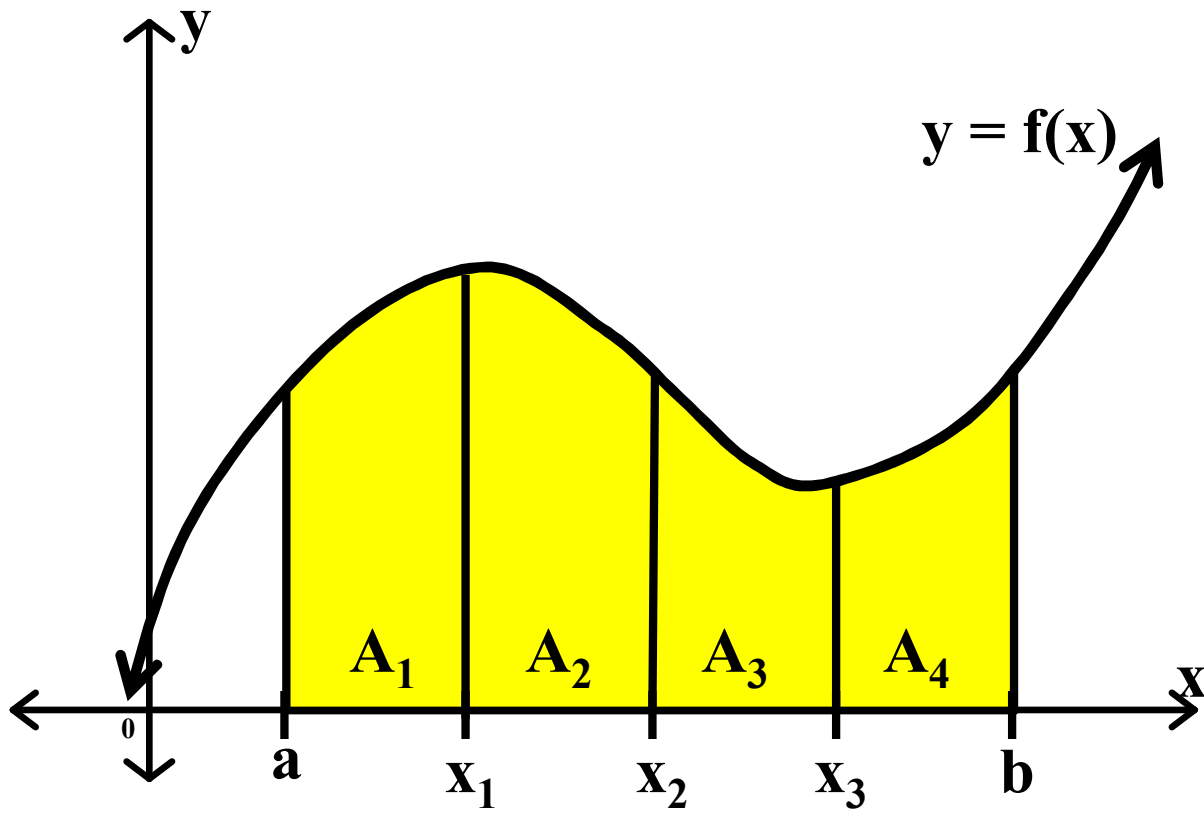
$$S_M = \sum_{i=1}^n f(x_i^*) \Delta x$$

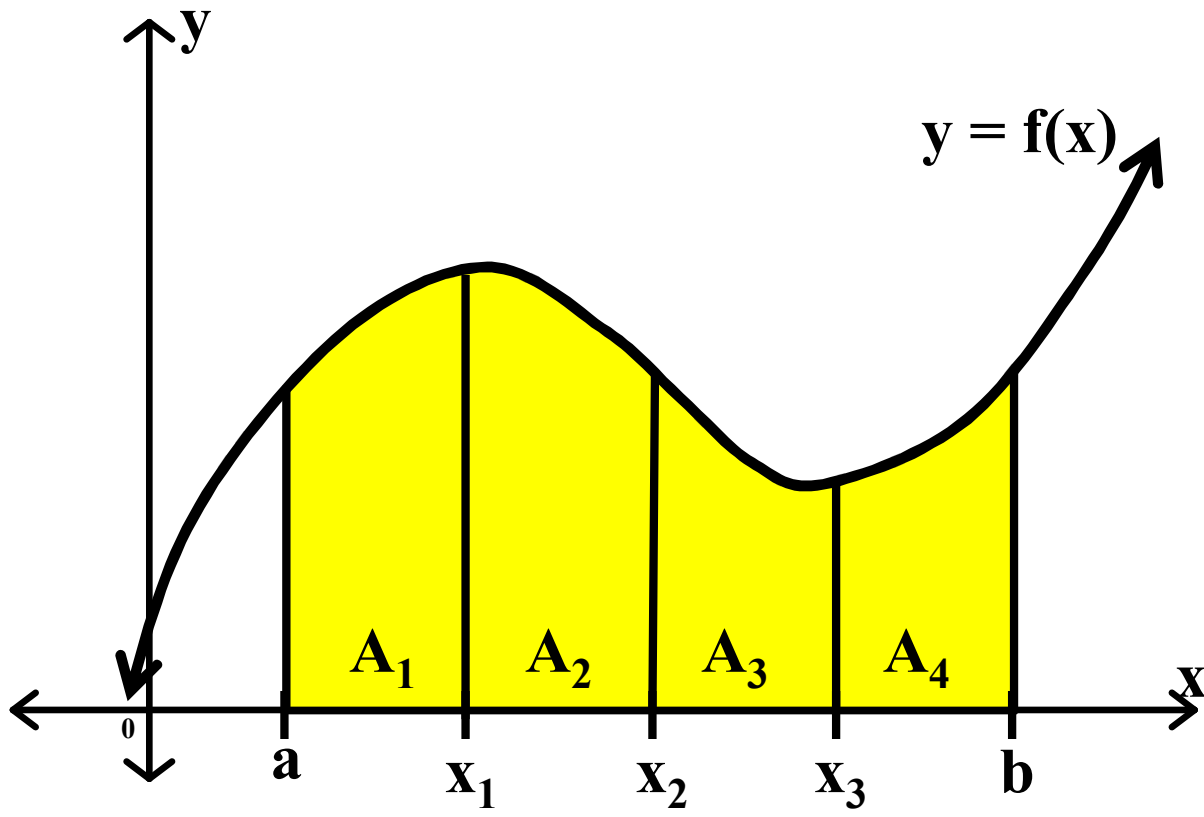
$$S_M = \sum_{i=1}^6 f(x_i^*) \Delta x$$

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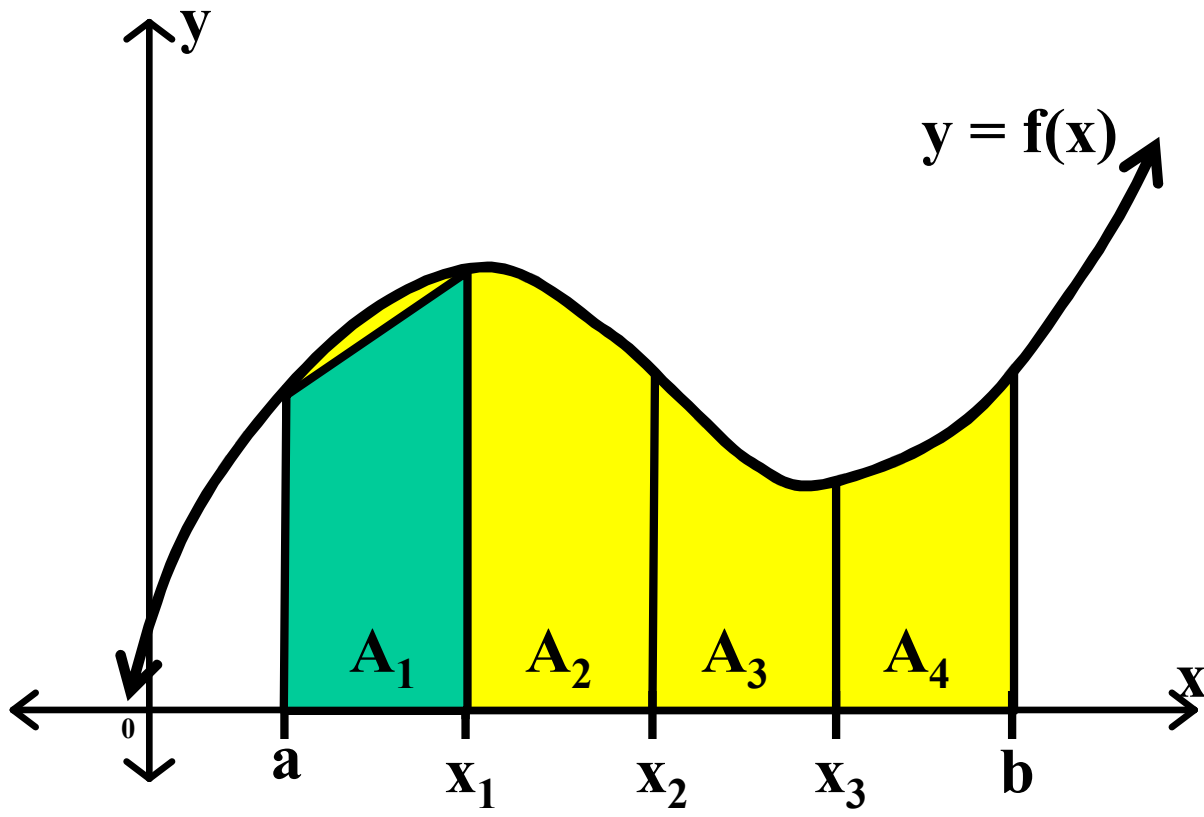
$$S_M = (\sqrt{2.25^3 - 3} + \sqrt{2.75^3 - 3} + \sqrt{3.25^3 - 3} + \sqrt{3.75^3 - 3} + \sqrt{4.25^3 - 3} + \sqrt{4.75^3 - 3})(.5)$$

$$S_M \approx 19.28$$

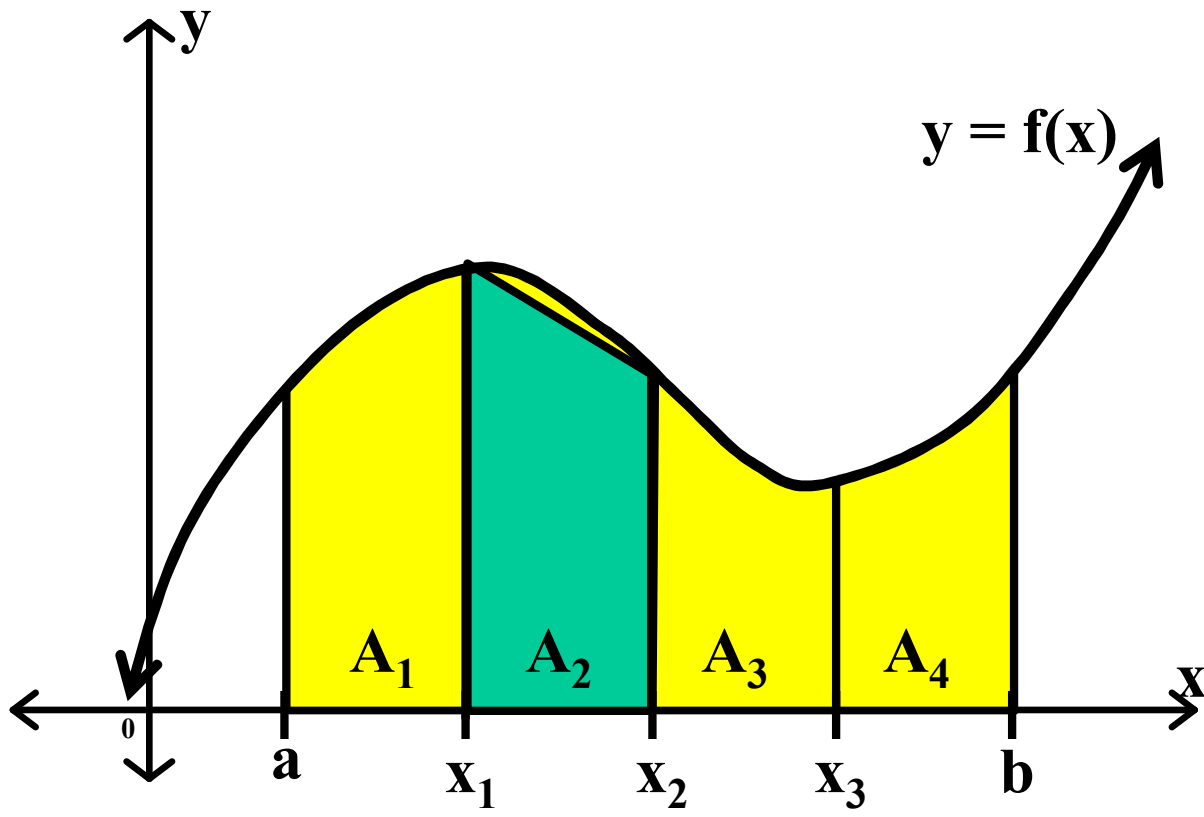




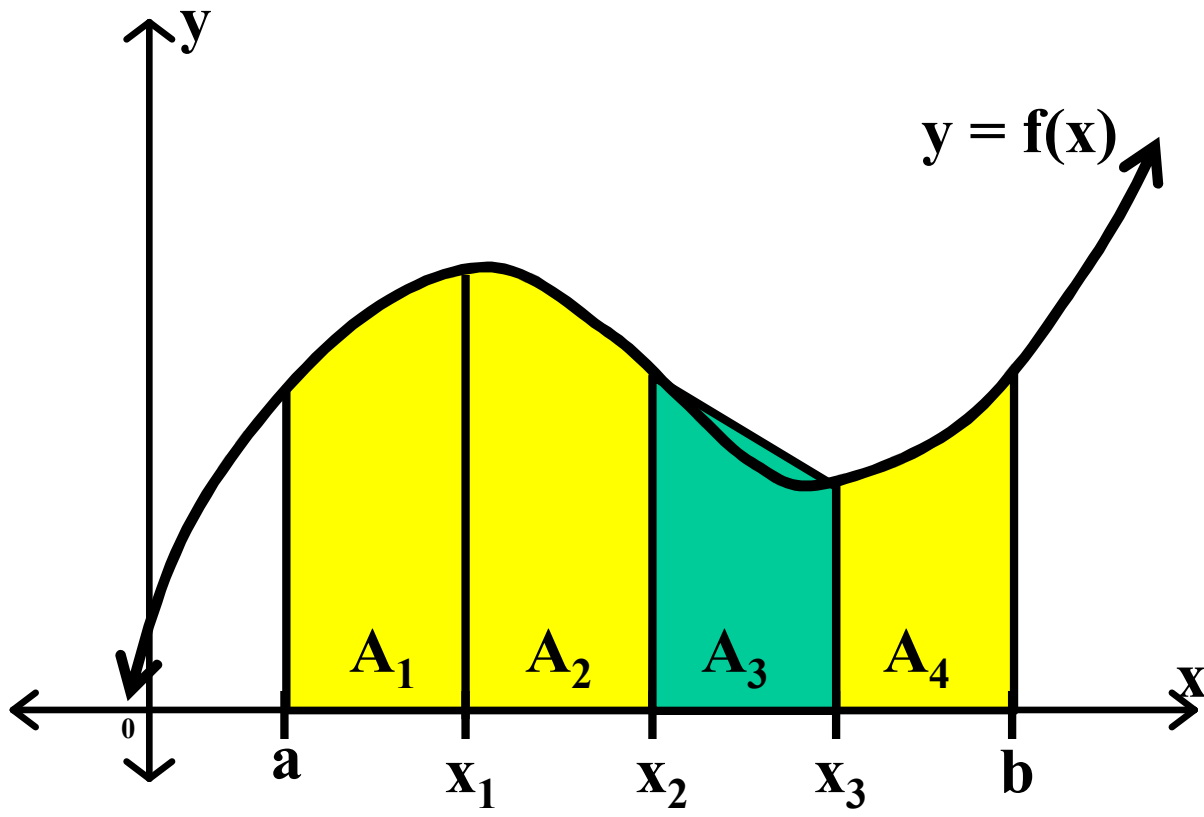
Trapezoids can also be used to approximate the area.



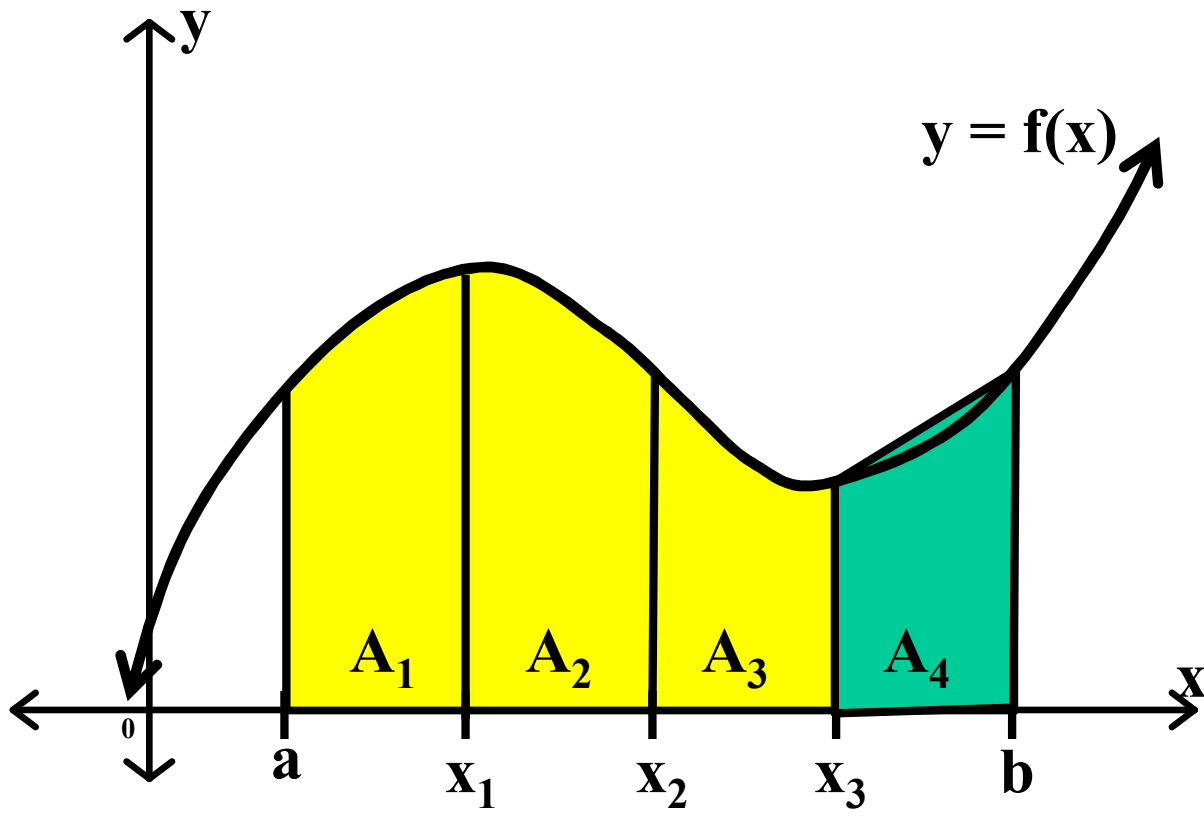
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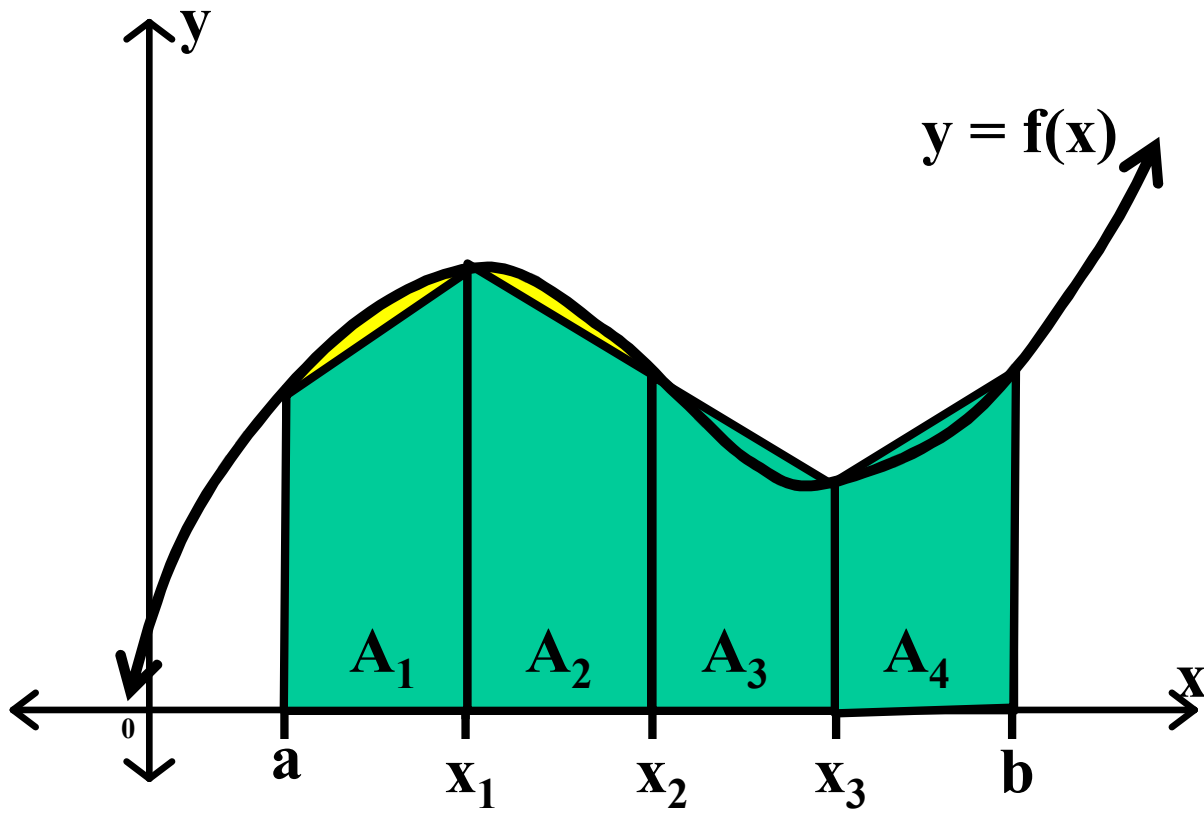
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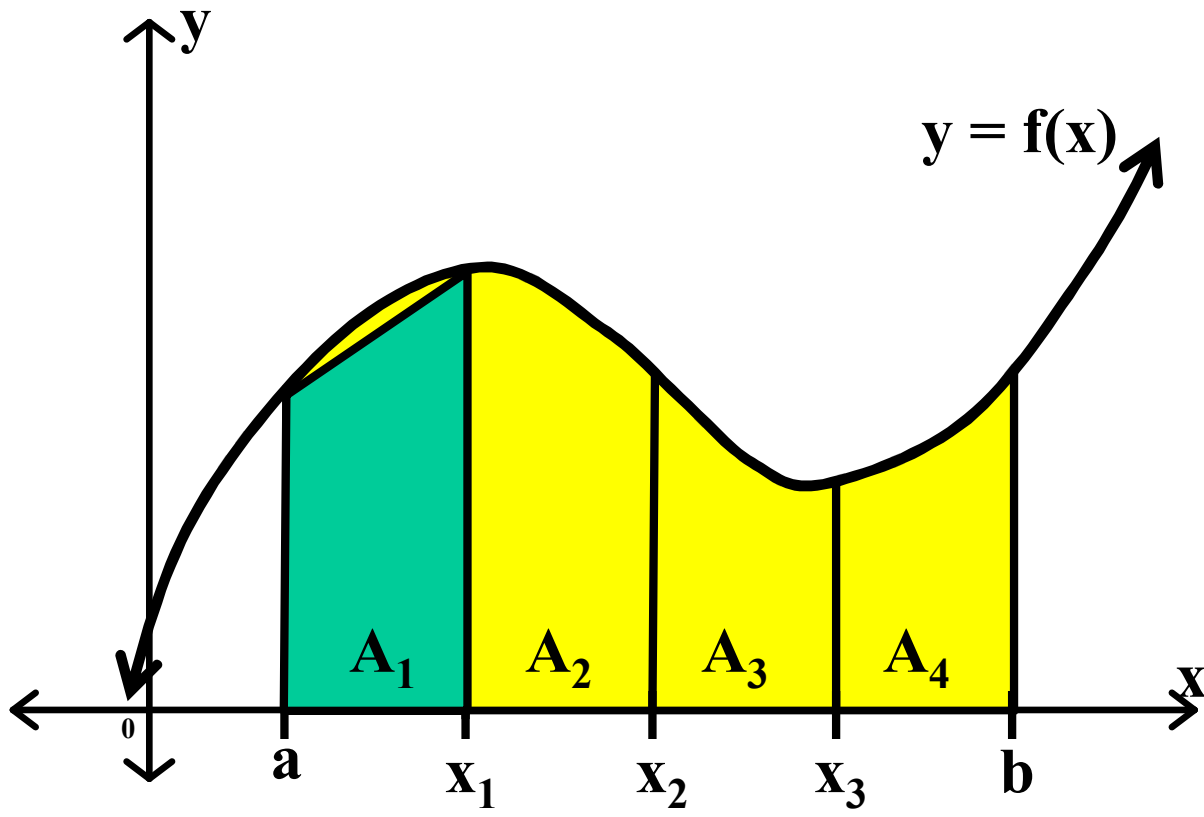


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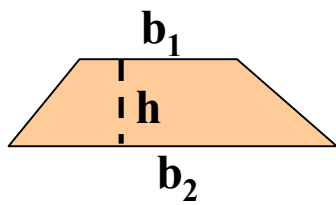


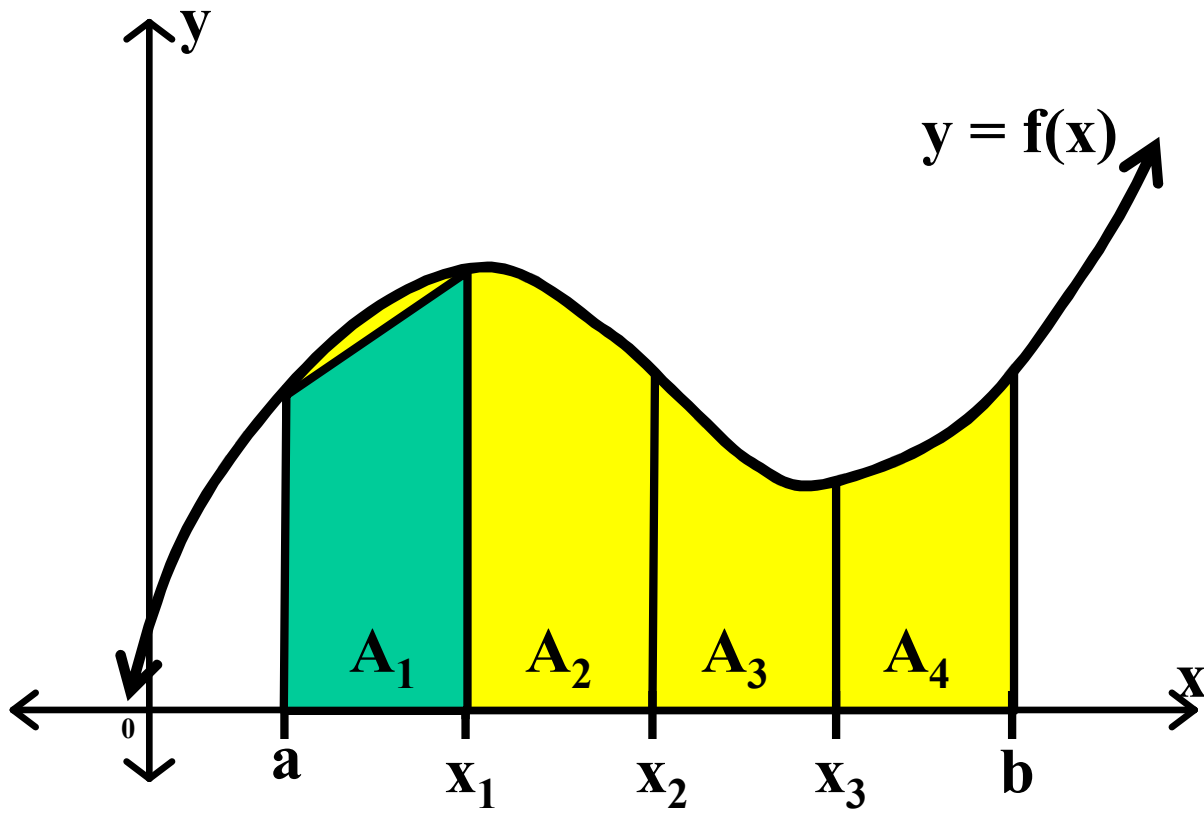
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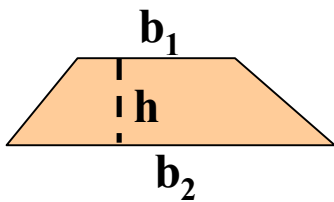


trapezoid

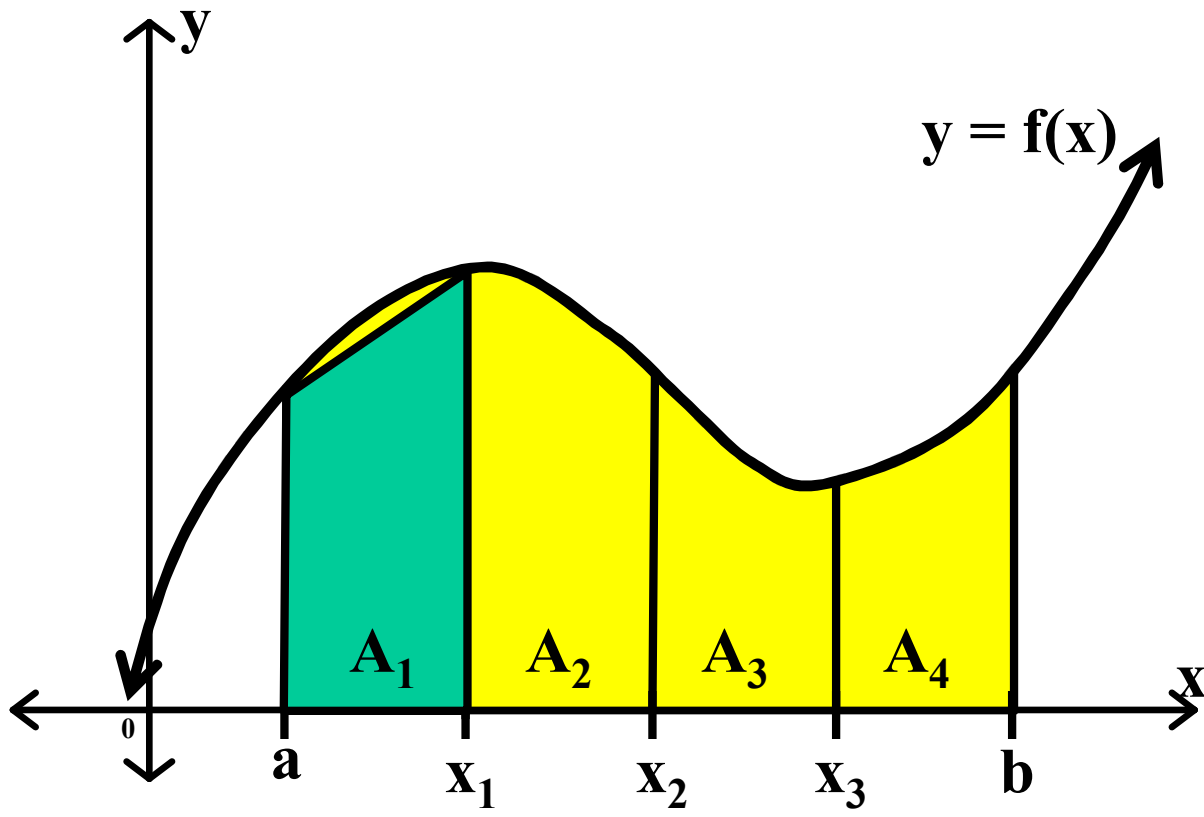




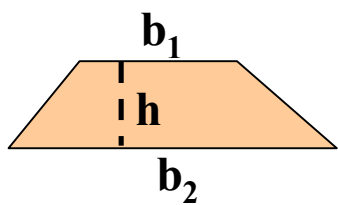
trapezoid



$$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$$

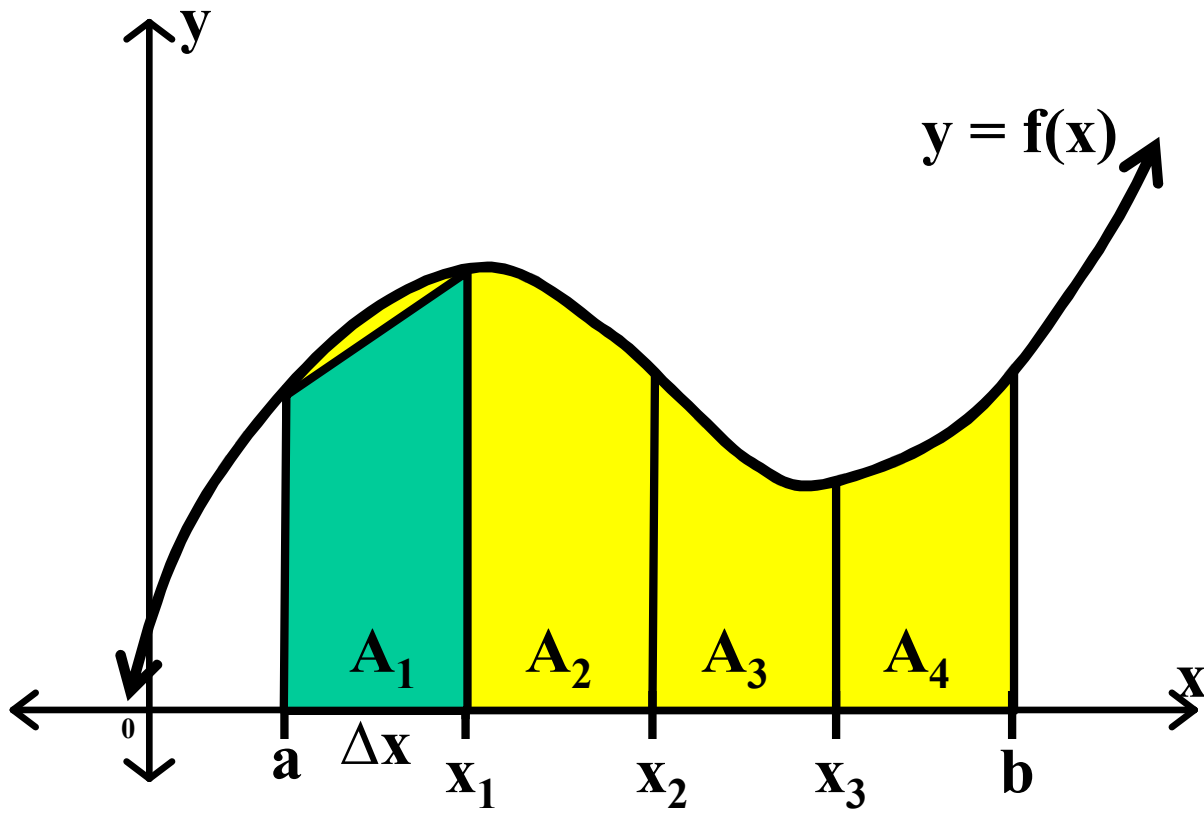


trapezoid

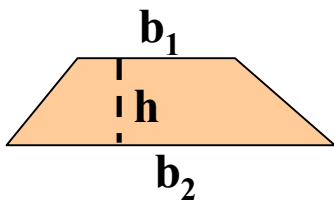


$A_1 \approx$

$Area = \frac{1}{2} * h * (b_1 + b_2)$



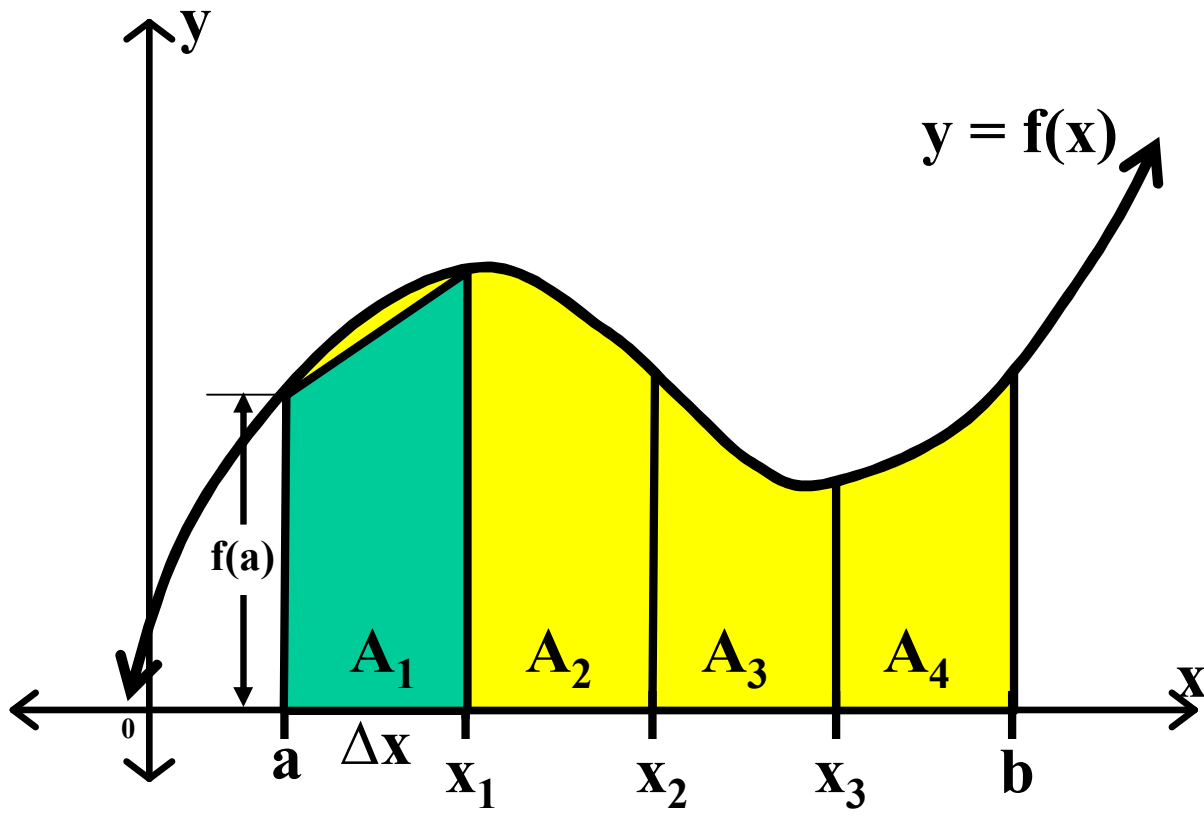
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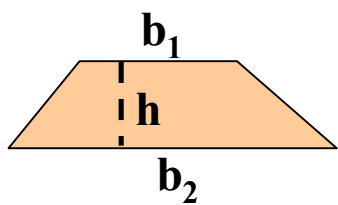
$$A_1 \approx$$

$$h = \Delta x$$

$$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$$



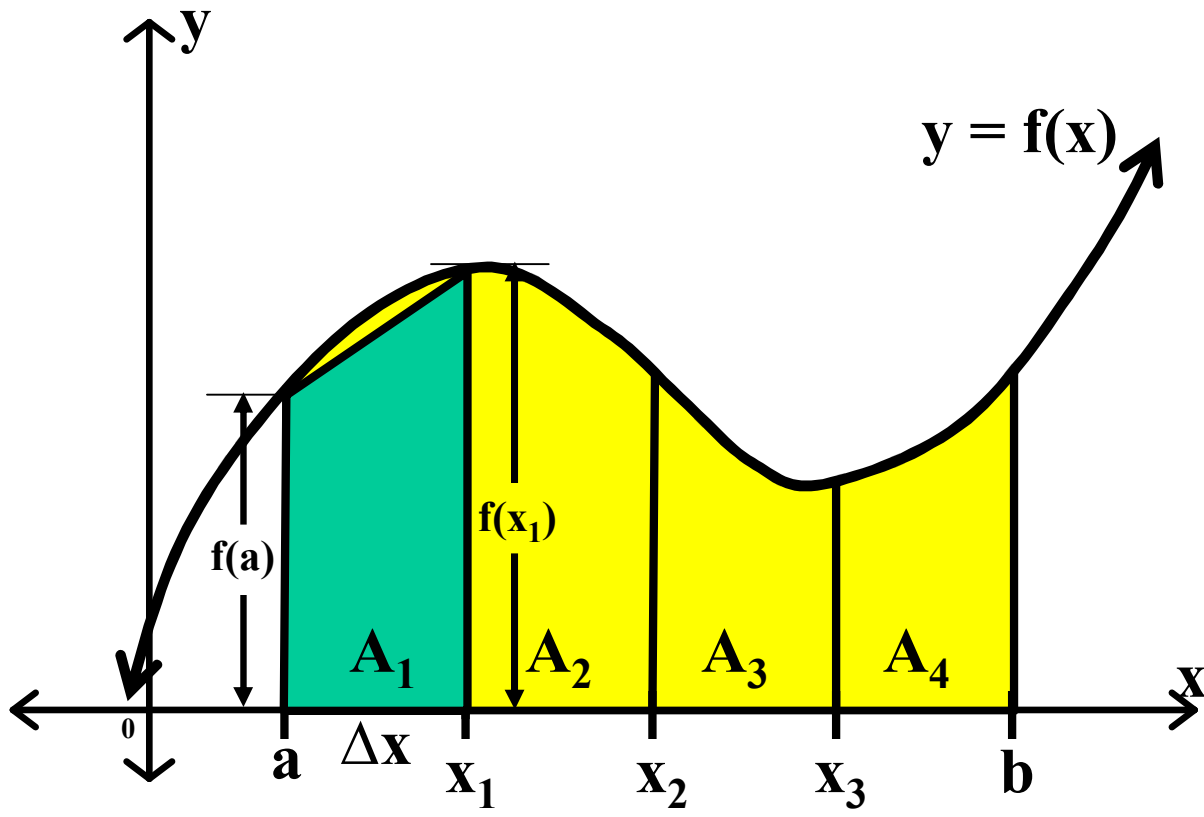
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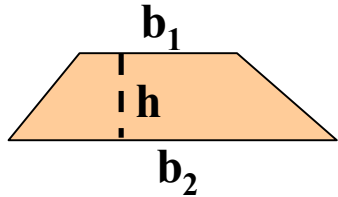
$$A_1 \approx$$

$$h = \Delta x \quad b_1 = f(a)$$

$$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$$



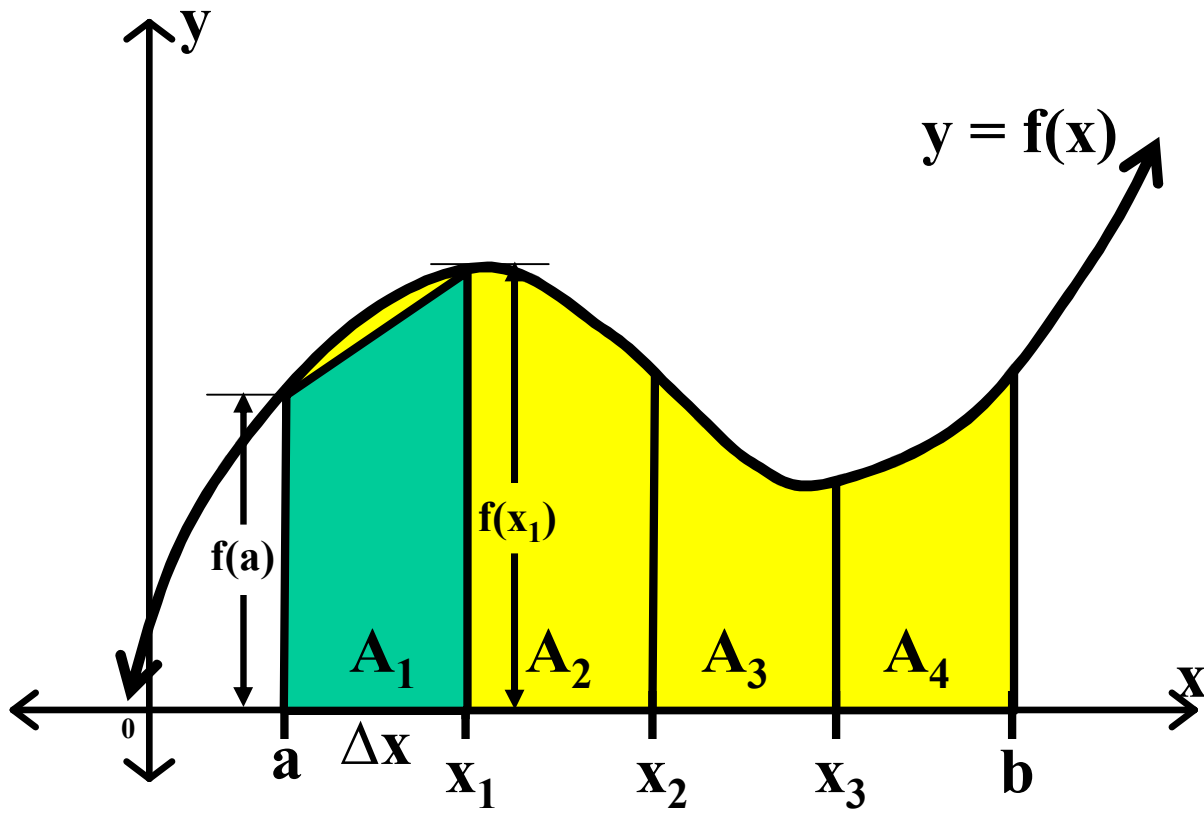
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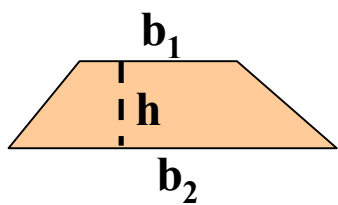
$$A_1 \approx$$

