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The Equations of an Ellipse

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ where $A \neq C$ and $AC > 0$

Standard Forms:

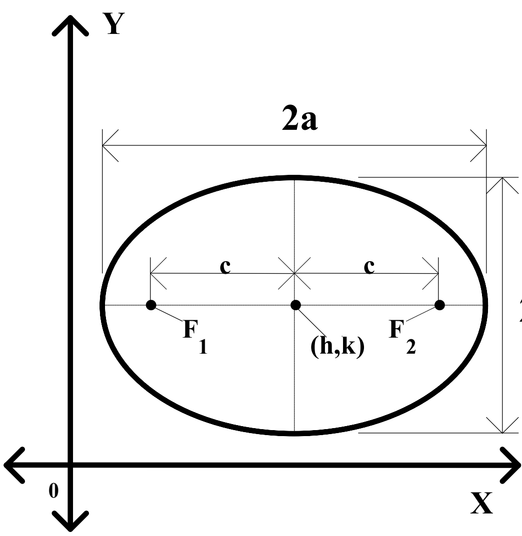
Type 1 (major axis horizontal)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

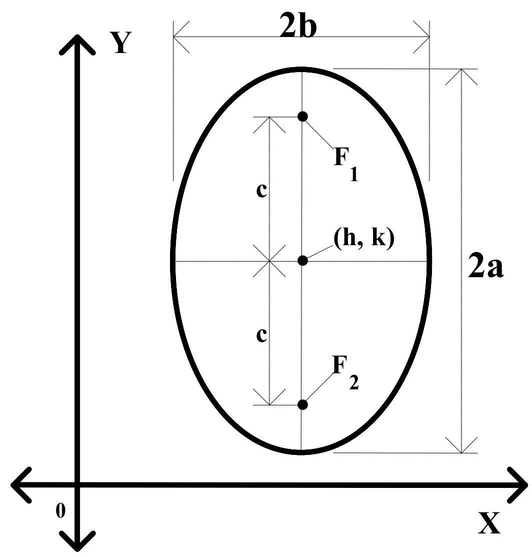
Type 2 (major axis vertical)

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Where the center of the ellipse is (h, k), the major axis is 2a units long, and the minor axis is 2b units long. (Note that $a > b > 0$, so $a^2 > b^2$.) Each ellipse has two focal points (foci). They are located on the major axis, c units from the center where $c^2 = a^2 - b^2$.



TYPE 1



TYPE 2

If P represents any point on the ellipse, then $PF_1 + PF_2 = 2a$. Each endpoint of the major axis is called a vertex of th ellipse.

Note: For type 1 ellipses, the foci are $F_1(h - c, k)$ and $F_2(h + c, k)$.
For type 2 ellipses, the foci are $F_1(h, k + c)$ and $F_2(h, k - c)$.

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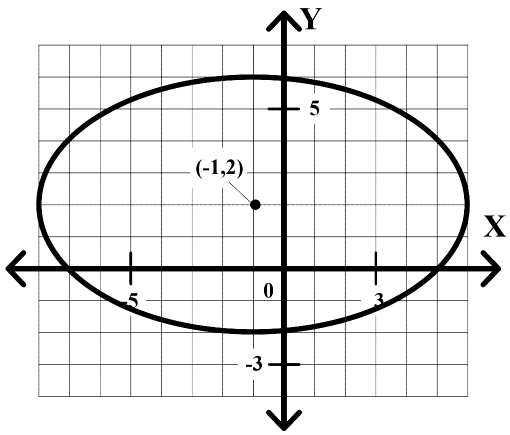
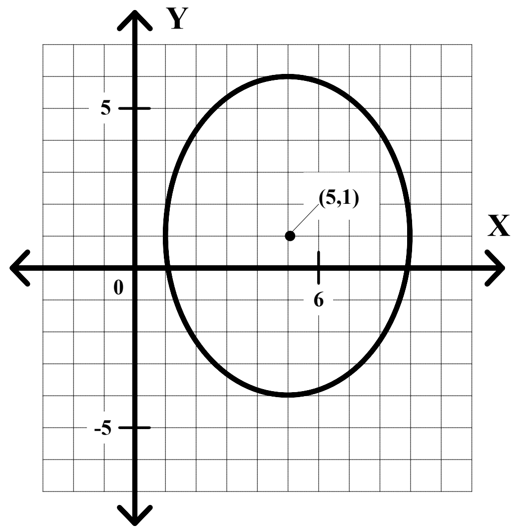
Equations of an ellipse (continued).

