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Equations of a Parabola

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ where $A = 0$ or $C = 0$ (not both)

Standard Forms:

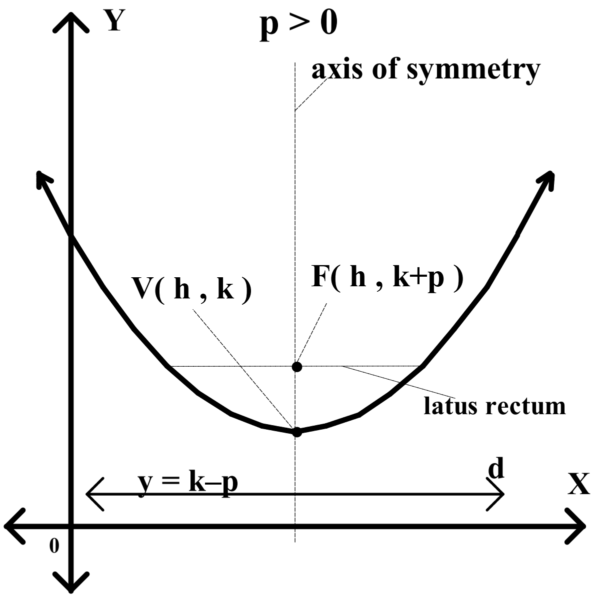
Type 1: Axis of Symmetry Vertical

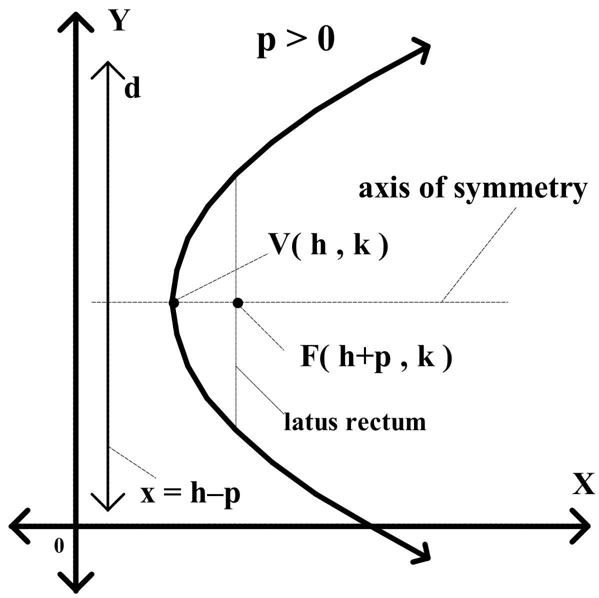
$y - k = a(x - h)^2$ where $a = 1/(4p)$

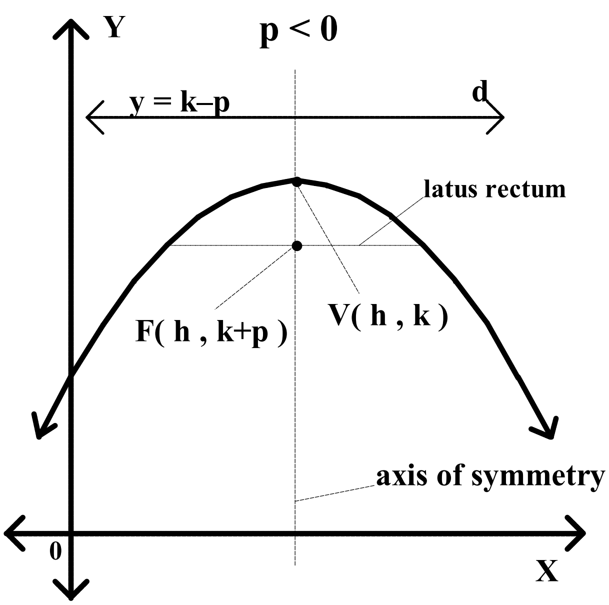
Type 2: Axis of Symmetry Horizontal

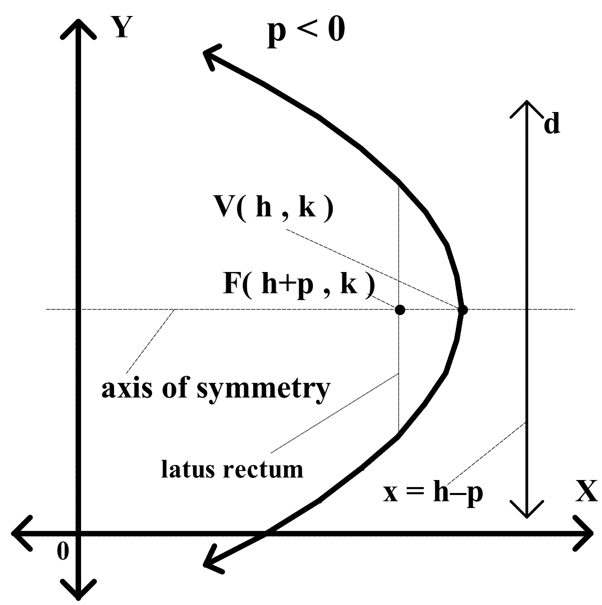
$x - h = a(y - k)^2$ where $a = 1/(4p)$

Given any line d (the directrix) and any point F (the focus) not on line d, a parabola is the set of all points in the plane that are equidistant from line d and point F. (See the graphs below.) The point (h, k) is called the vertex of the parabola. p is the directed distance from the vertex to the focus. The axis of symmetry is the line through the focus that is perpendicular to the directrix. The line segment that goes through the focus perpendicular to the axis of symmetry with its endpoints on the parabola is called the latus rectum. Its length is |4p|.