$$h = \Delta x \quad b_1 = f(a) \quad b_2 = f(x_1)$$

$$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$$



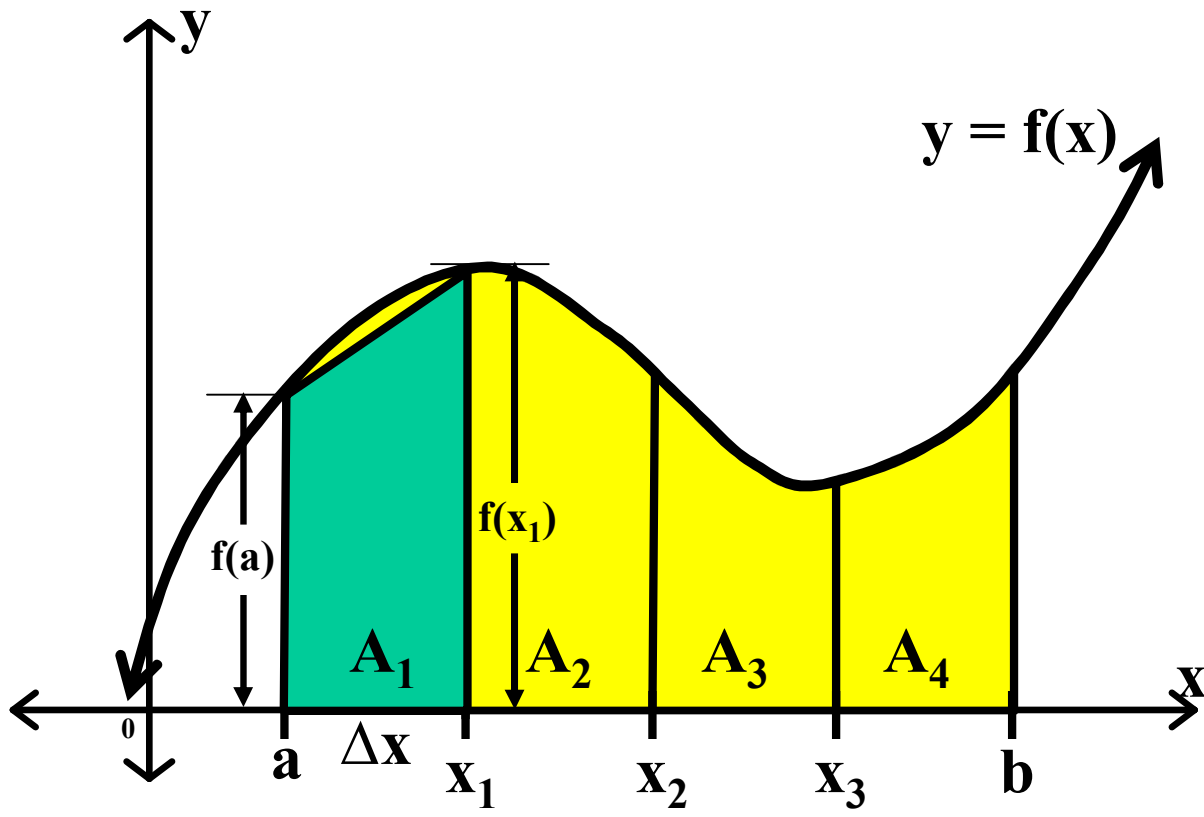
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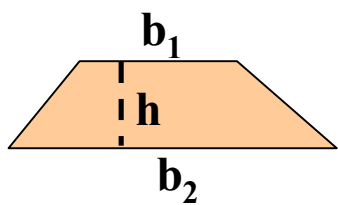
$$A_1 \approx \frac{1}{2} * \Delta x$$

$$h = \Delta x \quad b_1 = f(a) \quad b_2 = f(x_1)$$

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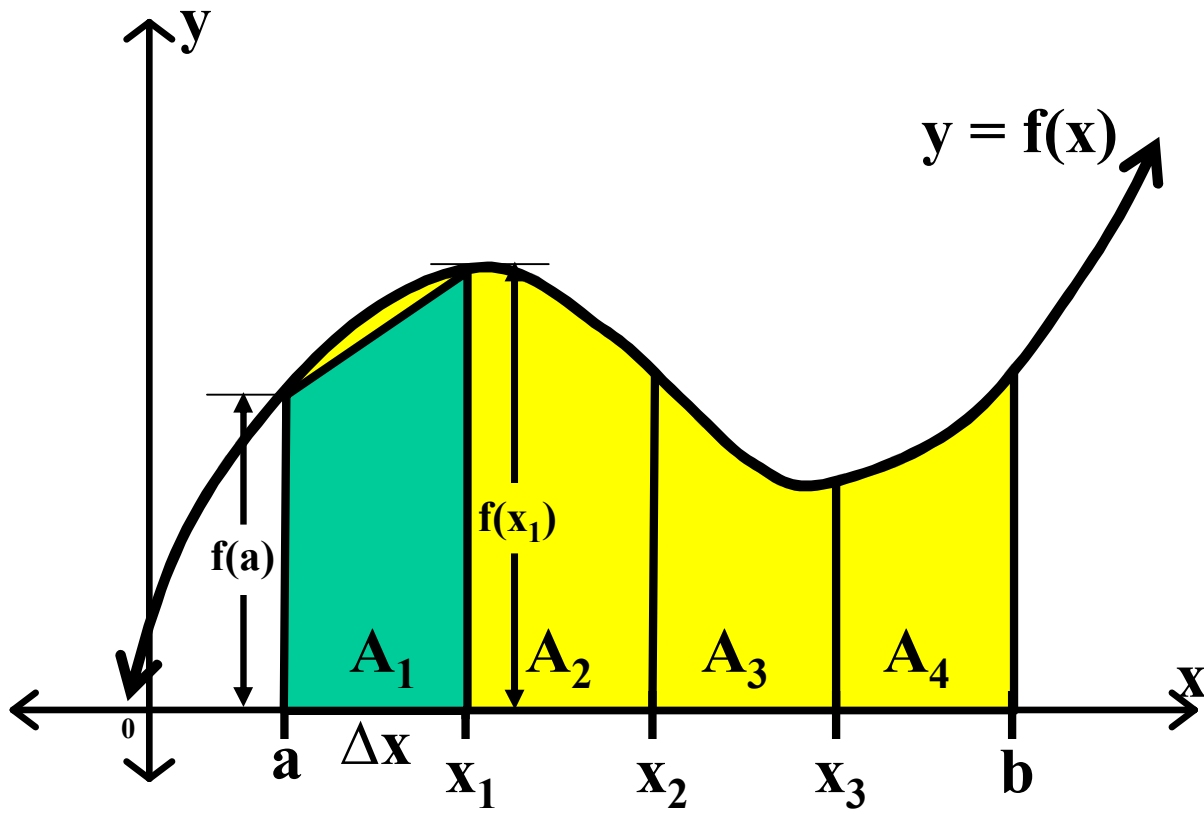
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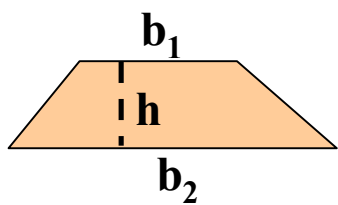
$$A_1 \approx \frac{1}{2} * \Delta x [f(a)]$$

$$h = \Delta x \quad b_1 = f(a) \quad b_2 = f(x_1)$$

$$\text{Area} = \frac{1}{2} * h (b_1 + b_2)$$



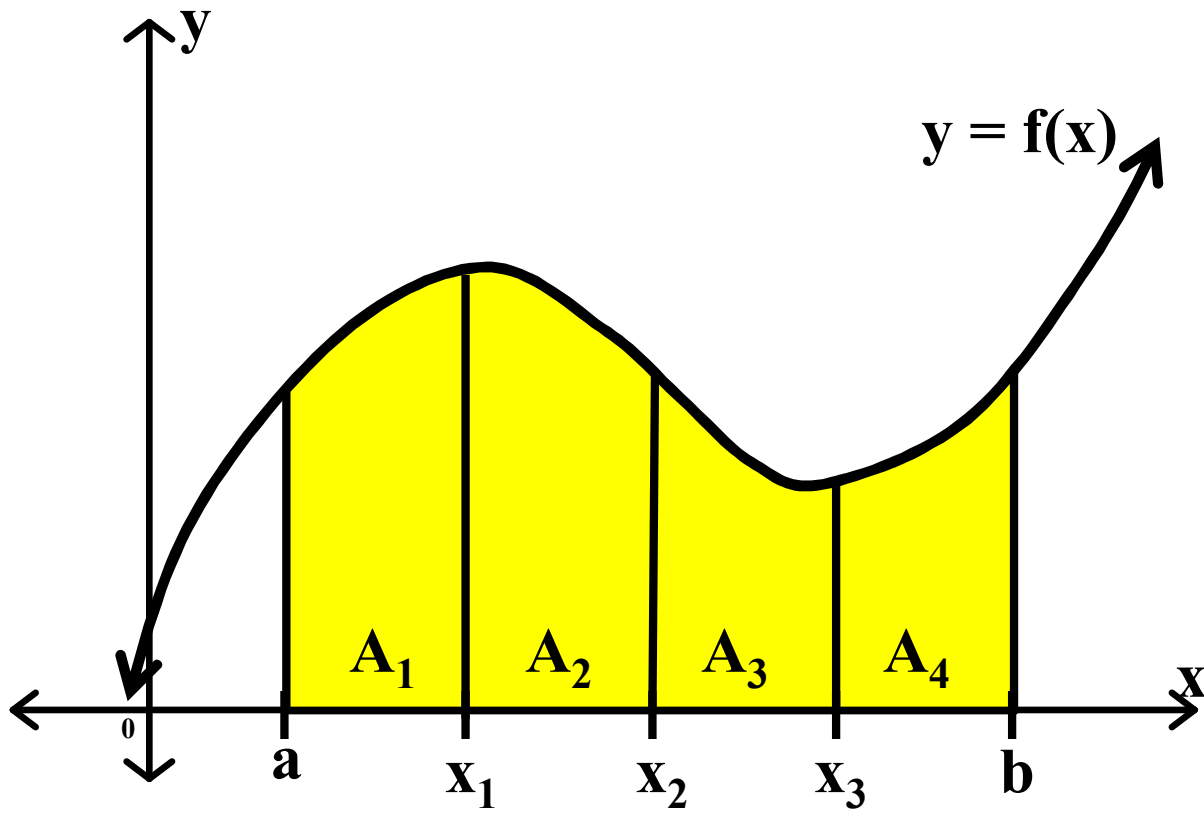
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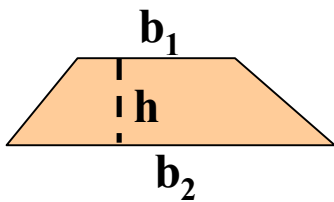
$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$$

$$h = \Delta x \quad b_1 = f(a) \quad b_2 = f(x_1)$$

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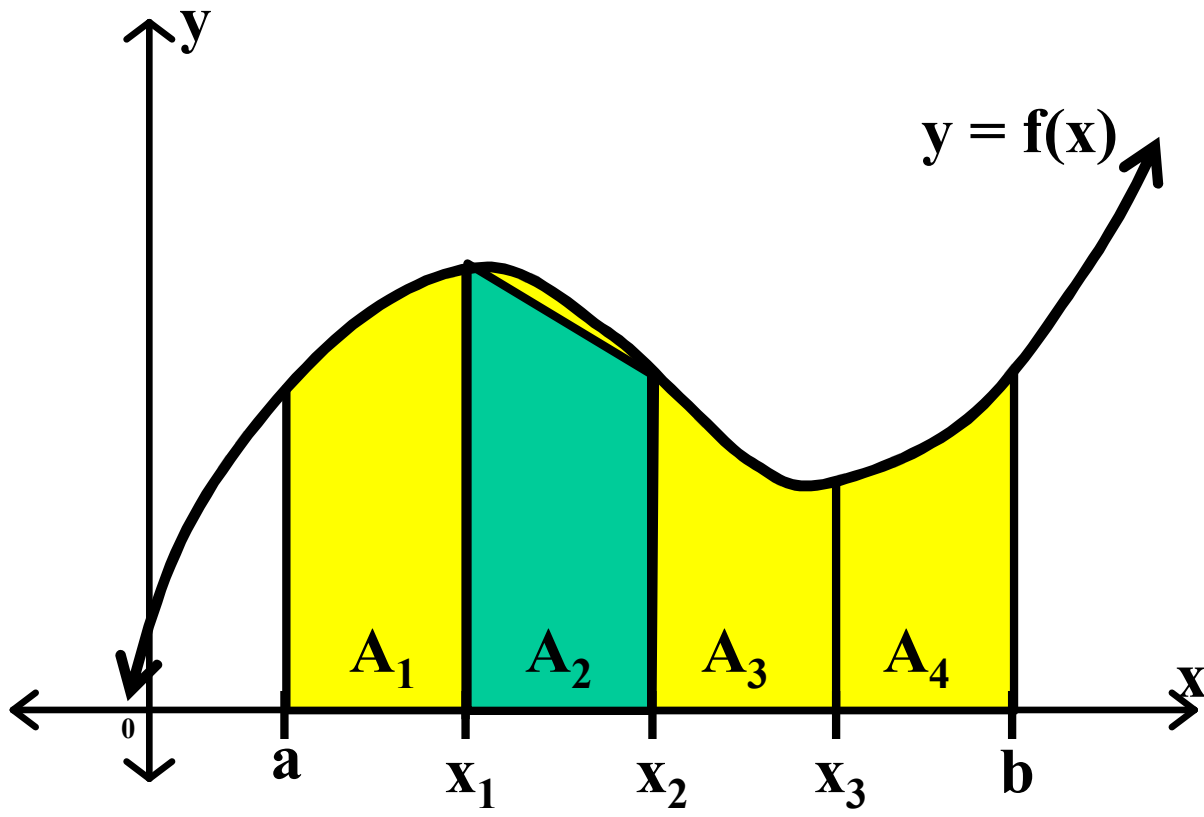


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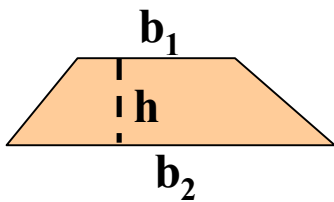


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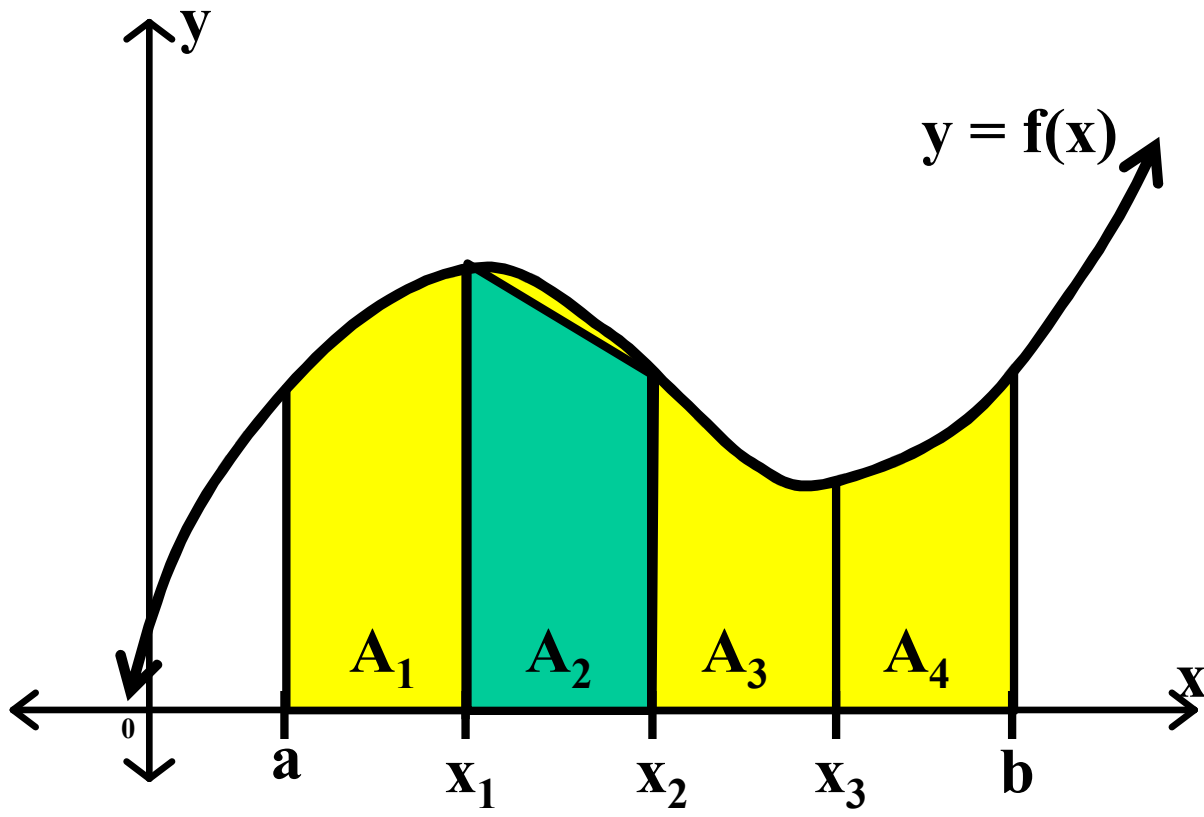


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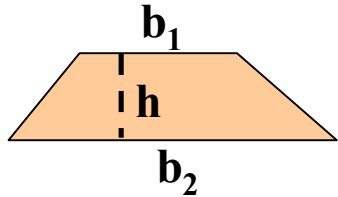


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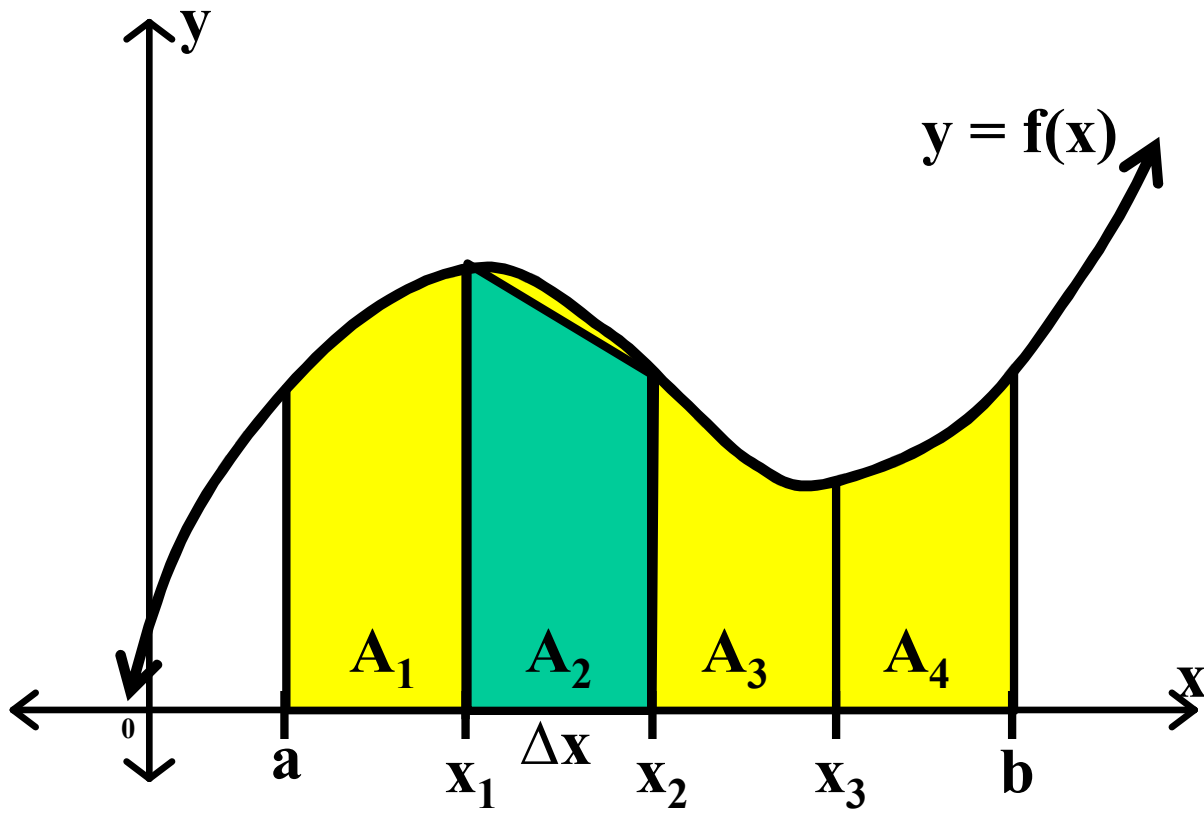
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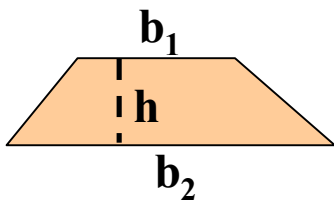
$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$

$A_2 \approx$



trapezoid

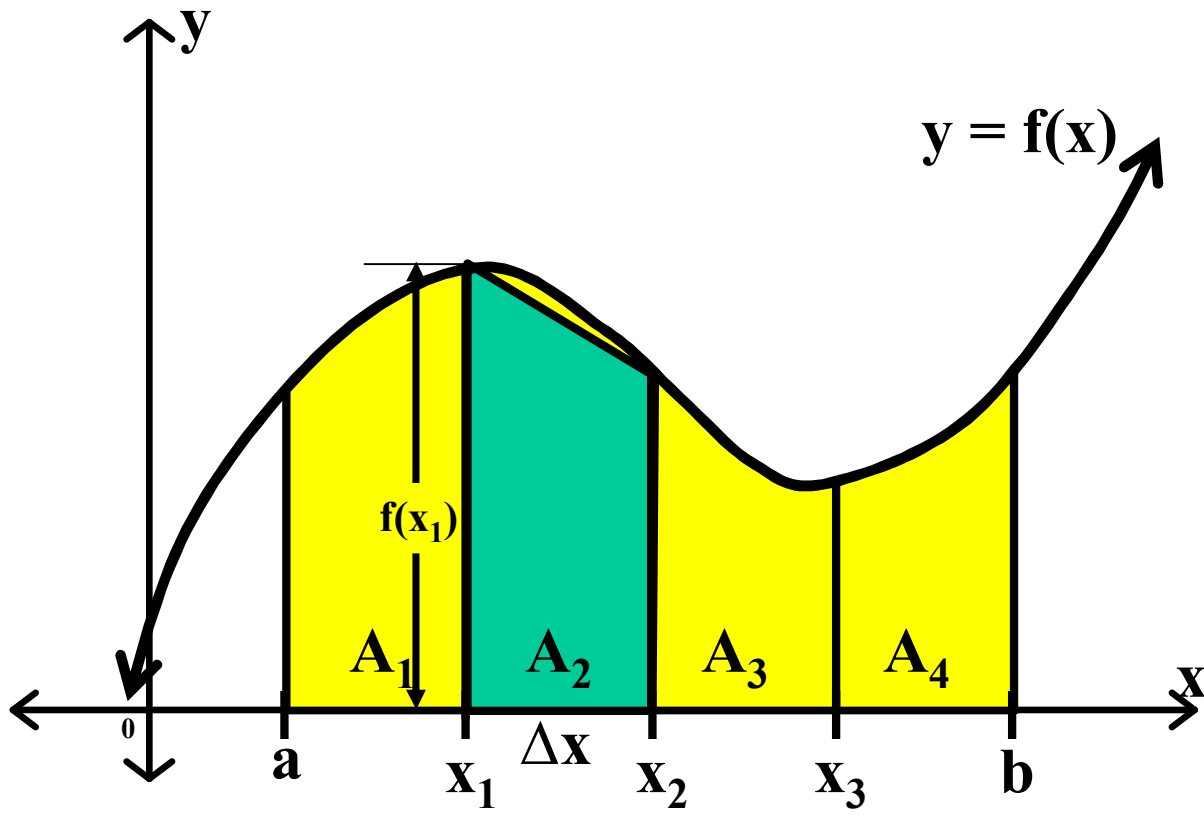


$$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$$

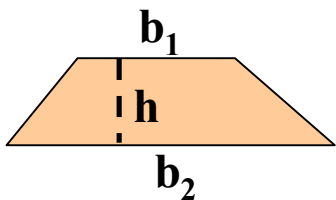
$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$$

$$A_2 \approx$$

$$h = \Delta x$$



trapezoid

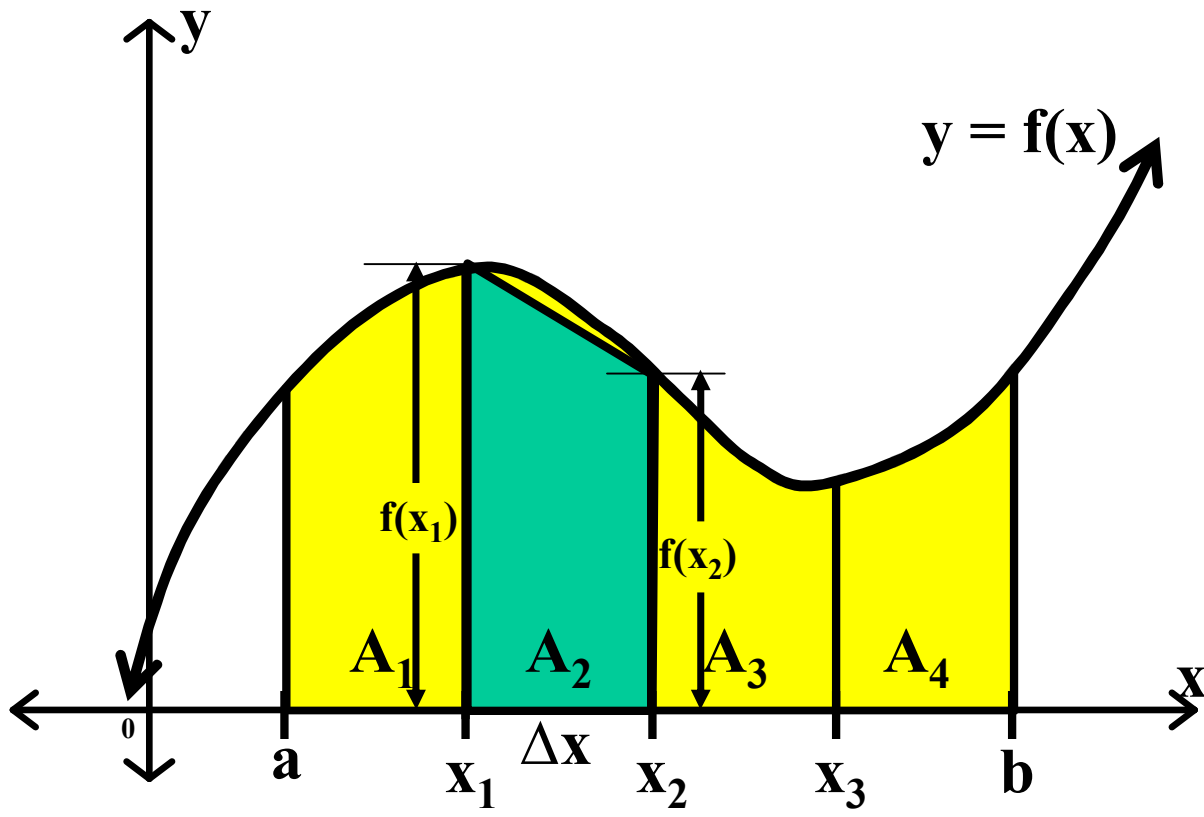


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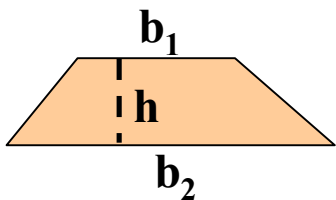
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$$A_2 \approx$$

$$h = \Delta x \quad b_1 = f(x_1)$$



trapezoid

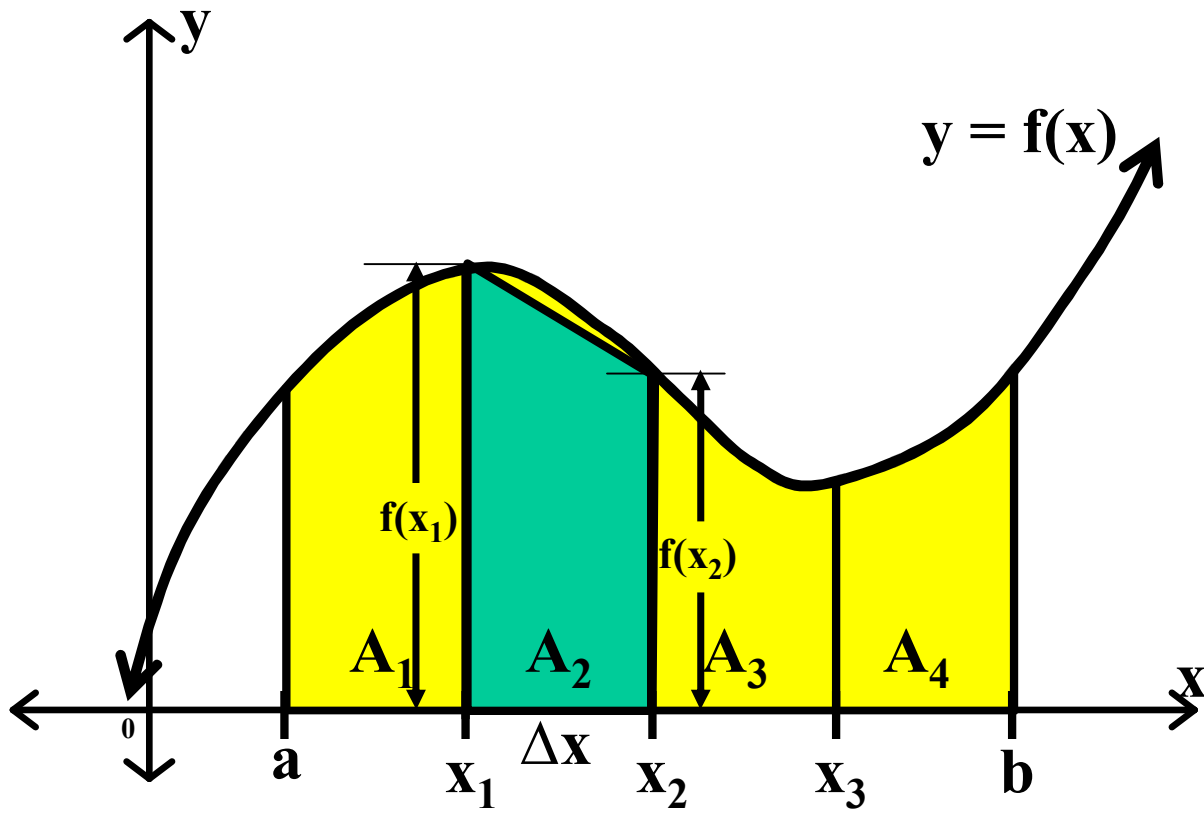


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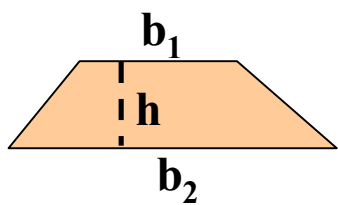
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trapezoid

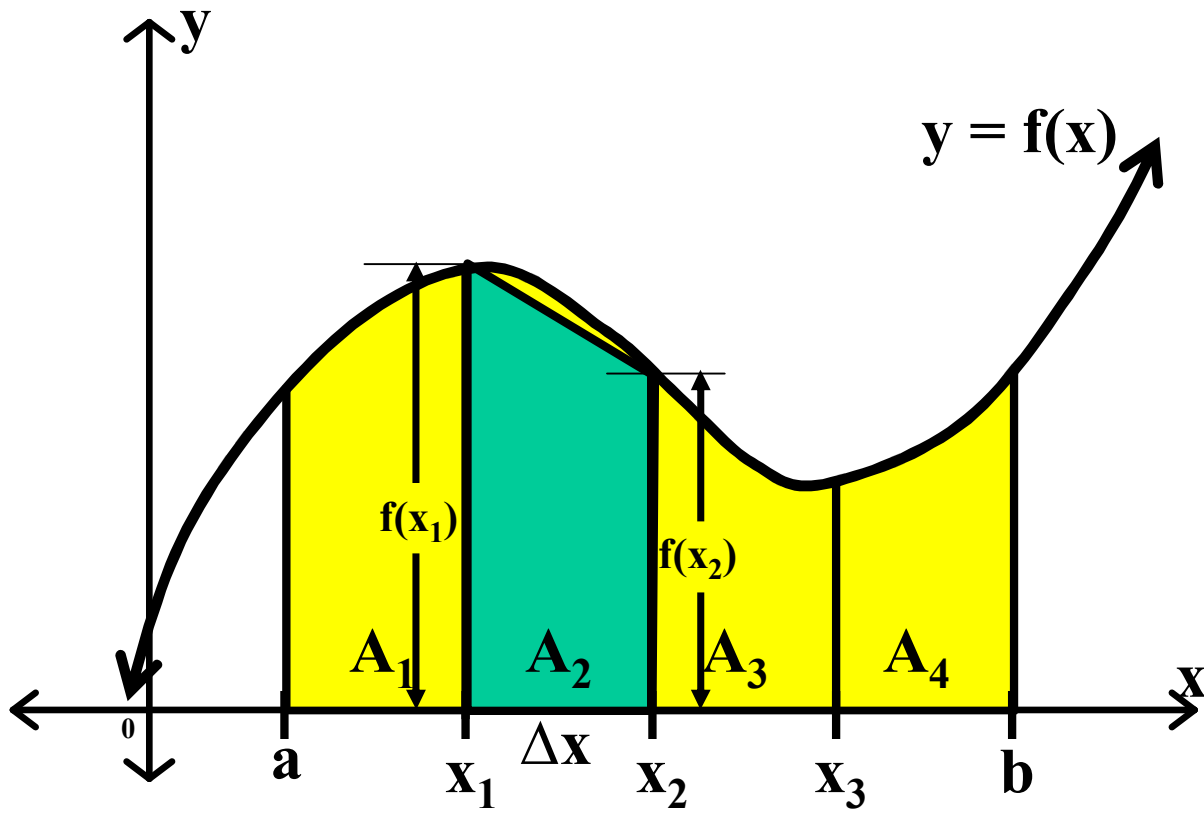


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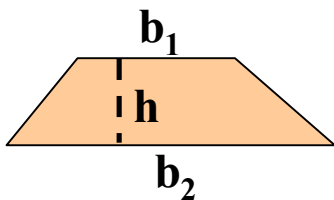
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trapezoid

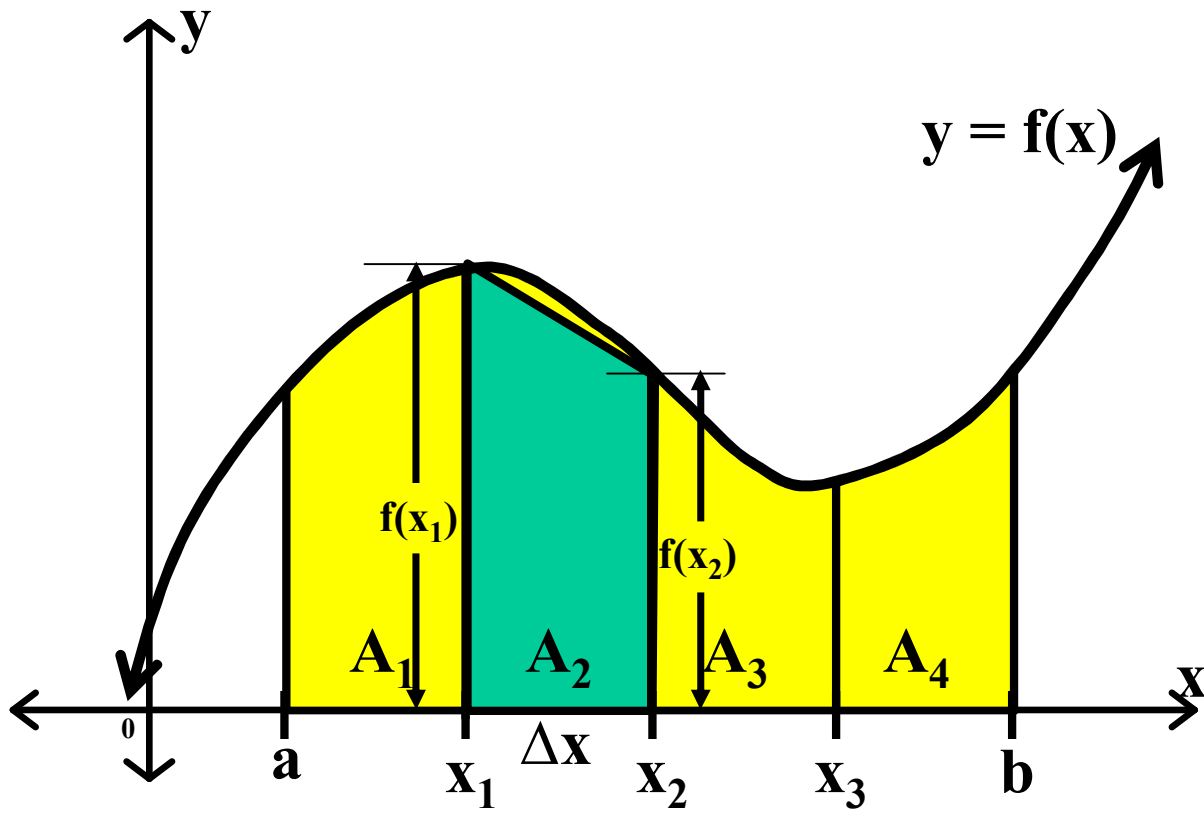


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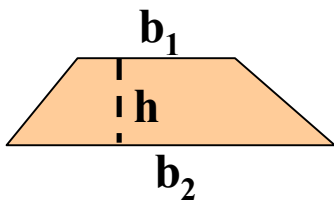
$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$$

$$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$$

$$h = \Delta x \quad b_1 = f(x_1) \quad b_2 = f(x_2)$$



trapezoid

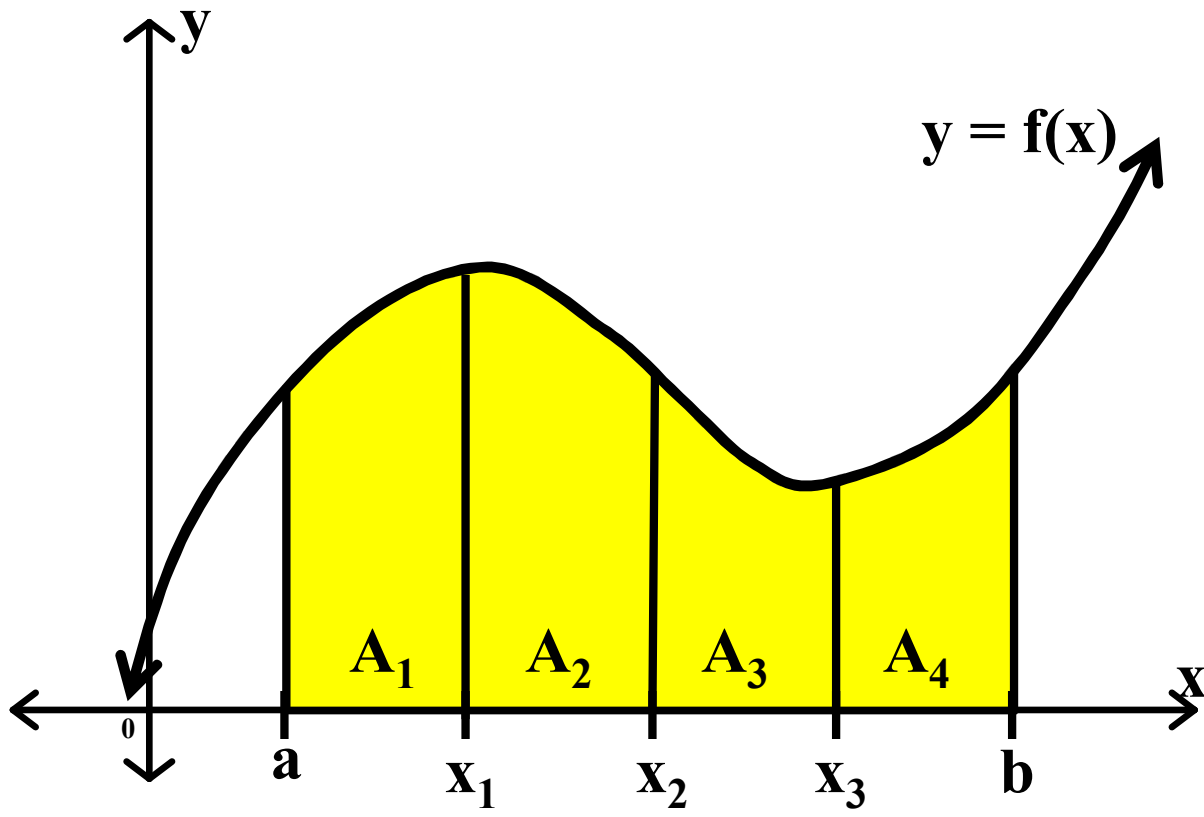


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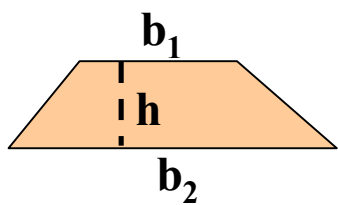
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$$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$$

$$h = \Delta x \quad b_1 = f(x_1) \quad b_2 = f(x_2)$$



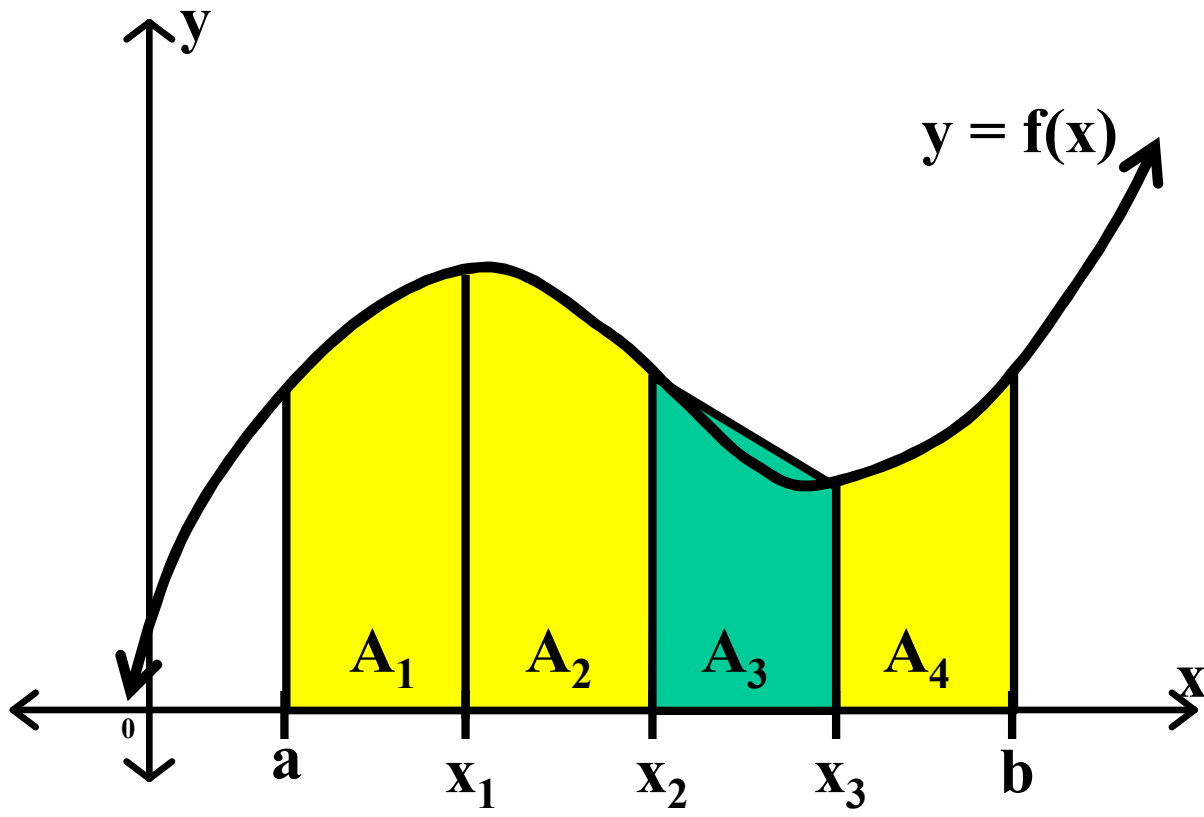
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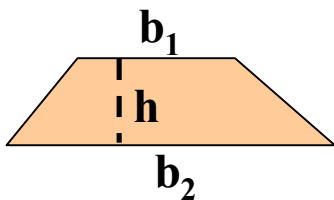
$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$