Find the standard form equation and the general form equation for each ellipse.

1.



2.



solutions

1. This is the first type of ellipse since the major axis is horizontal.
The center is at (-1, 2). h = -1 and k = 2.
The major axis is 14 units long and the minor axis is 8 units long. 2a = 14 and 2b = 8. a = 7 and b = 4.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x- -1)^2}{7^2} + \frac{(y-2)^2}{4^2} = 1$$

Standard: $$\boxed{\frac{(x+1)^2}{49} + \frac{(y-2)^2}{16} = 1}$$

$16(x+1)^2 + 49(y-2)^2 = (49)(16)$
$16(x^2 + 2x + 1) + 49(y^2 - 4y + 4) = 784$
$16x^2 + 32x + 16 + 49y^2 - 196y + 196 = 784$
$16x^2 + 49y^2 + 32x - 196y + 212 = 784$
General:
$16x^2 + 49y^2 + 32x - 196y - 572 = 0$

2. This is the second type of ellipse since the major axis is vertical.
The center is at (5, 1). h = 5 and k = 1.
The major axis is 10 units long and the minor axis is 8 units long. 2a = 10 and 2b = 8. a = 5 and b = 4.

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-5)^2}{4^2} + \frac{(y-1)^2}{5^2} = 1$$

Standard: $$\boxed{\frac{(x-5)^2}{16} + \frac{(y-1)^2}{25} = 1}$$

$25(x-5)^2 + 16(y-1)^2 = (16)(25)$
$25(x^2 - 10x + 25) + 16(y^2 - 2y + 1) = 400$
$25x^2 - 250x + 625 + 16y^2 - 32y + 16 = 400$
$25x^2 + 16y^2 - 250x - 32y + 641 = 400$
General:
$25x^2 + 16y^2 - 250x - 32y + 241 = 0$

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Equations of an ellipse (continued).

Locate the foci and the vertices of each ellipse in problem #1 and #2.

solutions: For an ellipse, $c^2 = a^2 - b^2$, where c is the distance from the center of the ellipse to each focus. (Note that the foci are on the major axis on opposite sides of the center.)

1. $a^2 = 49$ and $b^2 = 16.$
   $c^2 = 49 - 16 = 33$
   $c = \sqrt{33} \approx 5.7$

Since the center is at (-1, 2) , the foci are at
$F_1(-1 - c , 2)$ and $F_2(-1 + c , 2)$

For this ellipse, the approximate location of the foci are $F_1(-6.7 , 2)$ and $F_2(4.7 , 2)$.

2. $a^2 = 25$ and $b^2 = 16$
   $c^2 = 25 - 16 = 9$
   $c = \sqrt{9} = 3$

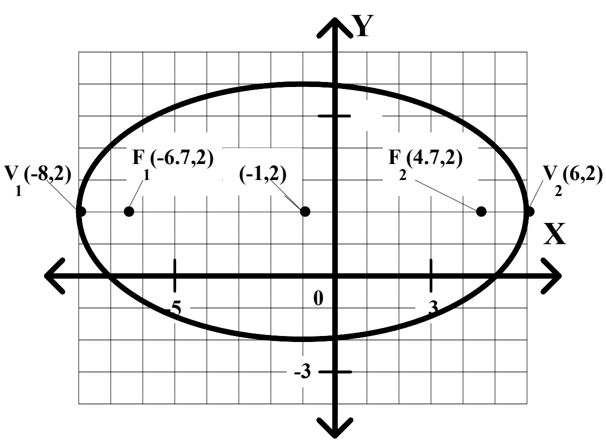
Since the center is at (5, 1), the foci are at
$F_1(5 , 1 + c)$ and $F_2(5 , 1 - c)$

