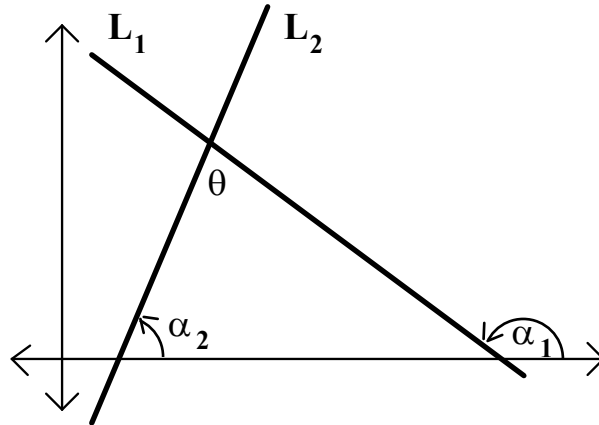


## Advanced Challenge Level 2 Problem #4

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This problem challenges you to find the angles formed when two lines intersect. Of course, as the diagram below shows, if the lines are not perpendicular, then there are two congruent acute angles and two congruent obtuse angles.

Consider two oblique lines  $L_1$  and  $L_2$  with inclinations  $\alpha_1$  and  $\alpha_2$  and slopes  $m_1$  and  $m_2$ , respectively. (Please see the diagram below.)



In this diagram,  $\alpha_1$  is an obtuse angle and  $\alpha_2$  is an acute angle. Of course, this is only one of many different situations. Assume that  $L_1$  and  $L_2$  are oblique, intersecting lines. For questions #1 and #2 below, you may make the following further assumptions.

- (a)  $L_1$  and  $L_2$  are not perpendicular to each other.
- (b)  $\alpha_1 > \alpha_2$ .

Answer the following questions. Describe your complete process neatly organized.

1. Find a relationship that gives angle  $\theta$  in terms of  $\alpha_1$  and  $\alpha_2$ . For the purpose of this question,  $\theta$  is an angle shown that is formed by the intersection of the lines. It may be acute or obtuse. Because of assumption (a) above,  $\theta$  is not a right angle.
2. Derive a relationship between  $\tan \theta$  and the slopes  $m_1$  and  $m_2$ .
3. Make the necessary adjustments in your equation for question 2 so that  $\theta$  is an acute angle formed at the intersection.
4. Investigate and explain the impact on your equation in #3 if  $\alpha_1 < \alpha_2$ .
5. Investigate and explain the impact on your equation in #3 if  $L_1$  and  $L_2$  are perpendicular.