$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$



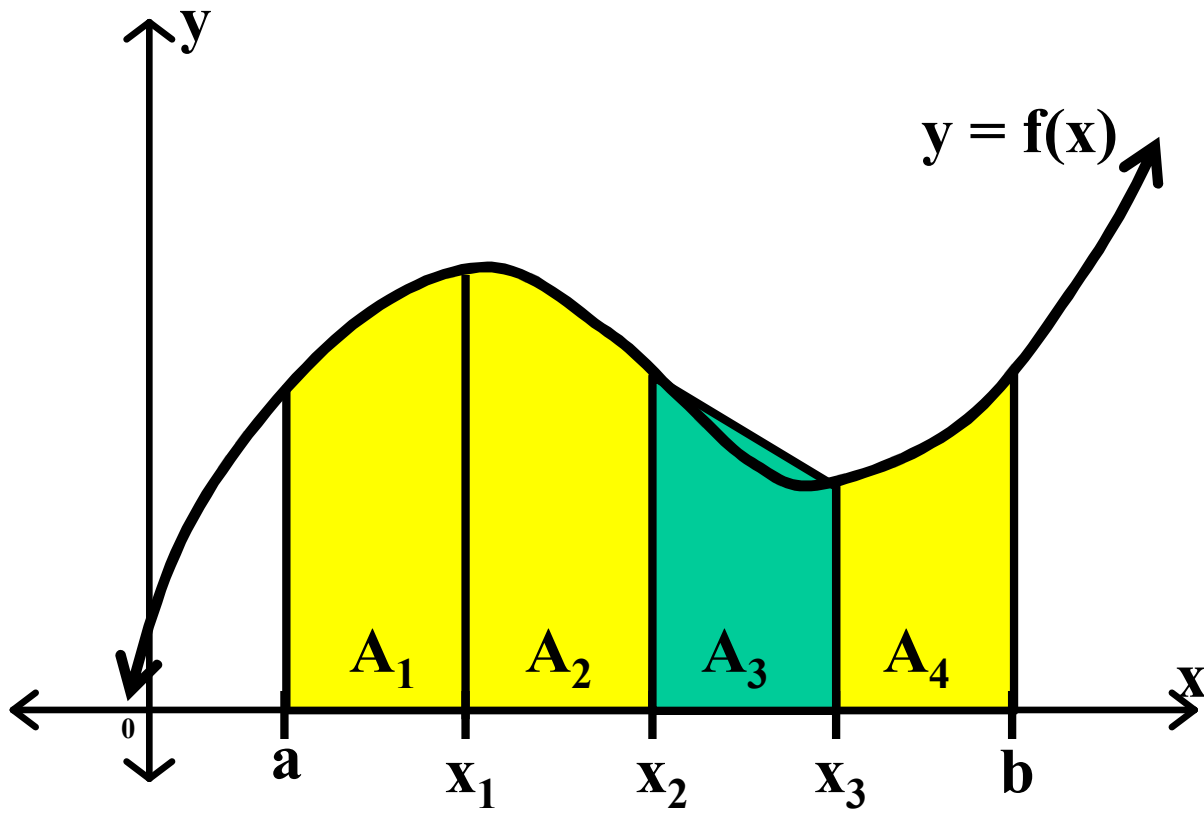
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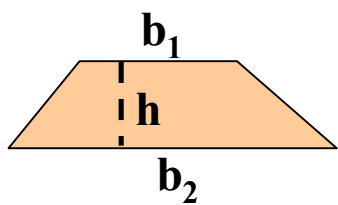
$$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$$

$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$$

$$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$$

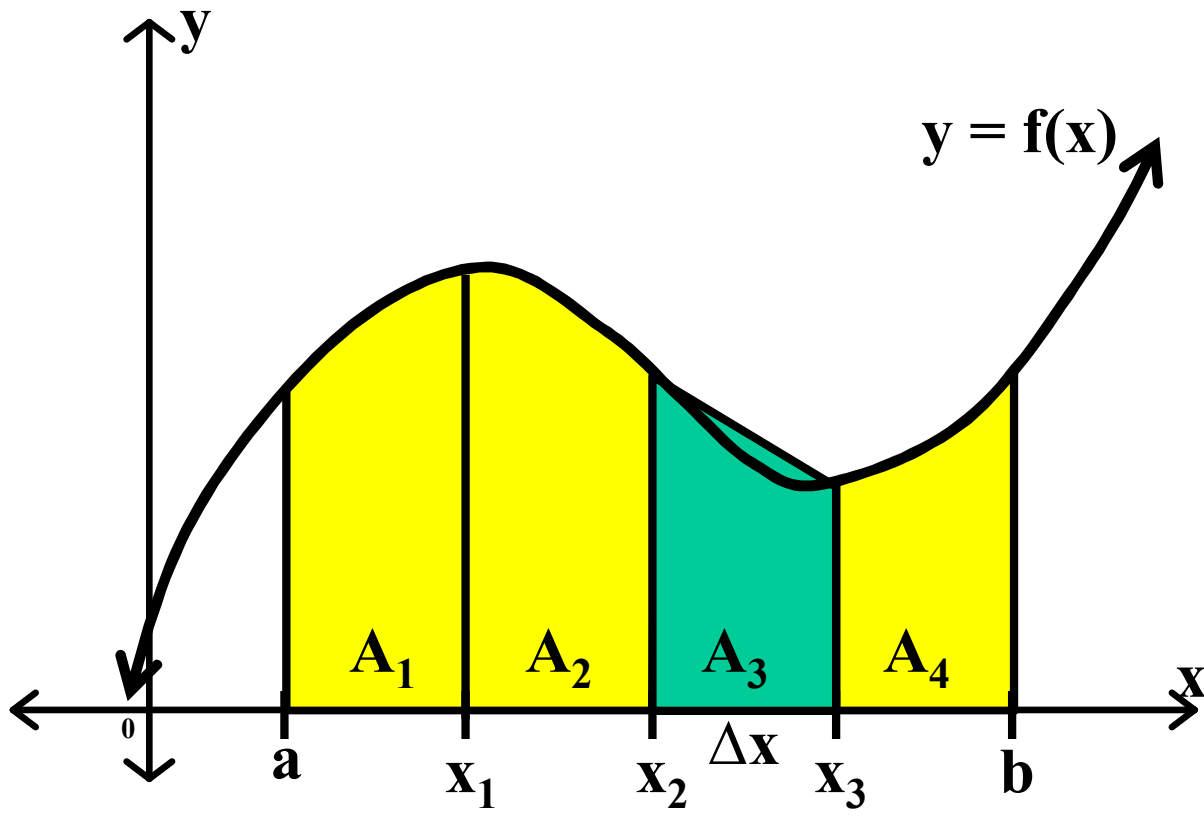


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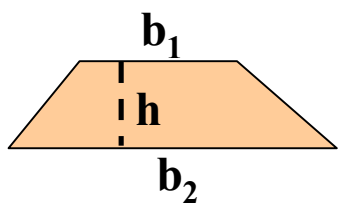


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$

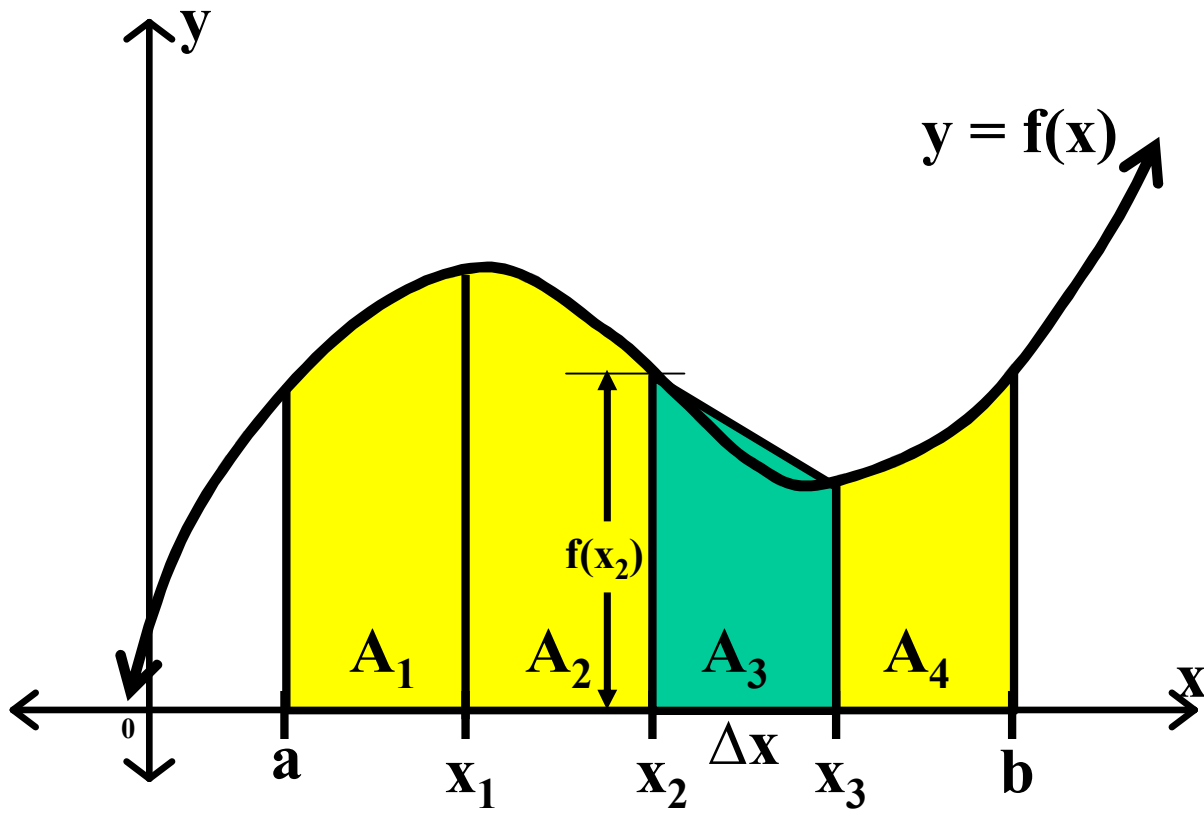


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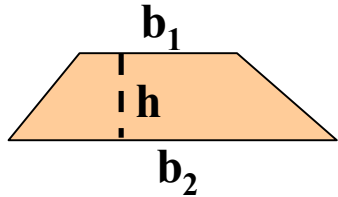


$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $h = \Delta x$

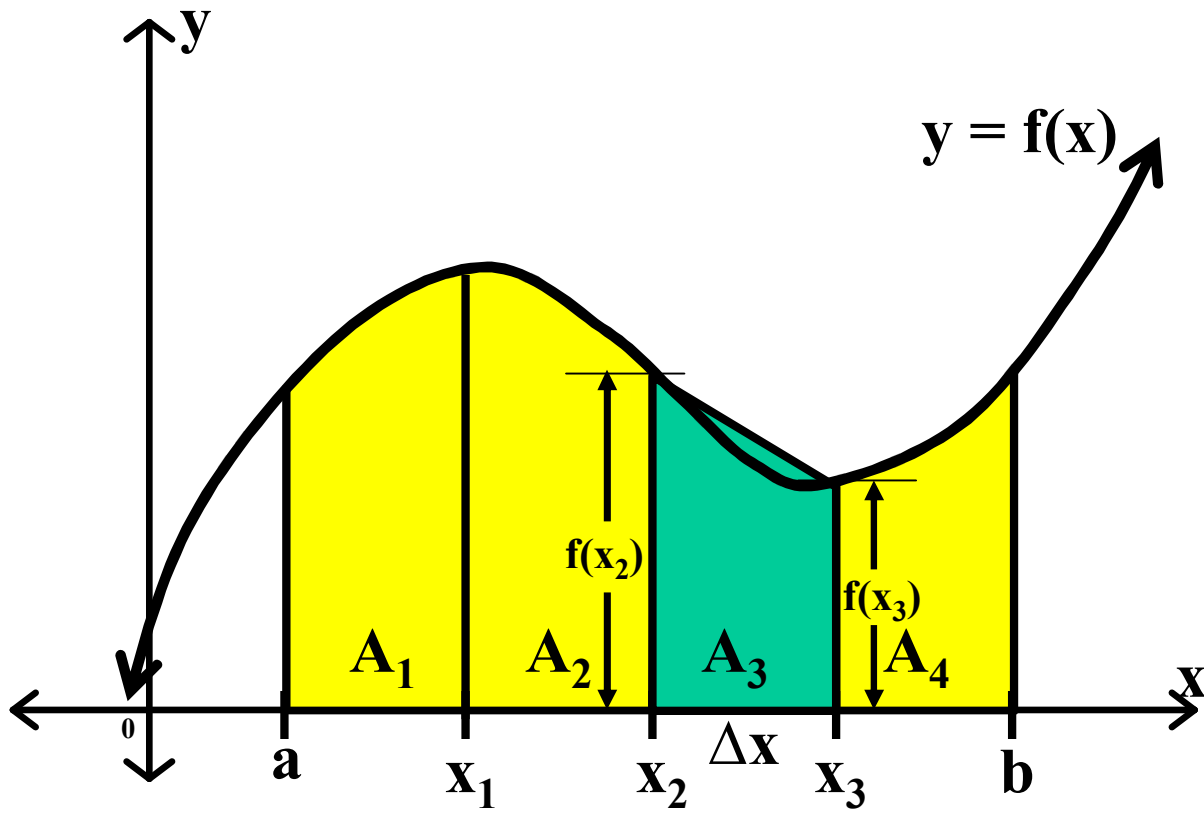


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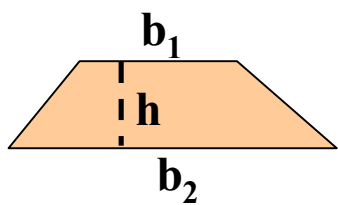


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $h = \Delta x$ $b_1 = f(x_2)$

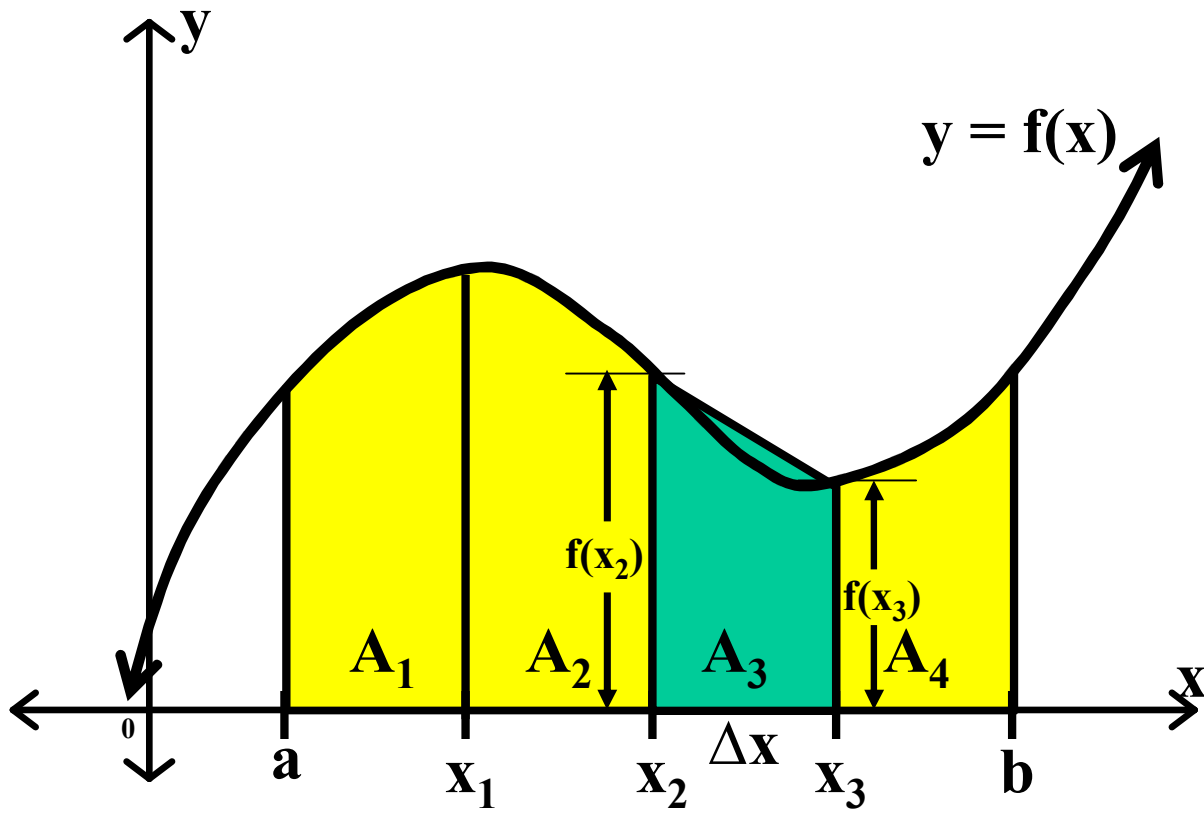


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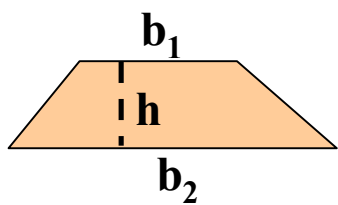


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $h = \Delta x$ $b_1 = f(x_2)$ $b_2 = f(x_3)$

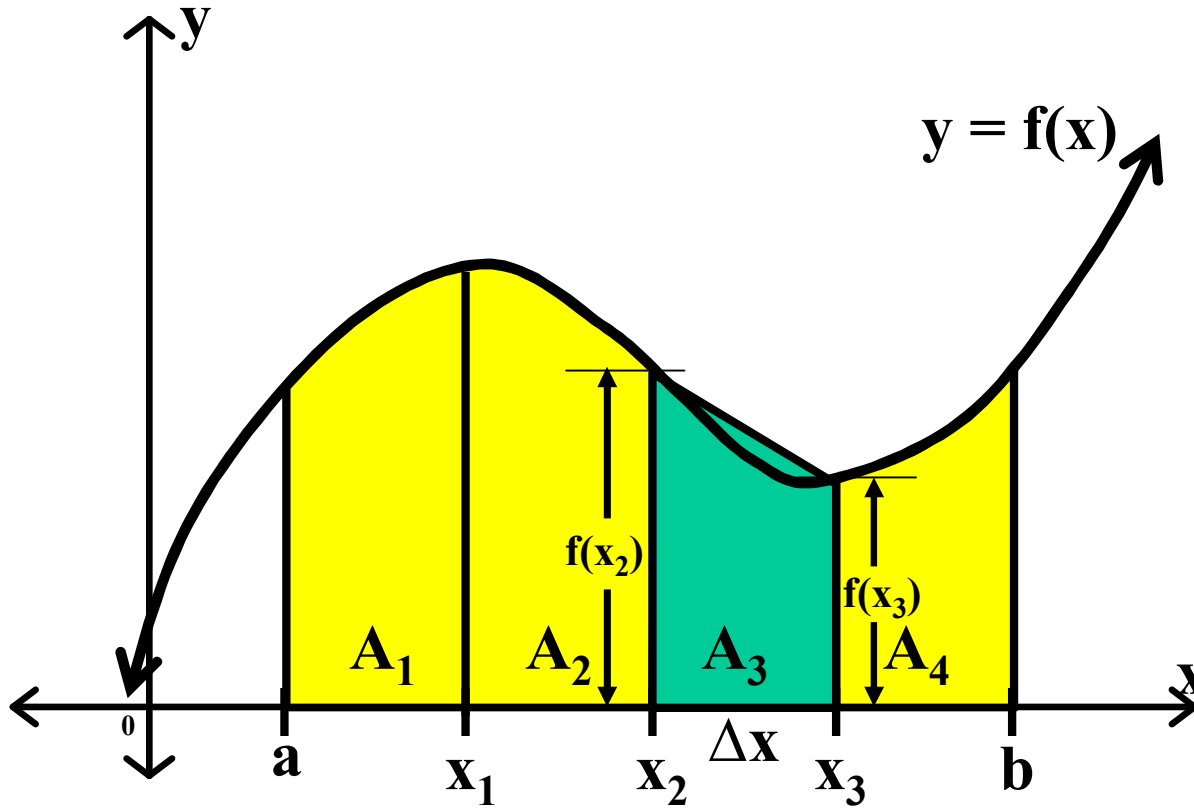


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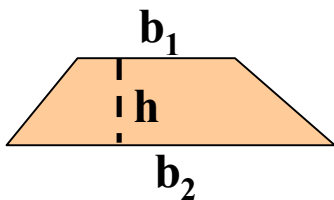


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $h = \Delta x$ $b_1 = f(x_2)$ $b_2 = f(x_3)$



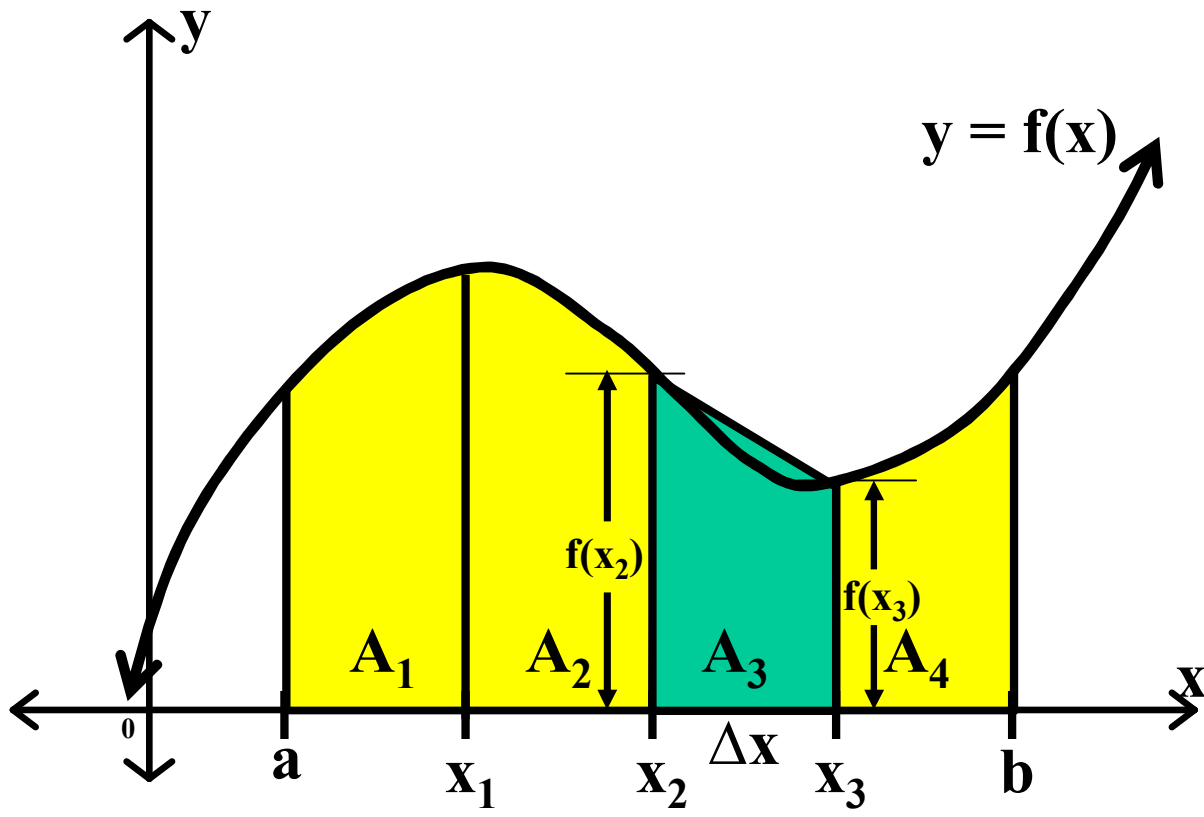
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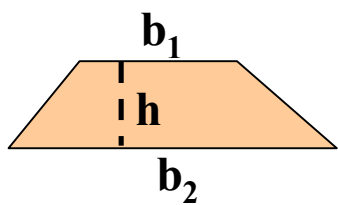
$$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$$

$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] \quad h = \Delta x \quad b_1 = f(x_2) \quad b_2 = f(x_3)$$

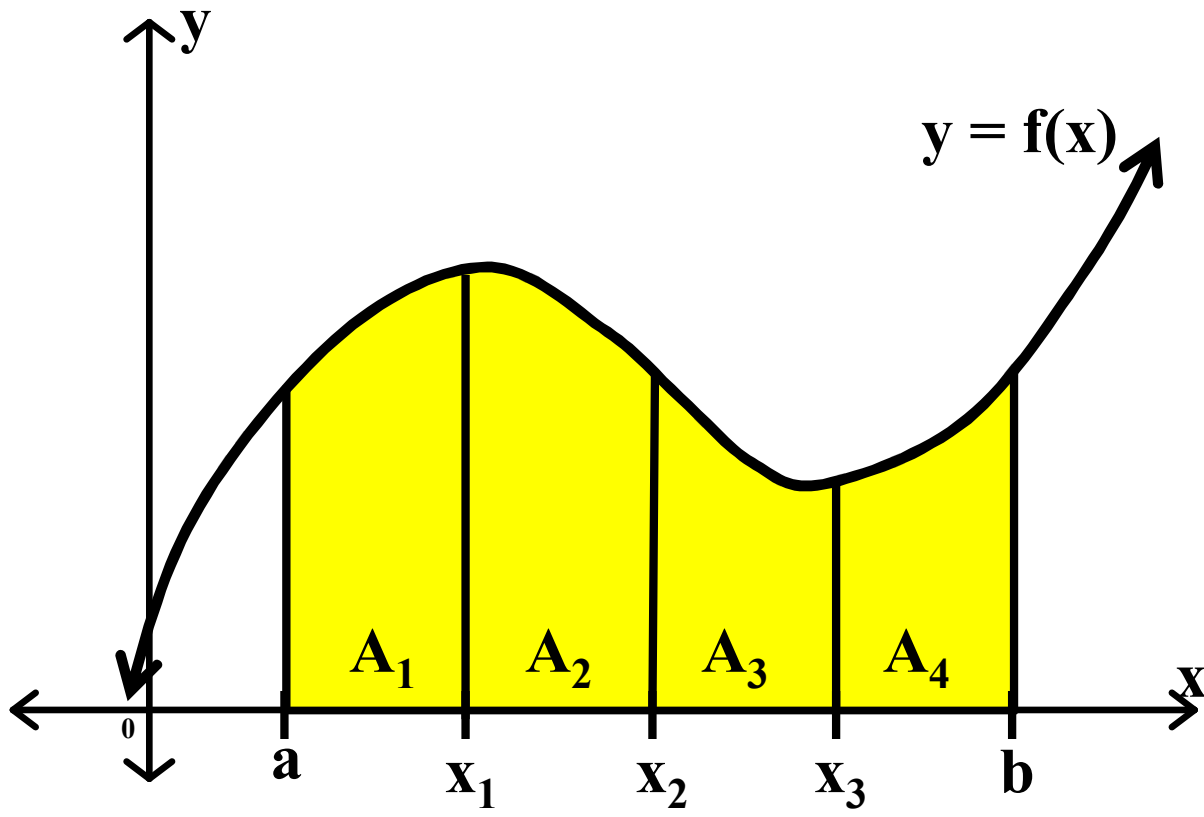


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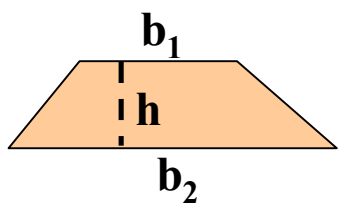


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $h = \Delta x$ $b_1 = f(x_2)$ $b_2 = f(x_3)$

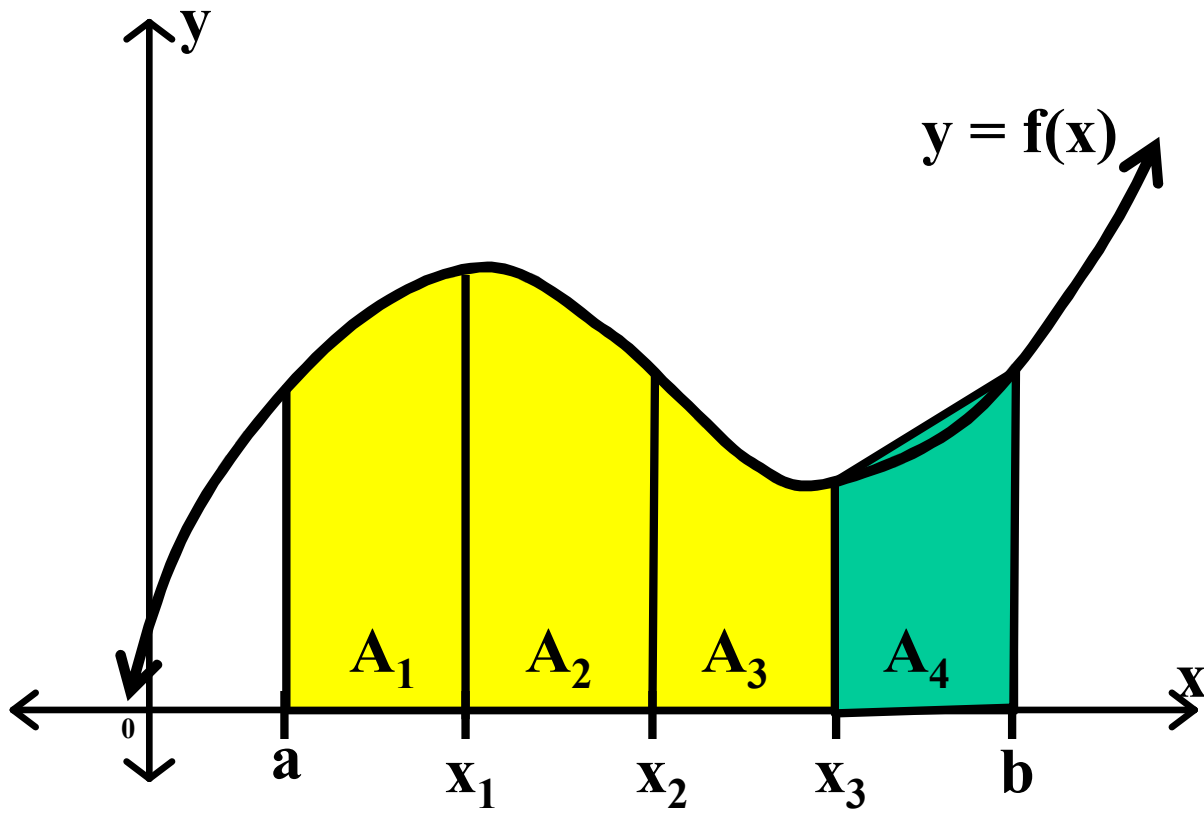


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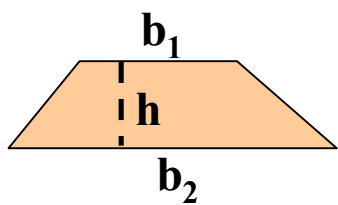


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$

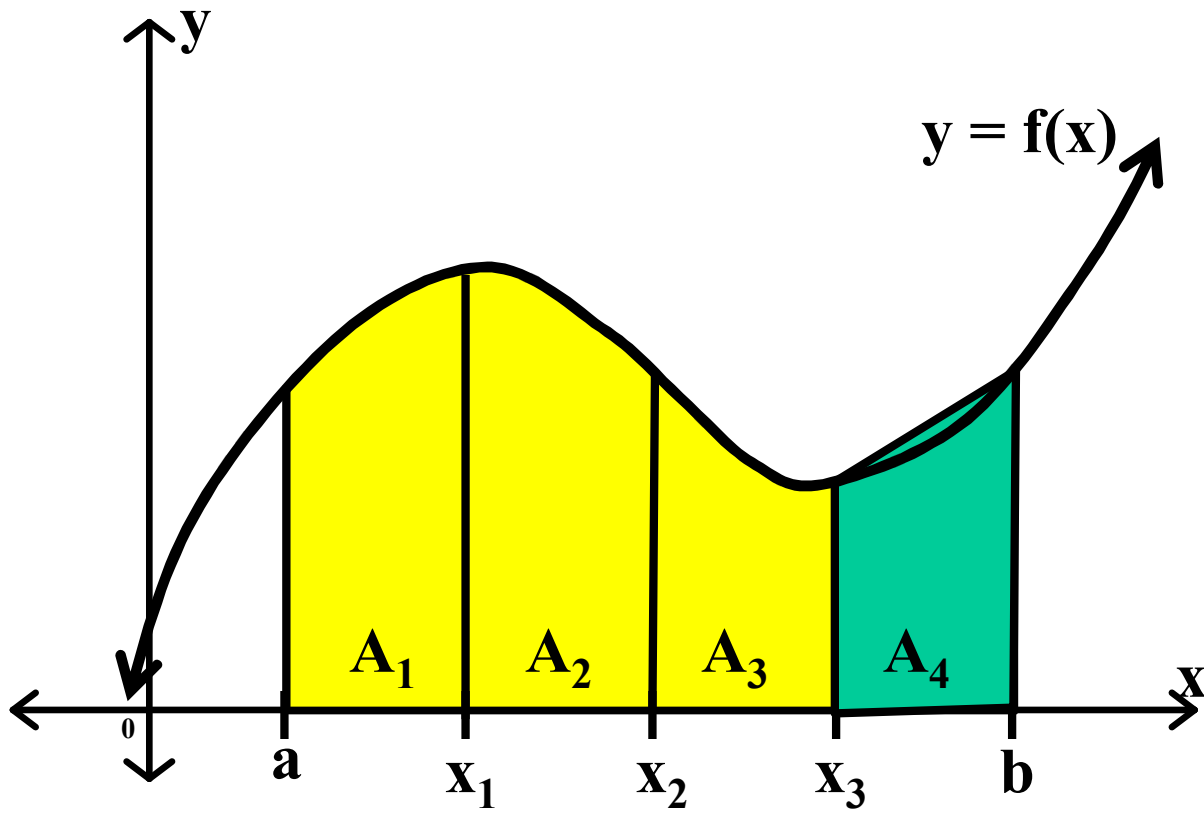


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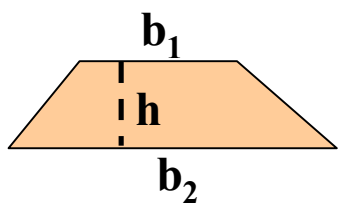


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$

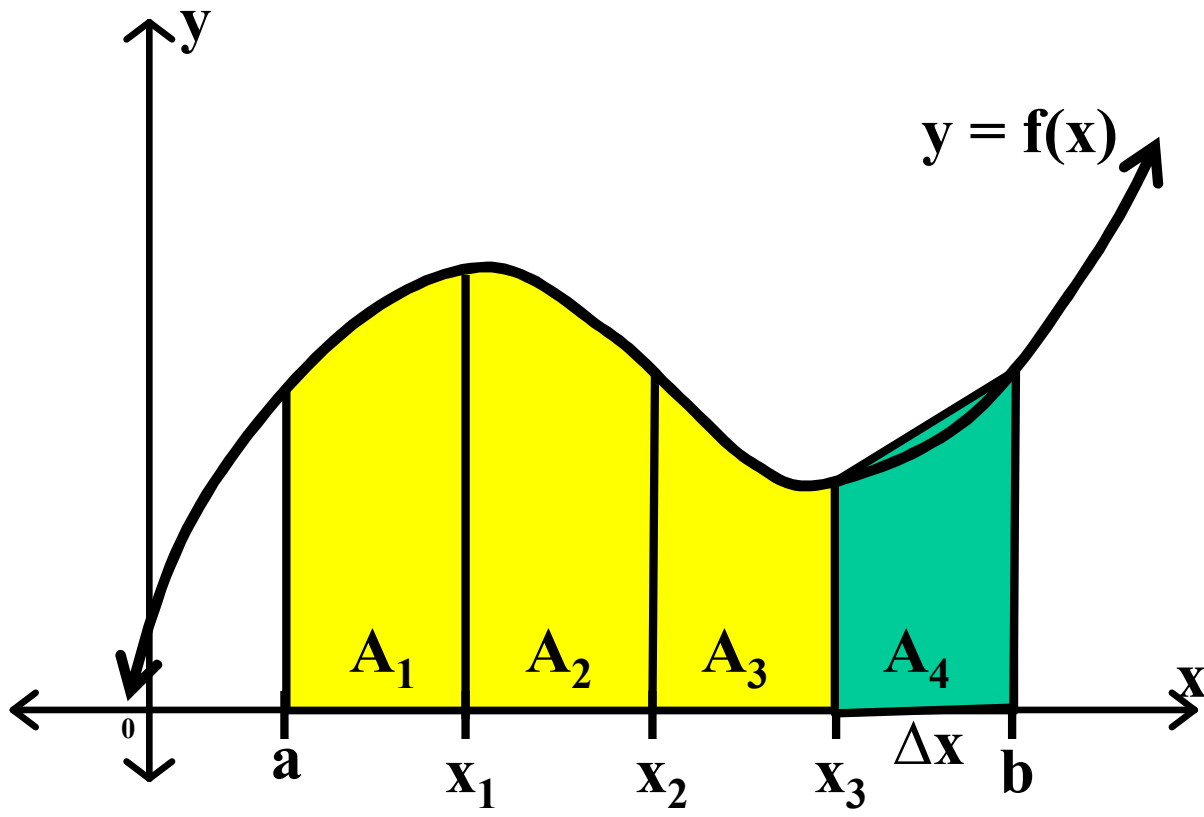


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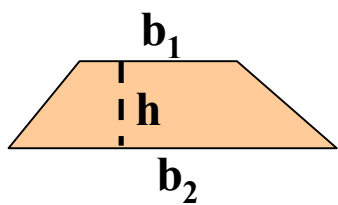


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
 $A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $A_4 \approx$

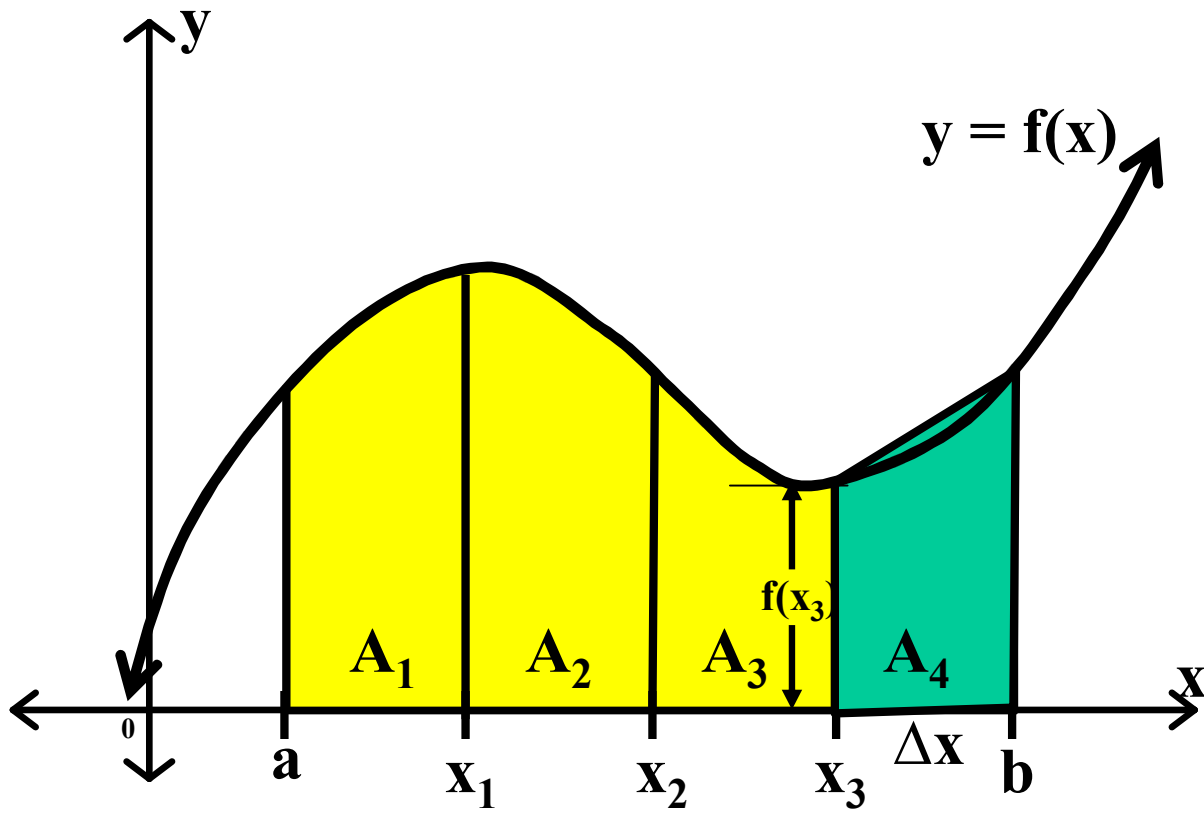


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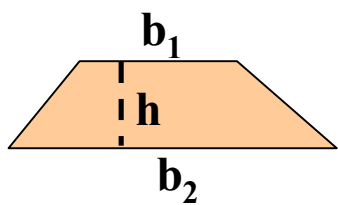


$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$	$A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$	$A_4 \approx$
	$h = \Delta x$

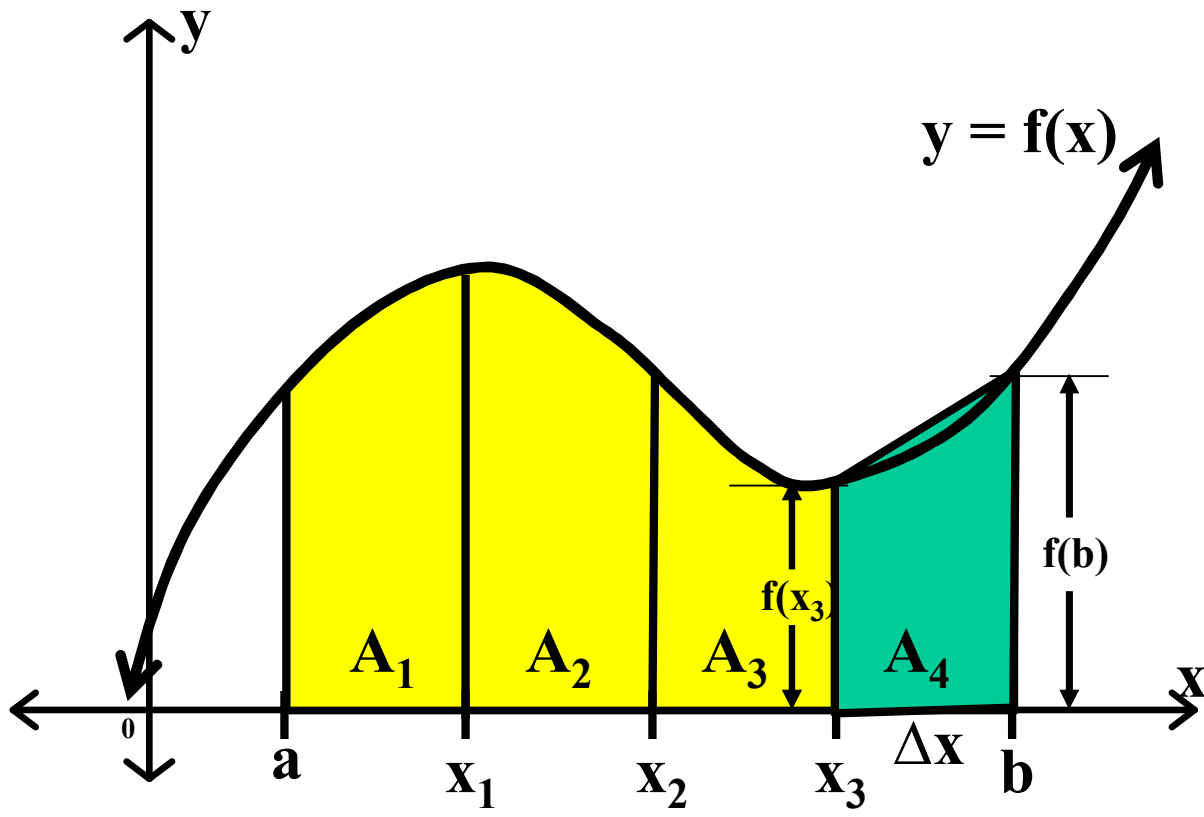


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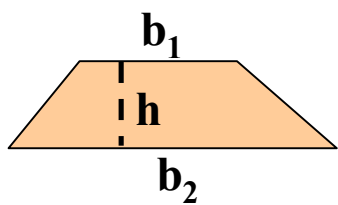


Area = $\frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$	$A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$	$A_4 \approx$
	$h = \Delta x \quad b_1 = f(x_3)$