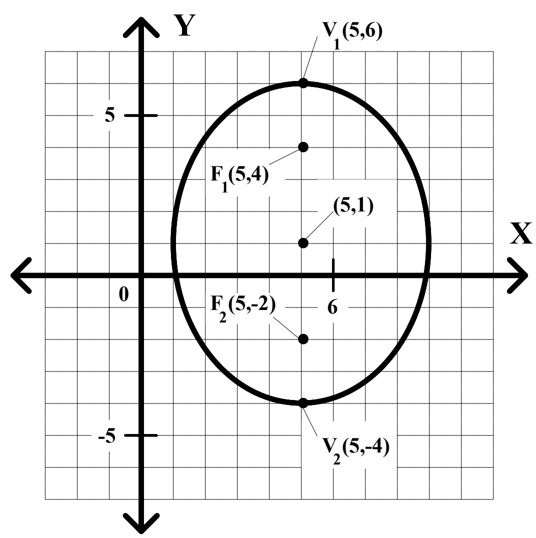
For this ellipse the foci are
$F_1(5 , 4)$ and $F_2(5 , -2)$.

The vertices are the endpoints of the major axis of the ellipse.

For #1, the vertices are
$V_1(-8 , 2)$ and $V_2(6 , 2)$.

For #2, the vertices are
$V_1(5 , 6)$ and $V_2(5 , -4)$.





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Equations of an ellipse (continued).
Find the standard form and graph the ellipse given by each equation. (The graph includes the foci.)

3. $9x^2 + 25y^2 + 36x - 189 = 0$

solutions

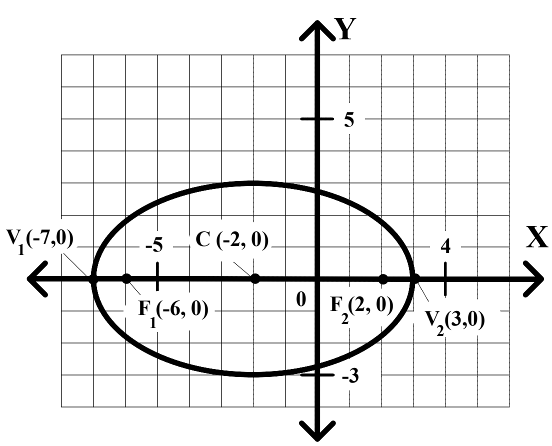
$$(9x^2 + 36x) + 25y^2 = 189$$
$$9(x^2 + 4x) + 25y^2 = 189$$
$$9(x^2 + 4x + 4) + 25y^2 = 189 + 36$$
$$9(x + 2)^2 + 25y^2 = 225$$

Standard: $\dfrac{(x+2)^2}{25} + \dfrac{y^2}{9} = 1$

This is the first type of ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 25$ and $b^2 = 9$. $a = 5$ and $b = 3$.
The major axis is horizontal and 10 units long. The minor axis is vertical and 6 units long.
$h = -2$ and $k = 0$. The center is $(-2, 0)$.
$c^2 = a^2 - b^2 = 25 - 9 = 16$. $c = 4$.
The foci are $F_1(-2 - c, 0)$ and $F_2(-2 + c, 0)$.
$F_1(-6, 0)$ and $F_2(2, 0)$.
The vertices are $V_1(-2-a, 0)$ and $V_2(-2+a, 0)$.
$V_1(-7, 0)$ and $V_2(3, 0)$.



4. $25x^2 + 16y^2 - 250x - 32y + 241 = 0$

$$(25x^2 - 250x) + (16y^2 - 32y) = -241$$
$$25(x^2 - 10x) + 16(y^2 - 2y) = -241$$
$$25(x^2 - 10x + 25) + 16(y^2 - 2y + 1) = -241 + 625 + 16$$
$$25(x - 5)^2 + 16(y - 1)^2 = 400$$

Standard: $\dfrac{(x-5)^2}{16} + \dfrac{(y-1)^2}{25} = 1$

This is the second type of ellipse.

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$a^2 = 25$ and $b^2 = 16$. $a = 5$ and $b = 4$.
The major axis is vertical and 10 units long. The minor axis is horizontal and 6 units long.
$h = 5$ and $k = 1$. The center is $(5, 1)$.
$c^2 = a^2 - b^2 = 25 - 16 = 9$. $c = 3$.
The foci are $F_1(5, 1 + c)$ and $F_2(5, 1 - c)$.
$F_1(5, 4)$ and $F_2(5, -2)$.
The vertices are $V_1(5, 1+a)$ and $V_2(5, 1-a)$.
$V_1(5, 6)$ and $V_2(5, -4)$.