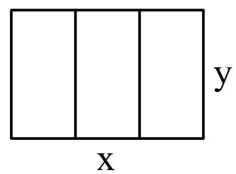
Algebra II Worksheet #8 Unit 8 Selected Homework Solutions

2. Sue wants to fence in a rectangular plot of land **and** to divide it into three equal areas using two lengths of fencing parallel to two opposite sides. If he has a total of 2000 feet of fencing to work with, then find the dimensions that will maximize the total area enclosed. What is the maximum area?



Consider the diagram shown. Let x represent the length of the rectangular plot of land. Let y represent its width. Clearly, the total amount of fencing required is $2x + 4y$.

Once again, to maximize the area, we must represent the area as a function of one variable.

$A = xy$ where $\quad \mathbf{2x + 4y = 2000}$

$$\mathbf{4y = -2x + 2000}$$
$$\mathbf{y = -.5x + 500}$$

Therefore, $\quad \mathbf{A = x(-.5x + 500)}$

$$\mathbf{A = -.5x^2 + 500x}$$

The maximum area corresponds to the vertex of this function.

$$\mathbf{A = -.5(x^2 - 1000x)}$$
$$\mathbf{A - 125{,}000 = -.5(x^2 - 1000x + 250{,}000)}$$
$$\mathbf{A - 125{,}000 = -.5(x - 500)^2}$$

The vertex is (500, <u>125,000</u>).

For maximum area, x = 500.

$$\mathbf{y = -.5(500) + 500 = -250 + 500 = 250.}$$

The plot with maximum area is 500 feet long and 250 feet wide.
The plot will have a maximum area of 125,000 square feet.