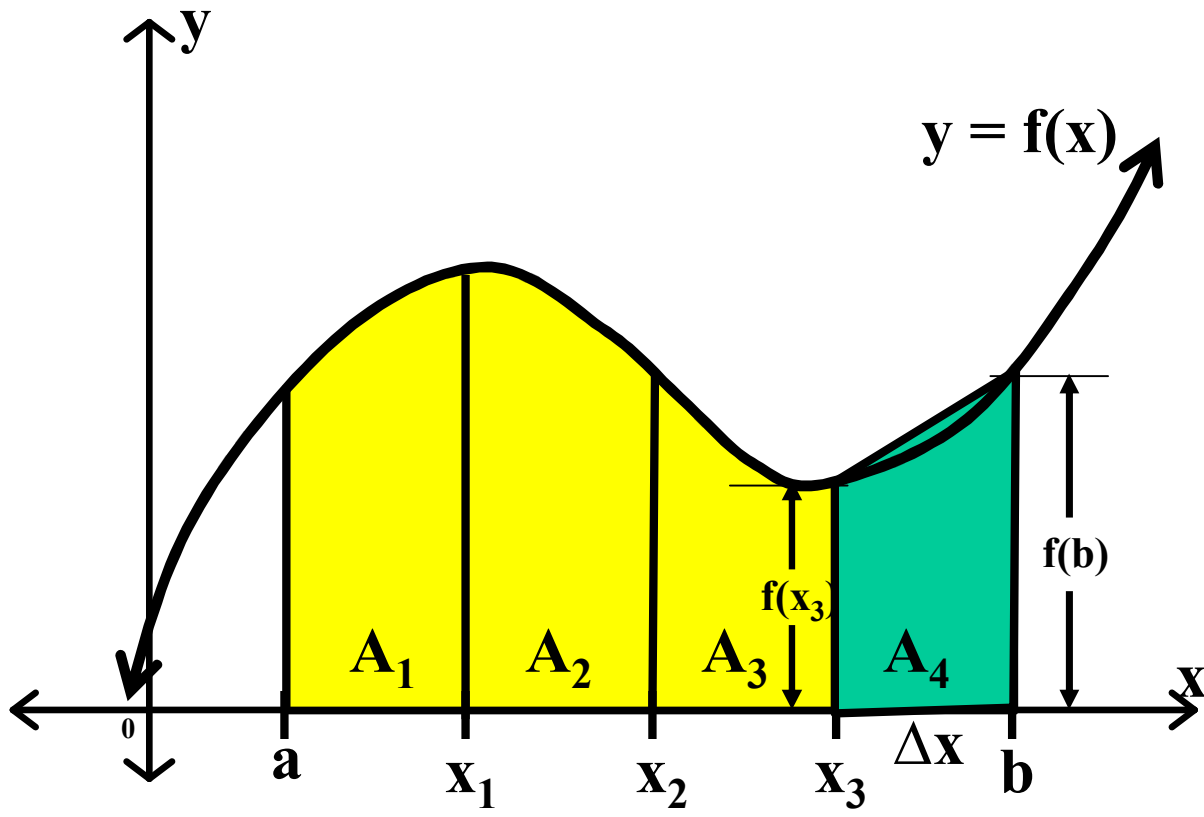


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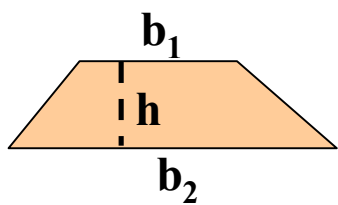


Area = $\frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$	$A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$	$A_4 \approx$
	$h = \Delta x \quad b_1 = f(x_3) \quad b_2 = f(b)$



trapezoid

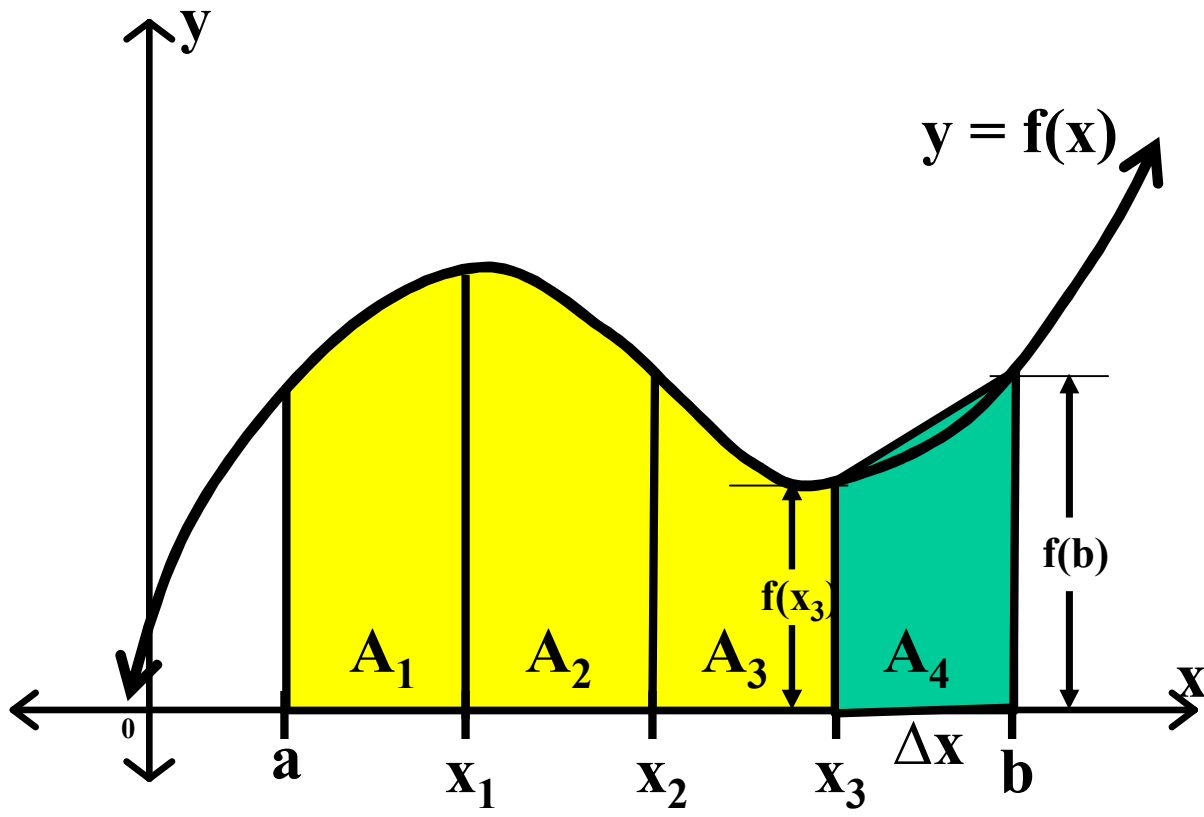


Area = $\frac{1}{2} * h * (b_1 + b_2)$

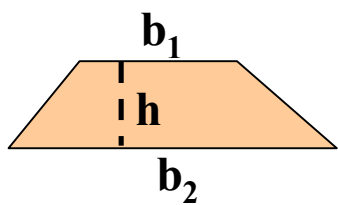
$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$ $A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$

$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$ $A_4 \approx \frac{1}{2} * \Delta x$

$h = \Delta x$ $b_1 = f(x_3)$ $b_2 = f(b)$

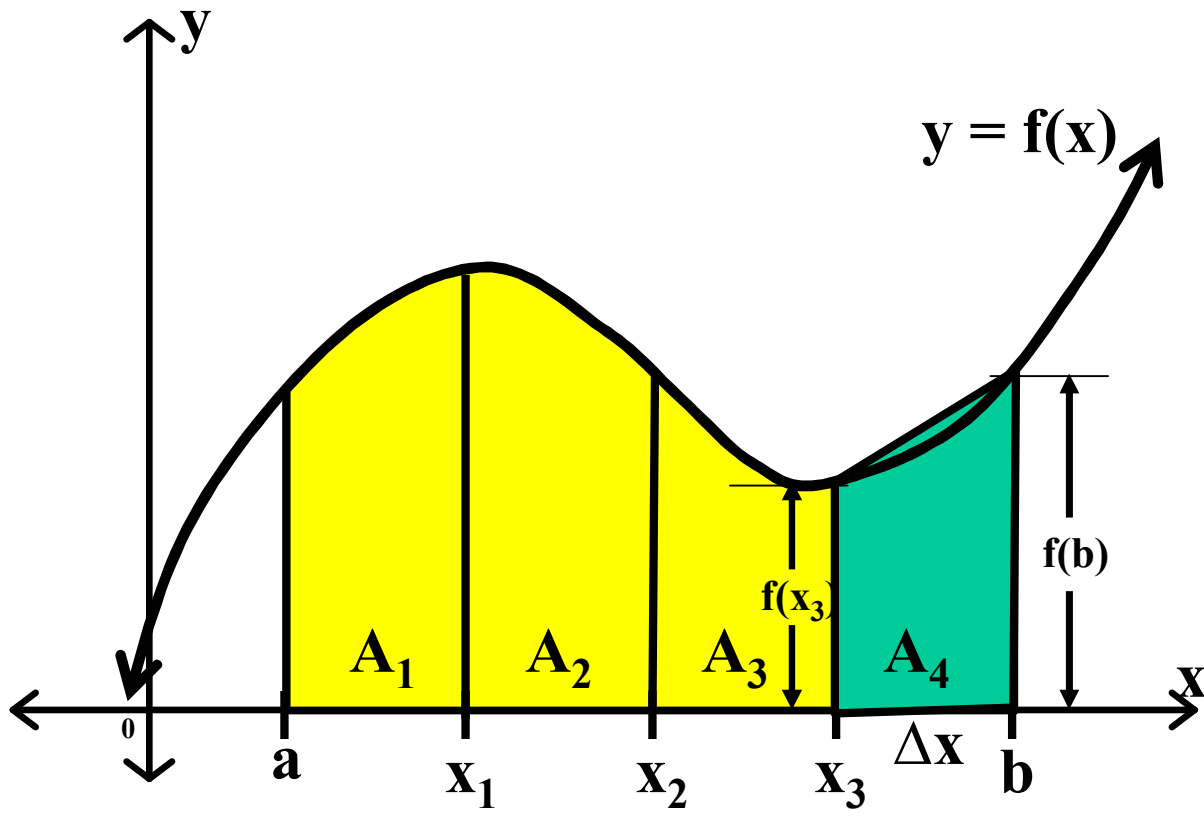


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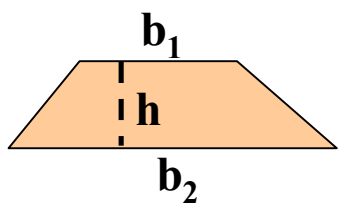


$\text{Area} = \frac{1}{2} * h * (b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$	$A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$	$A_4 \approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]$
$h = \Delta x \quad b_1 = f(x_3) \quad b_2 = f(b)$	

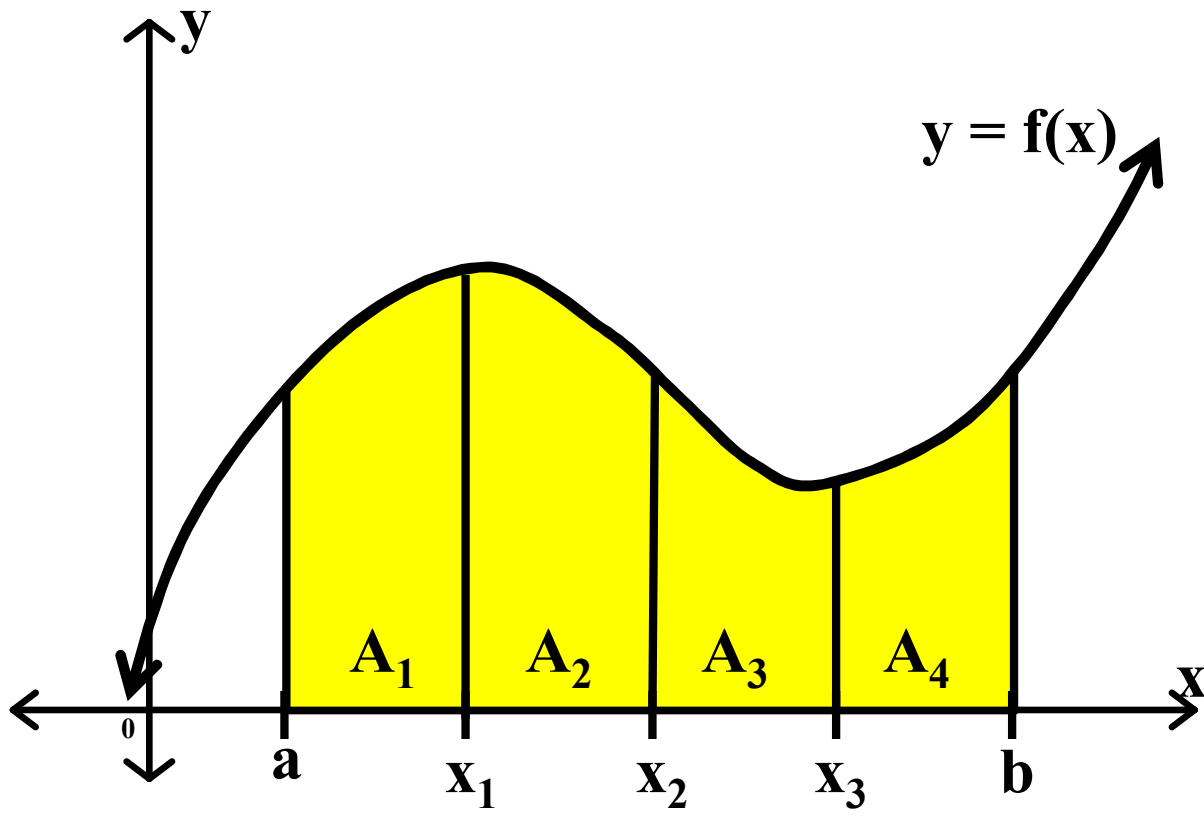


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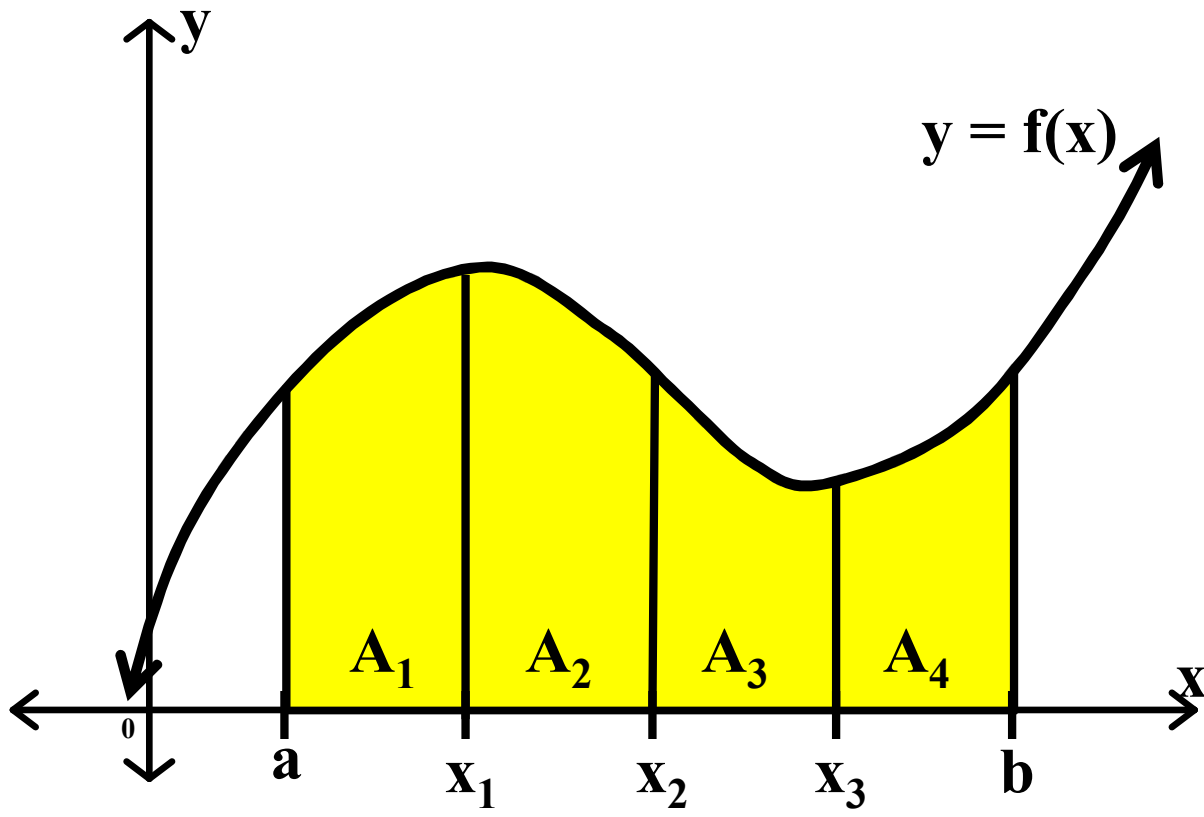


$\text{Area} = \frac{1}{2} * h(b_1 + b_2)$

$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$	$A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$
$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$	$A_4 \approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]$
$h = \Delta x \quad b_1 = f(x_3) \quad b_2 = f(b)$	



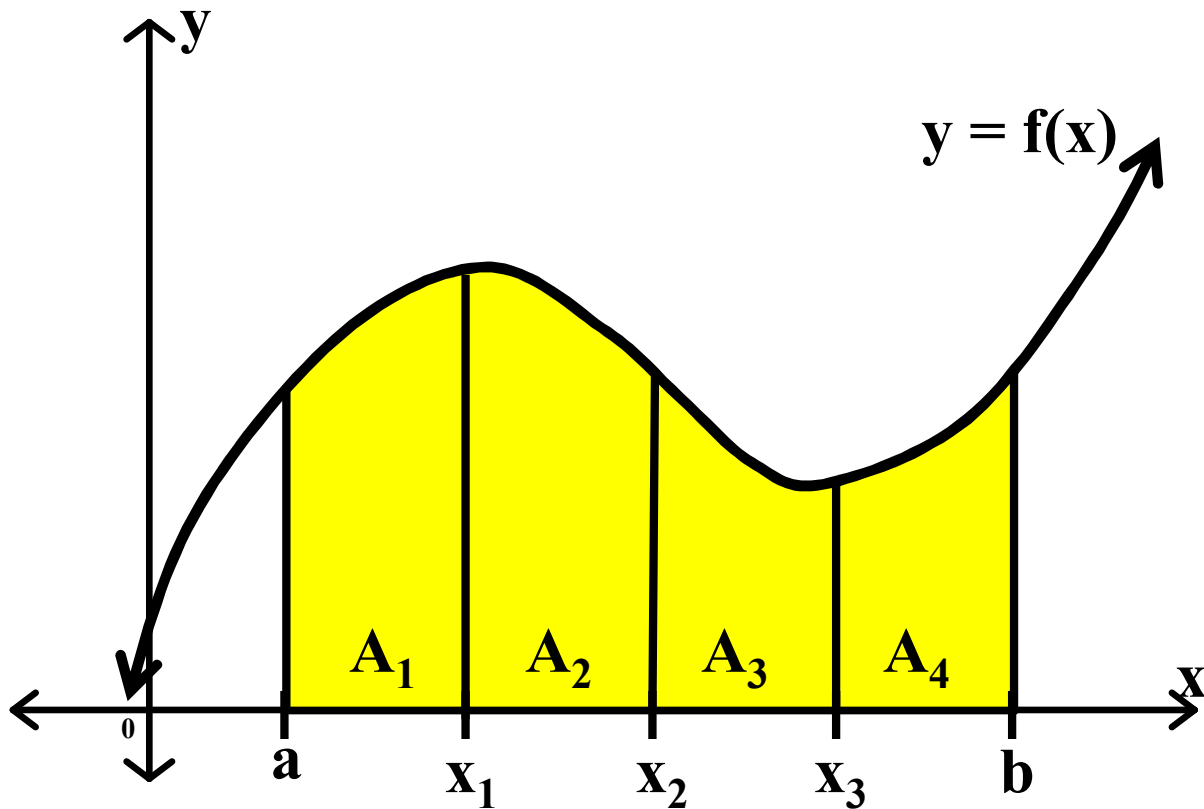
$$\begin{aligned}
 A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\
 A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$



$$A_1 \approx \frac{1}{2} \cdot \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \cdot \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \cdot \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \cdot \Delta x [f(x_3) + f(b)]$$

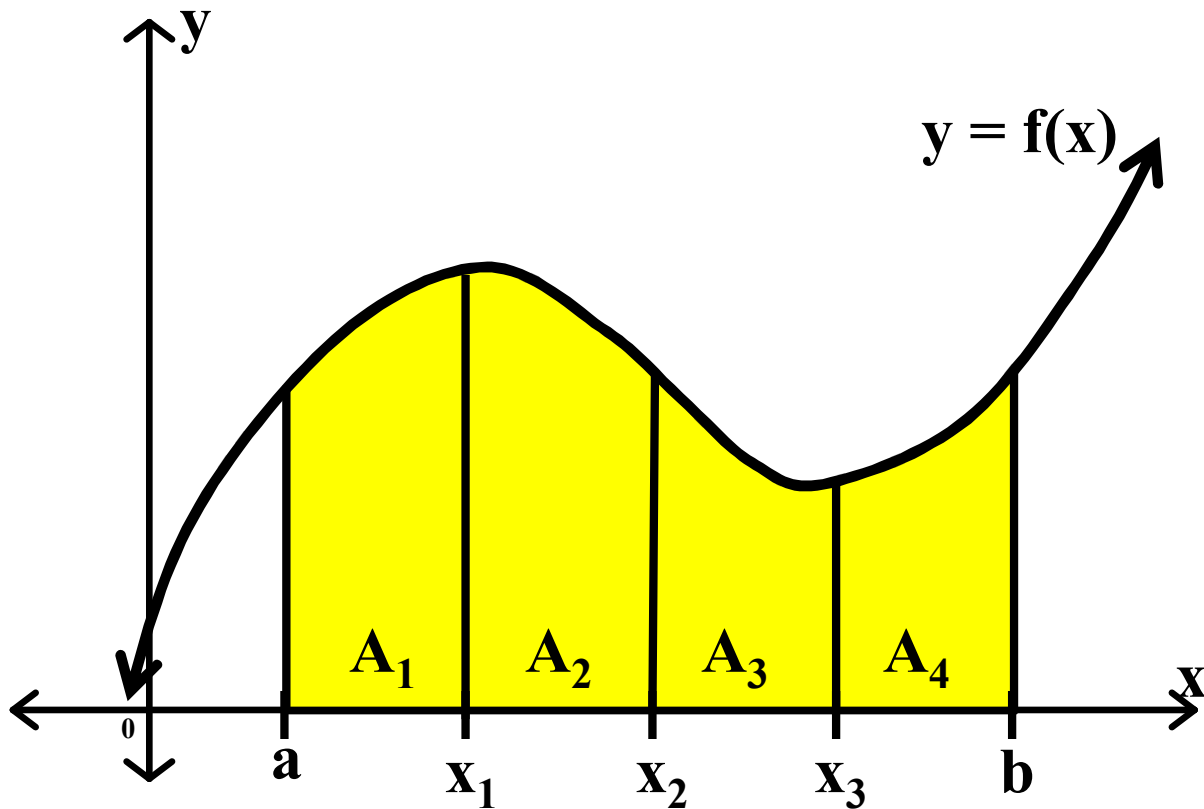
$$A = \int_a^b f(x) dx$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

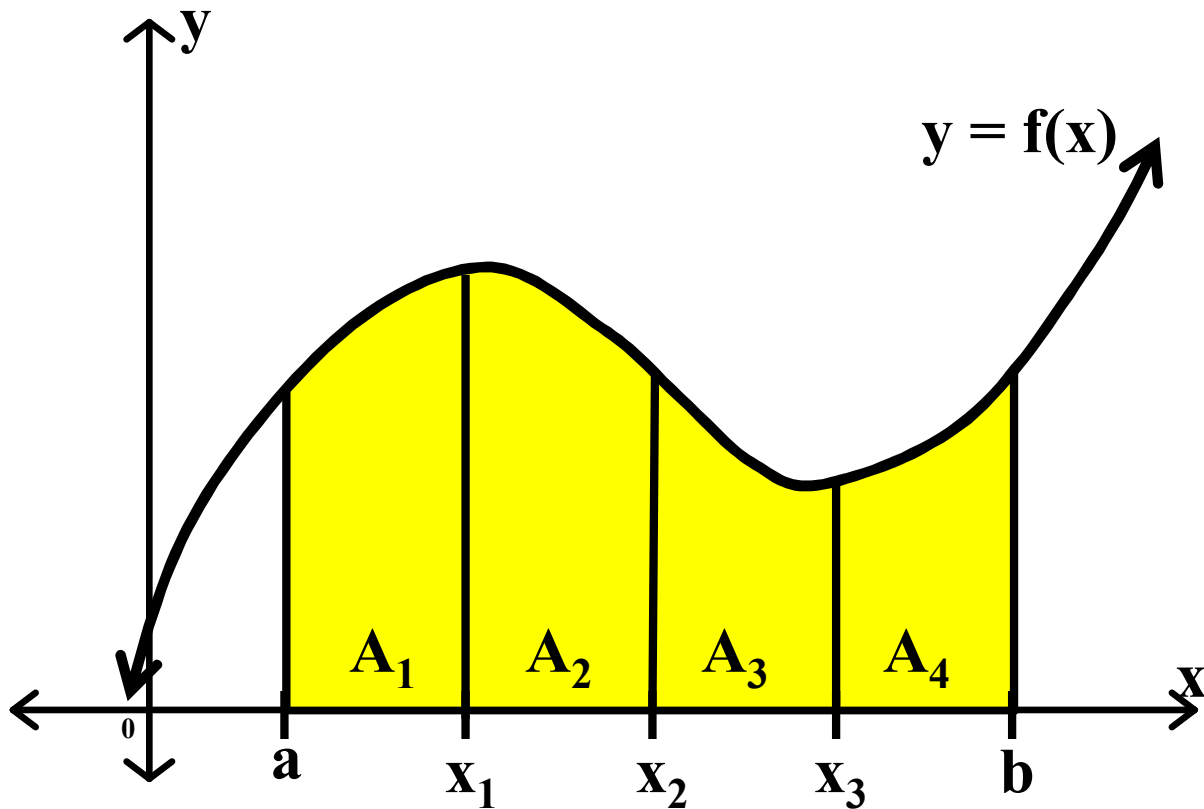
$$A = \int_a^b f(x) dx = A_1 + A_2 + A_3 + A_4$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

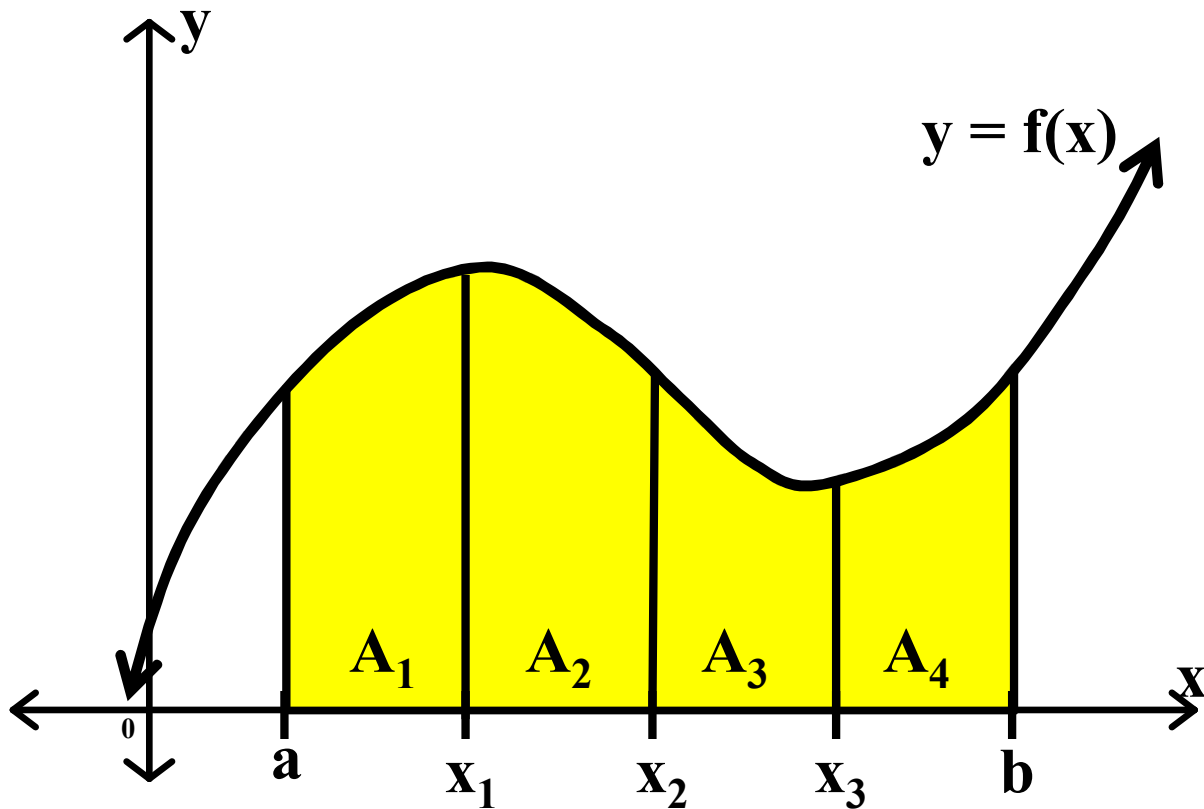
$$A = \int_a^b f(x) dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

$$A = \int_a^b f(x) dx = A_1 + A_2 + A_3 + A_4 = \sum_{i=1}^n A_i \quad (\text{In this case, } n = 4.)$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} \mathbf{A}_1 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{a}) + \mathbf{f}(\mathbf{x}_1)] & \mathbf{A}_3 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_2) + \mathbf{f}(\mathbf{x}_3)] \\ \mathbf{A}_2 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_1) + \mathbf{f}(\mathbf{x}_2)] & \mathbf{A}_4 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_3) + \mathbf{f}(\mathbf{b})] \end{aligned}$$

$$\mathbf{A} = \int_a^b \mathbf{f}(\mathbf{x}) d\mathbf{x} = \sum_{i=1}^n \mathbf{A}_i$$

$$\begin{aligned} \mathbf{A}_1 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{a}) + \mathbf{f}(\mathbf{x}_1)] & \mathbf{A}_3 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_2) + \mathbf{f}(\mathbf{x}_3)] \\ \mathbf{A}_2 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_1) + \mathbf{f}(\mathbf{x}_2)] & \mathbf{A}_4 &\approx \frac{1}{2} * \Delta \mathbf{x} [\mathbf{f}(\mathbf{x}_3) + \mathbf{f}(\mathbf{b})] \end{aligned}$$

$$\mathbf{A} = \int_a^b \mathbf{f}(\mathbf{x}) d\mathbf{x} = \sum_{i=1}^n \mathbf{A}_i$$

$$\mathbf{A} \approx$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] &+ \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &+ \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \end{aligned}$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

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$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$$

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‘Factor out’ the Δx factor from each of the four terms of the expression.

$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$$

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$$A \approx$$

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$$A \approx \Delta x$$

‘Factor out’ the Δx factor from each of the four terms of the expression.

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$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] +$$

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 \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

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 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}
 \end{aligned}$$

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$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

Now do the indicated multiplication.

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

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$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

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$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

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$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) \}$$

Now do the indicated multiplication.

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

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$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

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$$\begin{aligned} A &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

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Now combine like terms.

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

Now combine like terms.

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ &\quad + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ &\quad + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ &\quad + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ &\quad + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A &\approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ &\quad + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) +$$

$$\begin{aligned}
 A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\
 A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned}
 A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\
 + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}
 \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) +$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum f(x_i)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^n f(x_i)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i)]$$

$$A_1 \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] +$$

$$+ \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] +$$

$$+ \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) +$$

$$+ \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i)]$$

In this example, n = 4.

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned} A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \} \end{aligned}$$

$$\begin{aligned} A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \} \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) +$$

$$\begin{aligned}
 A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\
 A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned}
 A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\
 + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}
 \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) +$$

$$\begin{aligned}
 A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\
 A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

$$\begin{aligned}
 A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\
 + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}
 \end{aligned}$$

$$\begin{aligned}
 A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\
 + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}
 \end{aligned}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(b)]$$

$$\begin{aligned} A_1 &\approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] & A_3 &\approx \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] \\ A_2 &\approx \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] & A_4 &\approx \frac{1}{2} * \Delta x [f(x_3) + f(b)] \end{aligned}$$

$$A = \int_a^b f(x) dx = \sum_{i=1}^n A_i$$

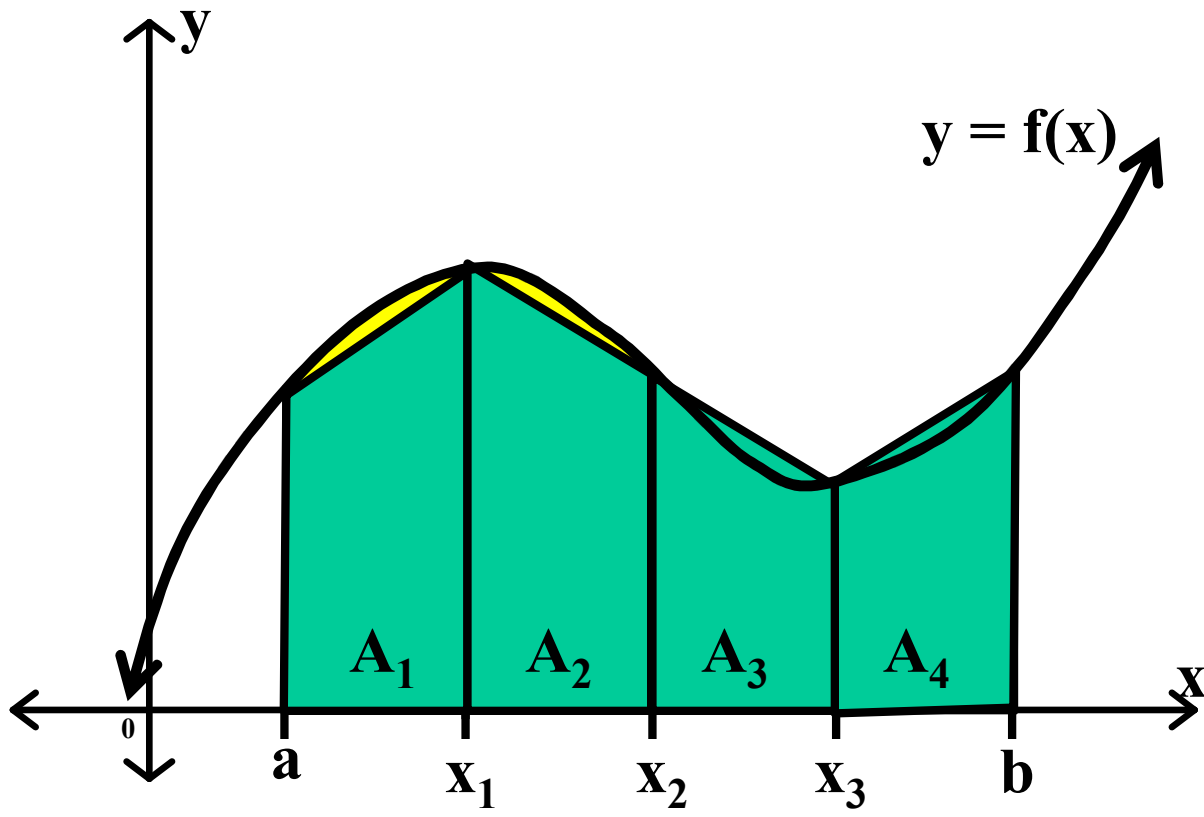
$$A \approx \frac{1}{2} * \Delta x [f(a) + f(x_1)] + \frac{1}{2} * \Delta x [f(x_1) + f(x_2)] + \\ + \frac{1}{2} * \Delta x [f(x_2) + f(x_3)] + \frac{1}{2} * \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \{ \frac{1}{2} [f(a) + f(x_1)] + \frac{1}{2} [f(x_1) + f(x_2)] + \\ + \frac{1}{2} [f(x_2) + f(x_3)] + \frac{1}{2} [f(x_3) + f(b)] \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_1) + \frac{1}{2} f(x_2) + \\ + \frac{1}{2} f(x_2) + \frac{1}{2} f(x_3) + \frac{1}{2} f(x_3) + \frac{1}{2} f(b) \}$$

$$A \approx \Delta x \{ \frac{1}{2} f(a) + f(x_1) + f(x_2) + f(x_3) + \frac{1}{2} f(b) \}$$

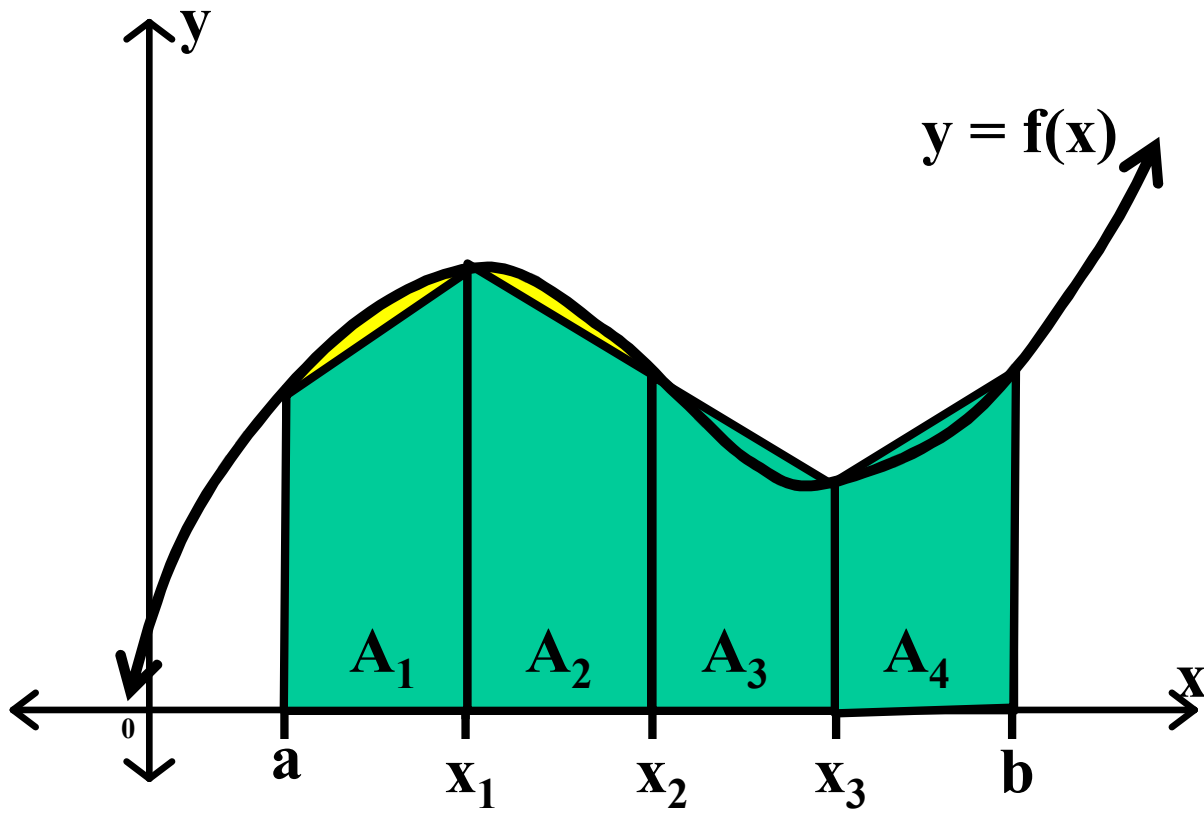
$$A \approx \Delta x [\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(b)]$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

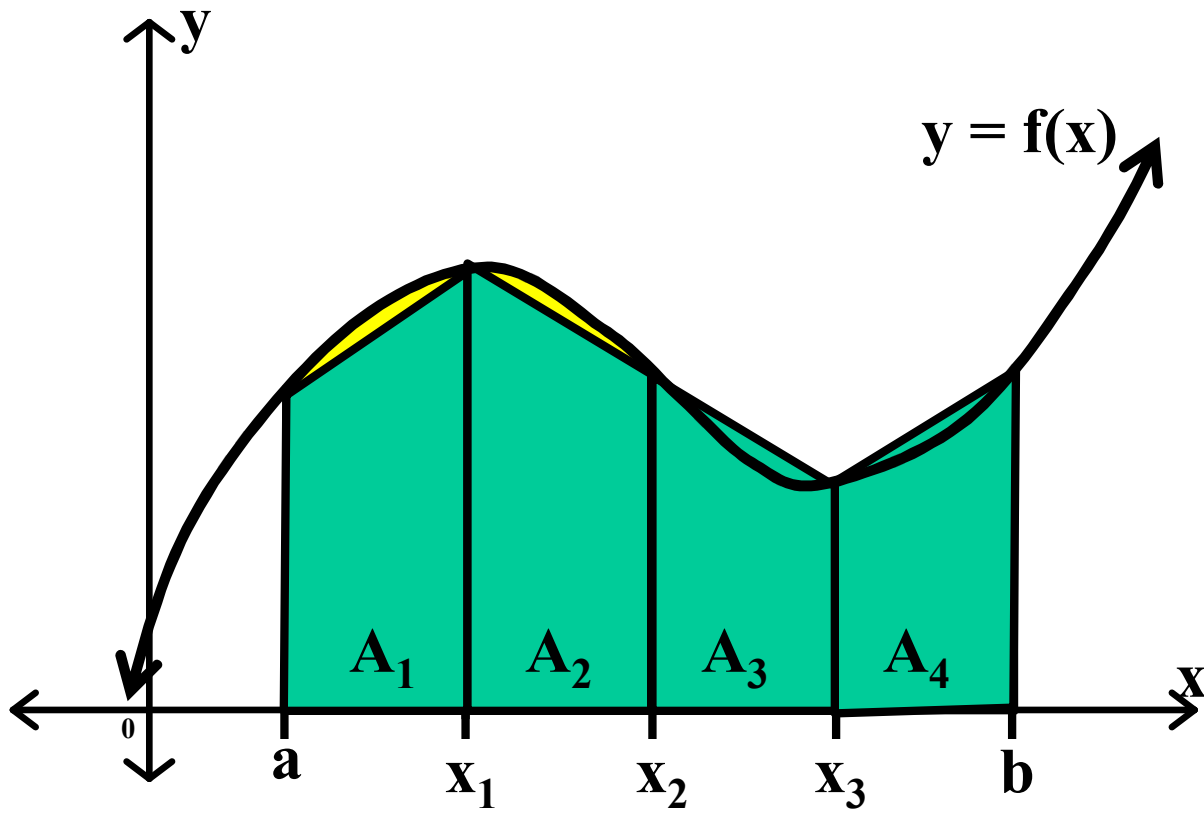
$$A \approx \Delta x \left[\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(b) \right]$$



$$A_1 \approx \frac{1}{2} \Delta x [f(a) + f(x_1)] \quad A_3 \approx \frac{1}{2} \Delta x [f(x_2) + f(x_3)]$$

$$A_2 \approx \frac{1}{2} \Delta x [f(x_1) + f(x_2)] \quad A_4 \approx \frac{1}{2} \Delta x [f(x_3) + f(b)]$$

$$A \approx \Delta x \left[\frac{1}{2} f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(b) \right] = S_T$$



$$S_T = \Delta x \left[\frac{1}{2}f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(b) \right]$$

The Trapezoidal Approximation

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) = \sqrt{122}$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$
$$S_T = \Delta x \left[\frac{1}{2}f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(b) \right]$$

$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
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$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$$\begin{array}{l} x_0 = a = 2 \\ x_1 = 2.5 \\ x_2 = 3 \\ x_3 = 3.5 \\ x_4 = 4 \\ x_5 = 4.5 \\ x_6 = b = 5 \end{array} \quad \begin{array}{l} f(x_0) = f(a) = f(2) = \sqrt{5} \\ f(x_1) = f(2.5) = \sqrt{12.625} \\ f(x_2) = f(3) = \sqrt{24} \\ f(x_3) = f(3.5) = \sqrt{39.875} \\ f(x_4) = f(4) = \sqrt{61} \\ f(x_5) = f(4.5) = \sqrt{88.125} \\ f(x_6) = f(b) = f(5) = \sqrt{122} \end{array} \quad S_T = \Delta x \left[\frac{1}{2}f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(b) \right]$$

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$$S_T = \Delta x \left[\frac{1}{2}f(a) + \sum_{i=1}^5 f(x_i) + \frac{1}{2}f(b) \right]$$

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$$S_T =$$

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$$S_T = (.5)[$$

Class Worksheet #5 Unit 11

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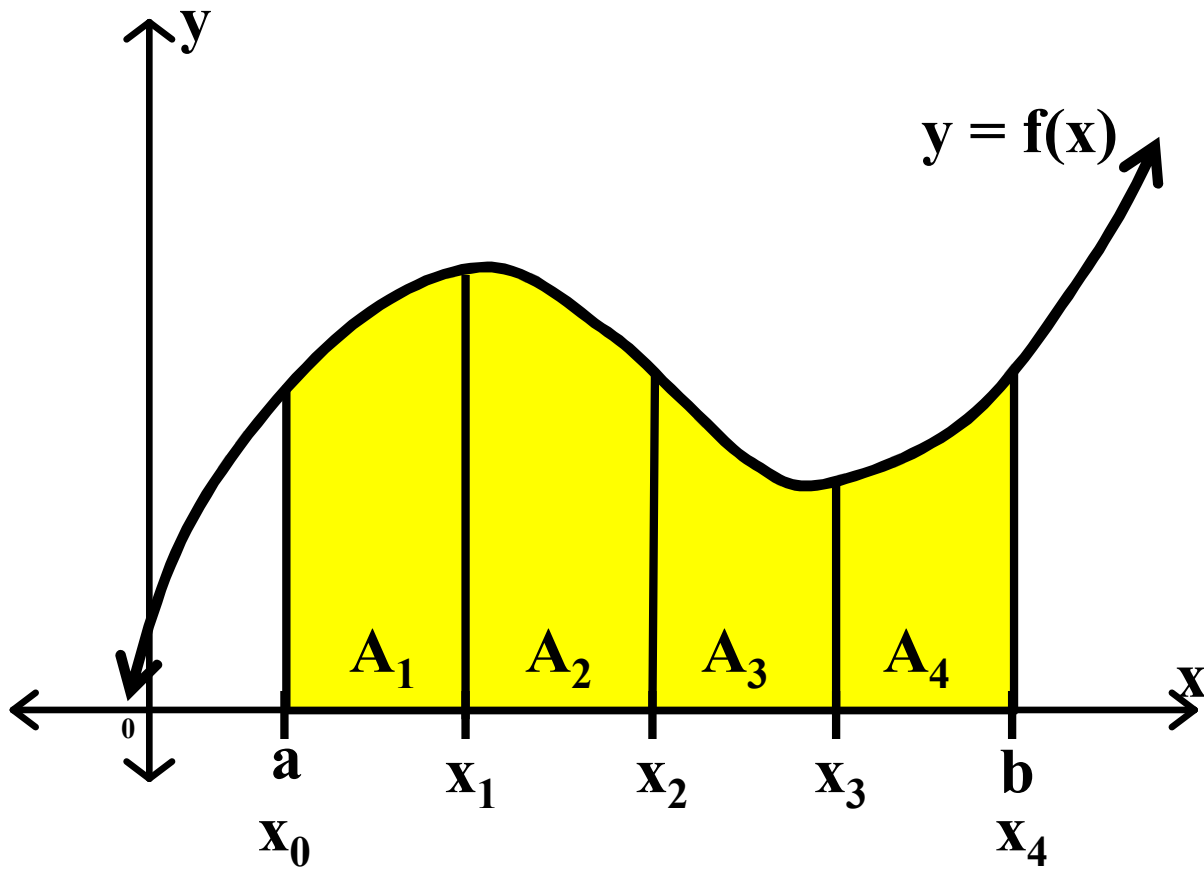
$$S_T = \Delta x \left[\frac{1}{2}f(a) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2}f(b) \right]$$

