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Equations of a Hyperbola

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ where $AC < 0$

Standard Forms:

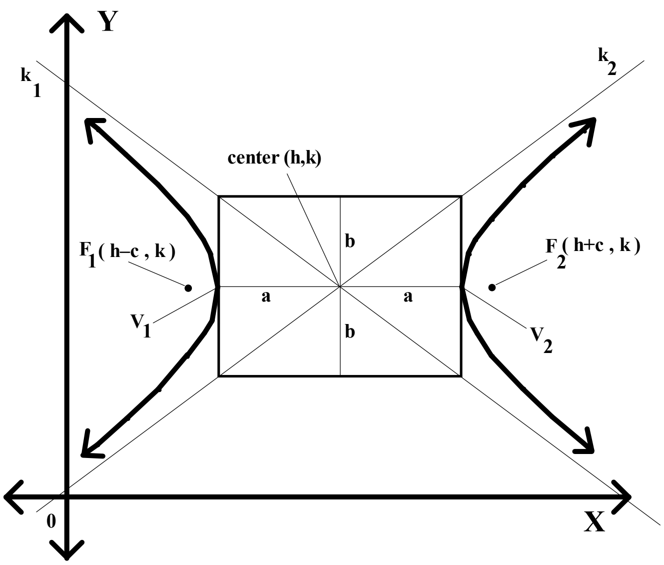
Type 1: Transverse Axis Horizontal

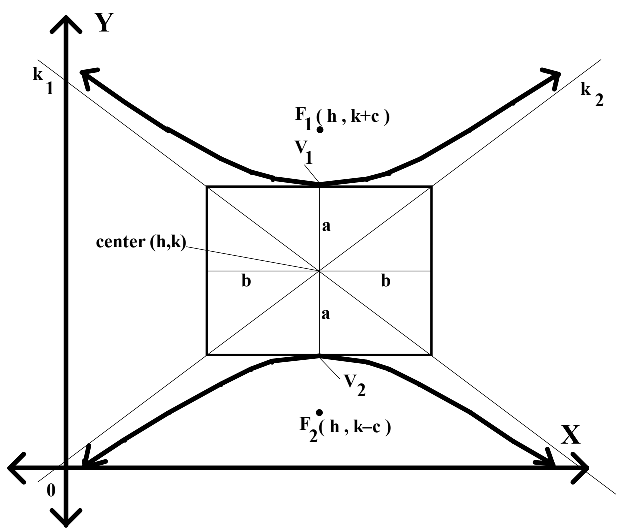
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Type 2: Transverse Axis Vertical

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The point (h, k) is called the center of the hyperbola. V_1 and V_2 are the vertices of the hyperbola and are the endpoints of the transverse axis, which is 2a units long. The conjugate axis is perpendicular to the transverse axis through the center and is 2b units long. Lines k_1 and k_2 are asymptotes of the hyperbola. A hyperbola has two focal points (foci) which are located on the line containing the transverse axis, on opposite sides of the center. Each focus is c units from the center where $c^2 = a^2 + b^2$. (Note that unlike the ellipse, a may be greater than b, equal to b, or less than b.)





If P represents any point on the ellipse, then $|PF_1 - PF_2| = 2a$.

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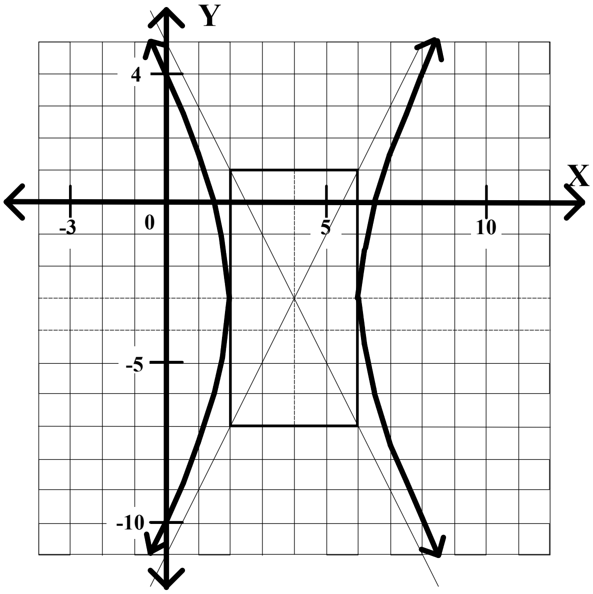
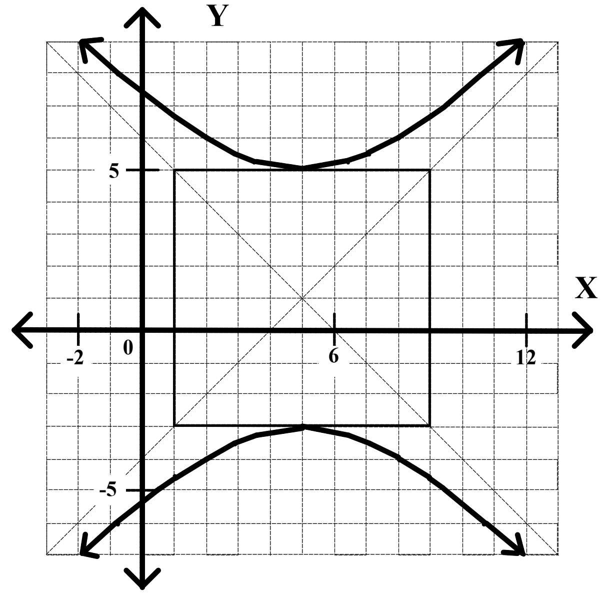
Equations of a Hyperbola

