Calculus Worksheet #5 Unit 10 Selected Solutions

Find the equations of the lines that are tangent and normal to the graph of the following function at the point on the graph with the given x-coordinate.

3. $f(x) = \dfrac{e^x}{\cos x}$; $x = 0$ $\qquad y = f(0) = \dfrac{e^0}{\cos 0} = 1$

$$f'(x) = \frac{(\cos x)(e^x) - (e^x)(-\sin x)}{\cos^2 x}$$

The point on the curve is (0, 1).

$$f'(x) = \frac{e^x(\cos x + \sin x)}{\cos^2 x}$$

$$m_t = f'(0) = \frac{e^0(\cos 0 + \sin 0)}{\cos^2 0} = 1$$

$$m_n = \frac{-1}{m_t} = -1$$

The equation of the line that is tangent to the graph at (0, 1) is $y = x + 1$.

The equation of the line that is normal to the graph at (0, 1) is $y = -x + 1$.

7. Find the area of the region bounded by the x-axis, the lines $x = \pi/6$ and $x = \pi/3$, and the curve $y = \tan x$.

$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x\,dx}{\cos x} = -\ln|\cos x|\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\ln(\cos\tfrac{\pi}{3}) + \ln(\cos\tfrac{\pi}{6}) \approx 0.549 \text{ sq. units}$$

11. Find the volume of the solid formed when the region bounded by the x-axis, the lines $x = \pi/6$ and $x = \pi/3$, and the curve $y = \tan x$ is revolved about the x-axis.

$$V = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2 x\,dx = \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x - 1)dx = \pi(\tan x - x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$$

$$= \pi[(\tan\tfrac{\pi}{3} - \tfrac{\pi}{3}) - (\tan\tfrac{\pi}{6} - \tfrac{\pi}{6})] \approx 1.98 \text{ cu. units}$$

17. Find the average value of the function $f(x) = e^x$ over the interval [0, ln 2].

$$\text{Ave. Value} = \frac{1}{\ln 2}\int_0^{\ln 2} e^x dx = \frac{1}{\ln 2}\,e^x\Big|_0^{\ln 2} = \frac{1}{\ln 2}[e^{\ln 2} - e^0] = \frac{1}{\ln 2}(2 - 1) = \frac{1}{\ln 2} \approx 1.44$$