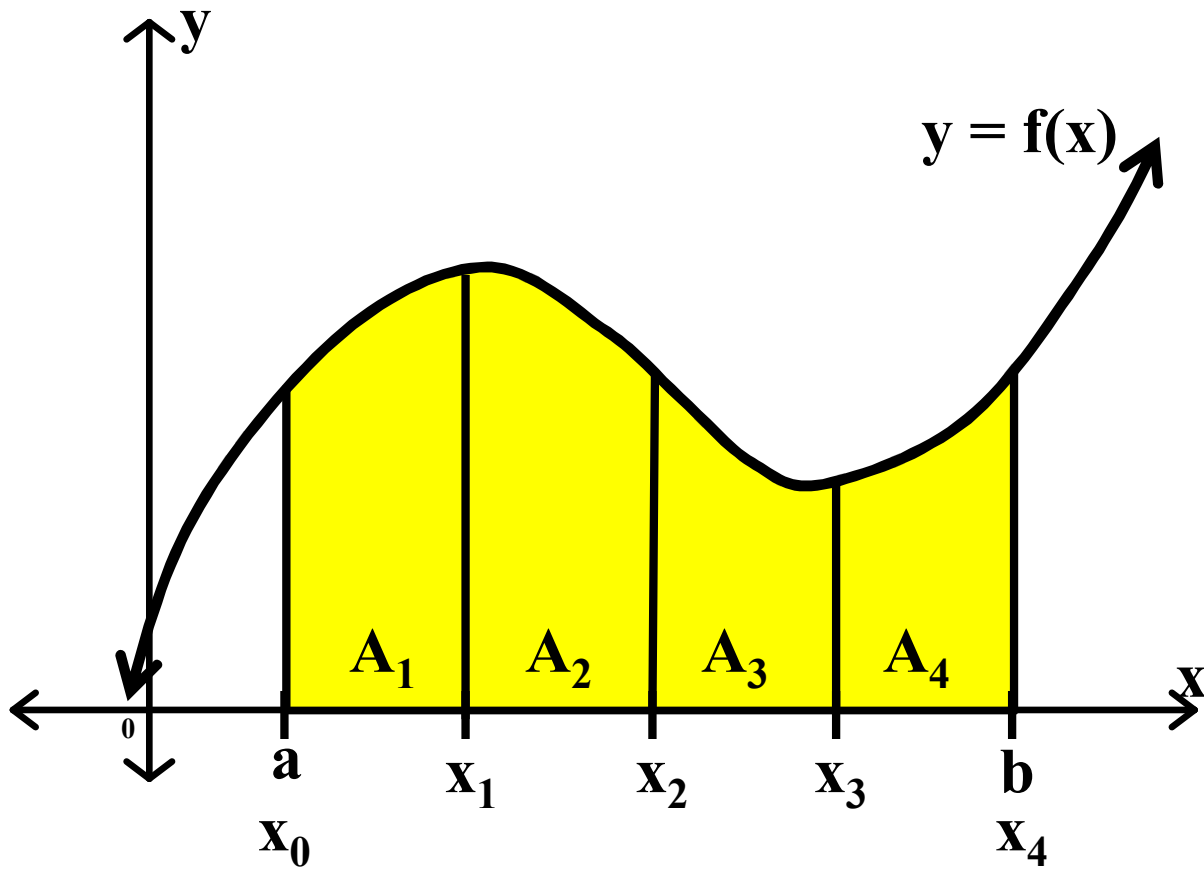
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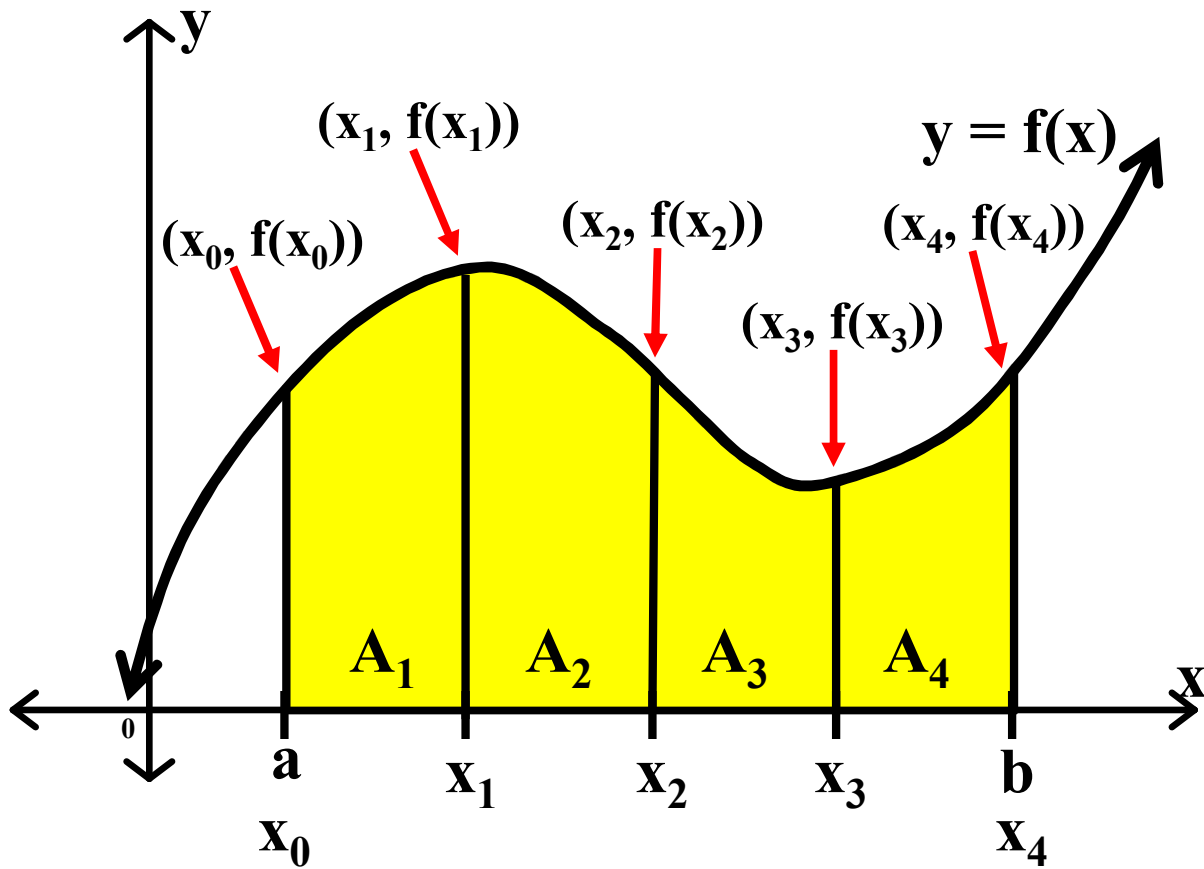
$$S_T = (.5) \left[\frac{1}{2}\sqrt{5} + \sqrt{12.625} + \sqrt{24} + \sqrt{39.875} + \sqrt{61} + \sqrt{88.125} + \frac{1}{2}\sqrt{122} \right]$$

$$S_T \approx 19.30$$

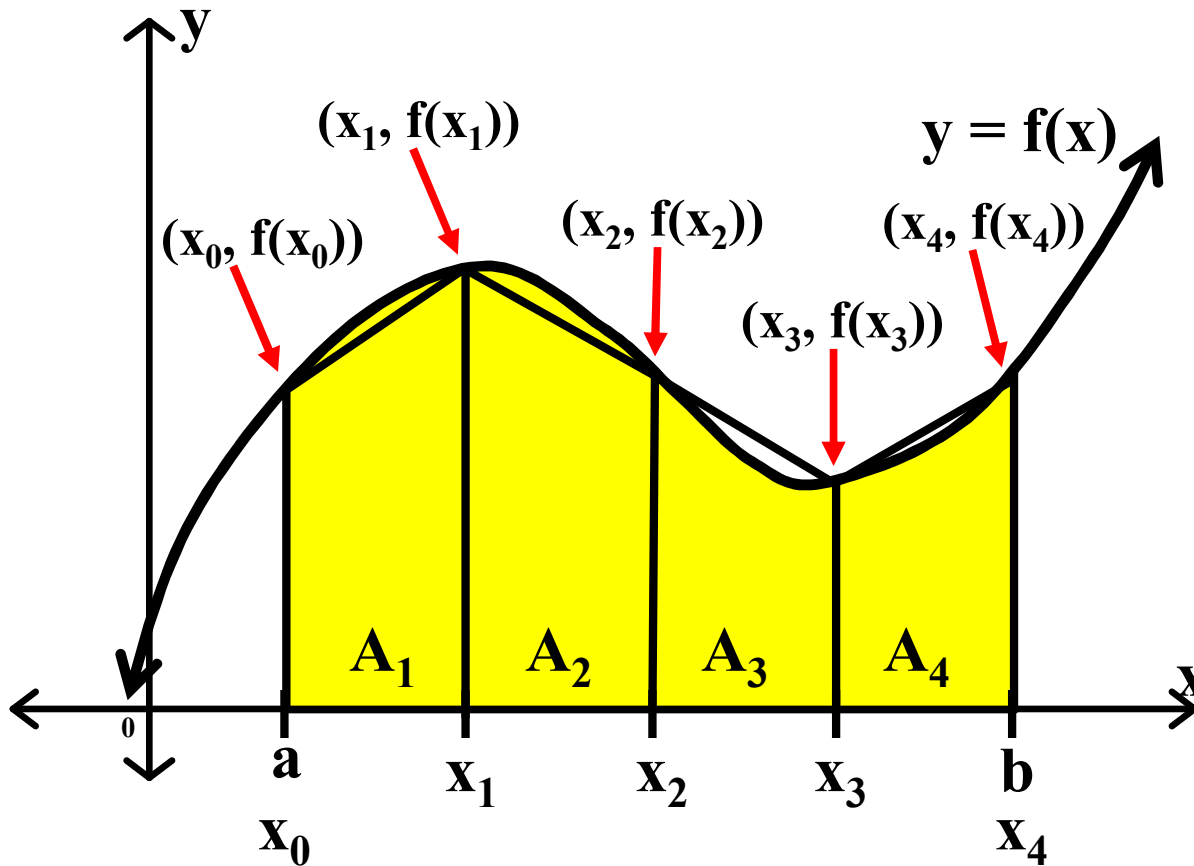




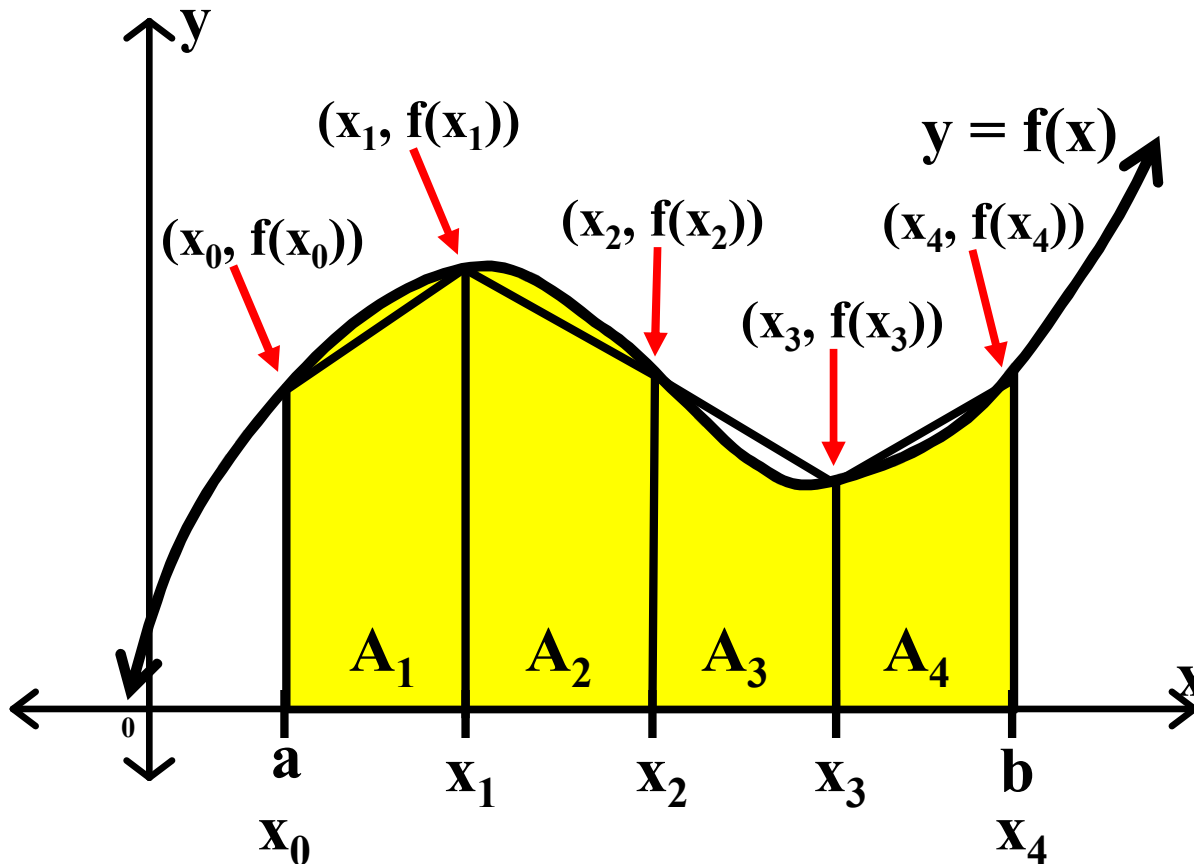
The trapezoidal approximation ‘connects the ‘key points’ on the graph of function f



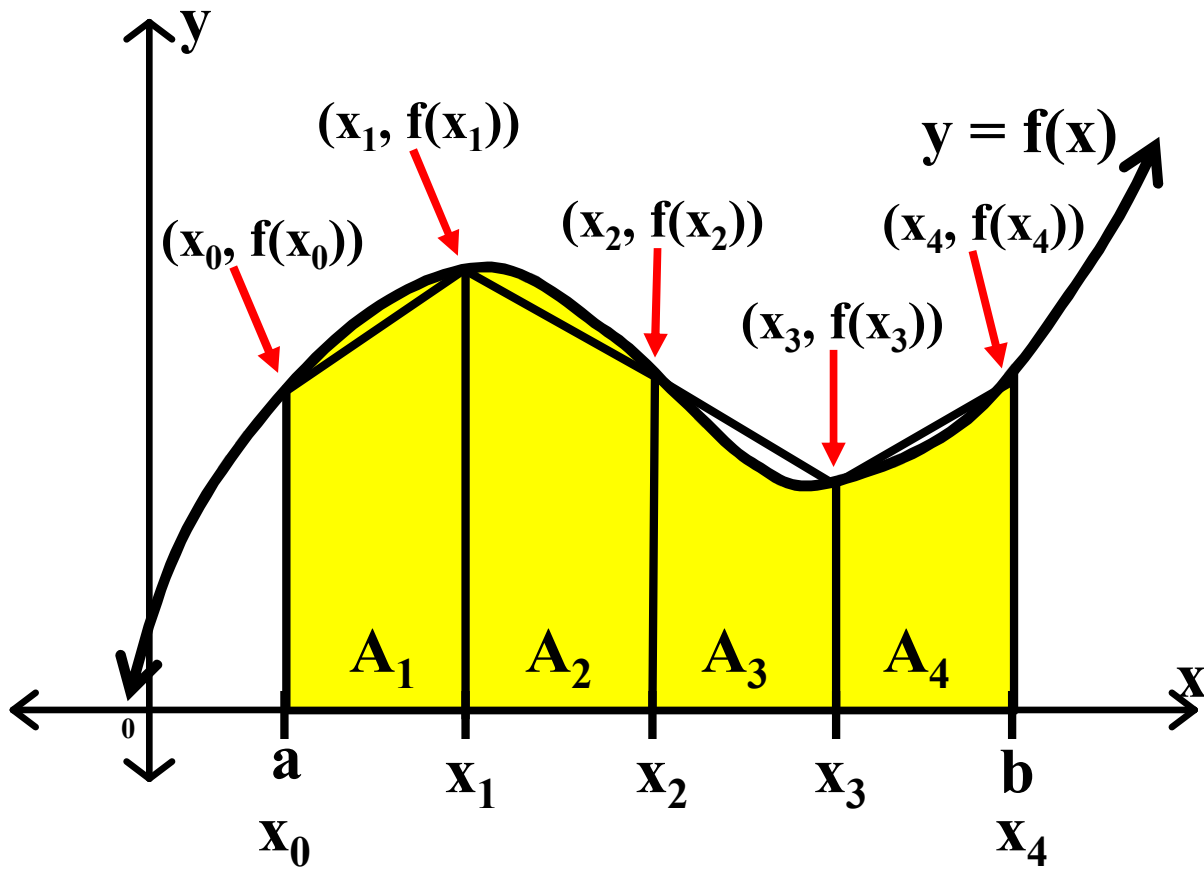
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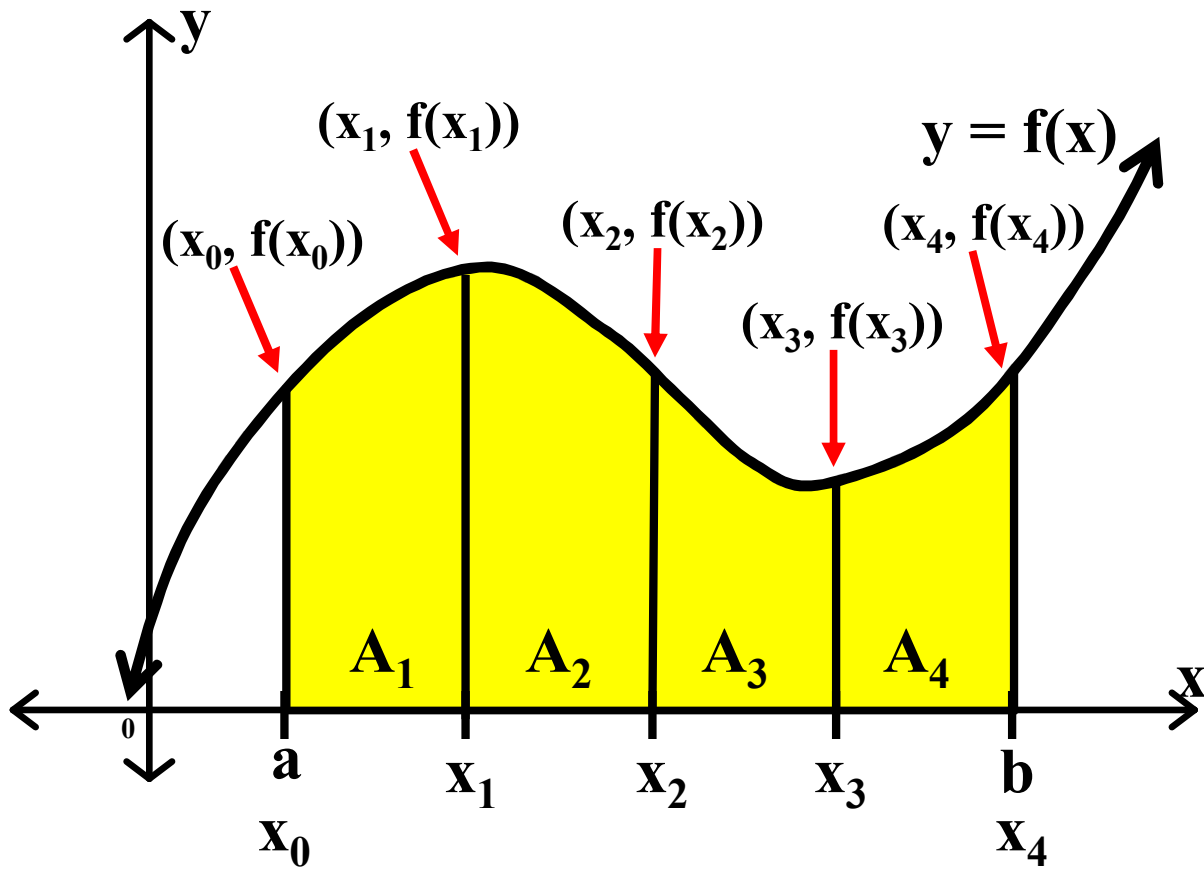


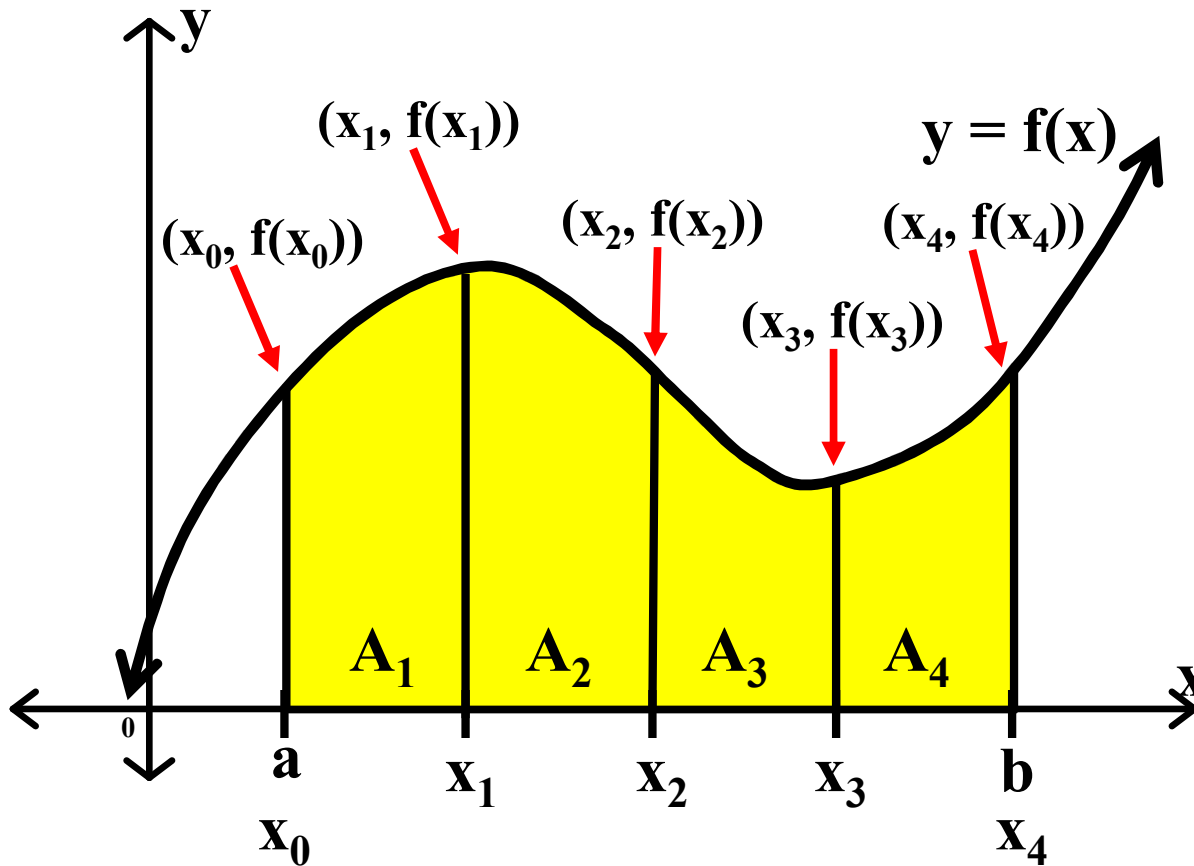
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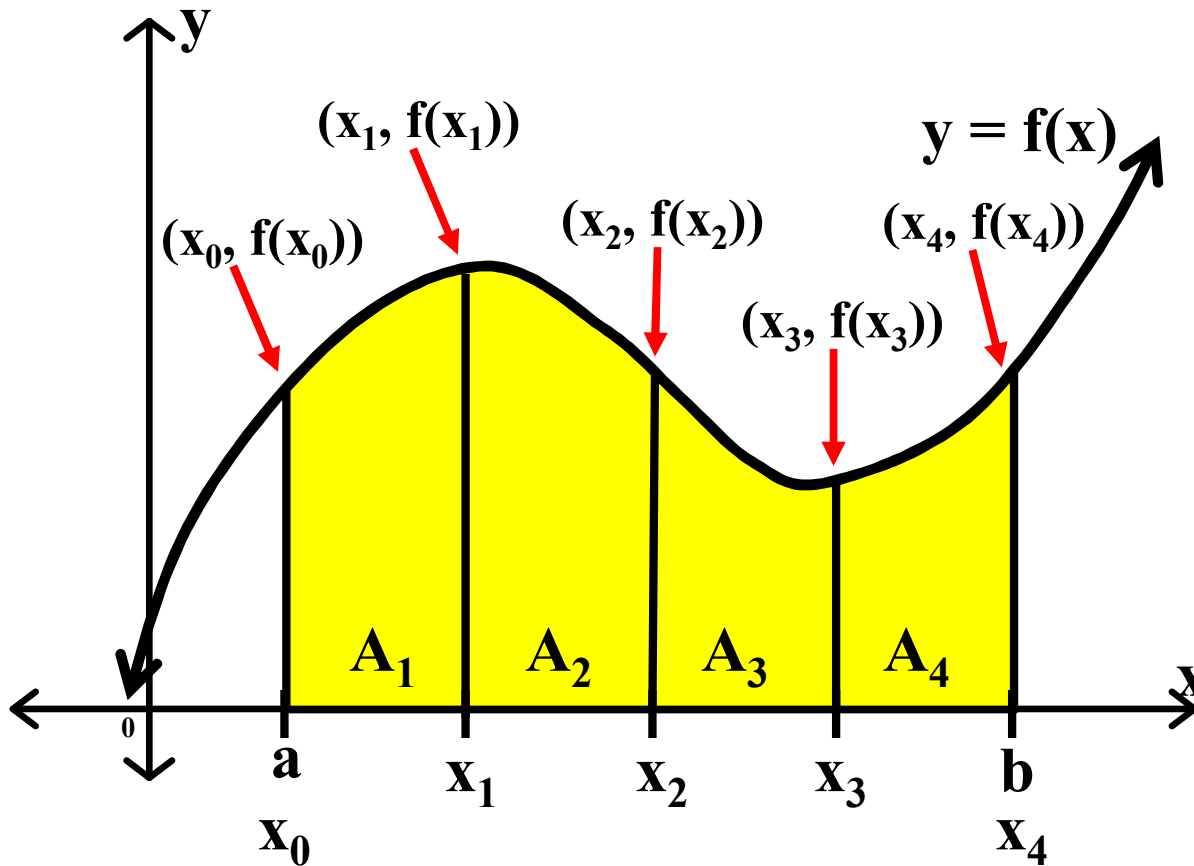
The trapezoidal approximation ‘connects the ‘key points’ on the graph of function f with a series of line segments forming the trapezoids. (This is called a polygonal path.)



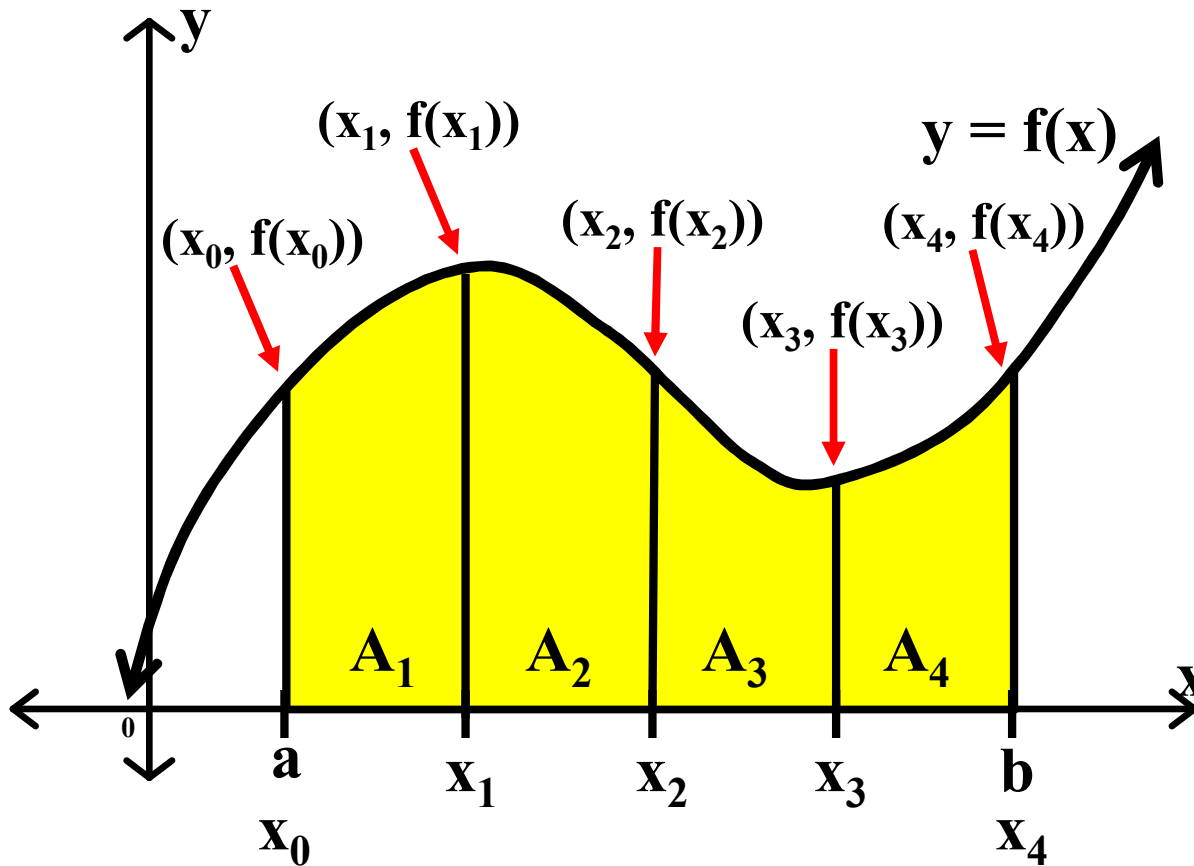




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This leads to the equation $4A - 2B + C = -8$

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Therefore, the parabola that would contain points P, Q, and R is

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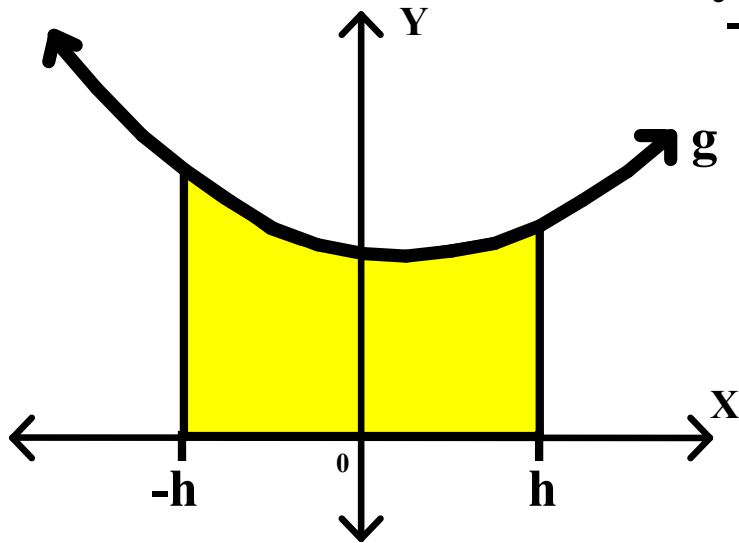
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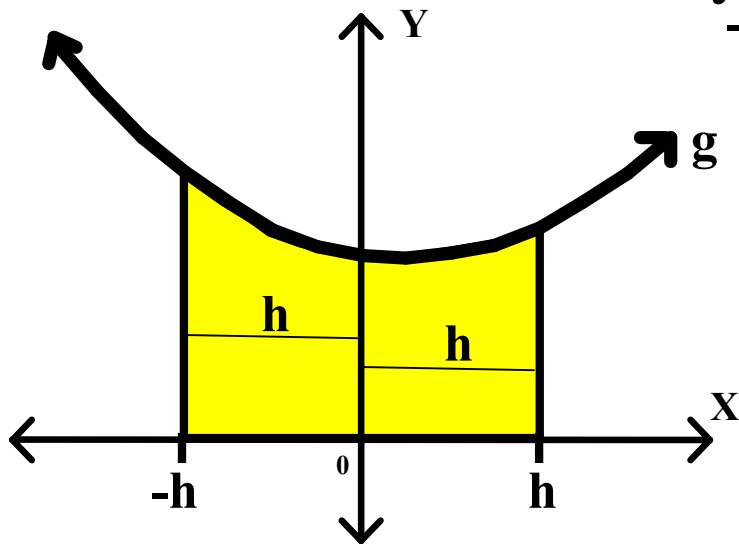
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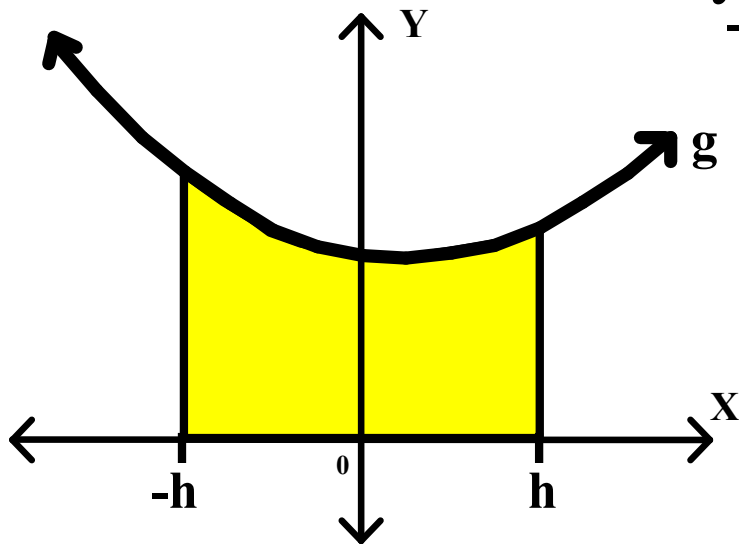
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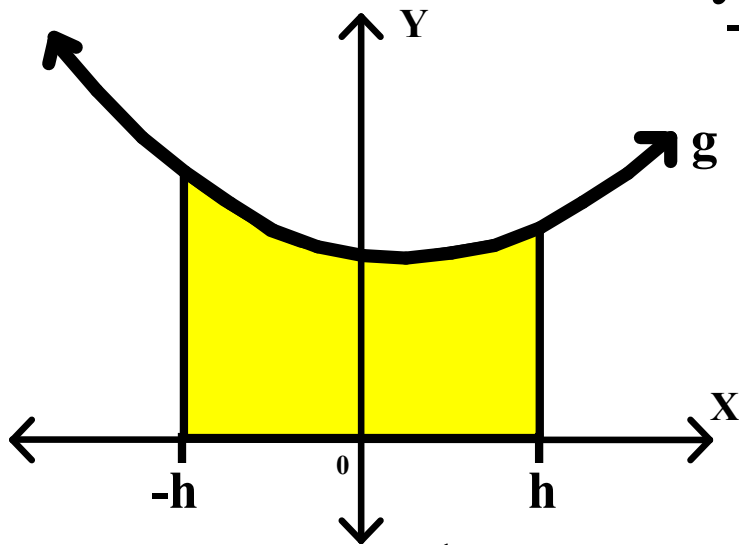
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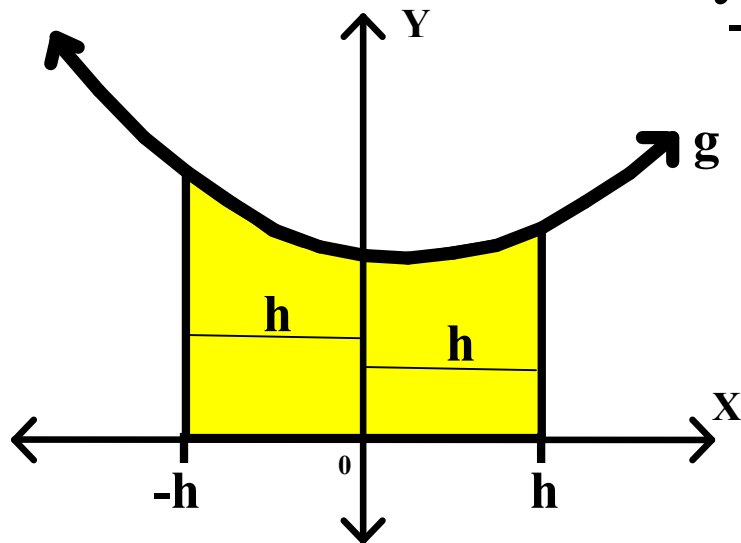
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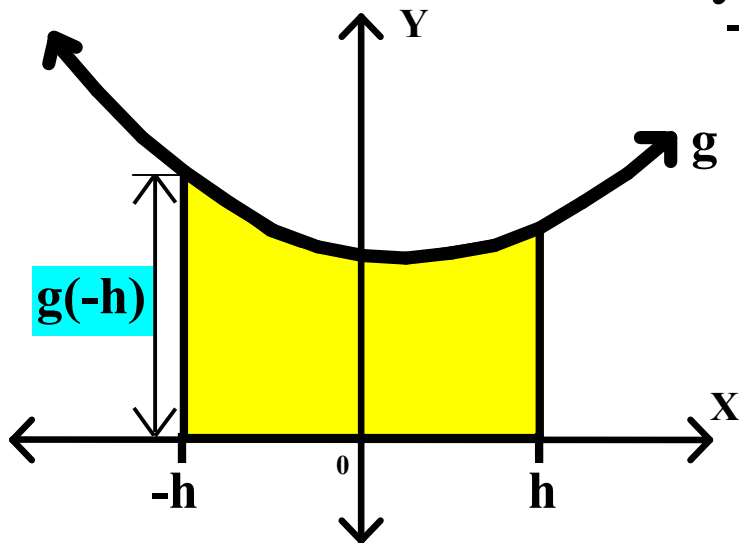
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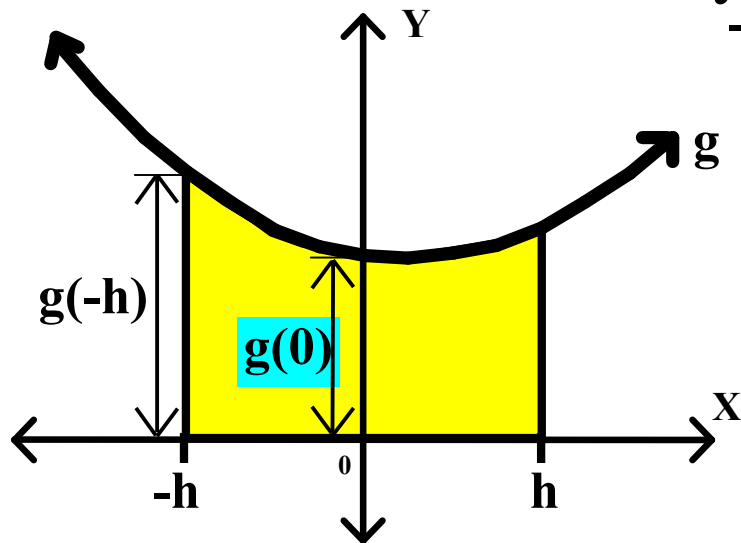
h is the width of each 'strip'.

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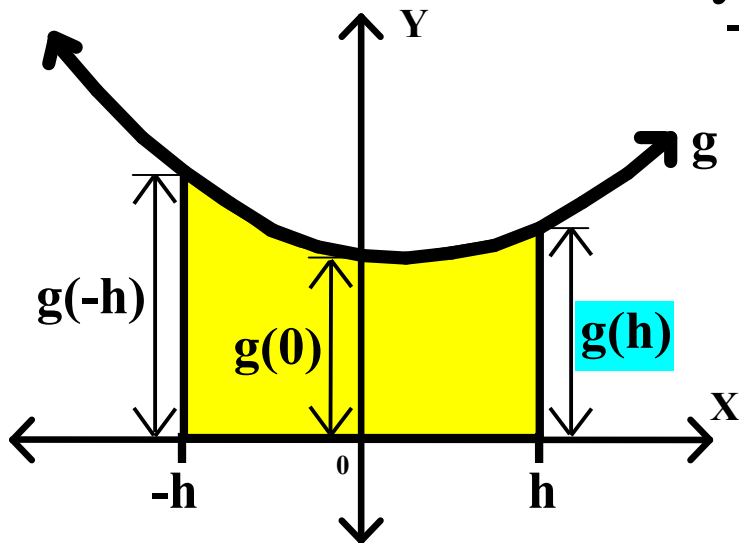
h is the width of each 'strip'. $g(-h)$ is the height of the left boundary.

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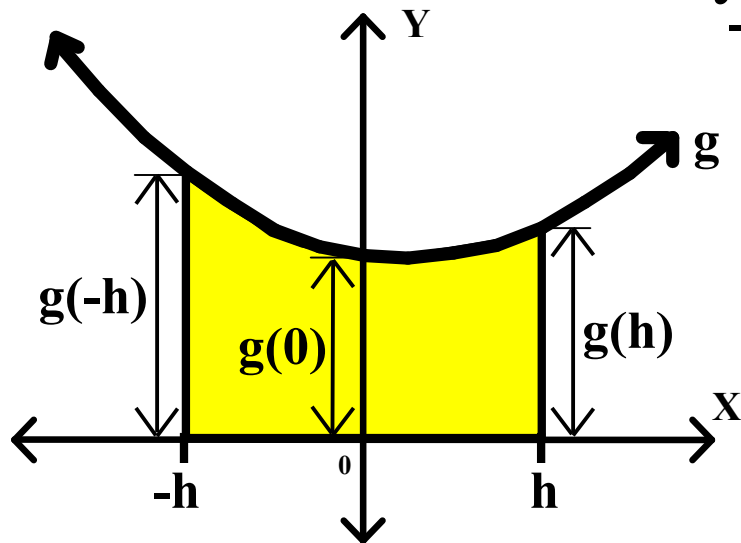
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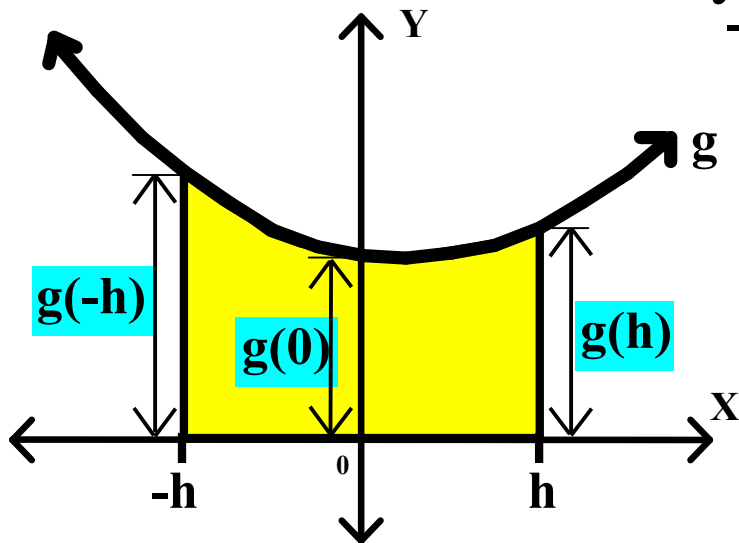
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Given any function g defined by the equation $g(x) = Ax^2 + Bx + C$ for any constants A , B , and C ,

we have proven that $\int_{-h}^h g(x)dx = \frac{1}{3}h[g(-h) + 4g(0) + g(h)]$.



Consider $\int_{-h}^h g(x)dx$.

This represents the shaded area shown here. Notice that this area is divided into two 'strips', each of width h .

Now consider $\frac{1}{3}h[g(-h) + 4g(0) + g(h)]$.