Find the standard form equation and the general form equation for each hyperbola graphed below. Also, locate the foci.

1.



2.



Solutions.

1. This is the first type of hyperbola.
   The center is (4 , -3). h = 4 and k = -3.
   The transverse axis is 4 units long.
   Since 2a = 4, a = 2.
   The congugate axis is 8 units long.
   Since 2b = 8, b = 4.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The standard form equation is:

$$\frac{(x-4)^2}{4} - \frac{(y+3)^2}{16} = 1$$

$$4(x-4)^2 - (y+3)^2 = 16$$

$$4(x^2 - 8x + 16) - (y^2 + 6y + 9) = 16$$

$$4x^2 - 32x + 64 - y^2 - 6y - 9 = 16$$

$$4x^2 - y^2 - 32x - 6y + 55 = 16$$

The general form equation is :

$$4x^2 - y^2 - 32x - 6y + 39 = 0$$

2. This is the second type of hyperbola.
   The center is (5, 1). h = 5 and k = 1.
   The transverse axis is 8 units long.
   Since 2a = 8, a = 4.
   The congugate axis is also 8 units long.
   Since 2b = 8, b = 4.

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The standard form equation is:

$$\frac{(y-1)^2}{16} - \frac{(x-5)^2}{16} = 1$$

$$(y-1)^2 - (x-5)^2 = 16$$

$$(y^2 - 2y + 1) - (x^2 - 10x + 25) = 16$$

$$y^2 - 2y + 1 - x^2 + 10x - 25 = 16$$

$$-x^2 + y^2 + 10x - 2y - 24 = 16$$

$$-x^2 + y^2 + 10x - 2y - 40 = 0$$

The general form equation is :

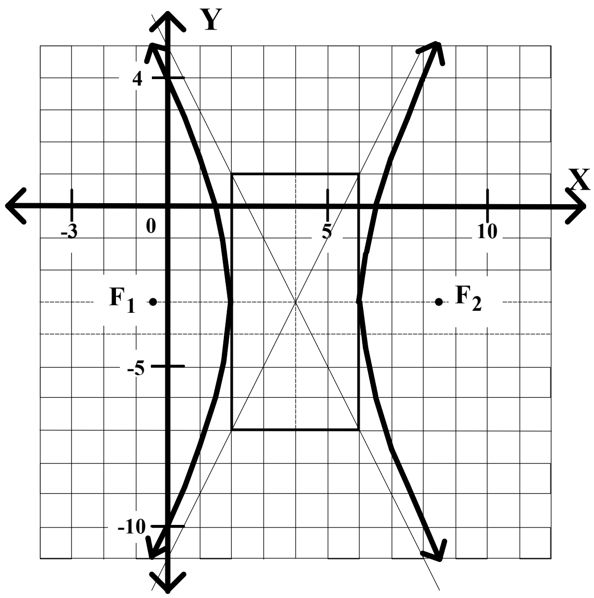
$$x^2 - y^2 - 10x + 2y + 40 = 0$$

Equations of a Hyperbola

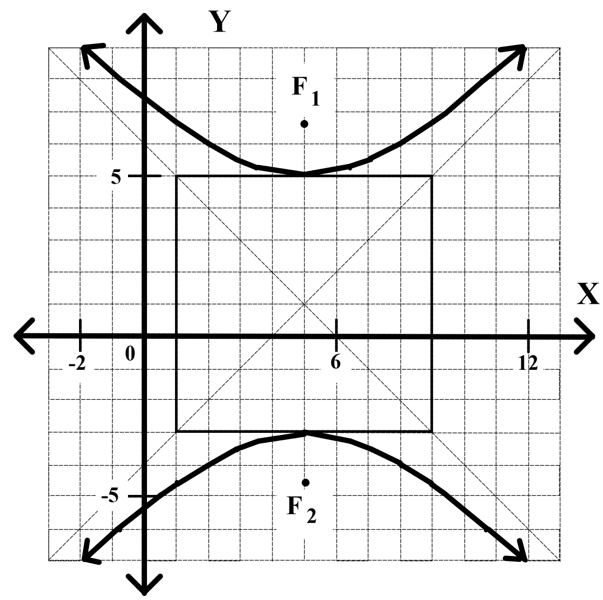
Solutions (continued)

The foci are each c units from the center of the hyperbola on the line containing the transverse axis. For the hyperbola $c^2 = a^2 + b^2$.