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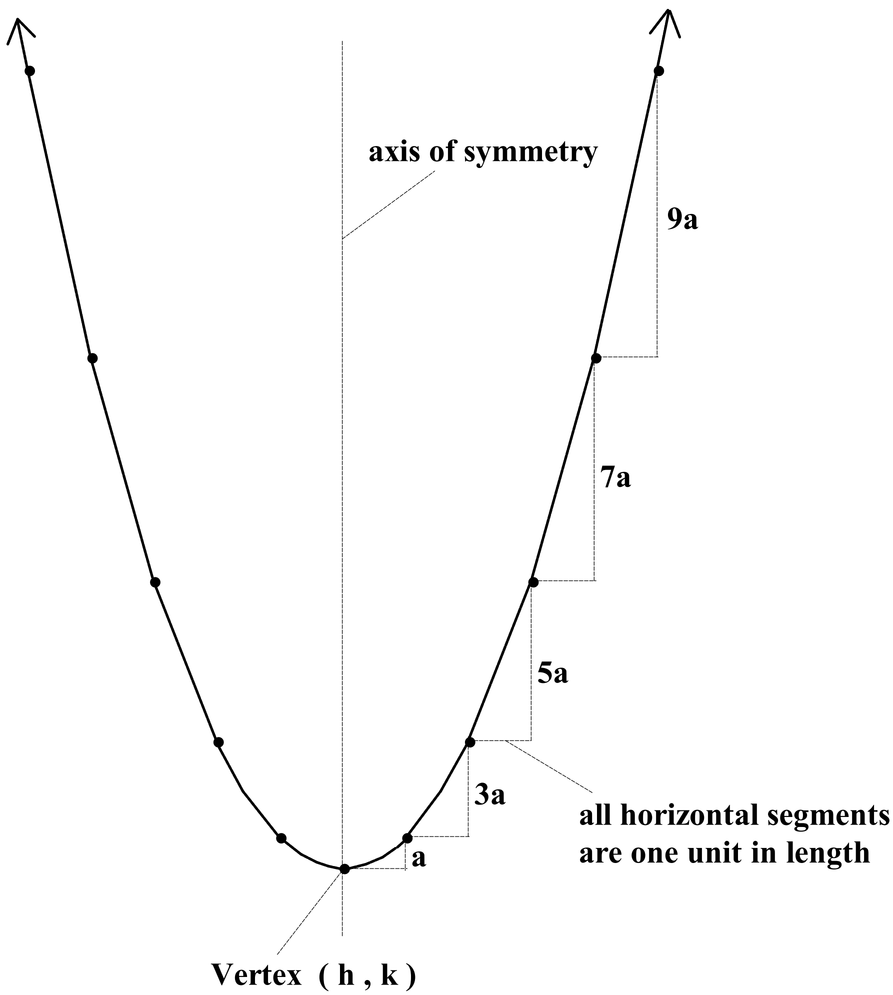
The basic shape of a parabola

The shape of any parabola is determined by the value of a in the standard form equation. Consider the equation $y - k = a(x - h)^2$. The vertex is (h , k). If $a > 0$, then the parabola opens upward. The shape is related to the value of a as shown below.



Once the vertex has been located, the value of a can be used to find several points on each side of the parabola. Then the parabola can be drawn through these points. Note how each horizontal step is one unit in length. However, the vertical steps are a, 3a, 5a, 7a, etc. If, in the equation $y - k = a(x - h)^2$, $a < 0$, then the vertical steps would be down instead of up and the parabola would open downward. (The shape would be the same.) For the type 2 parabolas with standard equation $x - h = a(y - k)^2$, the parabola would open to the right if $a > 0$ and to the left if $a < 0$. (To understand how this would look, rotate the above diagram 90° clockwise so that the axis of symmetry is horizontal.)