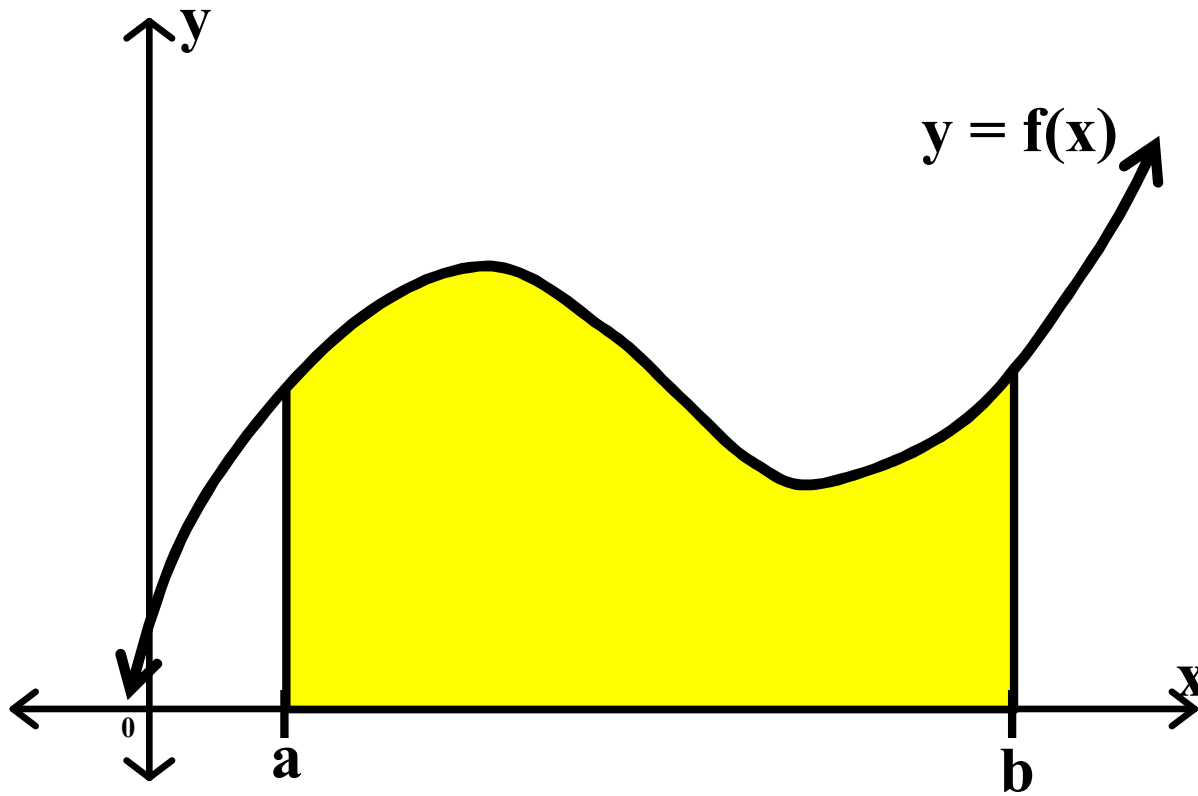
h is the width of each 'strip'.

$g(-h)$ is the height of the left boundary.

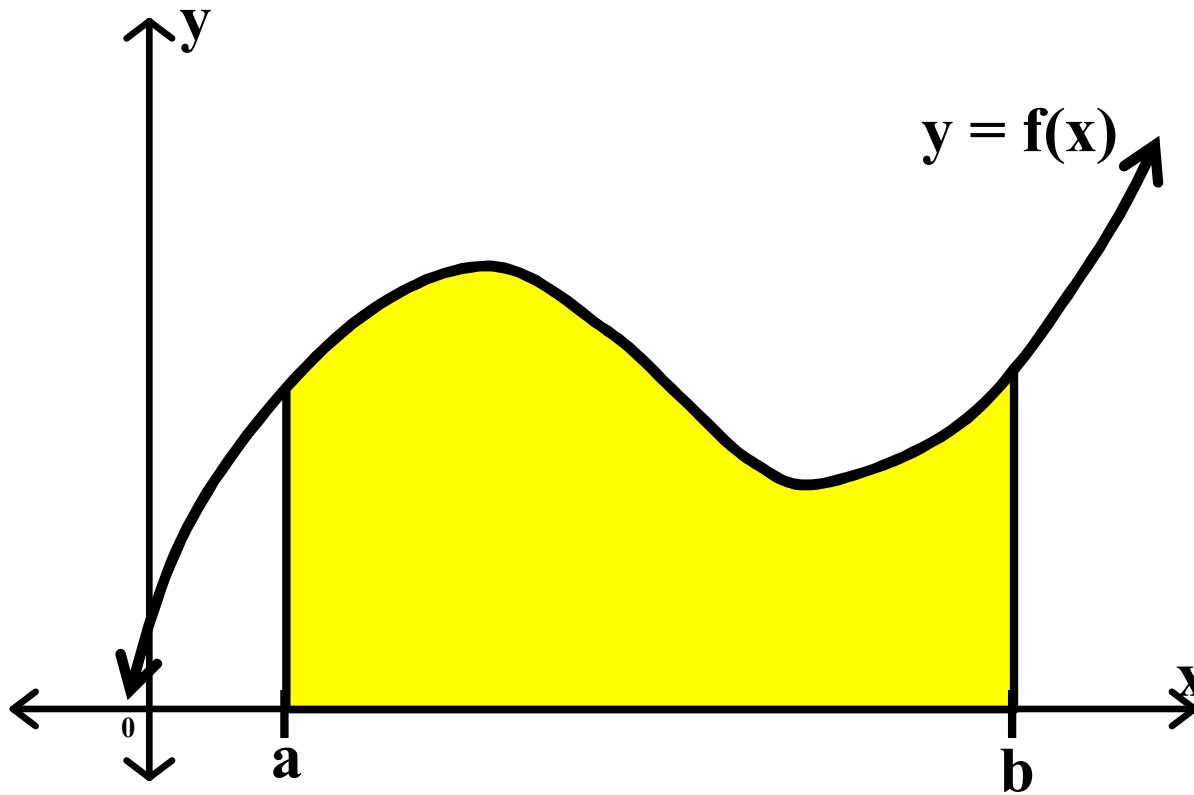
$g(0)$ is the height in the 'center'.

$g(h)$ is the height of the right boundary.

Simpson's Rule

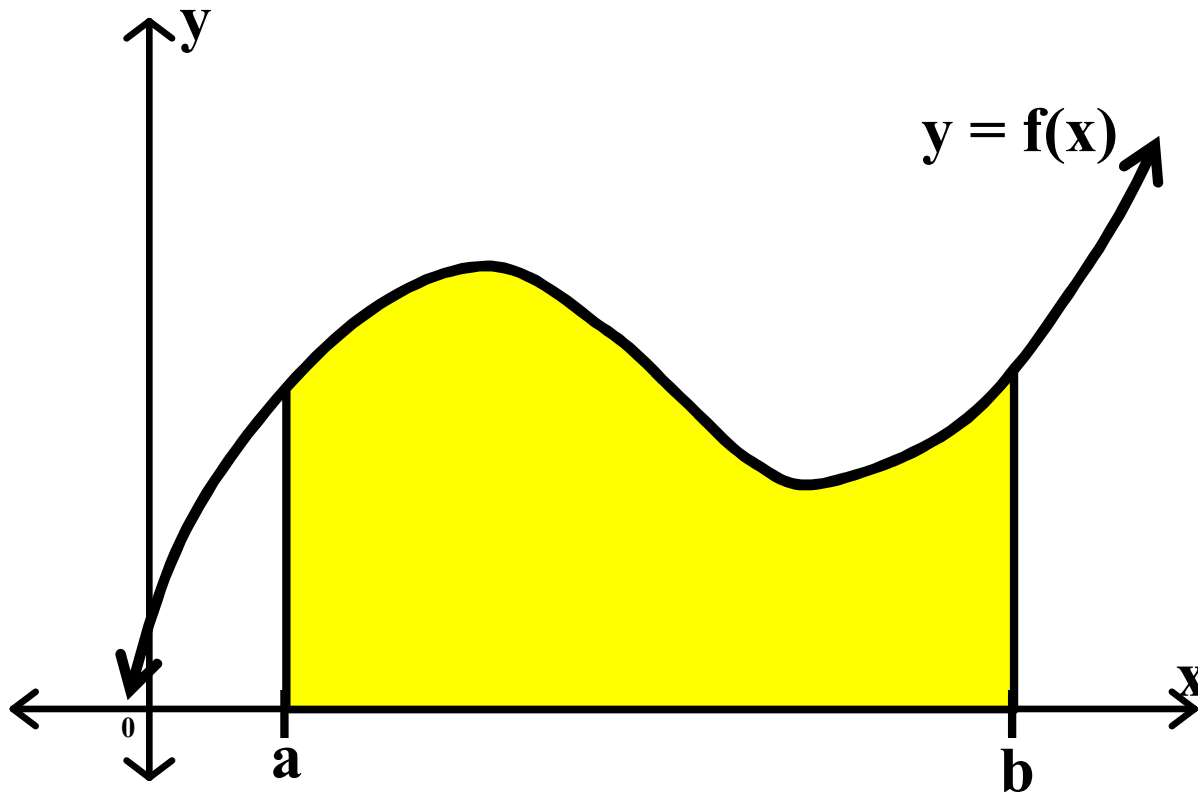


Simpson's Rule



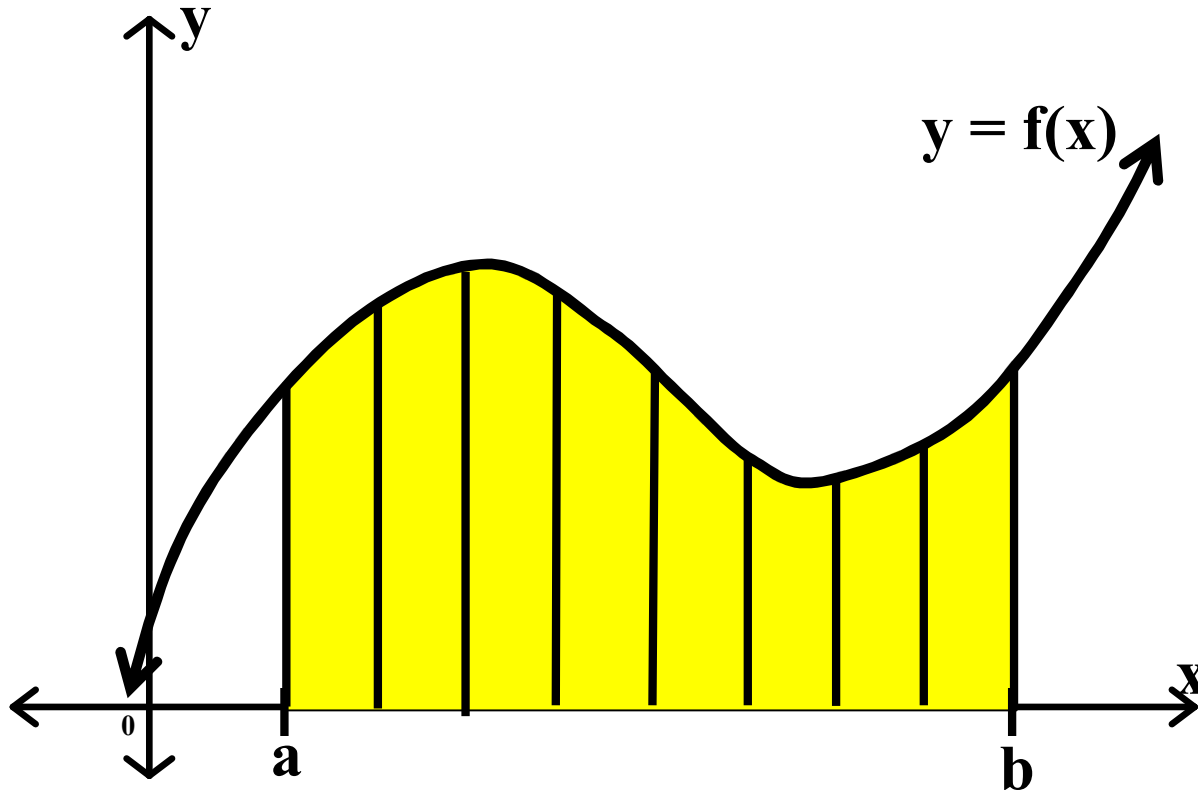
Divide the interval $[a, b]$ into $2n$ subintervals.

Simpson's Rule



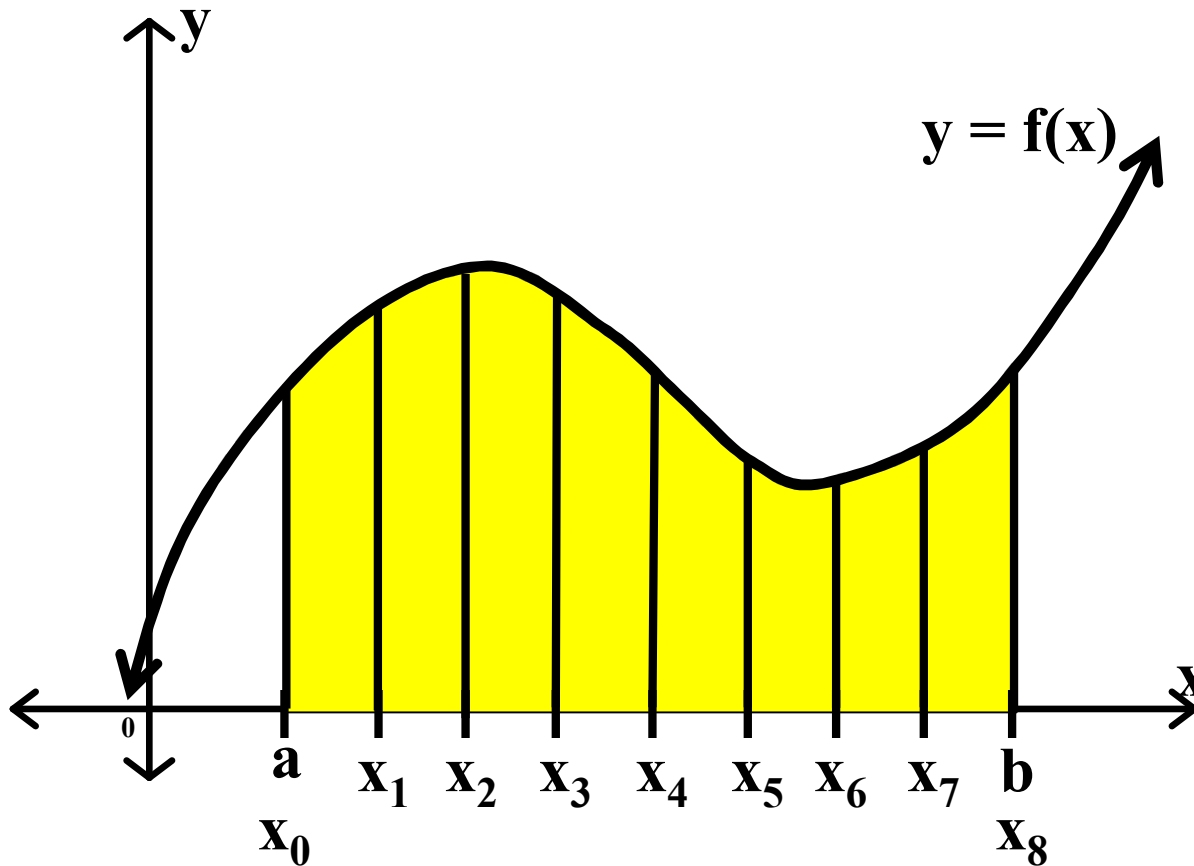
Divide the interval $[a, b]$ into $2n$ subintervals, each of width Δx .

Simpson's Rule



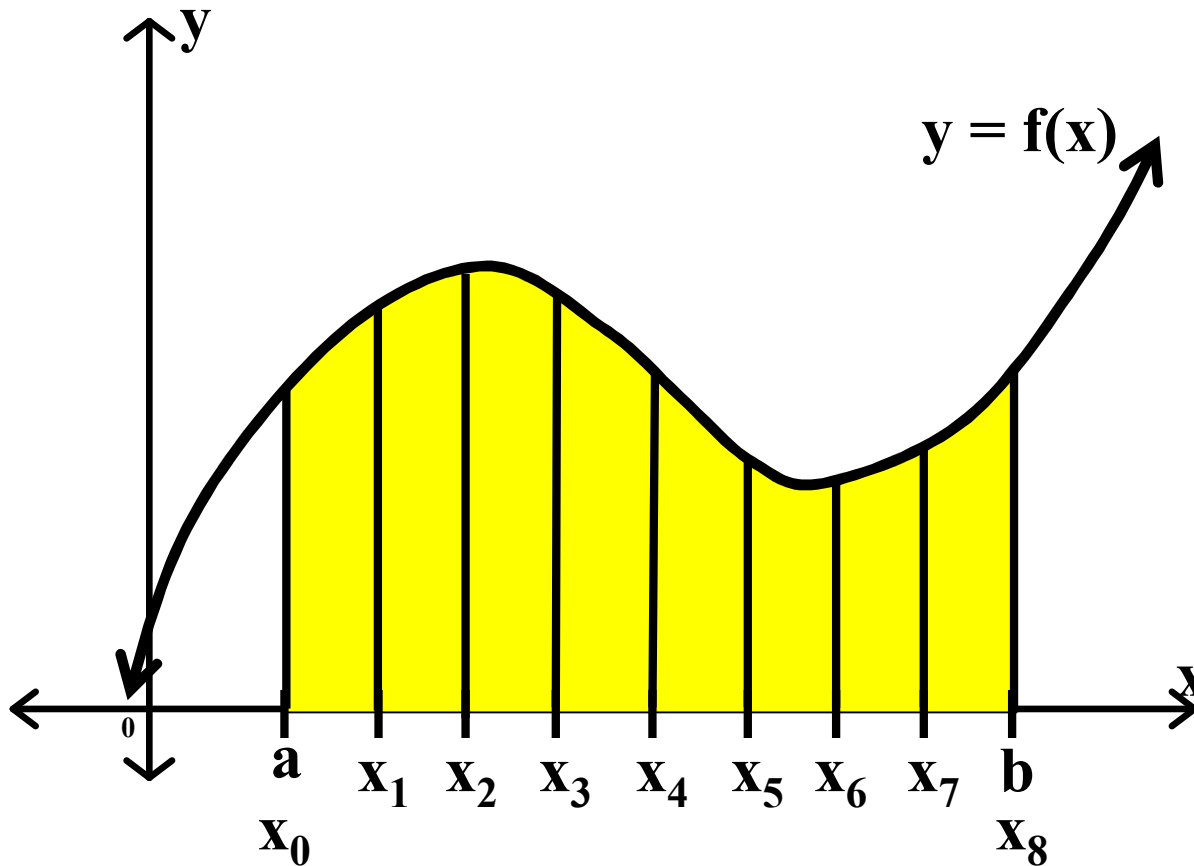
Divide the interval $[a, b]$ into $2n$ subintervals, each of width Δx .

Simpson's Rule



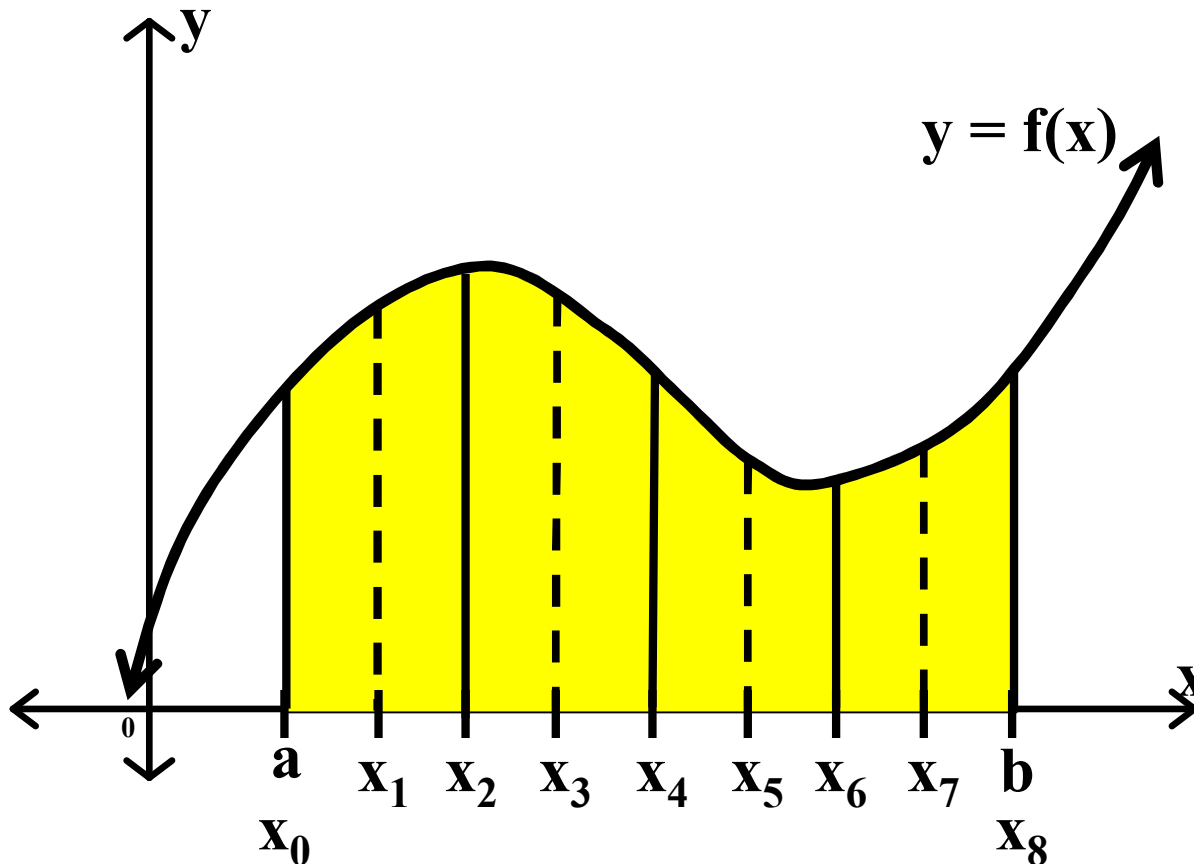
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Simpson's Rule



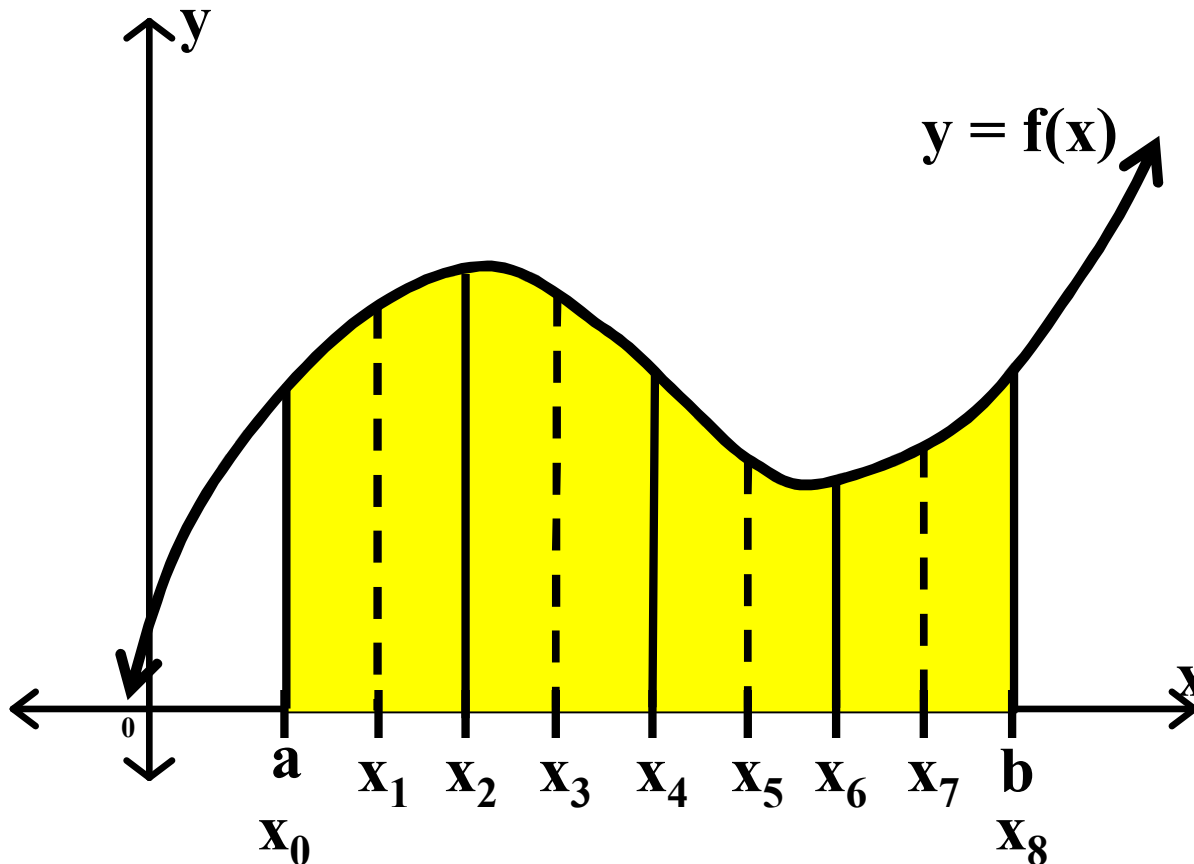
**Divide the interval $[a, b]$ into $2n$ subintervals, each of width Δx .
In this example, $2n = 8$.**

Simpson's Rule



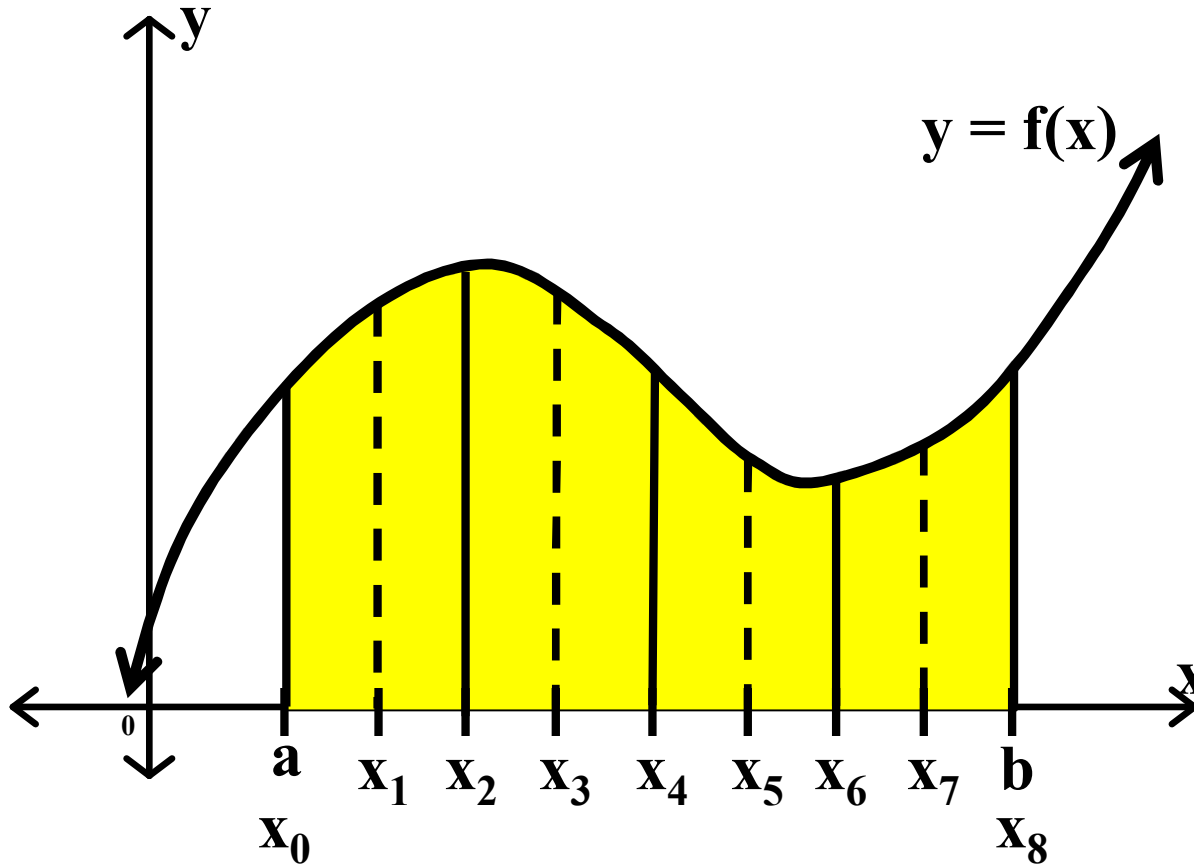
**Divide the interval $[a, b]$ into $\underline{2n}$ subintervals, each of width Δx .
In this example, $2n = 8$. Taking these strips, 2 at a time,**

Simpson's Rule



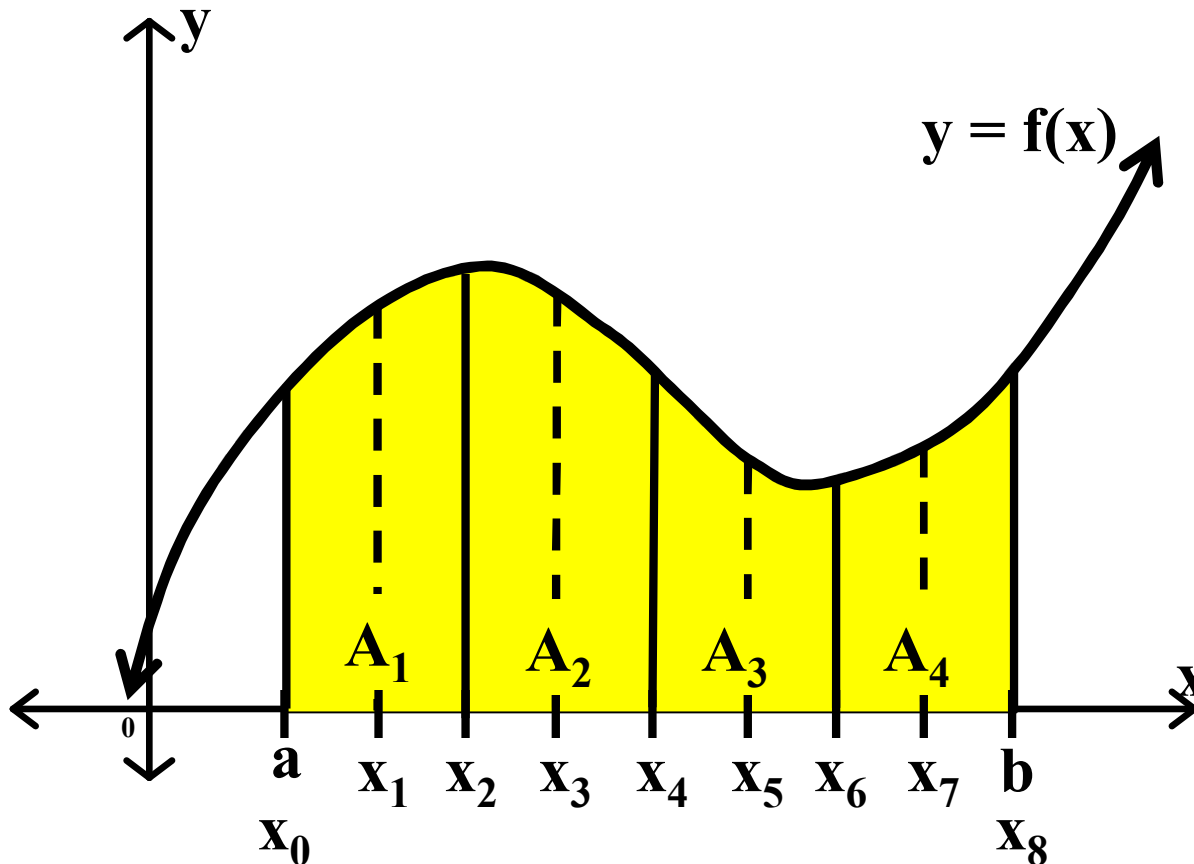
Divide the interval $[a, b]$ into $\underline{2n}$ subintervals, each of width Δx . In this example, $2n = 8$. Taking these strips, 2 at a time, we have n 'double strips' shown here.

Simpson's Rule



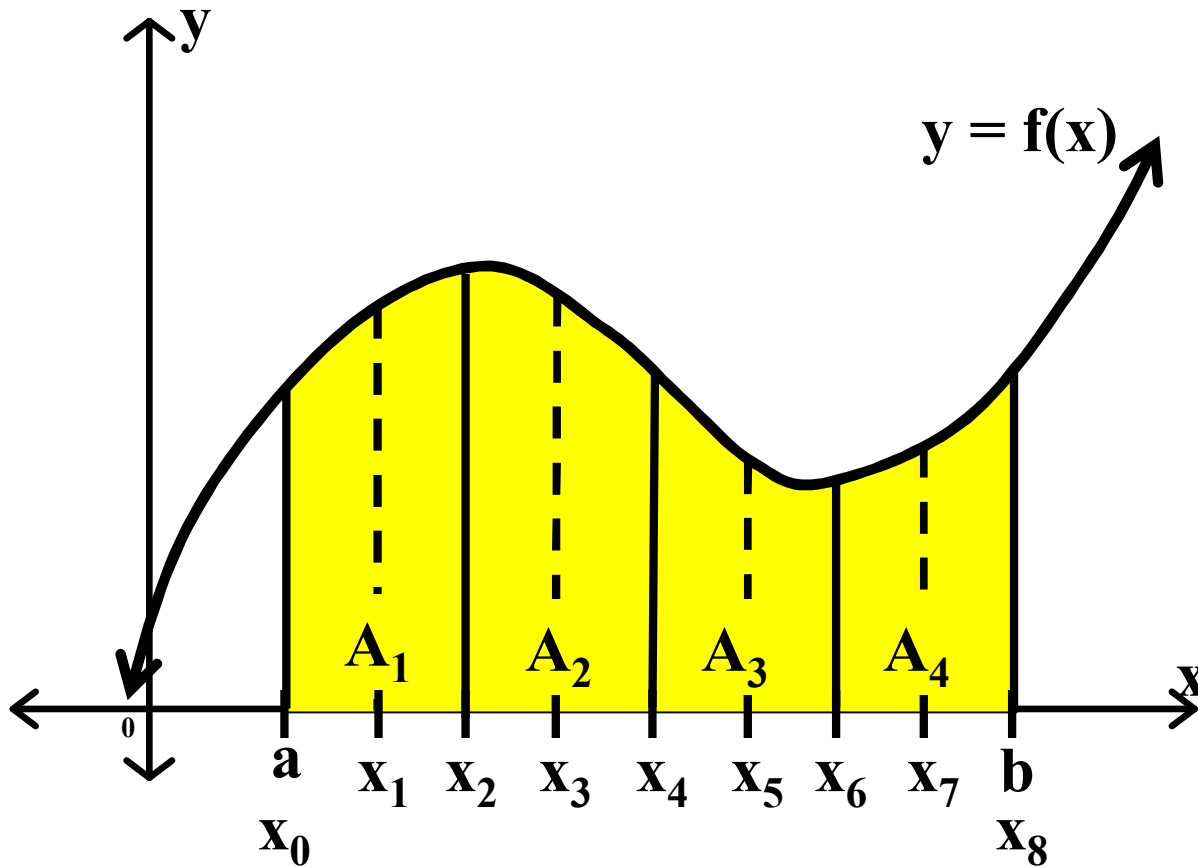
Divide the interval $[a, b]$ into $\underline{2n}$ subintervals, each of width Δx . In this example, $2n = 8$. Taking these strips, 2 at a time, we have n 'double strips' shown here. (In this case, $2n = 8$, so $n = 4$.)

Simpson's Rule

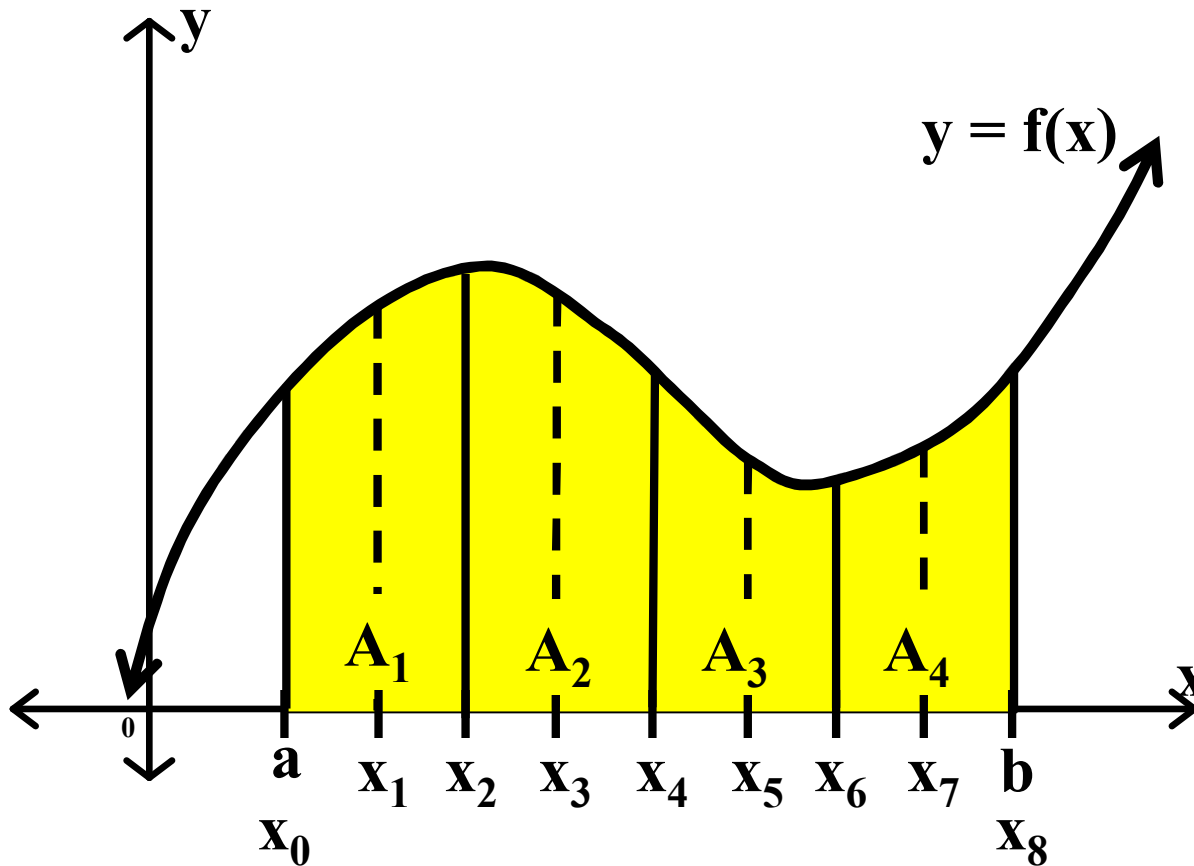


Divide the interval $[a, b]$ into $\underline{2n}$ subintervals, each of width Δx . In this example, $2n = 8$. Taking these strips, 2 at a time, we have n 'double strips' shown here. (In this case, $2n = 8$, so $n = 4$.) A_1, A_2, \dots, A_n represent the areas of these 'double strips'.

Simpson's Rule

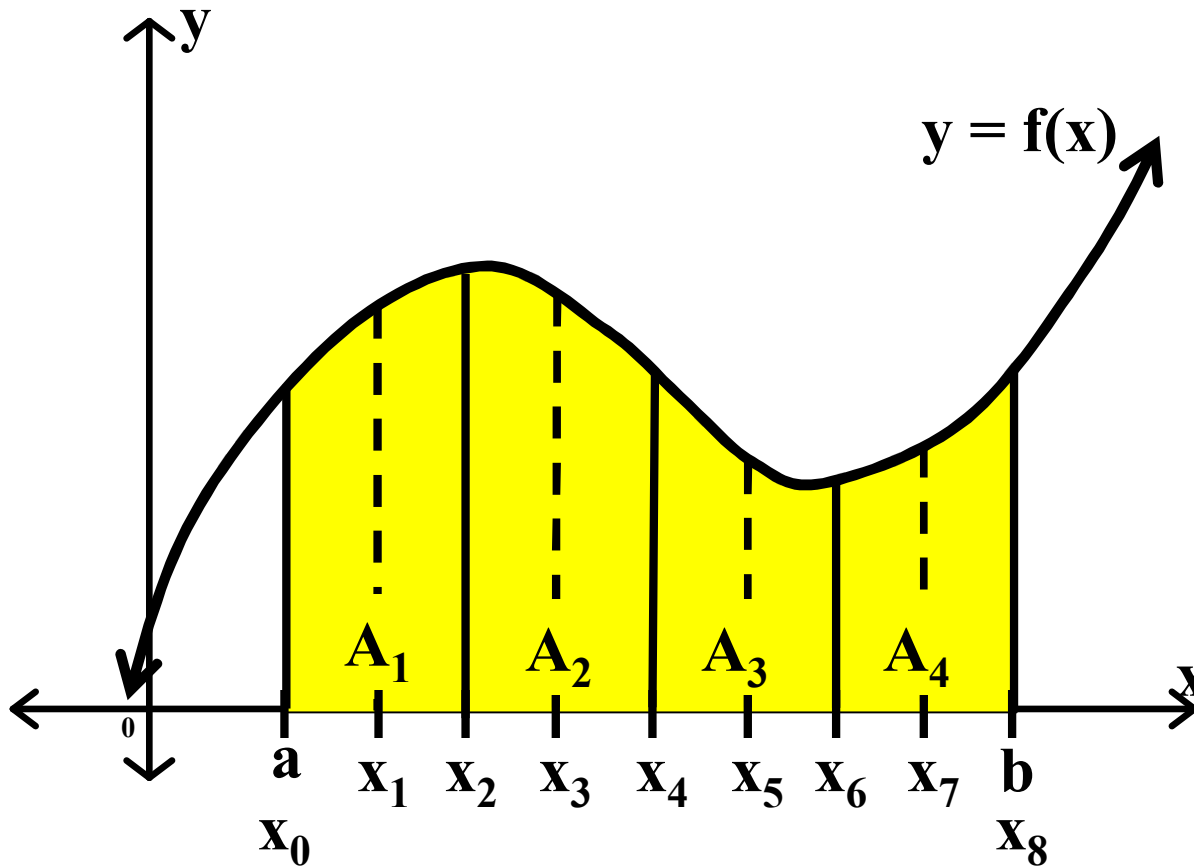


Simpson's Rule



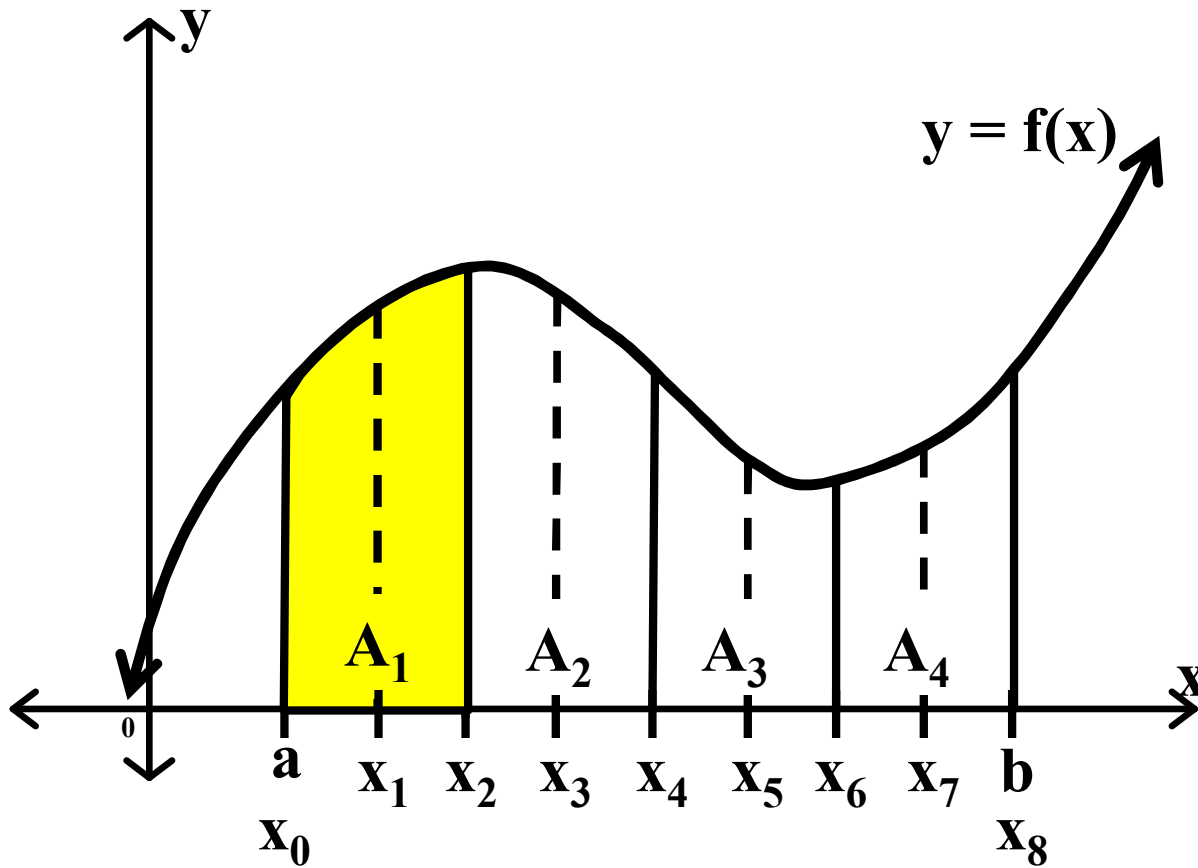
Simpson's Rule can be used to approximate the area of each of these 'double strips'.

