

## Algebra II Worksheet #6 Unit 9 Selected Homework Solutions

Solve each of the following problems. Show your work neatly organized.

1. Find the sum of the first 100 terms of an arithmetic sequence in which  $a_1 = 5$  and  $d = 3$ .

$$\begin{aligned} a_1 &= 5 & d &= 3 & n &= 100 & S_n &= \frac{n}{2}(a_1 + a_n) \\ a_{100} &= a_1 + 99d & S_{100} &= \frac{100}{2}(a_1 + a_{100}) \\ a_{100} &= 5 + 99(3) = 302 & S_{100} &= 50(5 + 302) = 50(307) = 15,350 \end{aligned}$$

3. Find the sum of the first 75 terms of the sequence defined by  $a_{n+1} = a_n + 6$  where  $a_1 = 20$ .

This is an arithmetic series.

$$\begin{aligned} a_1 &= 20 & d &= 6 & n &= 75 & S_n &= \frac{n}{2}(a_1 + a_n) \\ a_{75} &= a_1 + 74d & S_{75} &= \frac{75}{2}(a_1 + a_{75}) \\ a_{75} &= 20 + 74(6) = 464 & S_{75} &= 37.5(20 + 464) = 37.5(484) = 18,150 \end{aligned}$$

5. Show that the sum of the first  $n$  terms of the sequence 1, 3, 5, 7, ... is equal to  $n^2$ .

This is an arithmetic series.

$$\begin{aligned} a_1 &= 1 & d &= 2 & S_n &= \frac{n}{2}(a_1 + a_n) \\ a_n &= a_1 + (n-1)d & S_n &= \frac{n}{2}[1 + (2n-1)] \\ a_n &= 1 + (n-1)2 & S_n &= \frac{n}{2}(2n) = n^2 \\ a_n &= 1 + 2n - 2 = 2n - 1 \end{aligned}$$

7. Evaluate:  $\sum_{i=1}^{50} (2i + 3) = 5 + 7 + 9 + \dots + 103$

This is an arithmetic series.

$$\begin{aligned} n &= 50 & a_1 &= 5 & a_n &= 103 & S_n &= \frac{n}{2}(a_1 + a_n) \\ S_{50} &= \frac{50}{2}(a_1 + a_{50}) \\ S_{50} &= 25(5 + 103) = 25(108) = 2,700 \end{aligned}$$

9. An object accelerates in such a way that it travels 10 feet during the first second, 15 feet during the next second, and 20 feet during the third second. If this pattern continues, then how far will the object have moved during the first ten seconds?

This is an arithmetic series.

$$\begin{aligned} a_1 &= 10 & d &= 5 & n &= 10 & S_n &= \frac{n}{2}(a_1 + a_n) \\ a_{10} &= a_1 + 9d & S_{10} &= \frac{10}{2}(a_1 + a_{10}) \\ a_{10} &= 10 + 9(5) = 55 & S_{10} &= 5(10 + 55) = 5(65) = 325 \end{aligned}$$

It will have moved 325 feet.