Algebra II Worksheet #7 Unit 9 Selected Homework Solutions

Solve each of the following problems. Show your work neatly organized.

2. Find the sum of the first 8 terms of the sequence defined by $a_n = (-2)^n$.

geometric series

$a_1 = -2 \quad r = -2 \quad n = 8$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \qquad S_8 = \frac{-2[1 - (-2)^8]}{1 - (-2)}$$

$$S_8 = \frac{a_1(1 - r^8)}{1 - r} \qquad S_8 = \frac{-2(1 - 256)}{3} = 170$$

5. Evaluate the series $3 + 6 + 12 + 24 + 48 + ... + 3072$.

geometric series

$a_1 = 3 \quad r = 2 \quad a_n = 3072$

$$S_n = \frac{a_1 - a_n r}{1 - r}$$

$$S_n = \frac{3 - (3072)(2)}{1 - 2} = 6{,}141$$

6. Evaluate the infinite series $1 - .5 + .25 - .125 + ...$

infinite geometric series

$a_1 = 1 \quad r = -0.5$

$$S = \frac{a_1}{1 - r}$$

$$S = \frac{1}{1 - (-.5)} = \frac{1}{1.5} = \frac{10}{15} = \frac{2}{3}$$

Solve each of the following problems. Show your work neatly organized.

8. Evaluate: $\sum_{i=1}^{\infty} \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^{i-1} = \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^0 + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^1 + \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^2 + ...$

infinite geometric series

$a_1 = \frac{2}{3} \quad r = \frac{1}{2}$

$$S = \frac{a_1}{1 - r} \qquad S = \frac{\frac{2}{3}}{1 - \frac{1}{2}} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3}(2) = \frac{4}{3}$$

10. A ball is dropped from a height of 100 inches onto a concrete floor. On each bounce the ball rebounds to 80% of its previous height. What is the total vertical distance that the ball has traveled when its hits the floor for the tenth time? (Both cases below are geometric series.)

Downward

$100 + 80 + 64 + ...$

$a_1 = 100 \quad r = .8 \quad n = 10$

$$S_{10} = \frac{100(1 - .8^{10})}{1 - .8} \approx 446.31$$

Upward

$80 + 64 + 51.2 + ...$

$a_1 = 80 \quad r = .8 \quad n = 9$

$$S_{10} = \frac{80(1 - .8^9)}{1 - .8} \approx 346.31$$

$\text{Total} \approx 446.31 + 346.31 \approx 792.6$

The total vertical distance is about 792.6 inches.