Simpson's Rule

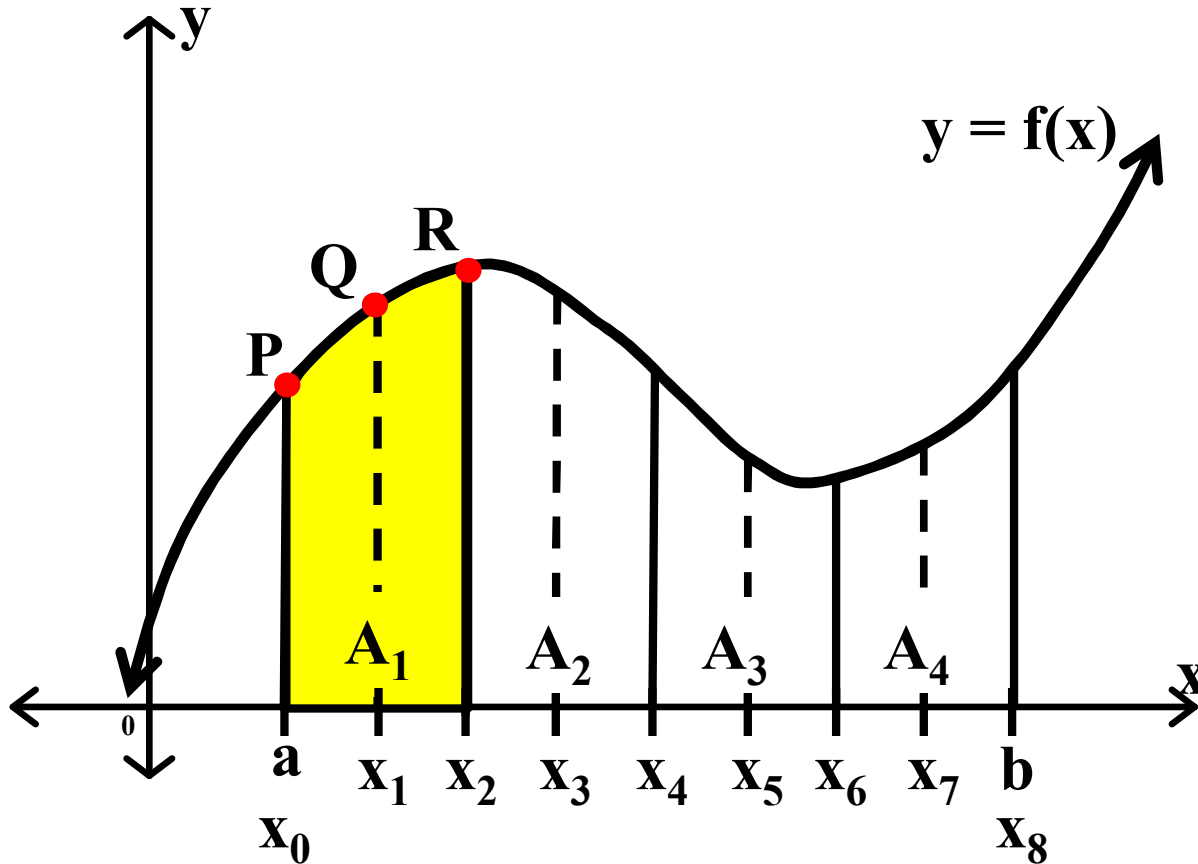


Simpson's Rule can be used to approximate the area of each of these 'double strips'. Consider area A_1 .

Simpson's Rule

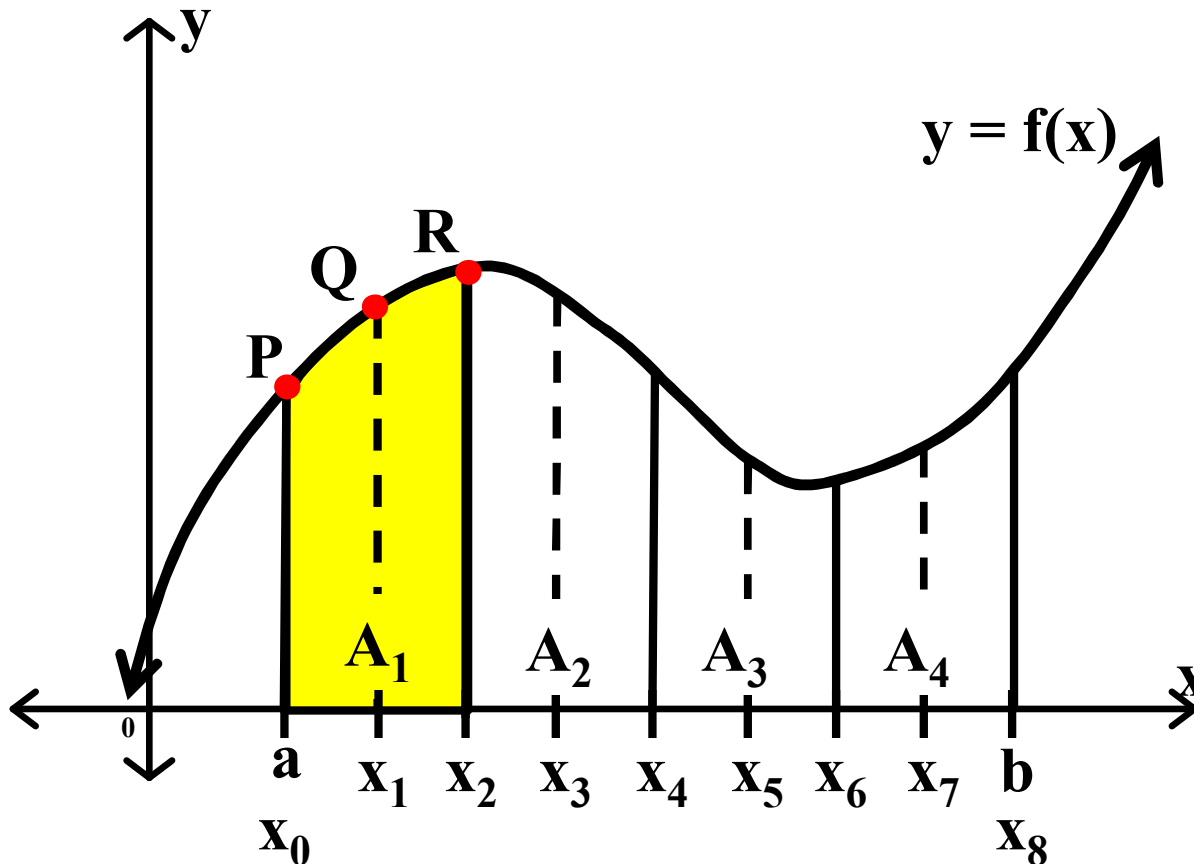


Simpson's Rule



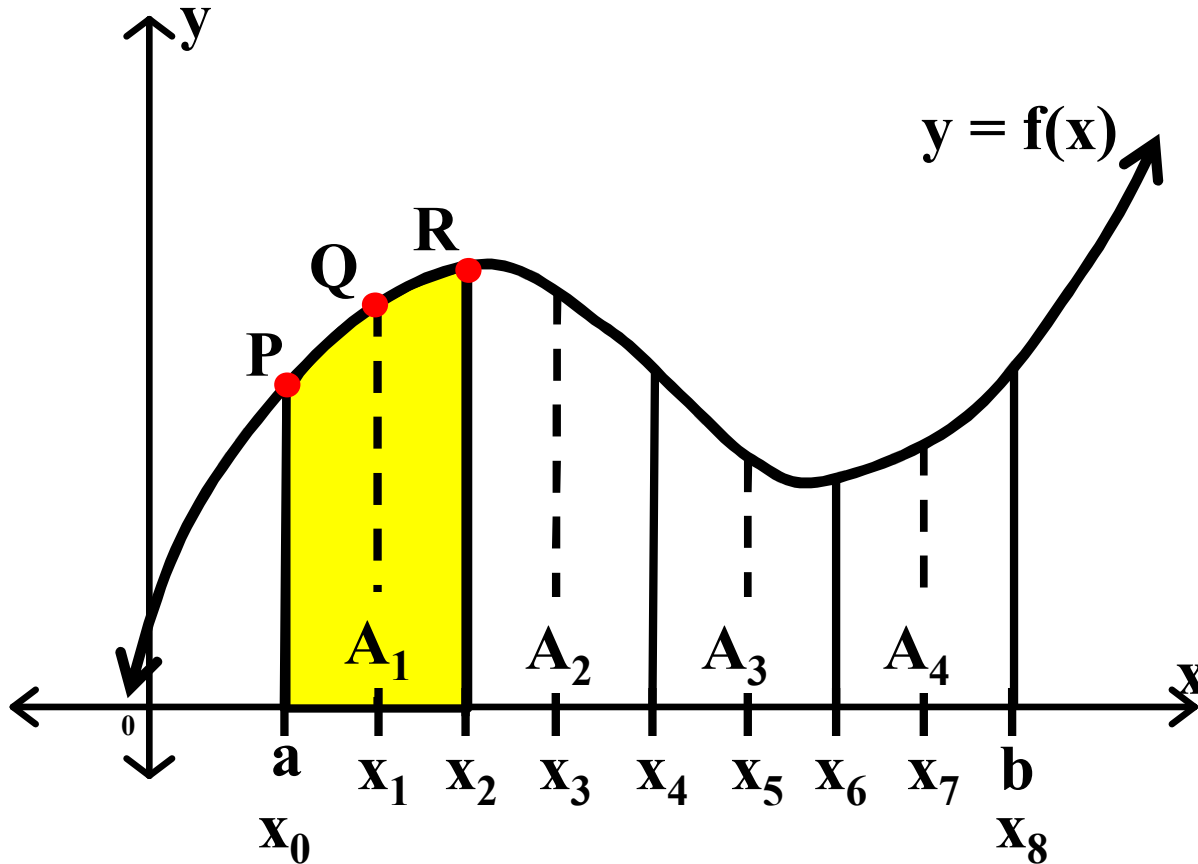
Given points P , Q , and R on the graph of function f ,

Simpson's Rule



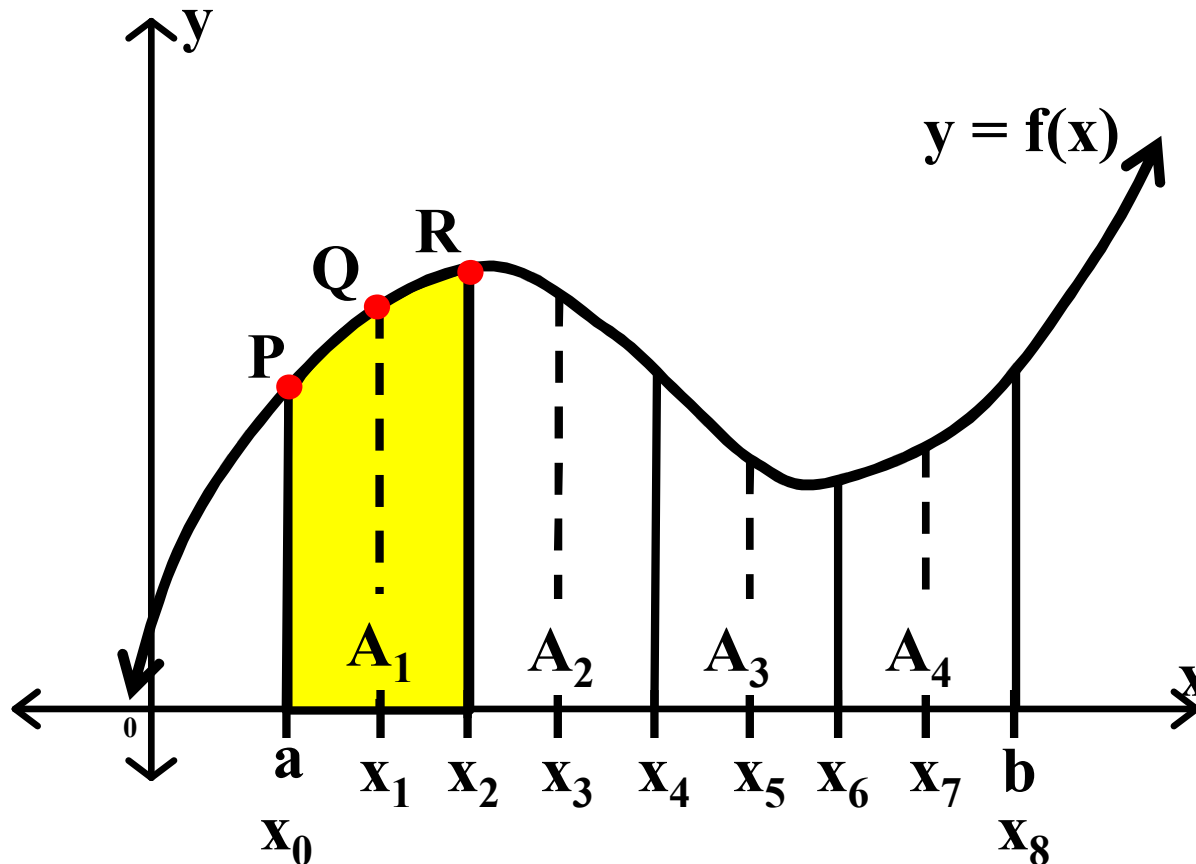
Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown),

Simpson's Rule



Given points P , Q , and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them.

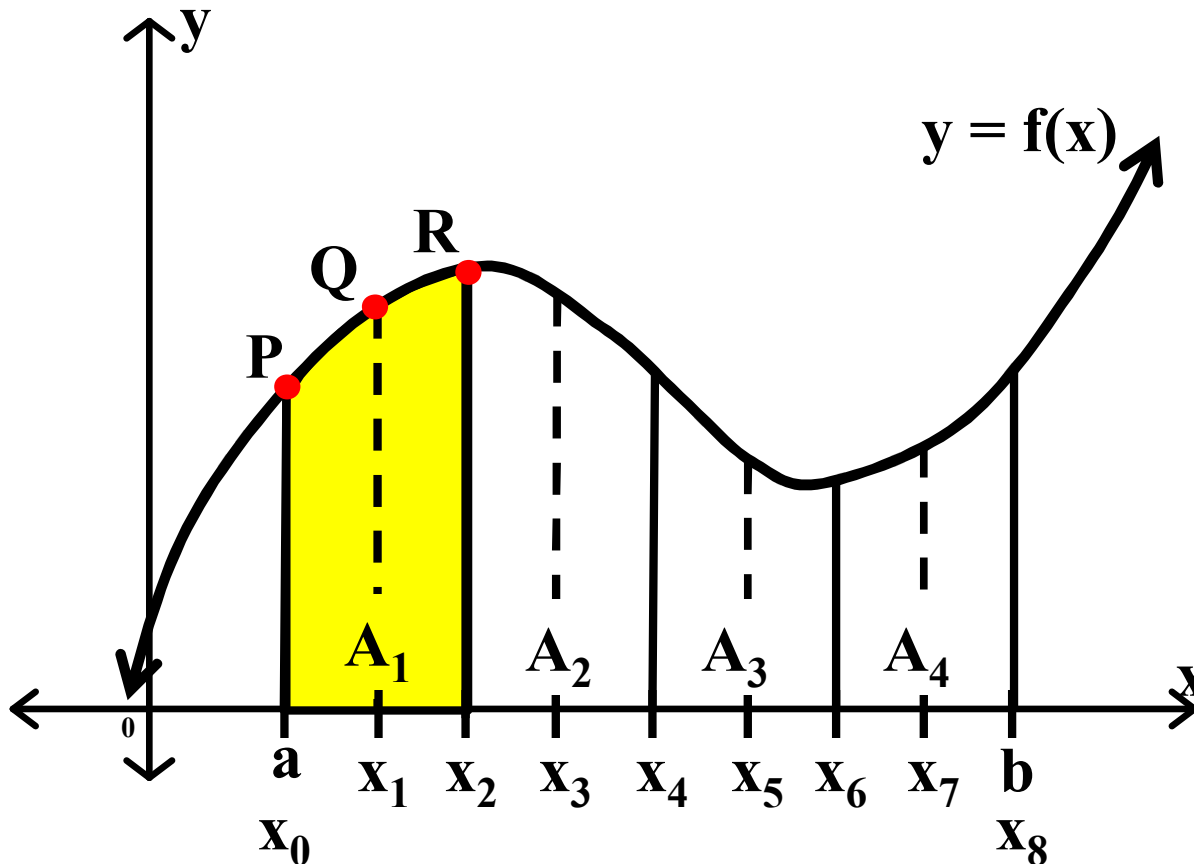
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx$$

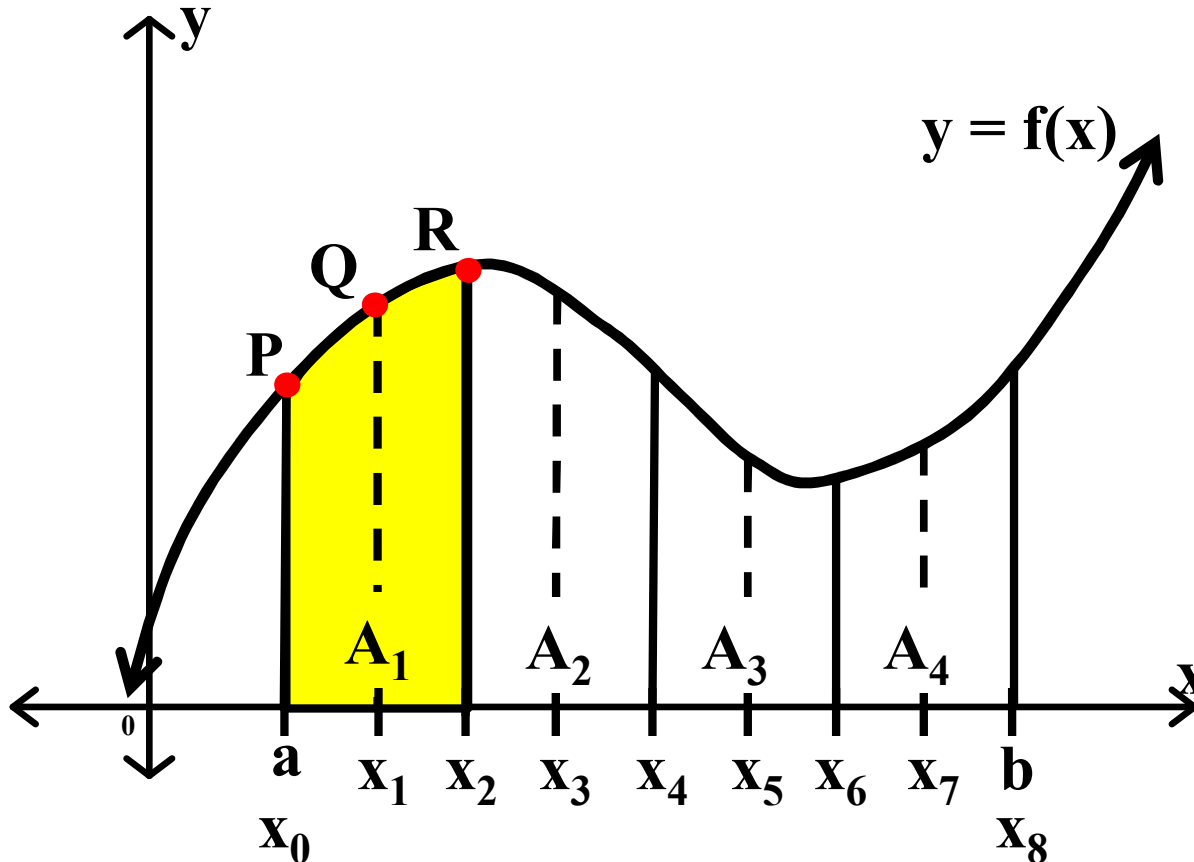
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x$$

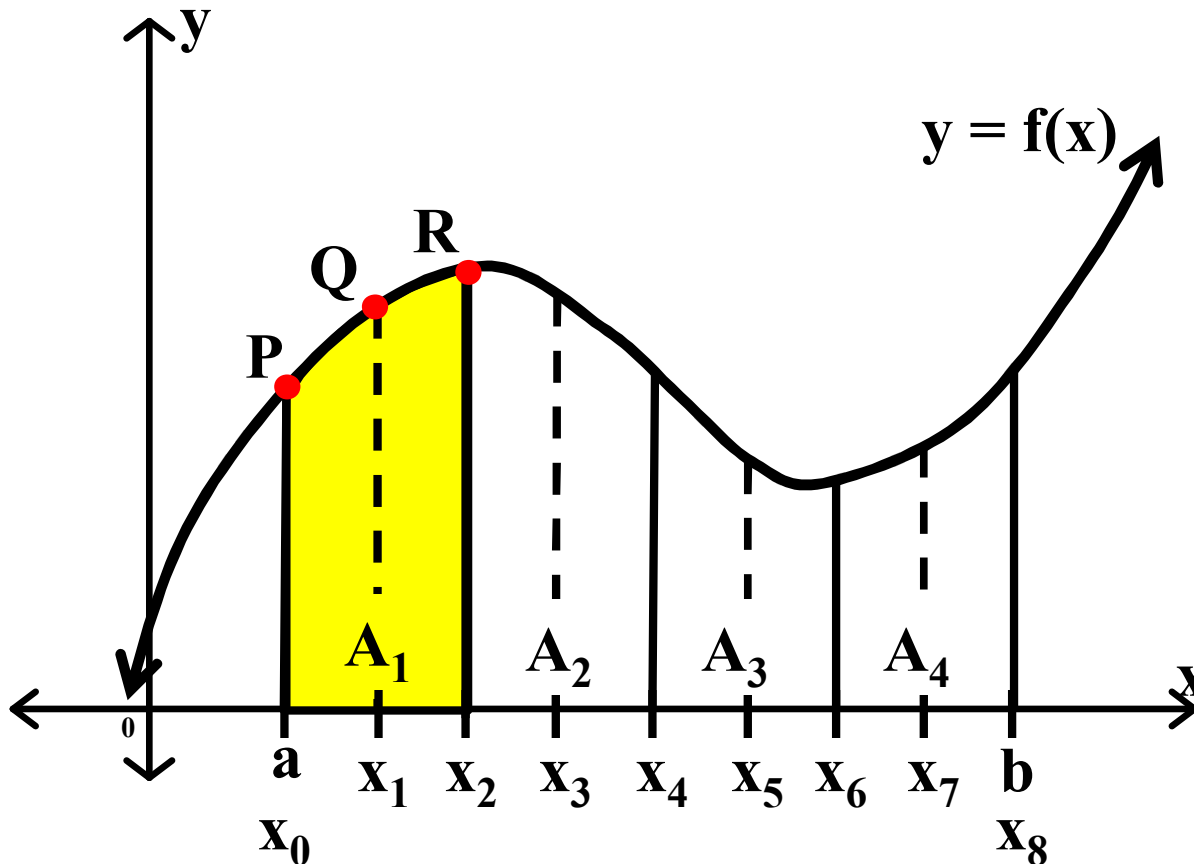
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3} \Delta x \leftarrow \text{the width of each strip}$$

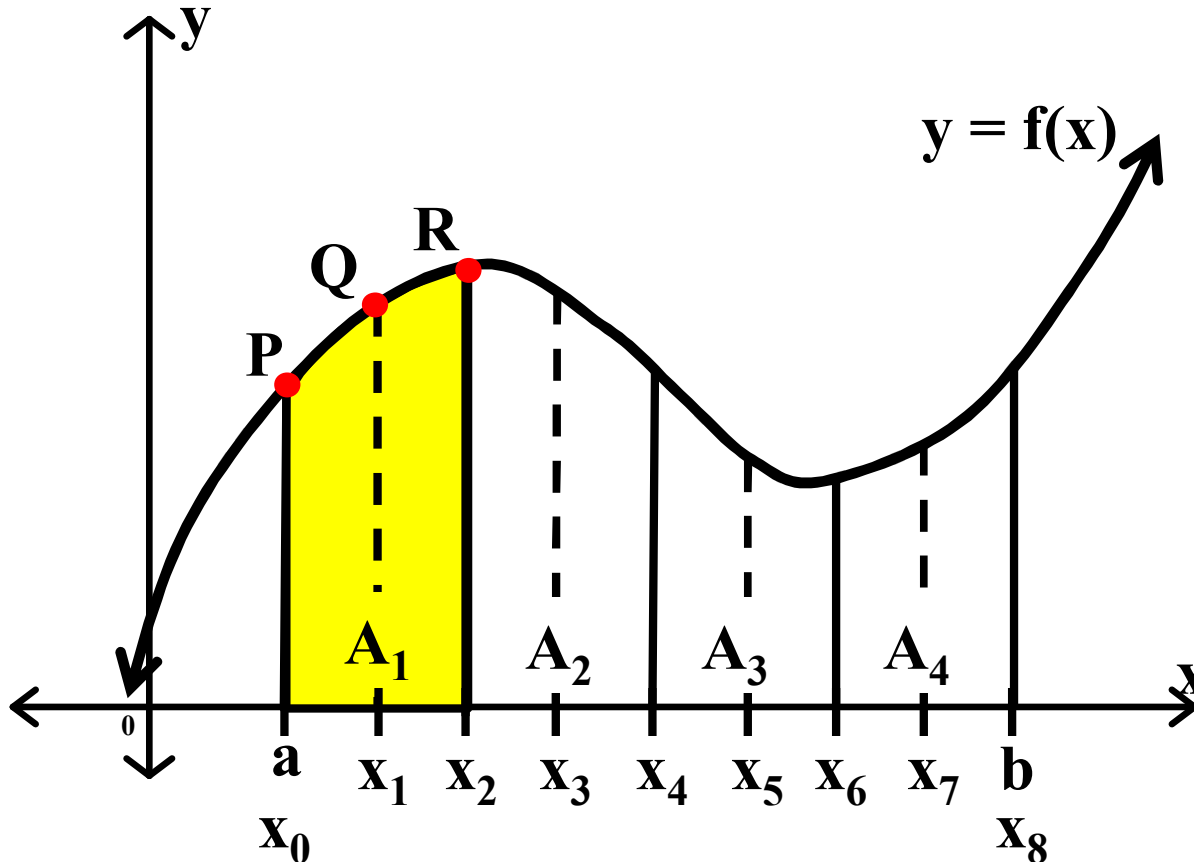
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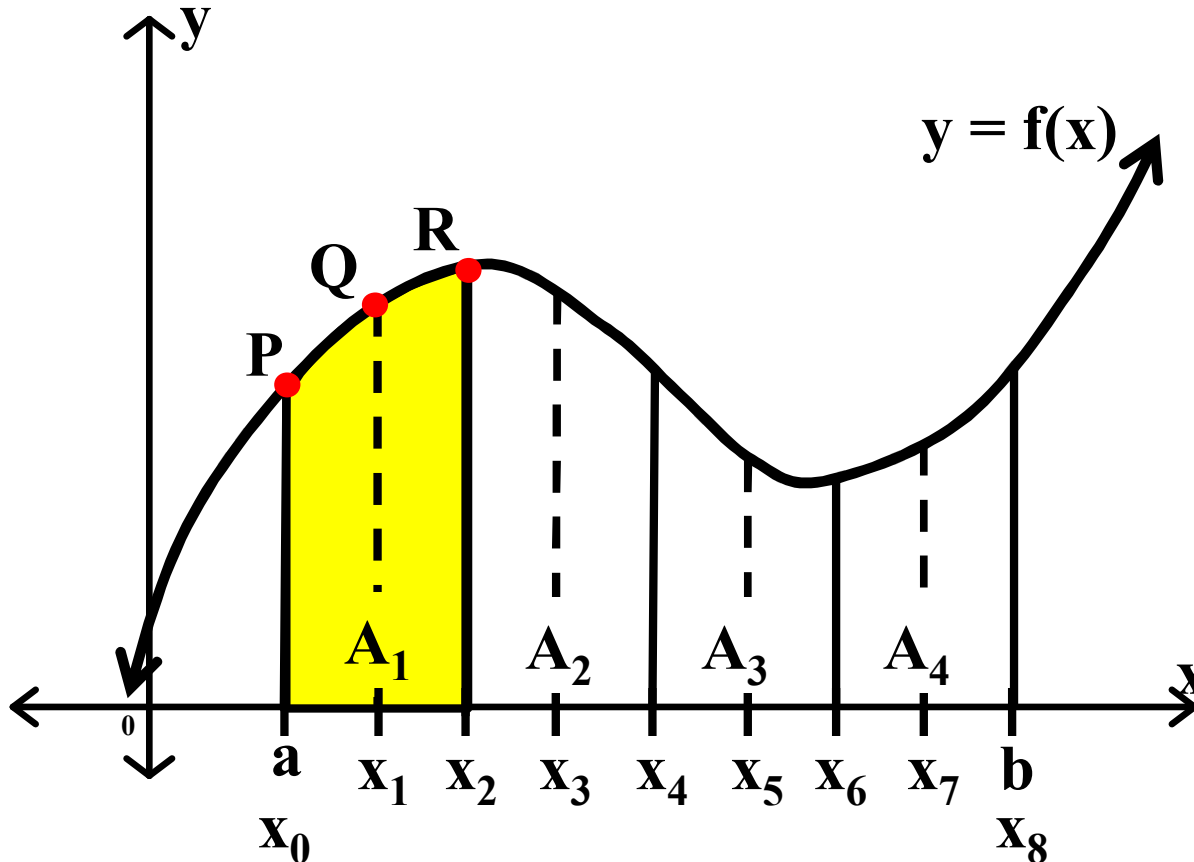
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Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3} \Delta x [f(a)]$$

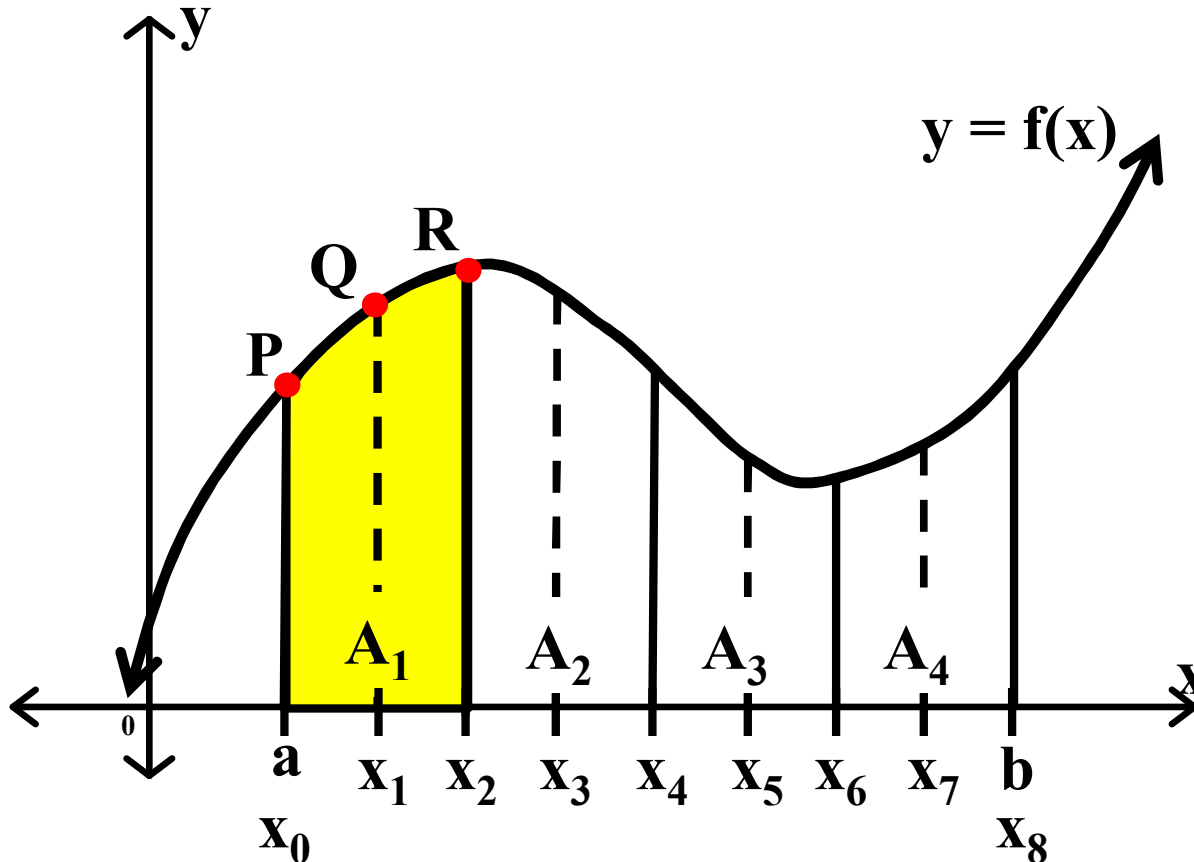
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3} \Delta x [f(a)] \quad \leftarrow \text{the height of the left boundary}$$

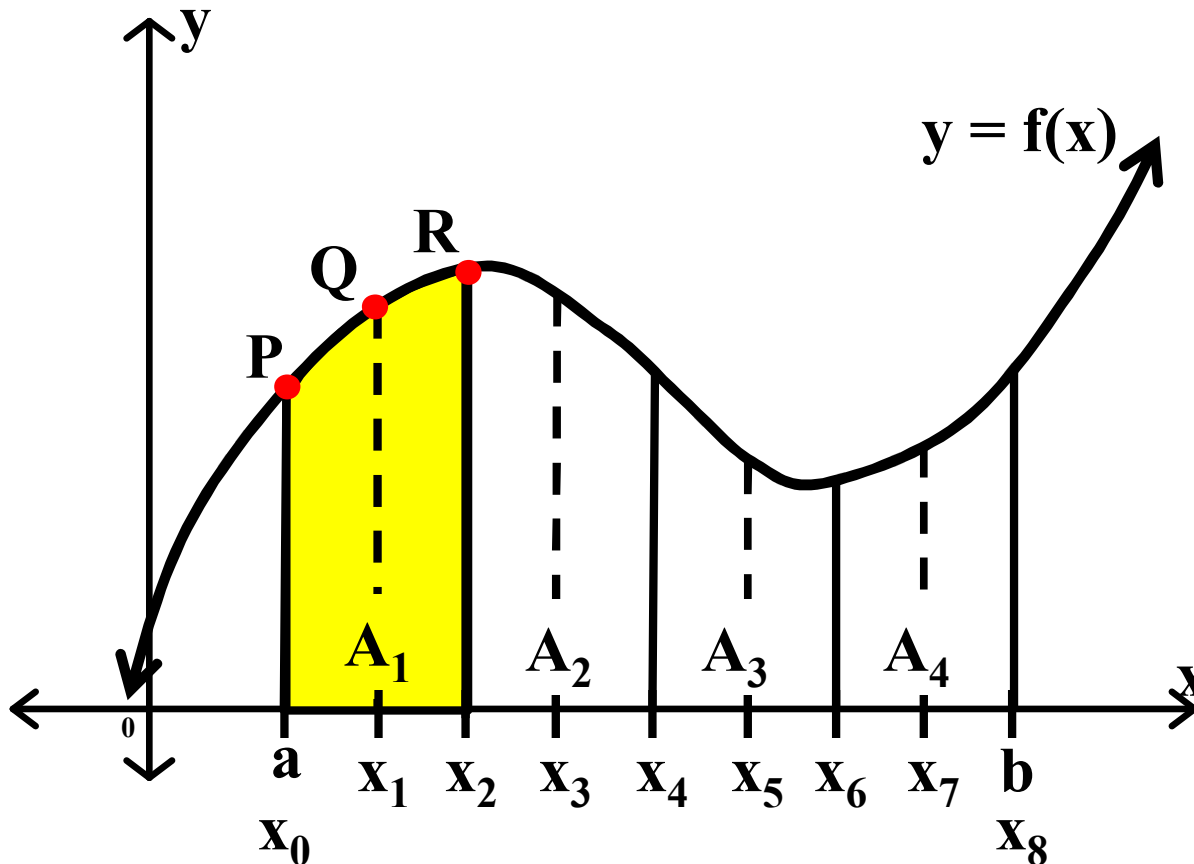
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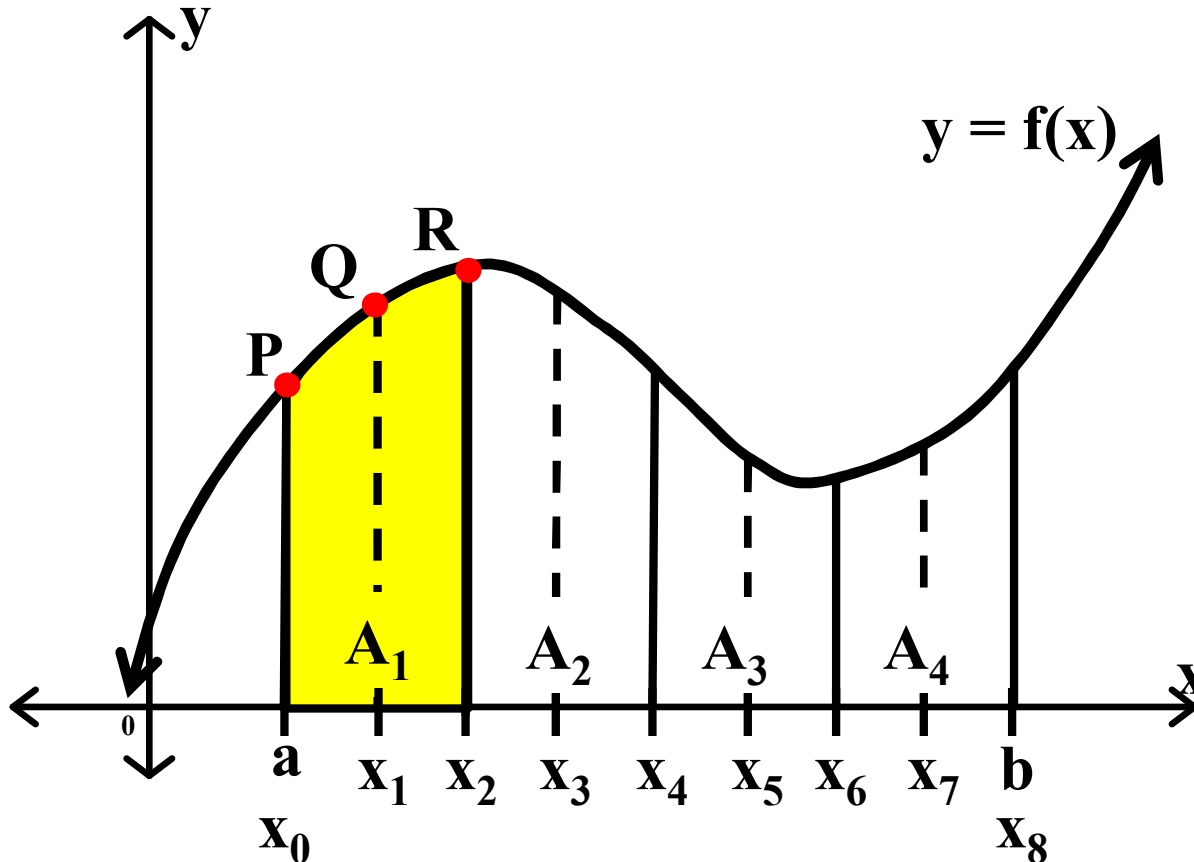
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$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1)]$$

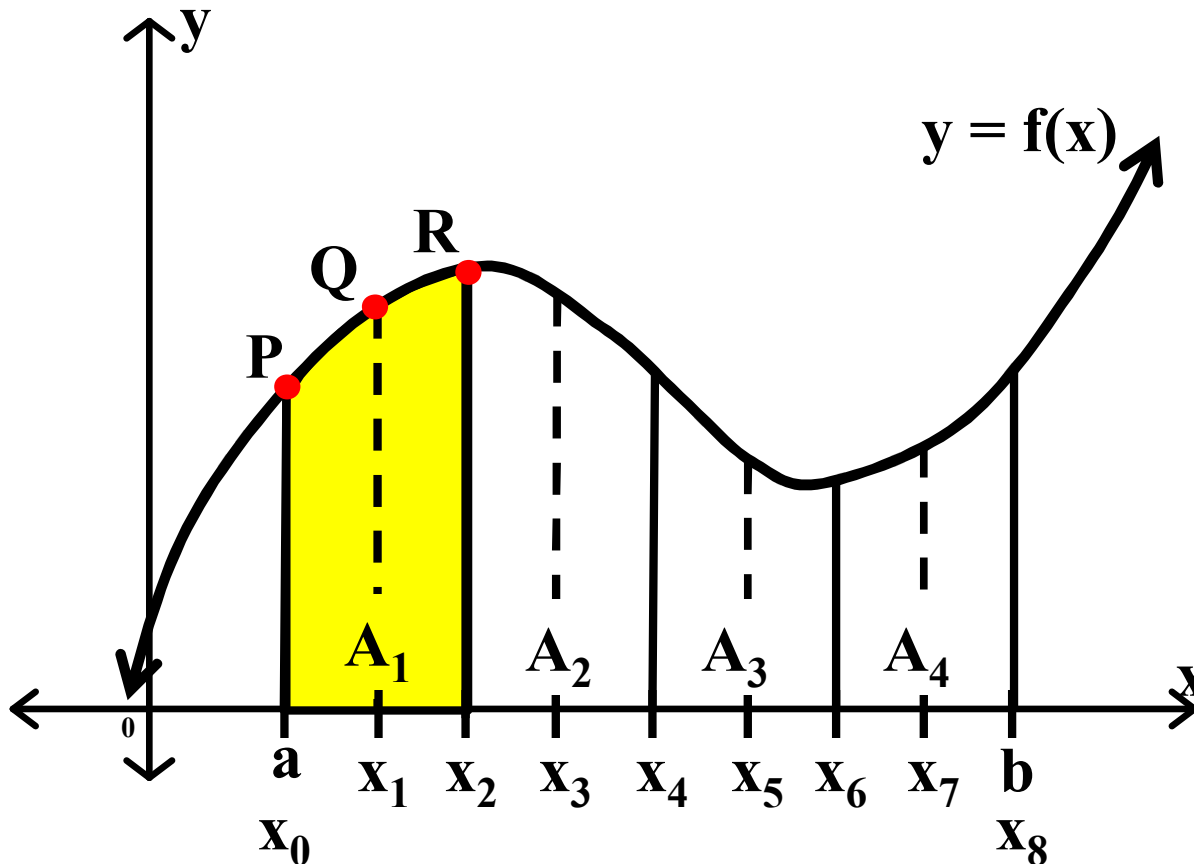
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) \leftarrow \text{the height in the center}]$$

