Algebra II   Class Worksheet #5   Unit 4 ____________________

Introduction to Linear Programming   page 1

Consider the following problem.

A farming family wishes to plant some barley and some wheat. They can plant a maximum of 100 acres of barley and a maximum of 80 acres of wheat. However, they only have 120 acres of land available for planting. Barley costs $20 per acre for seeds, and wheat costs $30 per acre for seeds. However, they only have $3000 available for seed costs. They expect a harvest of 1000 pounds per acre of barley and 3000 pounds per acre of wheat. How many acres of each crop should they plant to maximize their total harvest?

Solution:
Let x represent the number of acres of barley that they plant.
Let y represent the number of acres of wheat that they plant.
The following inequalities represent the constraints given in the problem concerning the number of acres planted.

$$x < 100$$
$$y < 80$$
$$x + y < 120$$

Since the seed cost is $20 per acre of barley, the total seed cost for the barley is 20x dollars.
Similarly, the total seed cost for the wheat is 30y dollars.
Since only $3000 is available,

$$20x + 30y < 3000.$$

Finally, since neither x nor y can be negative,

$$x \geq 0 \text{ and}$$
$$y \geq 0.$$

This set of six inequalities represents the system of constraints for the problem.
We can graph this system to find the set of feasible solutions for the problem.
To find the number of acres of each which should be planted in order to maximize the total production, we must first represent the total production as a function of x and y.
Since 1000 pounds of barley is expected for each acre planted, 1000x (pounds) represents to total harvest of barley expected. Similarly, 3000y (pounds) represents the total harvest of wheat expected. Therefore,

$$T = 1000x + 3000y \quad \text{(where T is the total harvest).}$$

This relationship is called the objective function.
The principle used to solve problems like this one is stated as follows.

If the set of feasible solutions is represented by a convex polygonal region, and the objective function is a linear expression in the two variables (ax + by) , then its maximum (or minimum) value will occur at a vertex of the region.

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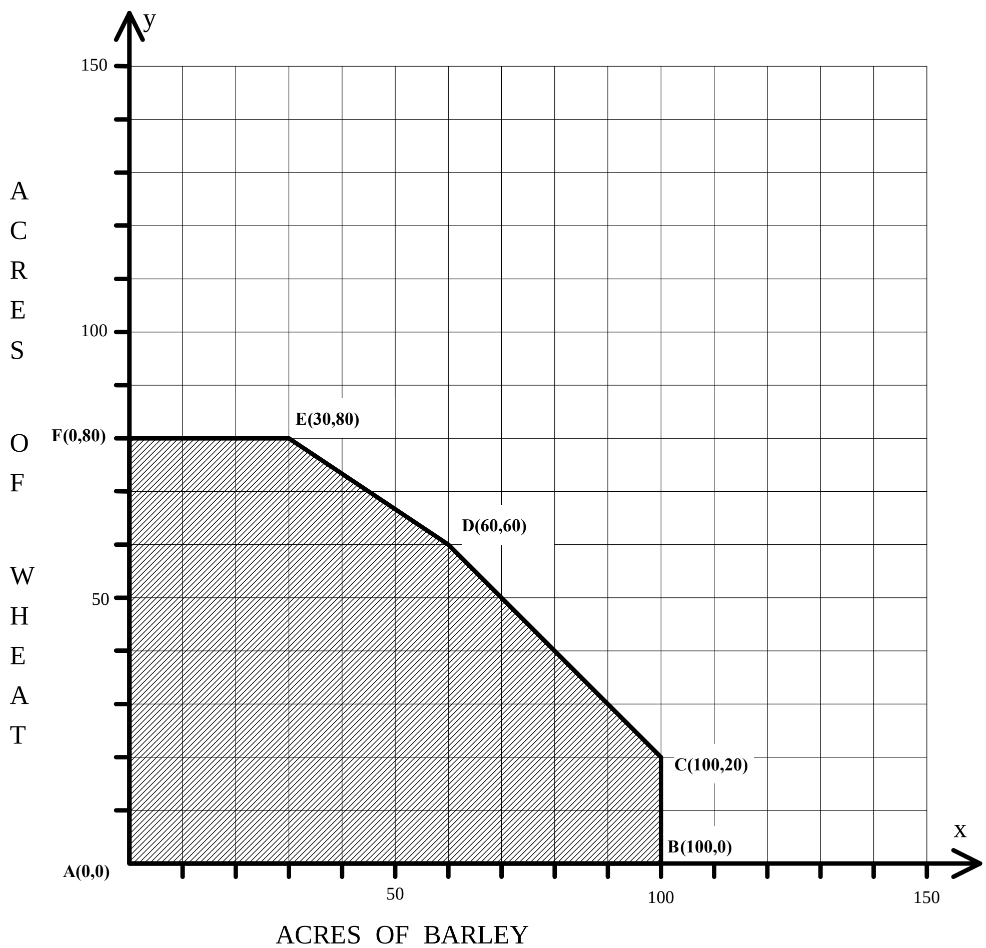
The Graphing Method for Solving a Linear Programming Problem

Step 1: Graph the system of constraints, thus showing the set of “feasible solutions”.
Step 2: Find the coordinates of each vertex of the region.
Step 3: Evaluate the objective function at each vertex.
Step 4: Use the vertex that corresponds to the maximum (or minimum) value to answer the question.

For the problem described for this lesson, the system of constraints was ...

$x < 100$	**$x < 100$**
$y < 80$	**$y < 80$**
$x + y < 120$	**$y < -x + 120$**
$x > 0$	**$x > 0$**
$y > 0$	**$y > 0$**

The graph of this region is below.



The vertices of the region of feasible solutions are:

A(0 , 0)
B(100 , 0)
C(100 , 20)
D(60 , 60)
E(30 , 80)
F(0 , 80)

The objective function T = 1000x + 3000y should be evaluated at each vertex.

At A, T = 1000(0) + 3000(0) = 0
At B, T = 1000(100) + 3000(0) = 100,000
At C, T = 1000(100) + 3000(20) = 160,000
At D, T = 1000(60) + 3000(60) = 240,000
At E, T = 1000(30) + 3000(80) = 270,000
At F, T = 1000(0) + 3000(80) = 240,000

Clearly, the maximum total production corresponds to vertex E(30, 80).

Therefore, the farming family should plant 30 acres of barley and 80 acres of wheat.

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Solve the following linear programming problem. Show all of your work neatly organized.

A small firm manufactures bracelets and necklaces. The total number of necklaces and bracelets it can manufacture per day is 24. Each bracelet requires 1 hour of labor to make, and each necklace requires .5 hours of labor to make. The total number of hours of labor available per day is 16. The profit on each bracelet is $4, and the profit on each necklace is $3. How many bracelets and how many necklaces should the company make per day in order to maximize its profits.