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The Equation of a Line

In this part of the unit, a line will be described. Your job will be to write its equation. We will start with a review of the three types of lines and their most common equations.

Type 1: **Horizontal Line** Equation: $\mathbf{y = k}$
Type 2: **Vertical Line** Equation: $\mathbf{x = k}$
Type 3: **Oblique Line** Equation: $\mathbf{y = mx + b}$ (slope-intercept equation)

When writing the equation of a line, first determine the type of line. Then find its equation. Horizontal and vertical lines should be fairly routine (especially once you have done a few practice problems). Oblique lines are the most challenging. Of course if both the slope and the y-intercept are known, then the slope-intercept equation can be written easily. In the event that the y-intercept is not given, however, there is another equation that proves to be very useful. It is called the point-slope equation. If you know a point on the line, represented by (x_1, y_1), and the slope of the line, represented by m, then an equation of the line is $\mathbf{y - y_1 = m(x - x_1)}$. This equation is called the point-slope equation. The slope-intercept equation can easily be derived from this equation.

Consider the following examples.

Find the equation of each line described. If the line is oblique, then write the slope-intercept equation.

1. the line through (3, -1) and (-2, -1)
Note that the two points have the same y-coordinates.
This tells us that the line is horizontal. $\mathbf{y = -1}$

2. the line through (3, -1) and (3, 5)
Note that the two points have the same x-coordinates.
This tells us that the line is vertical. $\mathbf{x = 3}$

3. the line through (-2, 5) with slope 0
The fact that the slope is 0 tells us that the line is horizontal.
The line contains the point (-2, 5). $\mathbf{y = 5}$

4. the line through (-2, 5) with ñno slopeò
The phrase ñno slopeò is commonly used to describe the slope of a vertical line. The slope of a vertical line is undefined.
The line contains the point (-2, 5). $\mathbf{x = -2}$

5. the line with slope 3 and y-intercept -1
This line is oblique. We are given that m = 3 and b = -1. $\mathbf{y = 3x - 1}$

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Find the equation of each line described. If the line is oblique, then write the slope-intercept equation.

6. the line with slope -2 through the point (0, 5)
This line is oblique. We are given that m = -2.
Since the line contains the point (0, 5), b = 5. **y = -2x + 5**
Note: The y-intercept, b, is the value of y when x = 0.

7. the line through (0, 1) and (2, 5)
This line is oblique. We must find the slope first.

$$\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{2 - 0} = \frac{4}{2} = 2}$$

Since the line contains the point (0, 1), b = 1. **y = 2x + 1**

8. the line with slope 4 through the point (3, 1)
The line is oblique. We are given that m = 4.
We are not given the y-intercept. We will use the point-slope equation. $\mathbf{y - y_1 = m(x - x_1)}$
Since the line contains the point (3, 1), $x_1 = 3$ and $y_1 = 1$.
The point-slope equation becomes **y – 1 = 4(x – 3)**.
Solving for y, we get **y – 1 = 4x – 12**
y = 4x – 11 **y = 4x – 11**

9. the line through (-2, 3) and (2, -5)
This line is oblique. We must find the slope first.

$$\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{2 - -2} = \frac{-8}{4} = -2}$$

We are not given the y-intercept. We will use the point-slope equation. $\mathbf{y - y_1 = m(x - x_1)}$
Since the line contains the point (2, -5), $x_1 = 2$ and $y_1 = -5$.
The point-slope equation becomes **y – -5 = -2(x – 2)**.
Solving for y, we get **y + 5 = -2x + 4**
y = -2x – 1 **y = -2x – 1**
Note: The point (-2, 3) would have worked as well.