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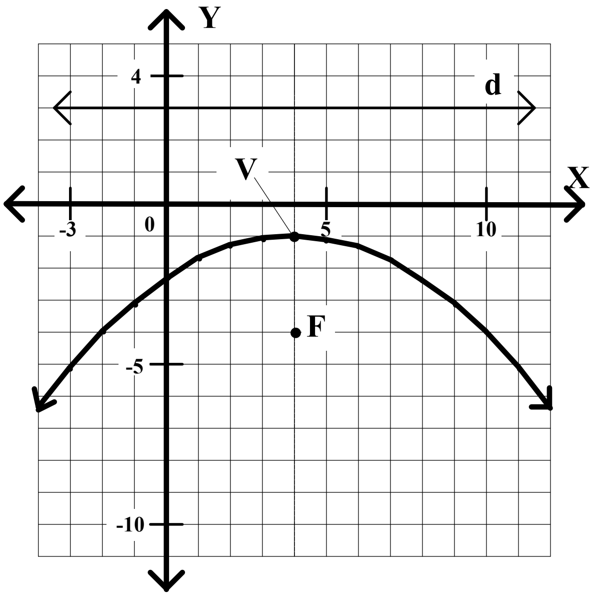
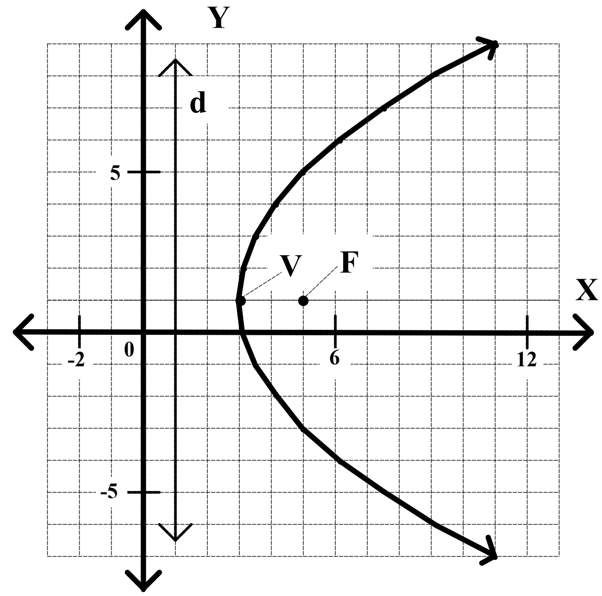
Find the standard form equation and the general form equation for each parabola graphed below.

1.



2.



Solutions

1. This is an example of the first type of parabola (axis vertical).
Since the vertex is (4, -1),
h = 4 and k = -1.
Since the focus is 3 units below the vertex, p = -3. a = 1/(4p) = -1/12

Standard: $y - k = a(x - h)^2$

$$y + 1 = \frac{-1}{12}(x - 4)^2$$

General: $-12(y + 1) = (x - 4)^2$

$$-12y - 12 = x^2 - 8x + 16$$

$$x^2 - 8x + 12y + 28 = 0$$

2. This is an example of the second type of parabola (axis horizontal).
Since the vertex is (3, 1),
h = 3 and k = 1.
Since the focus is 2 units right of the vertex, p = +2. a = 1/(4p) = 1/8

Standard: $x - h = a(y - k)^2$

$$x - 3 = \frac{1}{8}(y - 1)^2$$

General: $8(x - 3) = (y - 1)^2$

$$8x - 24 = y^2 - 2y + 1$$

$$y^2 - 8x - 2y + 25 = 0$$

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Write the standard form equation and sketch the graph of each of the following.

3. $2x^2 - y + 12x + 17 = 0$

Standard Form:

$2x^2 + 12x = y - 17$

$y - 17 = 2(x^2 + 6x)$

$y - 17 + 18 = 2(x^2 + 6x + 9)$

$y + 1 = 2(x + 3)^2$

axis of symmetry vertical (type 1)

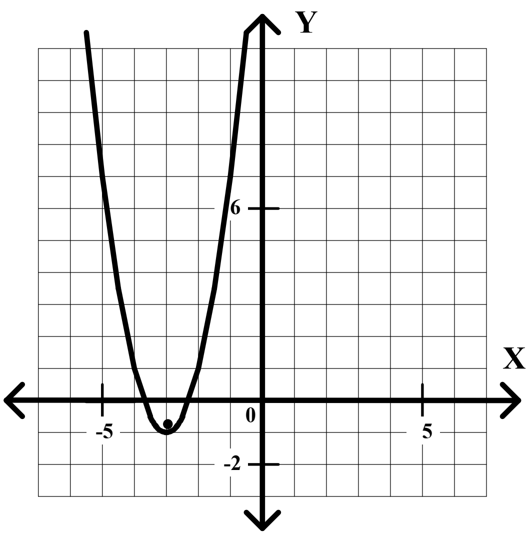
$h = -3$; $k = -1$; vertex $(-3,-1)$

$a = +2$ (opens up) steps 2, 6, 10, ...

Since $a = 1/(4p)$, $1/(4p) = 2$. $p = 1/8$

Therefore, the focus is $(-3, -7/8)$.

(1/8 unit above the vertex)



4. $y^2 + 4x + 2y - 11 = 0$

Standard Form:

$y^2 + 2y = -4x + 11$

$-4x + 11 + 1 = y^2 + 2y + 1$

$-4x + 12 = (y + 1)^2$

$-4(x - 3) = (y + 1)^2$

$x - 3 = \frac{-1}{4}(y + 1)^2$

axis of symmetry horizontal (type 2)

$h = 3$; $k = -1$; vertex $(3,-1)$

$a = -1/4$ (opens left) steps 1/4, 3/4, 5/4, ...

Since $a = 1/(4p)$, $1/(4p) = -1/4$. $p = -1$

Therefore, the focus is at $(2, -1)$.

(1 unit to the left of the vertex)