1. Since $a^2 = 4$ and $b^2 = 16$,
$c^2 = 4 + 16 = 20$. So c is about 4.5
For this hyperbola, one focus is located c units to the right of the center, and the other focus is located c units to the left of the center as shown below.



2. Since $a^2 = 16$ and $b^2 = 16$,
$c^2 = 16 + 16 = 32$. So c is about 5.7
For this hyperbola, one focus is located c units above the center, and the other focus is located c units below the center as shown below.



Equations of a Hyperbola

Given the general form equation of a hyperbola, (a) find the standard form equation and (b) graph the hyperbola.

3. $9x^2 - 16y^2 + 54x + 160y - 463 = 0$

solutions

(a) $9x^2 + 54x - 16y^2 + 160y = 463$

$9(x^2 + 6x\ ) - 16(y^2 - 10y\ ) = 463$

$9(x^2 + 6x + 9) - 16(y^2 - 10y + 25) = 463 + 81 - 400$

$$\frac{9(x+3)^2}{144} - \frac{16(y-5)^2}{144} = \frac{144}{144}$$

Standard Form Equation

$$\frac{(x+3)^2}{16} - \frac{(y-5)^2}{9} = 1$$

4. $9x^2 - 16y^2 + 36x - 32y + 164 = 0$

(a) $9x^2 + 36x - 16y^2 - 32y = -164$

$9(x^2 + 4x) - 16(y^2 + 2y) = -164$

$9(x^2 + 4x + 4) - 16(y^2 + 2y + 1) = -164 + 36 - 16$

$$\frac{9(x+2)^2}{-144} - \frac{16(y+1)^2}{-144} = \frac{-144}{-144}$$

Standard Form Equation

$$\frac{(y+1)^2}{9} - \frac{(x+2)^2}{16} = 1$$

3. $9x^2 - 16y^2 + 54x + 160y - 463 = 0$

solutions (continued)

The standard form equation is:

$$\frac{(x+3)^2}{16} - \frac{(y-5)^2}{9} = 1$$

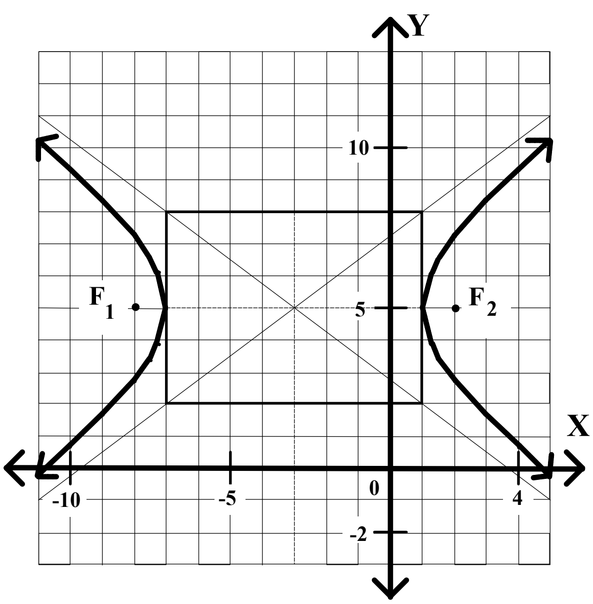
(b) This is the first type of hyperbola.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Since h = -3 and k = 5, the center is (-3, 5). Since $a^2 = 16$, a = 4. The transverse axis is 2a = 8 units long (horizontal).

Since $b^2 = 9$, b = 3. The congugate axis is 2b = 6 units long (vertical).

The foci are each c units from the center where $c^2 = a^2 + b^2$. $c^2 = 16 + 9 = 25$. c = 5. One focus is 5 units left of the center. The other is 5 units right of the center.



4. $9x^2 - 16y^2 + 36x - 32y + 164 = 0$

The standard form equation is:

$$\frac{(y+1)^2}{9} - \frac{(x+2)^2}{16} = 1$$

(b) This is the second type of hyperbola.

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Since h = -2 and k = -1, the center is (-2,-1). Since $a^2 = 9$, a = 3. The transverse axis is 2a = 6 units long (vertical).

Since $b^2 = 16$, b = 4. The congugate axis is 2b = 8 units long (horizontal).

The foci are each c units from the center where $c^2 = a^2 + b^2$. $c^2 = 9 + 16 = 25$. c = 5. One focus is 5 units above the center. The other focus is 5 units below the center.