

Advanced Challenge Level 1 Problem #54

A projectile is defined as a fired, thrown, or otherwise projected object, having no capacity for self-propulsion. Consider a stone that is thrown or propelled in some way. It will follow a parabolic path until it hits the ground. (I am ignoring any possible effect of wind or other objects that might be in the way.) The motion of the stone can be broken down into two components, vertical and horizontal. The vertical component gives the height of the stone, above the level ground. The height can be found using an equation of the form $y = at - 16t^2$ for some specific value of a . In this equation, y is the height in feet t seconds after the stone is 'thrown'. The horizontal component gives the horizontal distance the stone has traveled. This can be found using an equation of the form $x = bt$ for some specific value of b . In this equation, x represents this distance in feet t seconds after the stone is 'fired'. The path that the stone will follow depends on the initial speed of the stone and the angle at which it is 'fired'. Suppose a stone is propelled with an initial speed of 100 feet per second. The following table gives the values of a and b for several different angles (θ).

θ	10°	20°	30°	40°	45°	50°	60°	70°	80°	90°
a	17.36	34.2	50	64.28	70.71	76.6	86.6	93.97	98.48	100
b	98.48	93.97	86.6	76.6	70.71	64.28	50	34.2	17.36	0

For this challenge, you are to answer each of the following questions for each of the (10) angles given above.

1. How long will it take the stone to hit the ground?
2. How far from its starting point will it be when it hits the ground? (This is called the range.)
3. What is the maximum height attained by the stone in its flight?

In addition to this, you must make an accurate, scale drawing showing the flight path of the stone for one of the angles given above. The angle that you are responsible for can be determined from the chart below.

Birthday	Jan 1 to Feb 10	Feb 11 to Mar 23	Mar 24 to Apr 30	May 1 to June 10	June 11 to July 21	July 22 to Aug 31	Sept 1 to Oct 11	Oct 12 to Nov 21	Nov 22 to Dec 31
Angle	10°	20°	30°	40°	45°	50°	60°	70°	80°