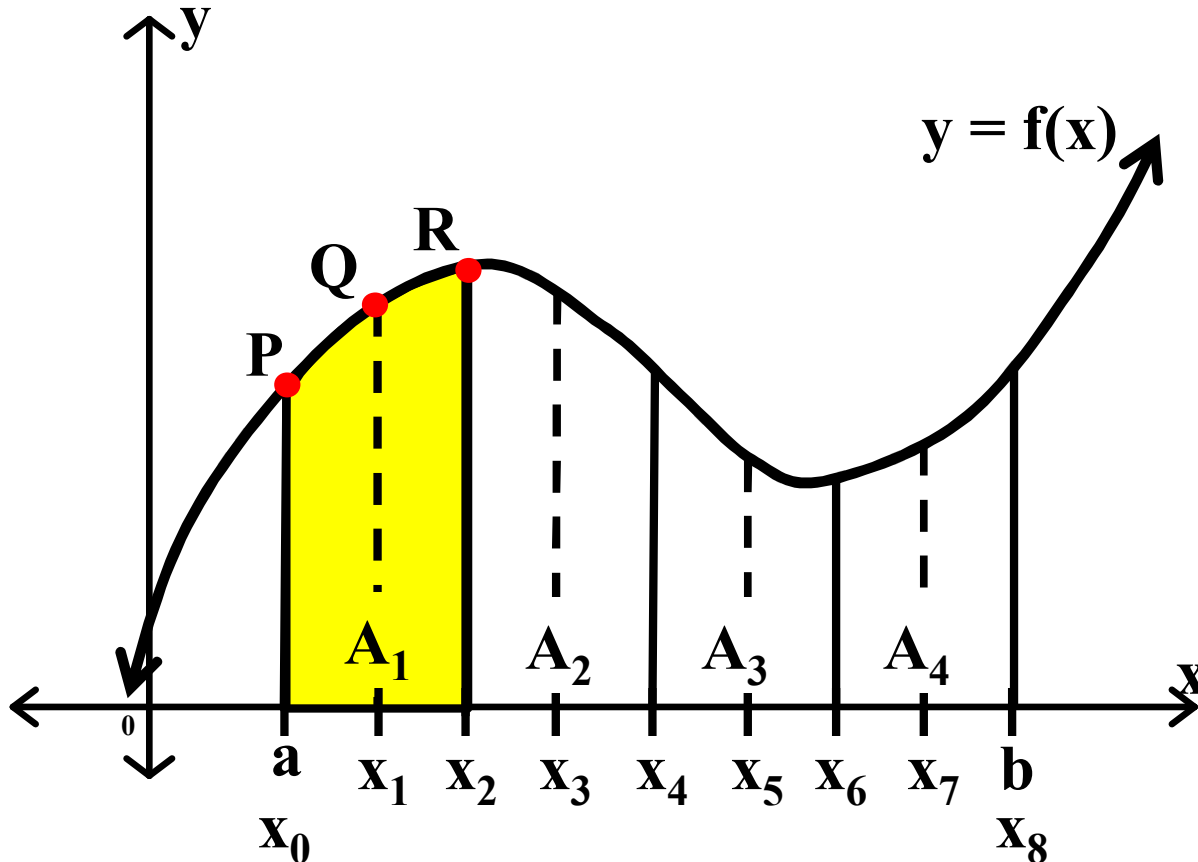
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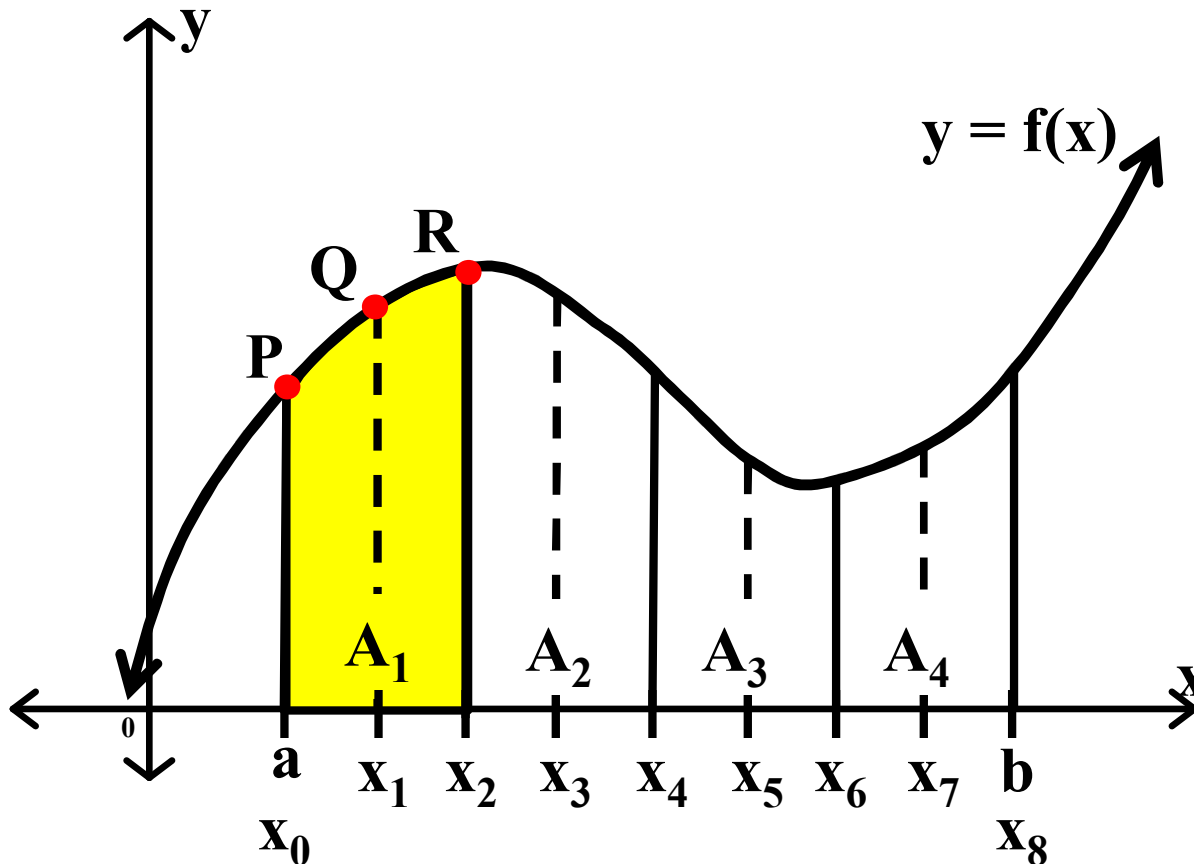
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$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

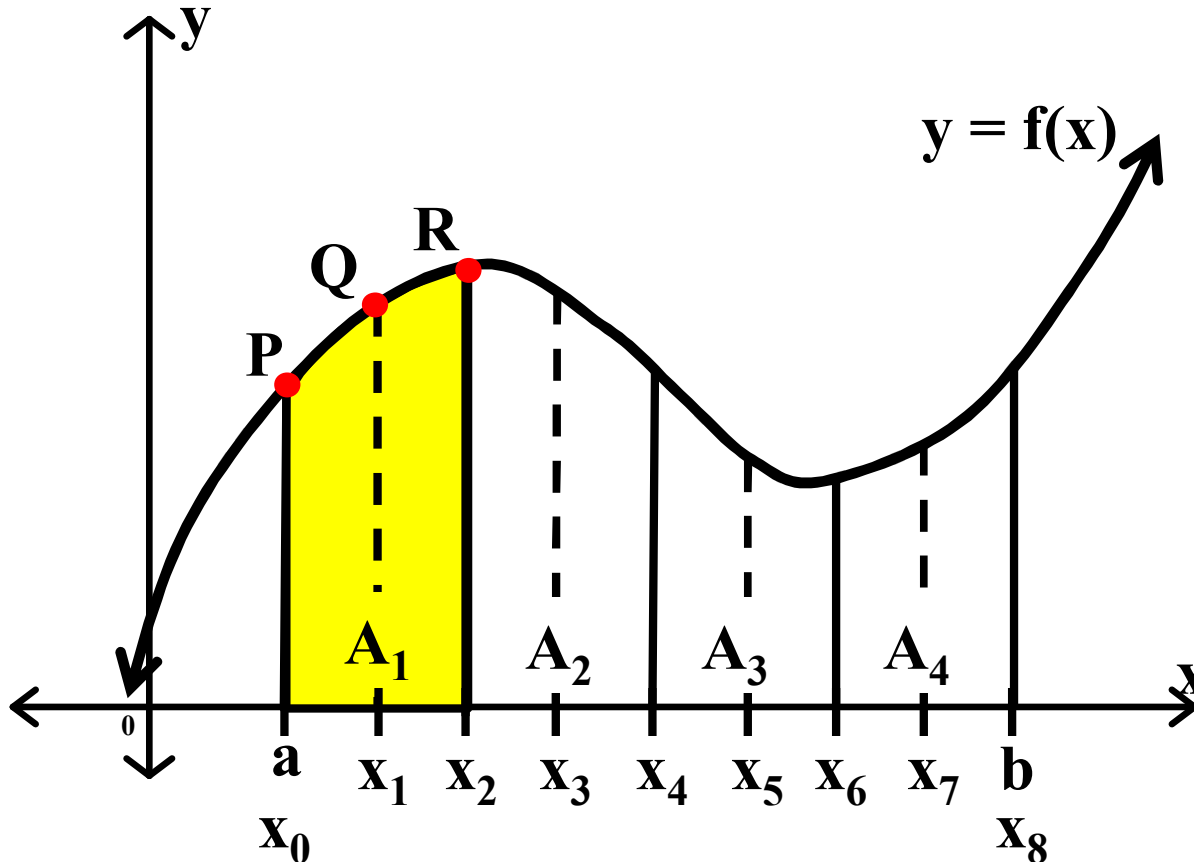
Simpson's Rule



Given points P, Q, and R on the graph of function f , there exists a second degree function, the arc of a parabola (not shown), that contains them. According to Simpson's Rule

$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) + f(x_2)] \leftarrow \text{the height of the right boundary}$$

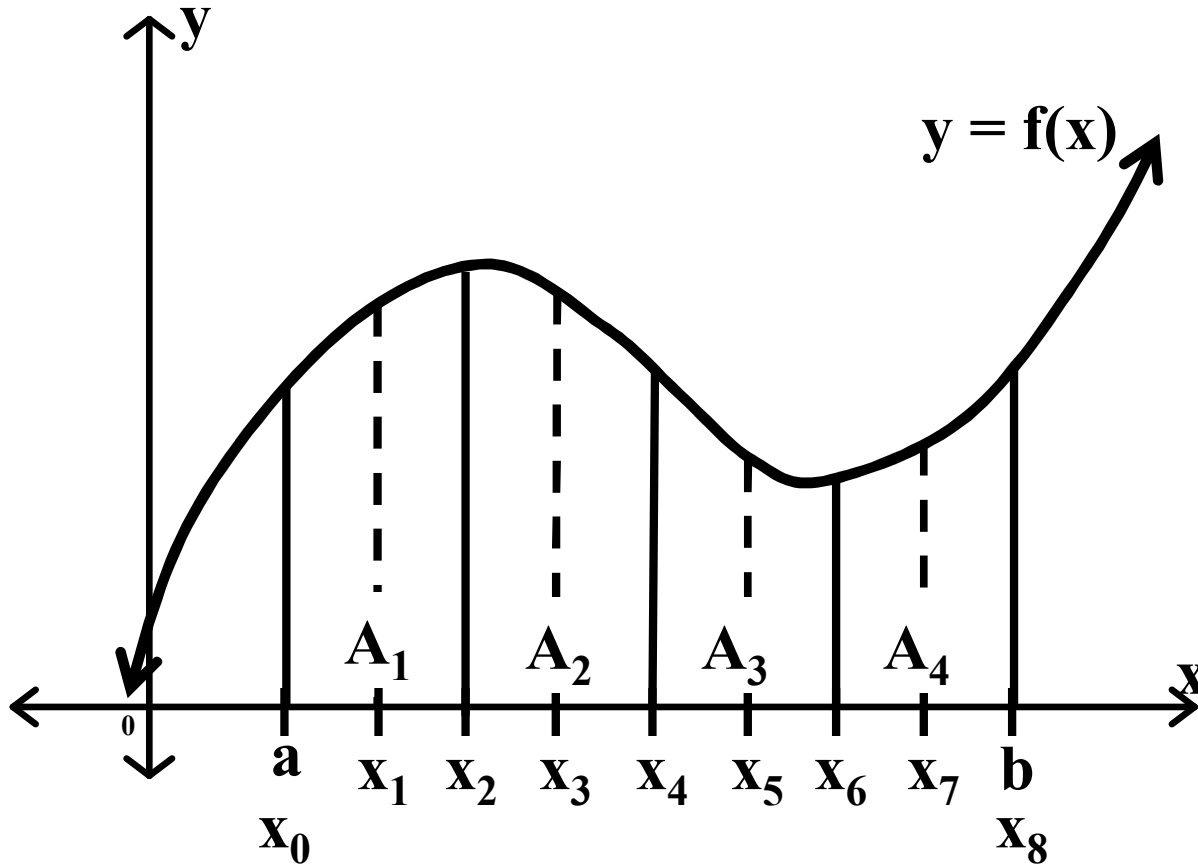
Simpson's Rule



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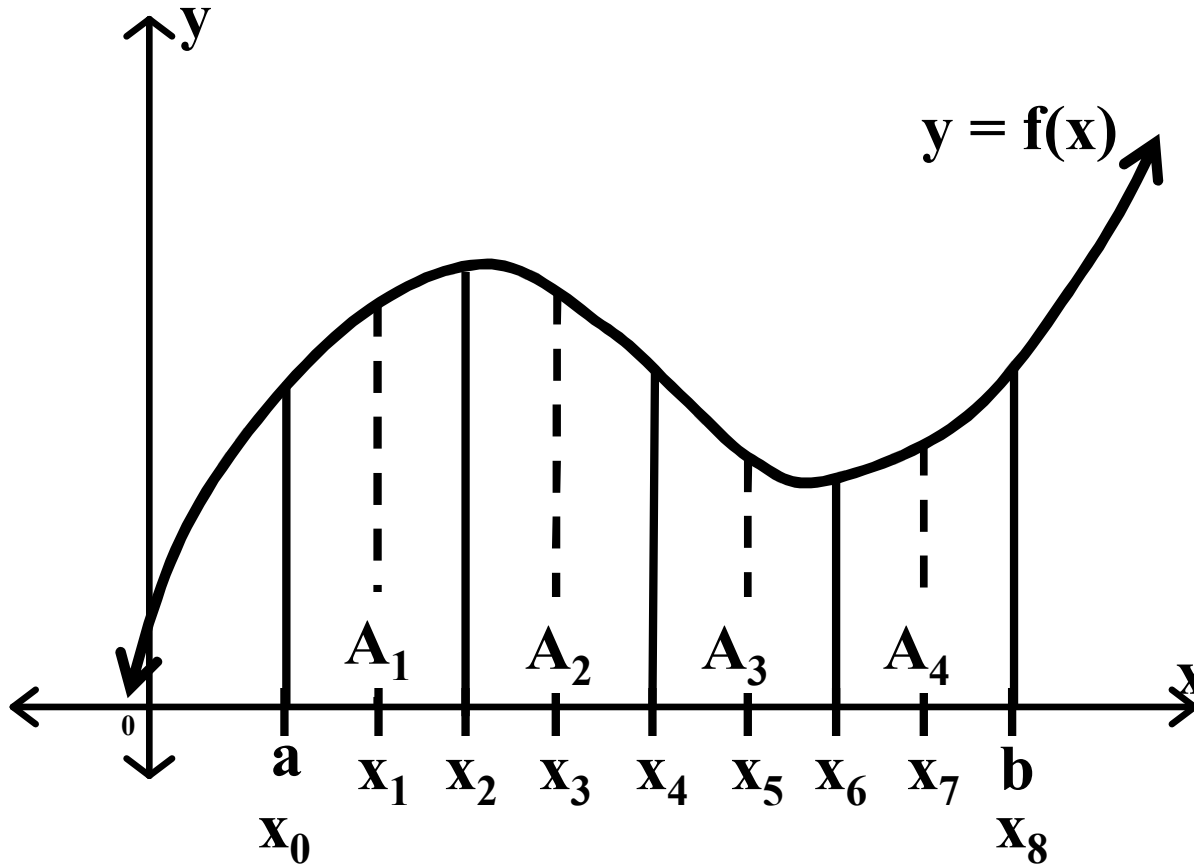
$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) + f(x_2)].$$

Simpson's Rule



$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) + f(x_2)]$$

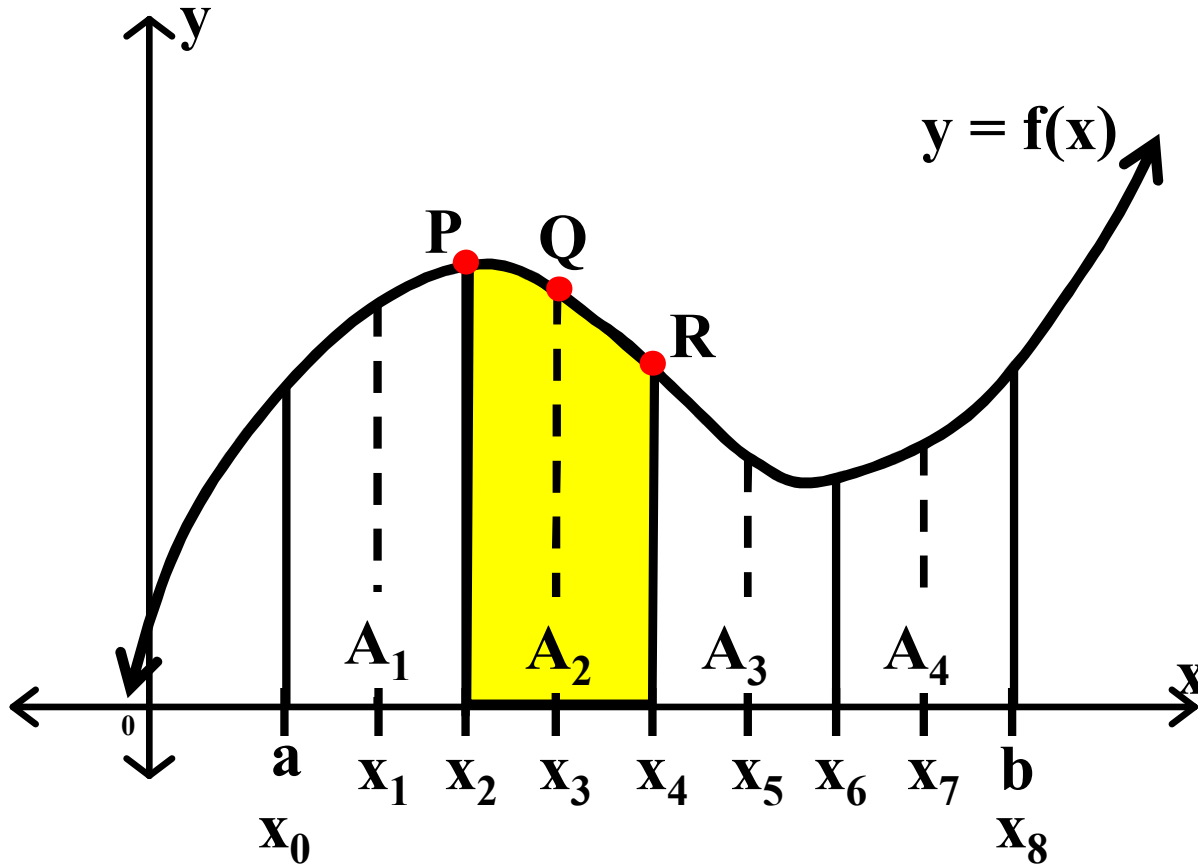
Simpson's Rule



$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) + f(x_2)]$$

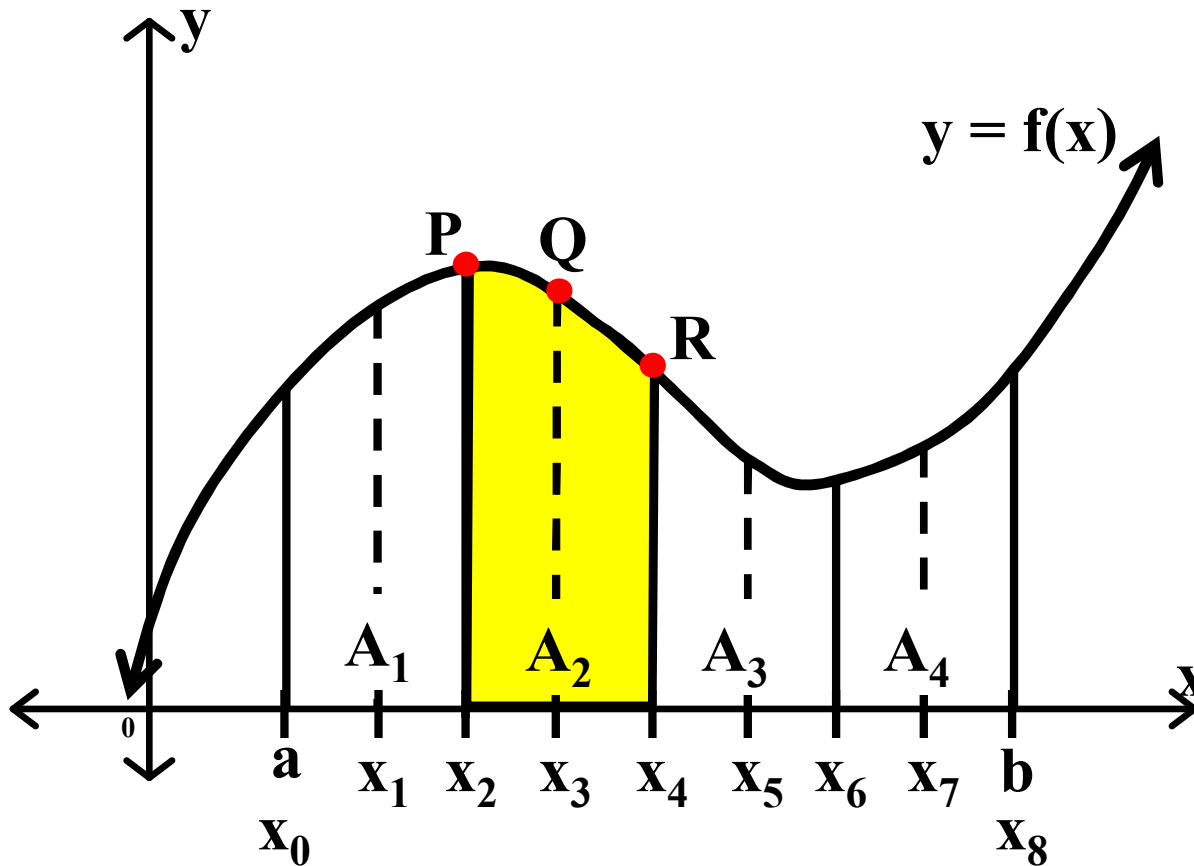
In the same way, the areas of the other 'double strips' can be approximated.

Simpson's Rule



$$A_1 \approx \frac{1}{3} \Delta x [f(a) + 4f(x_1) + f(x_2)]$$

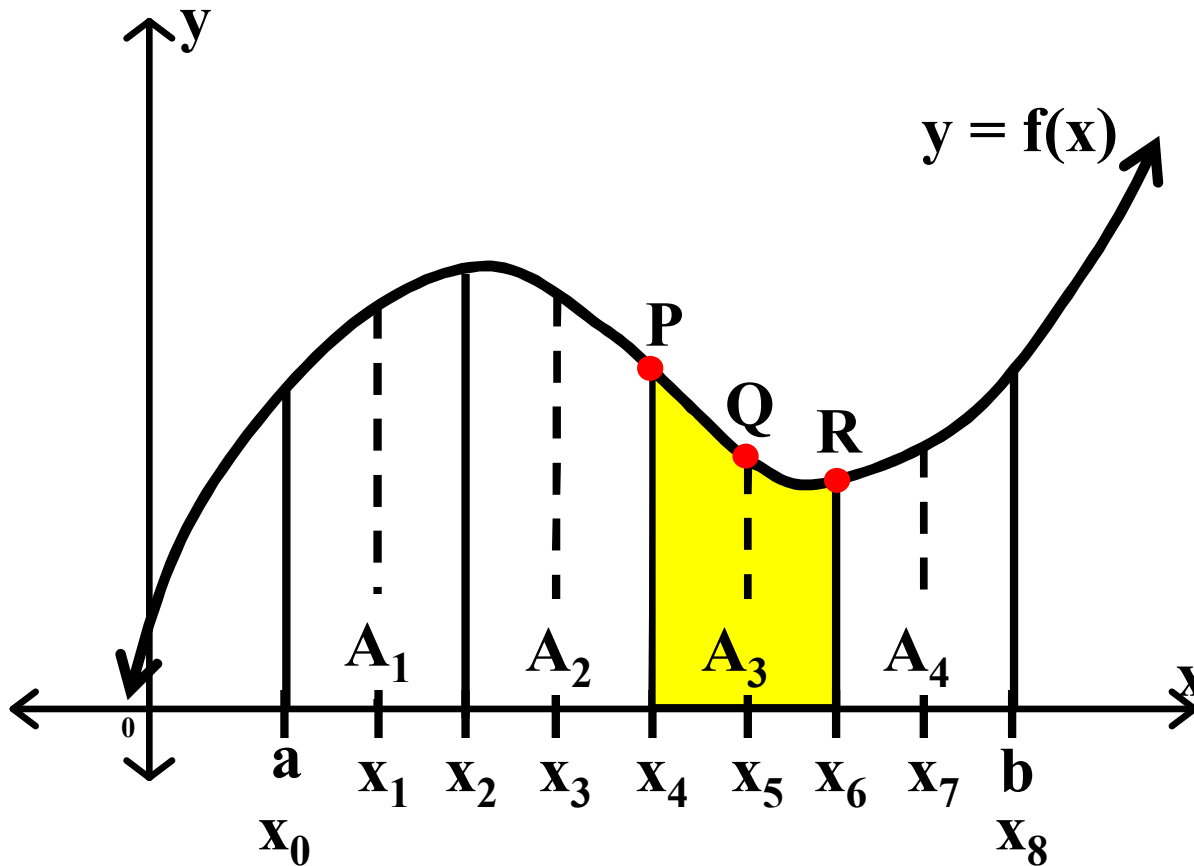
Simpson's Rule



$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)]$$

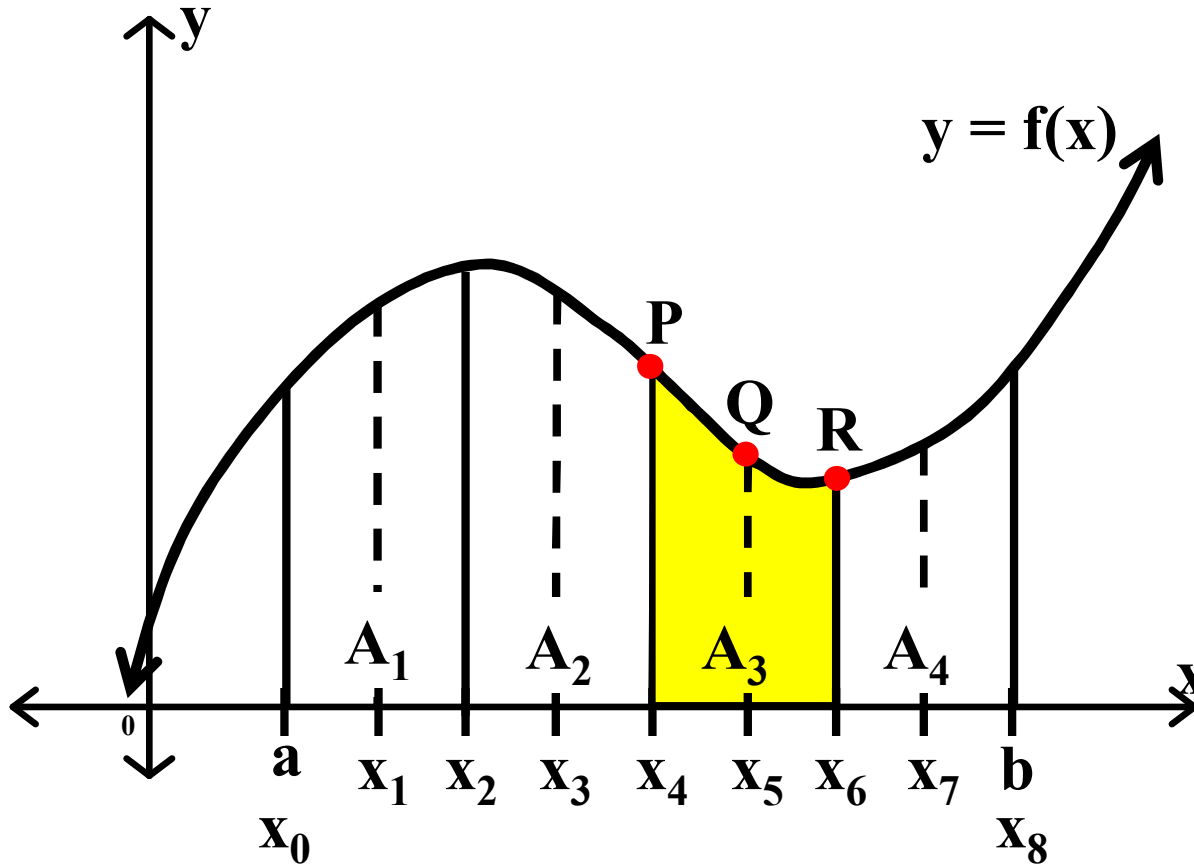
Simpson's Rule



$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

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Simpson's Rule

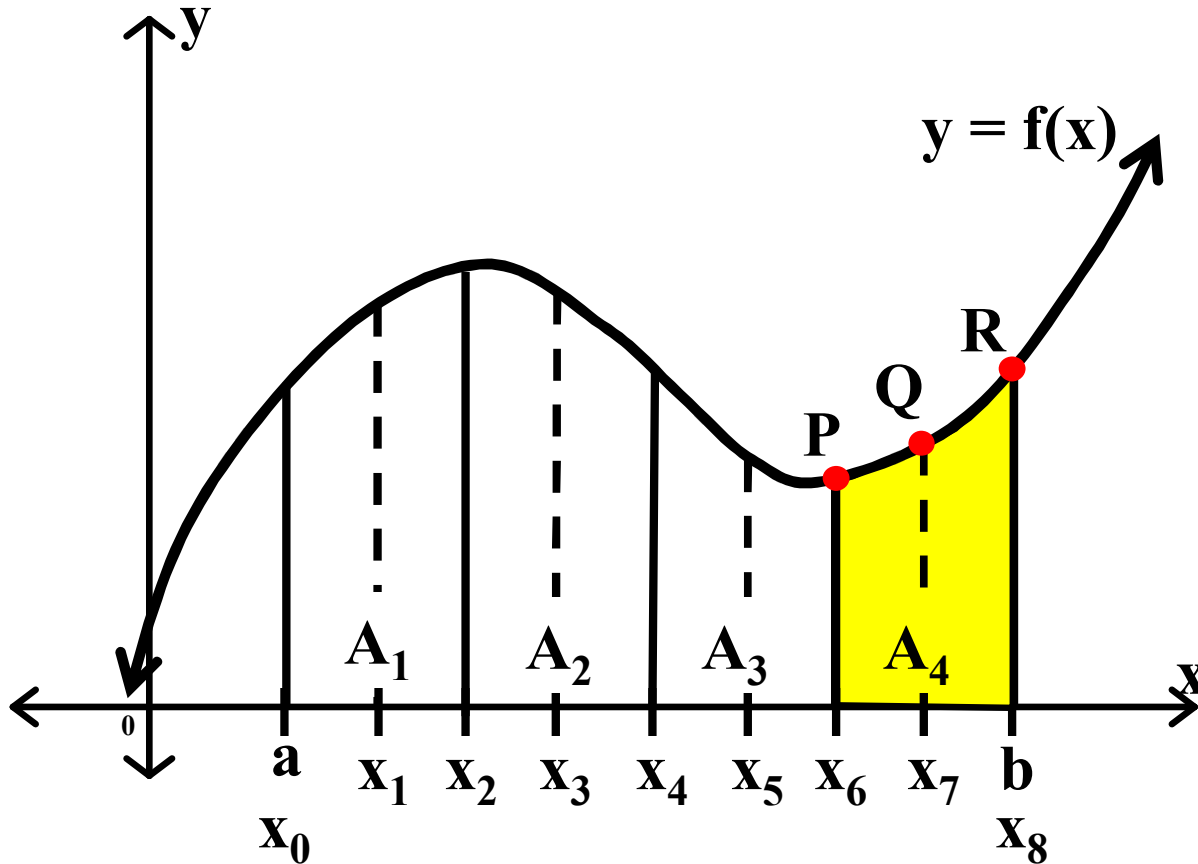


$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

$$A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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Simpson's Rule

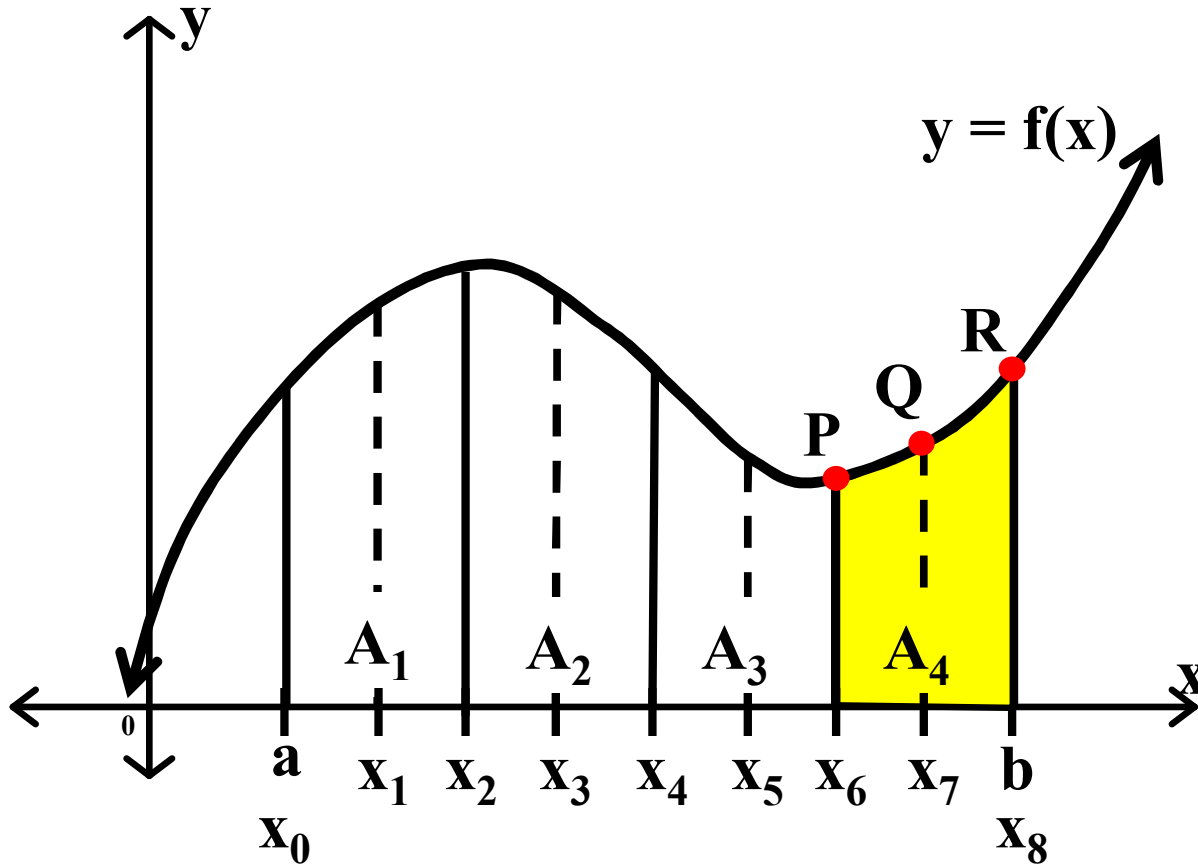


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Simpson's Rule



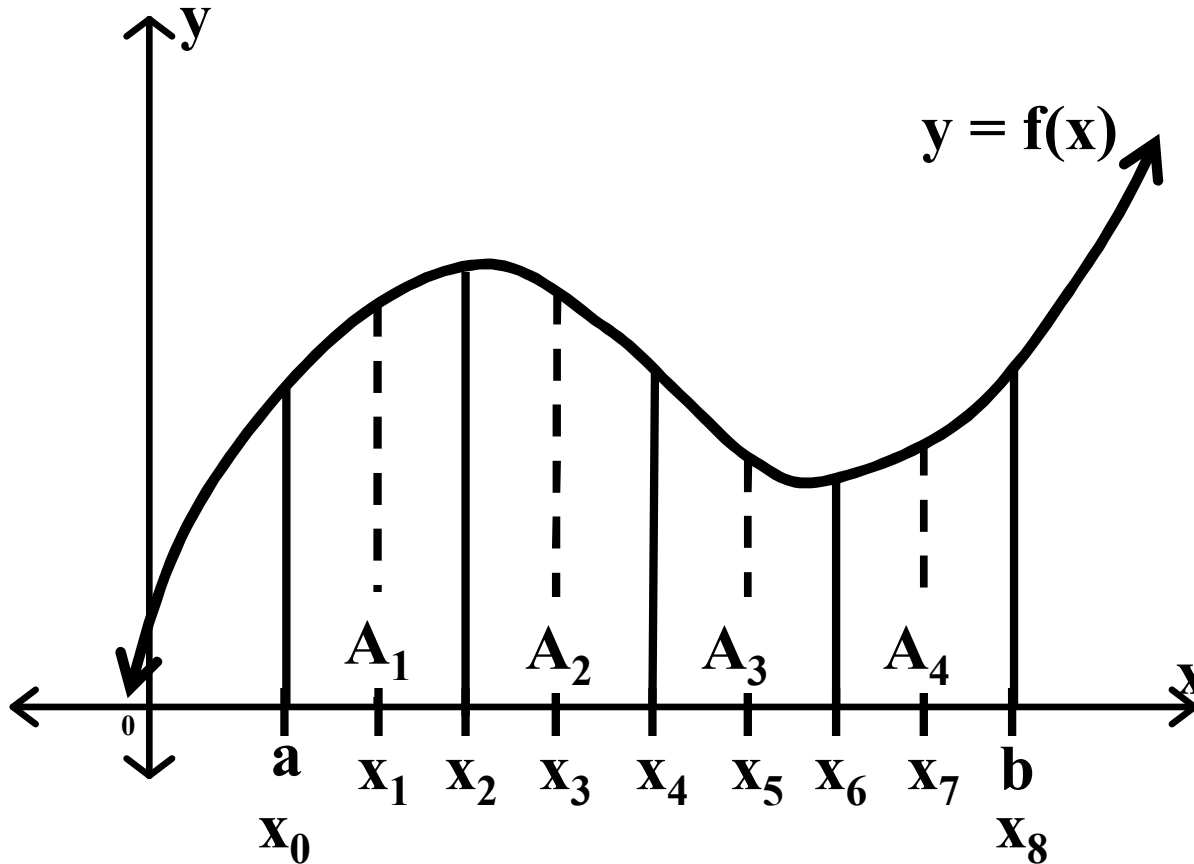
$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

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$$A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

Simpson's Rule



$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)]$$

$$A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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Simpson's Rule

$$\begin{aligned} A_1 &\approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] & A_3 &\approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] \\ A_2 &\approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] & A_4 &\approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)] \end{aligned}$$

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$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i$$

Simpson's Rule

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$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

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$$A \approx$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
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$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] +$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
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$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] + \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)]$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
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$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

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Factor $\frac{1}{3}\Delta x$ from each term.

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Rearrange and combine like terms.

Simpson's Rule

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Simpson's Rule

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$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] + \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] +$$
$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$
$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a)$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

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$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$
$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

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$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

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$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$

$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$

$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
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$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$
$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] + \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] +$$
$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$
$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1})]$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

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Simpson's Rule

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$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
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$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$
$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1}) + 2\sum$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

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$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1}) + 2\sum_{i=1}^{n-1}$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1}) + 2\sum_{i=1}^{n-1} f(x_{2i})$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$

$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

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Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

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$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

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$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1}) + 2\sum_{i=1}^{n-1} f(x_{2i}) + f(b)]$$

Simpson's Rule

$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$
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$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$
$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

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$$A_1 \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] \quad A_3 \approx \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)]$$

$$A_2 \approx \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] \quad A_4 \approx \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

$$A = \int_a^b f(x)dx = \sum_{i=1}^n A_i \quad \text{In our example, } n = 4.$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2)] + \frac{1}{3}\Delta x[f(x_2) + 4f(x_3) + f(x_4)] +$$

$$+ \frac{1}{3}\Delta x[f(x_4) + 4f(x_5) + f(x_6)] + \frac{1}{3}\Delta x[f(x_6) + 4f(x_7) + f(b)]$$

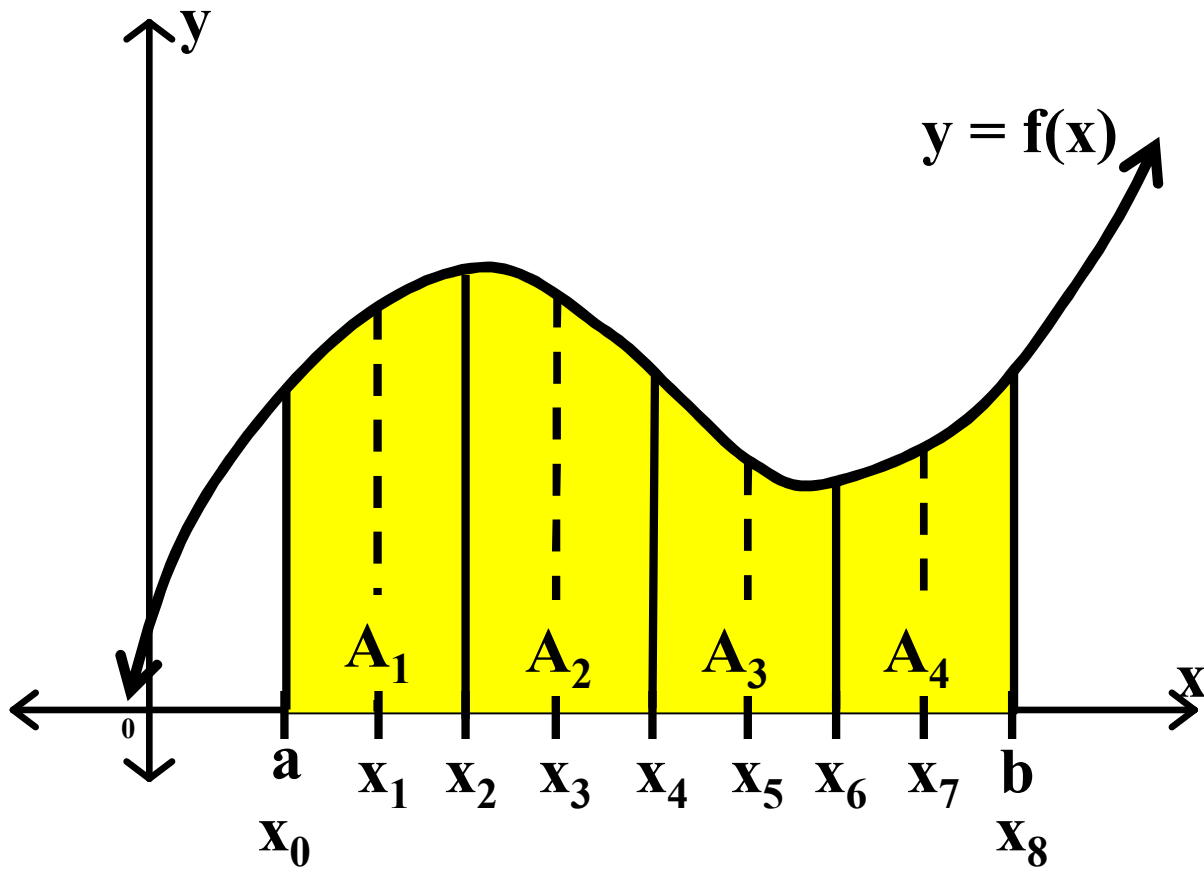
$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + f(x_2) + f(x_2) + 4f(x_3) + f(x_4) +$$

$$+ f(x_4) + 4f(x_5) + f(x_6) + f(x_6) + 4f(x_7) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4f(x_1) + 4f(x_3) + 4f(x_5) + 4f(x_7) +$$

$$+ 2f(x_2) + 2f(x_4) + 2f(x_6) + f(b)]$$

$$A \approx \frac{1}{3}\Delta x[f(a) + 4\sum_{i=1}^n f(x_{2i-1}) + 2\sum_{i=1}^{n-1} f(x_{2i}) + f(b)] = S_S$$



$$S_S = \frac{1}{3} \Delta x [f(a) + 4 \sum_{i=1}^n f(x_{2i-1}) + 2 \sum_{i=1}^{n-1} f(x_{2i}) + f(b)]$$

Simpson's Rule Approximation

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals.

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a = 2$	$f(x_0) = f(a) = f(2) = \sqrt{5}$
$x_1 = 2.5$	$f(x_1) = f(2.5) = \sqrt{12.625}$
$x_2 = 3$	$f(x_2) = f(3) = \sqrt{24}$
$x_3 = 3.5$	$f(x_3) = f(3.5) = \sqrt{39.875}$
$x_4 = 4$	$f(x_4) = f(4) = \sqrt{61}$
$x_5 = 4.5$	$f(x_5) = f(4.5) = \sqrt{88.125}$
$x_6 = b = 5$	$f(x_6) = f(b) = f(5) = \sqrt{122}$

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$$S_S = \frac{1}{3} \Delta x [f(a) + 4 \sum_{i=1}^3 f(x_{2i-1}) + 2 \sum_{i=1}^2 f(x_{2i}) + f(b)]$$

$$S_S = \frac{1}{3} \Delta x [f(a) + 4\{f(x_1) + f(x_3) + f(x_5)\} + 2\{f(x_2) + f(x_4)\} + f(b)]$$

$$S_S = \frac{1}{3} (.5) [\sqrt{5} + 4\{\sqrt{12.625} + \sqrt{39.875} + \sqrt{88.125}\} + 2\{\sqrt{24} + \sqrt{61}\} + \sqrt{122}]$$

$$S_S \approx 19.29$$

Class Worksheet #5 Unit 11

Approximate the following definite integral using each of the following approximation methods.

(a) S_L (Left Rectangular), (b) S_R (Right Rectangular), (c) S_M (Midpoint Rectangular), (d) S_T (Trapezoidal), and (e) S_S (Simpson's).

Show your complete solutions neatly organized. In every case, divide the interval $[a, b]$ into 6 sub-intervals. $2n = 6 \rightarrow n = 3$

$$\int_2^5 \sqrt{x^3 - 3} \, dx \quad \Delta x = \frac{b - a}{n} = \frac{5 - 2}{6} = 0.5 \quad f(x) = \sqrt{x^3 - 3}$$

$x_0 = a$ **Good luck on your homework !!** $x_6 = b$

$$x_2 = 3 \quad f(x_2) = f(3) = \sqrt{24}$$

$$x_3 = 3.5 \quad f(x_3) = f(3.5) = \sqrt{39.875}$$

$$x_4 = 4 \quad f(x_4) = f(4) = \sqrt{61}$$

$$x_5 = 4.5 \quad f(x_5) = f(4.5) = \sqrt{88.125}$$

$$x_6 = b = 5 \quad f(x_6) = f(b) = f(5) = \sqrt{122}$$

$$S_S = \frac{1}{3} \Delta x [f(a) + 4 \sum_{i=1}^3 f(x_{2i-1}) + 2 \sum_{i=1}^2 f(x_{2i}) + f(b)]$$

$$S_S = \frac{1}{3} \Delta x [f(a) + 4\{f(x_1) + f(x_3) + f(x_5)\} + 2\{f(x_2) + f(x_4)\} + f(b)]$$

$$S_S = \frac{1}{3} (.5) [\sqrt{5} + 4\{\sqrt{12.625} + \sqrt{39.875} + \sqrt{88.125}\} + 2\{\sqrt{24} + \sqrt{61}\} + \sqrt{122}]$$

$$S_S \approx 19.29$$

